

Abstract

RAO, SUNIL, MURALI. Tchebycheff Method-based Evolutionary Algorithm for Multi-objective Optimization (Under the direction of Dr. Ranji Ranjithan)

In the operations research literature, the Tchebycheff method has been demonstrated to be a useful approach for exploring the non-dominated solutions for multiobjective optimization problems. While this method has been investigated with mathematical programming-based solution approaches, its application with modern heuristic search procedures is lacking. As heuristic search procedures continue to show promise as practical solution approaches for realistic engineering problems typically with multiple design objectives, the need for their applications in multiobjective optimization is becoming increasingly important. This paper investigates a new evolutionary algorithm-based multiobjective optimization procedure that builds upon the Tchebycheff method. By embedding a beneficial seeding approach, the efficiency of the algorithm is expectedly enhanced. This Tchebycheff Method-based Evolutionary Algorithm (TMEA) is tested and evaluated using a suite of 2-objective test problems, representing a range of complexities in the decision space as well as in the objective space. The performance of TMEA with those of other multiobjective evolutionary algorithms are compared using several performance metrics that are reported in the literature. For the problems considered in this paper, TMEA performs relatively well in generating non-dominated solutions that are close to the known Pareto set and are well distributed in the non-inferior space.

TCHEBYCHEFF METHOD-BASED EVOLUTIONARY ALGORITHM FOR MULTIOBJECTIVE OPTIMIZATION

BY
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A THESIS SUBMITTED TO THE GRADUATE FACULTY OF
NORTH CAROLINA STATE UNIVERSITY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

DEPARTMENT OF CIVIL, CONSTRUCTION AND ENVIRONMENTAL ENGINEERING

RALEIGH
JULY 2003

APPROVED BY:

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Acknowledgments

I would like to thank my adviser and committee chairman Dr. S. R. Ranjithan for his invaluable guidance and constant encouragement throughout the course of this work. I am also grateful Dr. Downey Brill and Dr. John Baugh Jr. for being on my committee and providing valuable comments on my thesis.

I would also like to thank my office mates, Emily , Siva and Parthee for their wonderful friendship and also for sharing their ideas, opinions and experiences.

Lastly I would like to express my gratitude towards my parents, Dr. B. K. Murali and Mrs. Saraswati Murali, for their love, support and for being a constant source of inspiration throughout my education.

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Chapter 1

Introduction

Most real world engineering optimization problems require consideration of more than one objective. These objectives portray diverse design criteria and are often conflicting. The goal in single objective optimization is to find the best solution with respect to the objective considered. In multiobjective optimization the goal is to find a set of non-dominated solutions (also called the Pareto optimal set) that represent the non-inferior tradeoff among the multiple objectives being considered.

The concept of *Pareto optimum* was formulated by Vilfredo Pareto in 1896 [6] and is considered as the origin of research in multiobjective optimization. Several mathematical programming approaches like the ε -constraint method [3], Compromise programming [19], Min-Max techniques [1], Goal programming [4] emerged during the first half of the twentieth century. The Tchebycheff interactive vector space reduction method has been successfully used in the mathematical programming for a variety of applications [22].

As evolutionary algorithms (EAs) continue to be used in solving real world optimization problems and show promise for successful application in a variety of domains areas [9], interest in extending their capabilities to multiobjective analysis is rapidly growing. Since the development of the Vector Evaluated Genetic Algorithm (VEGA) by Schaffer [23], there has been a growing interest in the use EAs for solving multiobjective optimization problems (MOPs). The state-of-the-art in multiobjective evolutionary algorithms (MOEAs) is well represented by Deb [10] and Coello Coello et al [7]. The more

commonly cited among them are the Nondominated Sorting Genetic Algorithm (NSGA-II) [12], Strength Pareto Evolutionary Algorithm (SPEA-II) [27], Multiobjective Genetic Algorithm (MOGA) [15], Niched Pareto Genetic Algorithm (NPGA) [17]. All these techniques are termed as Pareto based approaches [16] i.e., they converge towards the Pareto front in a single execution of the algorithm. These techniques mainly differ in the way they assign fitness to the individuals within a population. Alternatively, there is a class of EAs that generates the Pareto front by iteratively solving a series of single objective optimization problems. The ε -constraint method-based evolutionary algorithm (CMEA) [20] [21] and Subdivision Method [2] fall into this class. These algorithms generate comparable results to the population-based searches and are relatively easier to implement.

This paper presents a new MOEA that utilizes the Tchebycheff method iteratively to generate a set of non-dominated solutions. It uses the concepts of beneficial seeding [20] to improve convergence during the intermediate iterations of the algorithm. It is tested using standard test problems having convex, concave and discontinuous Pareto surfaces, and evaluated using several performance metrics commonly reported in the MOEA literature. Its performance is compared with that of CMEA and other population based search algorithms.

The next section describes the mathematical foundation for the Tchebycheff method, followed by a description of the new EA-based approach. Descriptions of test problems, results, and performance comparisons of the new algorithm with other contemporary MOEAs are provided in the subsequent sections.

Chapter 2

The Tchebycheff Method

The Tchebycheff method is an iterative algorithm where preference information from the decision maker is taken progressively as the algorithm proceeds to find a set of non-dominated solutions. The preference information is characterized via two user-specified inputs. One is an utopian objective vector or ideal solution (Z^{ref}) to the MOP, and the other is a weight vector ($\gamma_i; i = 1, 2, \dots, k$) that assigns user's relative preference for the k objectives. The algorithm then proceeds to find the non-dominated solution that is closest to Z^{ref} by solving the following mathematical model.

$$\text{Minimize} \quad \text{Max}_{i \in k} [\gamma_i |Z_i^{ref} - Z_i|] \quad (2.1)$$

$$\text{Subject to} \quad g_j(x) \leq 0 \quad \forall j = 1, 2, \dots, m \quad (2.2)$$

$$\gamma_i \in \mathcal{R}^k | \gamma_i \in \{0, 1\} \quad (2.3)$$

$$\sum_{i \in k} \gamma_i = 1 \quad (2.4)$$

$$x \in \mathcal{X}$$

where, Z_i is one of the k -objectives being maximized and Z_i^{ref} is the corresponding i^{th} utopian objective vector value, γ_i is i^{th} weight from the weight vector. $g_j(x)$ is the j^{th} constraint for the original problem, m is the total number of the constraints, x represents the decision vector and \mathcal{X} represents the decision space. Most often Z^{ref} is defined by incrementing each individual optimum $f_i(x)^*$ by an arbitrarily small amount ϵ_i $\{i = 1, 2, \dots, k\}$.

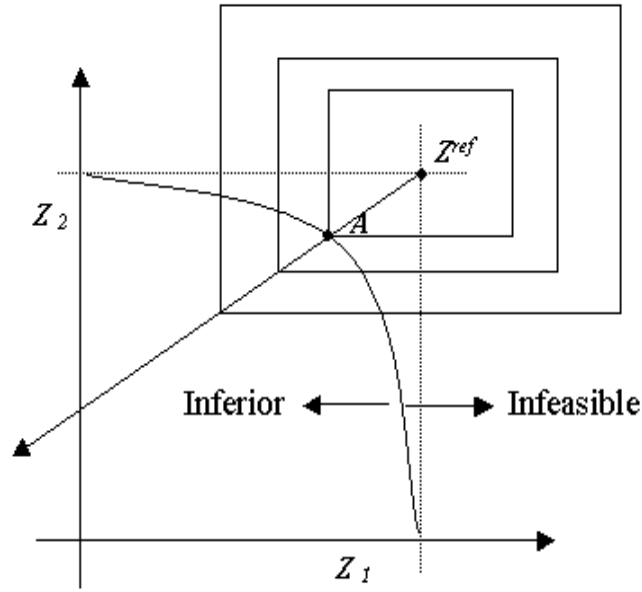


Figure 2.1: Illustration for a two objective problem

Figure 2.1 shows for an illustrative two objective case, different contours of the Tchebycheff metric (2.1) for a given set of γ_i . The innermost rectangle represents the minimum contour, and the optimal solution to the above model is point A. As no feasible solution is present in the northeast quadrant (for the maximization problem), no other solution can dominate this optimal solution. Thus, solution A found by solving model (1)-(2), by definition, is a non-dominated solution.

A different weight vector will result in a different set of rectangular contours and a corresponding new optimal solution, yielding another non-dominated solution. By varying $\{\gamma_i\}$ and solving model (1)-(2), different non-dominated solutions are generated until the decision maker is satisfied with a solution. Alternatively, this procedure can be applied to estimate the entire noninferior tradeoff among the objectives by iterating through the full range of values for the weight vector.

By the structure of the algorithm, this method can be applied to problems for which the

Pareto front is convex and concave in shape, discontinuous, or discrete. A few examples are illustrated in Figures 2.2 and 2.3

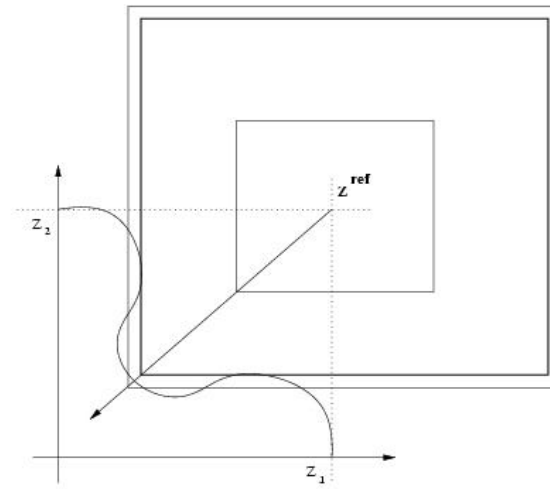


Figure 2.2: Illustration for a two-objective problem with concave and convex non-inferior front

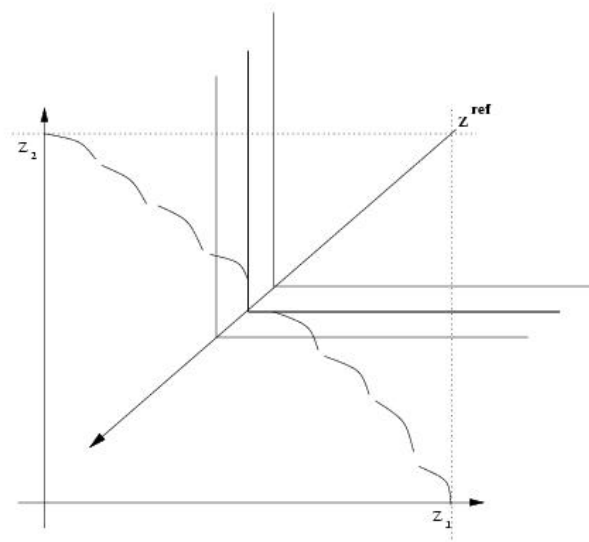


Figure 2.3: Illustration for a two-objective problem with discontinuous non-inferior front

Chapter 3

Tchebycheff Method-based EA

For engineering problems where the Tchebycheff formulation of the multiobjective optimization model (1)-(2) cannot be solved using a mathematical programming approach, heuristic search procedures, including evolutionary algorithms, can be applied. The Tchebycheff method-based evolutionary algorithm (TMEA) presented in this paper is a procedure that integrates the above Tchebycheff method within an evolutionary computation framework.

TMEA is described here in the context of generating the entire non-inferior tradeoff. Starting at an extreme of the non-inferior set, this procedure solves the model (1)-(2) corresponding to that γ weight vector via an evolutionary algorithm. This weight vector is incrementally changed and the updated model is resolved to find adjacent non-inferior points, eventually exploring the entire non-inferior space.

This procedure can become computationally intensive as it involves solving a series of single objective problems. TMEA uses the concepts of beneficial seeding to improve convergence within the intermediate iterations by seeding the starting solutions with the converged solutions obtained in the previous generations. This not only achieves faster convergence but also improves the search. This method was successfully implemented within the CMEA approach for multiobjective optimization [20].

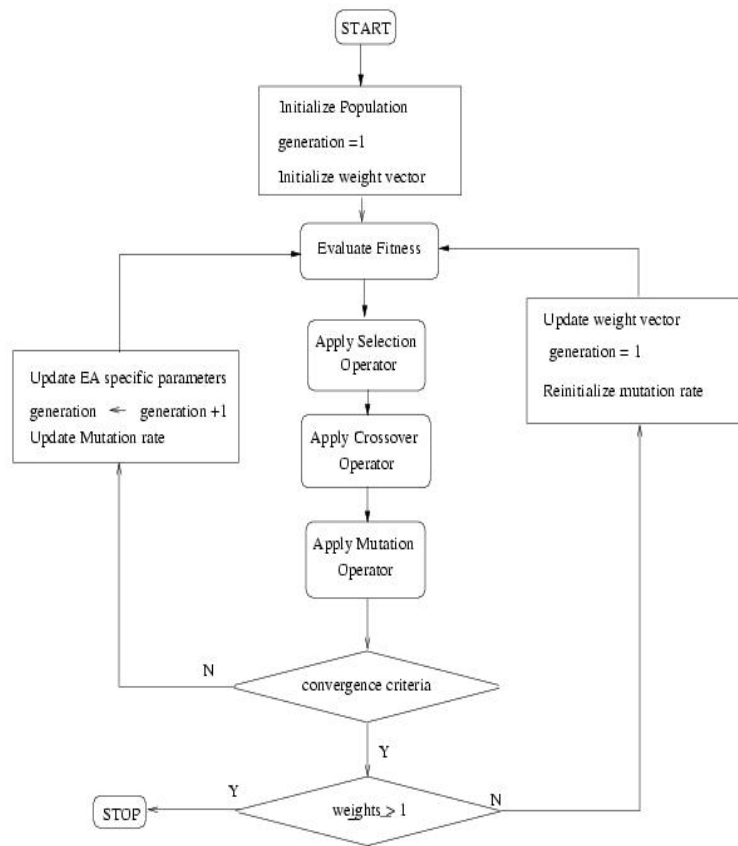
The flowchart in Figure 3.1 shows the sequence of execution of the algorithm. The inner loop of the flow chart corresponds to the execution of any EA with typical operators

like selection, recombination and mutation. The stopping criterion for the inner loop is determined either by exceeding maximum generations or a specified number of generations without improvement in the best solution. The outer loop of the flow chart corresponds to the change in the γ vector. From Equation 2.1 this corresponds to a shift in the weight placed on the objectives and consequentially a change in the direction of the search process. The number of non-dominated solutions being obtained depends on the step-size chosen for incrementing the γ vector. For each execution of the outer loop, the EA in the inner loop solves a particular instance of the problem described by Equations (1)-(2) and generates a non-inferior solution. After each execution of the outer loop, the γ vector is incremented. The solutions generated through sequential execution of the outer loop will represent the non-dominated (or Pareto optimal) set.

An adaptive mutation parameter defined in Equation 3.1 is used to introduce higher diversity at the start of a new iteration of the outer loop. The mutation operator is applied adaptively, starting with a higher rate initially and reducing it exponentially as generations proceed in the inner loop. The higher rate at the beginning increases the diversity in the previously converged population of solutions potentially avoiding premature convergence.

$$\text{Mutation Rate} = \begin{cases} 0.1 & \text{if } (t = 1), \\ \text{Max}(0.01, 0.1 \times \exp(-t/10)) & \text{otherwise} \end{cases} \quad (3.1)$$

where t is the number of generations

**Figure 3.1:** Flowchart for TMEA

Chapter 4

Evaluation of TMEA

Apart from the development of newer and improved algorithms for MOPs, researchers have also concentrated on developing test functions to validate and test MOEAs. Veldhuizen [25] summarized many test functions that have been suggested by different researchers. Deb et al [11] presented the Tunable Test Problem generator that can be used to generate problems of varying complexity, and Deb et al [13] suggested a test suite of MOPs with more than two objectives. More recently Knowles and Corne [8] introduced a test suite based on the Quadratic Assignment Problem (QAP).

The TMEA was tested on the following test problems, each with different degrees of difficulty and different characteristics with respect to convexity and continuity in the non-inferior set, discreteness in the decision space, and degree of constraints. The set of constrained test problems (CTPs) by Deb [11], the extended 0/1 knapsack problem presented by Zitzler [27] and two of the commonly used unconstrained test problems are used to test TMEA. A complete list and details of the test problems that were used in this study is provided in Table 4.1. The parameter settings used for all the algorithms is provided in the Table 4.2. All objective and constraint functions values were normalized, and correspondingly the γ weight values were kept to a $[0, 1]$ range. The constraints, where applicable, were handled using the constraint violation-based selection approach suggested by Deb et al [12]. Each test function was solved for a 50 random trials to test the robustness of the algorithm.

Table 4.1: Test problems used in this study. The objective functions are denoted by $Z_l(x)$, $1 \leq l \leq k$, where k denotes the number of objectives and N the number of decision variables.

Problem	N	Domain	Objective Functions	Constraints
CONSTR	2	$x_1 \in [0.1, 1]$ $x_2 \in [0, 5]$	$Z_1(x) = x_1$ $Z_2(x) = (1 + x_2)/x_1$	$c_1(x) = x_2 + 9x_1 \geq 6$ $c_2(x) = -x_2 + 9x_1 \geq 1$
SRN	2	$x_i \in [-20, 20]$ $i = 1, 2$	$Z_1(x) = (x_1 - 2)^2 + (x_2 - 1)^2 + 2$ $Z_2(x) = 9x_1 - (x_2 - 1)^2$	$c_1(x) = x_1^2 + x_2^2 \leq 225$ $c_2(x) = x_1 - 3x_2 \leq -10$
TNK	2	$x_i \in [0, \pi]$ $i = 1, 2$	$Z_1(x) = x_1$ $Z_2(x) = x_2$	$c_1(x) = -x_1^2 - x_2^2 + 1 + 0.1 \cos(16 \arctan(x_1/x_2)) \leq 0$ $c_2(x) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 \leq 0.5$
SCH	1	$x_i \in [-5, 7]$ $i = 1, 2$	$Z_1(x) = x^2$ $Z_2(x) = (x - 2)^2$	Unconstrained
ZDT3	30	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	$Z_1(x) = x_1$ $Z_2(x) = g(x)[1 - \sqrt{x_1/g(x)} - x_1/g(x) \sin(10\pi_1)]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n - 1)$	Unconstrained
Knapsack Problem	750	$\{0, 1\}^N$	$Z_l(x) = \sum_{j=1}^n p_{l,j} x_j$ $\forall l = 1, 2, \dots, k$	$\sum_{j=1}^n w_{l,j} x_j \leq c_l$
CTP Problem	5	$x_1 \in [0, 1]$ $x_i \in [-5.12, 5.12]$ $i = 2, \dots, n$	$Z_1(x) = x_1$ $Z_2(x) = g(x)(1 - \frac{Z_1(x)}{g(x)})$ $g(x) = 10n + \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i))$	$c(x) \equiv \cos(\theta)(Z_2(x) - e) - \sin(\theta)Z_1(x) \geq a \sin(b\pi (\sin(\theta)(Z_2(x) - e) + \cos(\theta)Z_1(x))^c) ^d$
CTP2: $\theta = -0.2\pi, a = 0.2, b = 10, c = 1, d = 6, e = 1$				
CTP3: $\theta = -0.2\pi, a = 0.1, b = 10, c = 1, d = 0.5, e = 1$				
CTP6: $\theta = 0.1\pi, a = 40, b = 0.5, c = 1, d = 2, e = -2$				
CTP7: $\theta = -0.05\pi, a = 40, b = 5, c = 1, d = 6, e = 0$				

Table 4.2: TMEA parameters and settings used in solving the test problems

Problem	Variable Type	TMEA parameters			
		Weight intervals	Pop. size	Encoding	Crossover
SCH	Real	100	50	Real	Uniform
ZDT3	Real	100	70	Real	Uniform
CONSTR	Real	100	100	Real	Uniform
TNK	Real	100	100	Real	Uniform
CTP Problems	Real	100	100	Real	Uniform
Knapsack	Binary	100	100	Binary	Uniform

Chapter 5

Performance Assessment

The results generated by TMEA and other MOEAs are compared with respect to three performance metrics: accuracy, spread, and coverage.

- *Accuracy*: This metric is used to compare the degree of dominance of the non-dominated set of solutions obtained by one method over those obtained by another. The *S*-factor defined by Zitzler and Thiele [26] is computed to characterize this metric in the comparisons made in this paper. For a set of maximization objectives, a larger value represents a better performance. A more detailed description about the *S*-factor with proofs can be found in [14].
- *Spread*: This metric is used to compare how much of the range of the non-inferior surface is covered by a set of non-dominated solutions. The spread parameter reported by Chetan [5] and Ranjithan et al. [20] is used to characterize this metric in the comparisons made in this paper. Using the illustration in Figure 5.1, points A and B refer to the two extreme noninferior solutions (corresponding to the single objective optima for each objective). The maximum range covered by the MOEA generated non-dominated solutions represented by the ordered set $C = \{C_h, \forall h \in \{1, \dots, q\}\}$ is $(Z_1^{C_q} - Z_1^{C_1})$ and $(Z_2^{C_1} - Z_2^{C_q})$ in Z_1 and Z_2 objective space, respectively. The spread metrics in objective space Z_1 and Z_2 are defined as $(Z_1^{C_q} - Z_1^{C_1}) / (Z_1^B - Z_1^A)$ and $(Z_2^{C_1} - Z_2^{C_q}) / (Z_2^A - Z_2^B)$, respectively. A larger value of this metric indicates better performance.

- Coverage:** This metric is used to compare how well the non-dominated solutions obtained by an MOEA are distributed in the objective space. Coverage is calculated by measuring the Euclidean distance between adjacent solutions to measure the distribution in objective space. Two coverage metrics V1 and V2 are defined ([5],[20]) to characterize the coverage within the range of non-inferior region defined by (1) the extreme noninferior solutions A and B, and (2) by the extreme solutions (C_1 and C_q) generated by that MOEA, respectively. Using the notations from Figure 5.1, V1 is defined as $\text{Max}\{d_h, \forall h \in \{0, 1, \dots, q\}\}$, and V2 is defined as $\text{Max}\{d_h, \forall h \in \{1, \dots, q-1\}\}$. A smaller value of V1 (or V2) implies more closely spaced noninferior solutions, thus indicating better coverage.

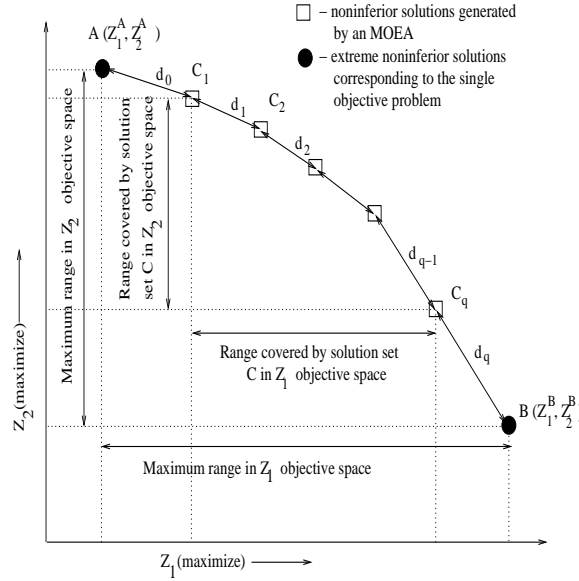


Figure 5.1: An example two-objective nondominated tradeoff to illustrate the computation of *Spread* and *Coverage* metrics.

Chapter 6

Results

For the test problems summarized in Table 4.1, the sets of non-dominated solutions obtained using TMEA are presented in graphs showing the non-inferior surface in the objective space. While the results were generated for 50 random trials, the results for only a representative run are reported here since the results exhibited robust behavior. Where available, non-dominated solutions reported in the literature are also shown for comparison. Further these solutions are compared using the performance metrics described above.

6.1 Unconstrained Test Problems

Schaffer's F2 problem [23] and the ZDT3 [13] problem are two unconstrained test problems that were used to evaluate TMEA. While F2 problem represents a continuous Pareto surface, the ZDT3 problem consists of discontinuities. The sets of non-dominated solutions obtained using TMEA are shown in Figures 6.1 and 6.2.

6.2 Constrained Test Problems with Continuous Decision Variables

Several constrained problems with continuity in the decision space were used to evaluate TMEA. Figures 6.3 and 6.4 show the non-dominated solutions obtained by TMEA for the

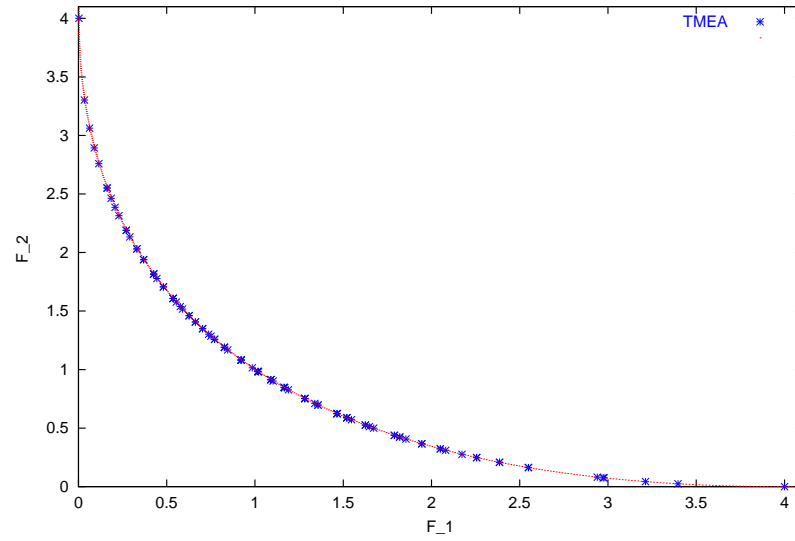


Figure 6.1: Non dominated solutions for the F2 problem

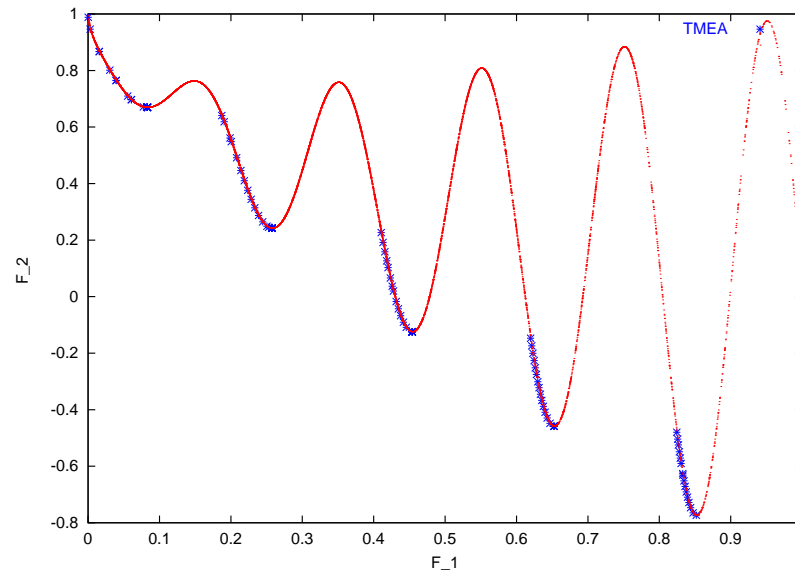


Figure 6.2: Non dominated solutions for the ZDT3 problem

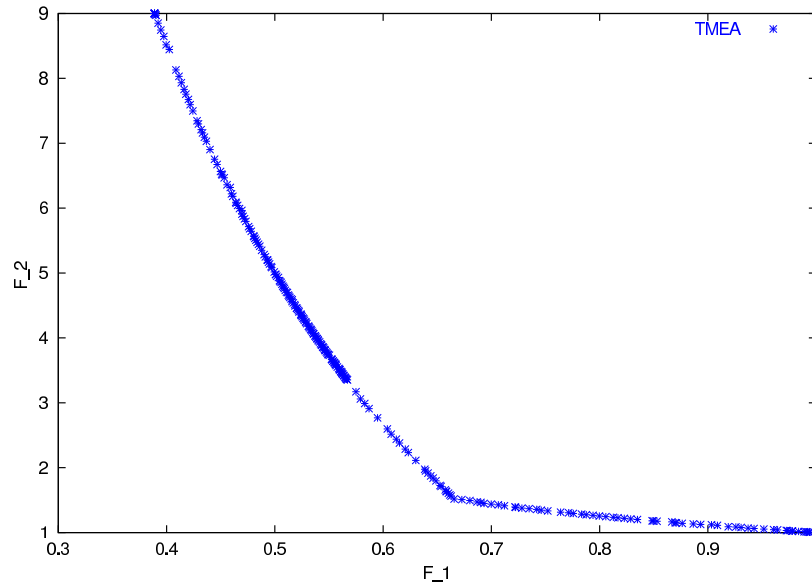


Figure 6.3: Non dominated solutions for the CONSTR problem

CONSTR problem and the SRN problem, both described by Deb et al [11].

Figure 6.5 shows the non-dominated solutions for the TNK problem that was proposed by Tanaka [24]. While the Pareto fronts for the CONSTR and SRN problems are continuous in the objective space, the Pareto front for the TNK problem includes discontinuities. In all these cases, the results indicate that TMEA was able to generate non-dominated solutions that represent the true Pareto front accurately with a uniform coverage.

The following set of constrained test problems (CTP2, CTP3, CTP6, and CTP7) was used to compare the results obtained using TMEA with those obtained using other MOEAs. Different instances of CTP, representing sharp discontinuities in the objective space, were proposed by Deb et al [11], and have been used frequently in testing and comparisons in the MOEA literature. The Figures 6.6 - 6.8 show the solutions obtained by TMEA for the CTPs along with those obtained by NSGA-II and CMEA. The results for NSGA-II and CMEA are obtained from the results reported by Kumar [18].

The Pareto front for CTP2 has a number of discontinuous regions, making the search for non-dominated solutions that represent these regions relatively more difficult. As seen

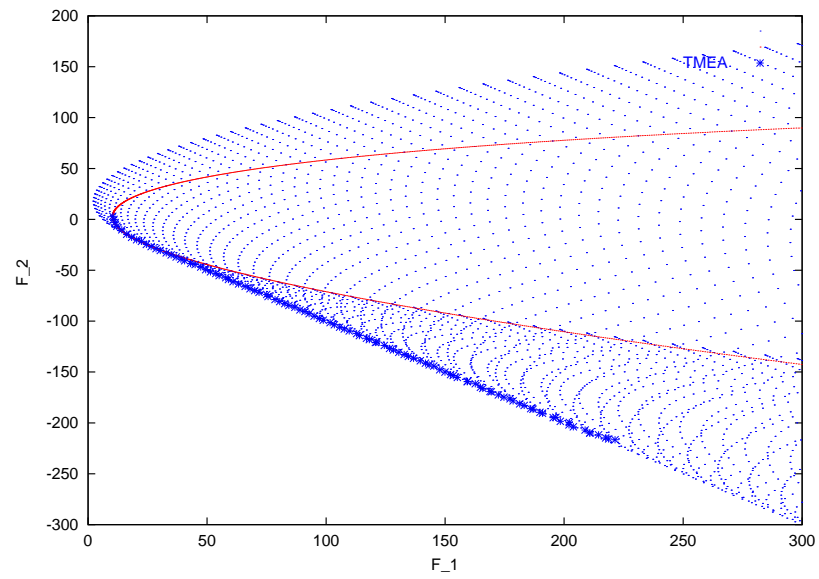


Figure 6.4: Non dominated solutions for the SRN problem

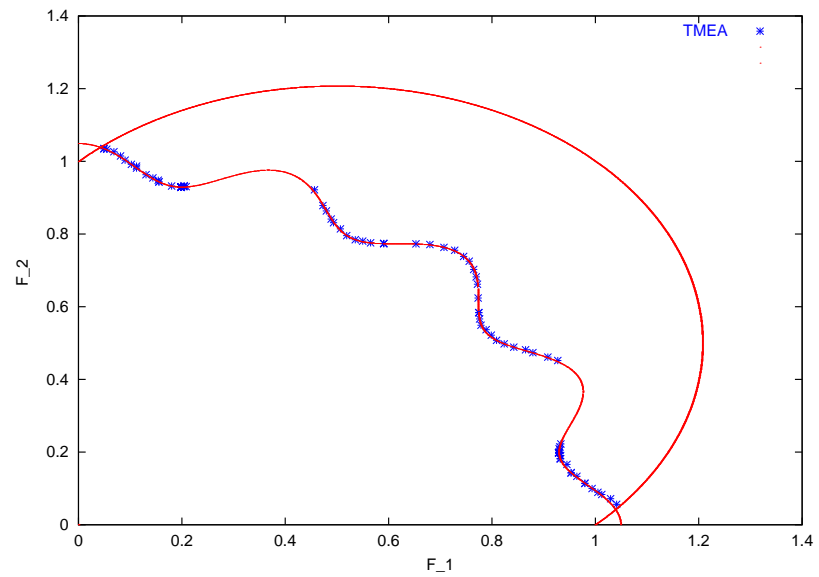


Figure 6.5: Non dominated solutions for the TNK problem

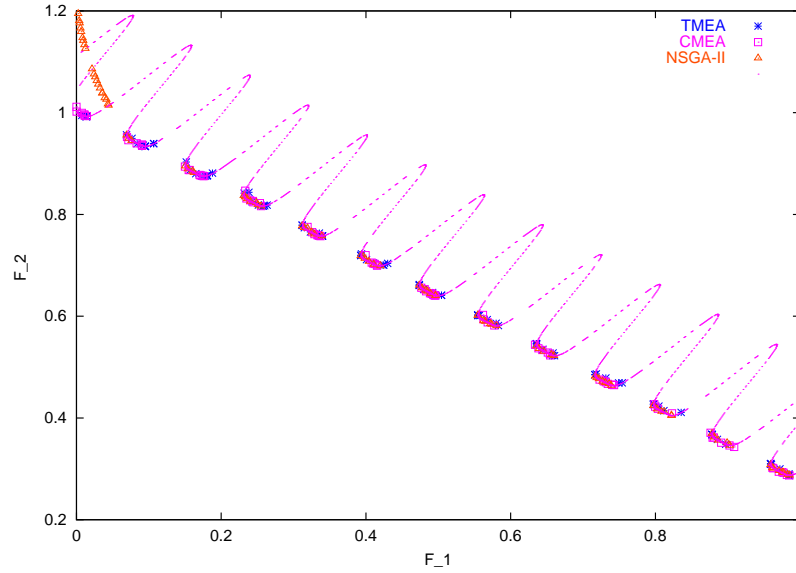


Figure 6.6: Non dominated solutions for the CTP2 Problem

in the Figure 6.6, TMEA was able to find successfully non-dominated solutions in all the disconnected regions. For this problem, TMEA performs relatively well in comparison to the solutions obtained by the other algorithms. The performance metrics are compared in Tables 6.1, 6.2 and 6.3.

CTP3 and CTP7 (Figures 6.7 - 6.8) consist of discrete regions in the Pareto front where one solution represents the non-inferior solution in each discontinuous region. For CTP3 (Figure 6.7), TMEA performs well to identify a non-dominated solution in each discrete region, but does not do as well as CMEA in several regions.

For CTP7 (Figure 6.8), TMEA again is able to identify non-dominated solutions in several of the discrete regions, and does better than the other algorithms in those regions. It misses, however, to identify non-dominated solutions in all discrete regions while CMEA was able to have a better distribution of non-dominated solutions. Again, the numeric values of the performance metrics are compared in Tables 6.1, 6.2 and 6.3. CTP6 (Figure 6.9) represents a problem with discrete bands of feasible regions in the objective space with one band of solutions corresponding to the Pareto set. TMEA performs comparatively well for

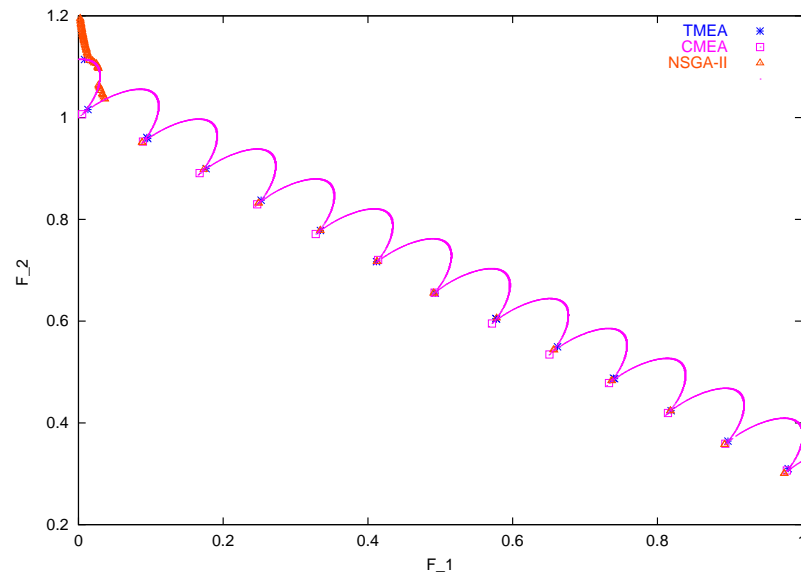


Figure 6.7: Non dominated solutions for the CTP3 Problem

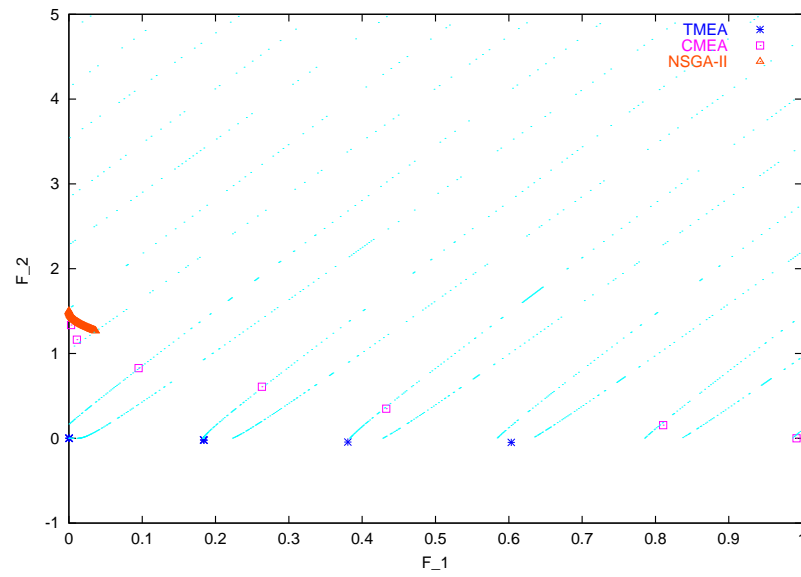


Figure 6.8: Non dominated solutions for the CTP7 Problem

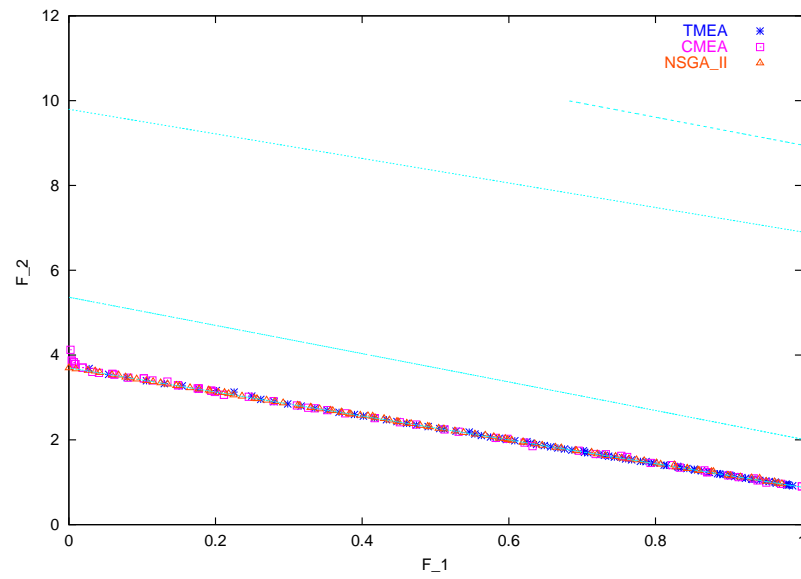


Figure 6.9: Non dominated solutions for the CTP6 Problem

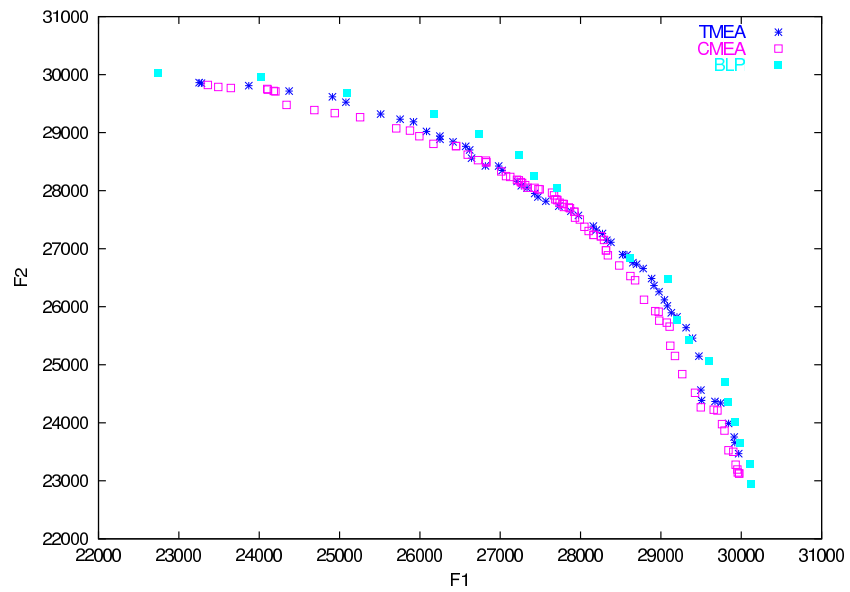


Figure 6.10: Non dominated solutions for the Multiobjective Knapsack Problem

this problem.

6.3 Constrained Test Problem with Discrete Decision Variables

Performance of TMEA was also tested and compared using the extended multiobjective knapsack problem proposed by Zitzler and Thiele [26]. This is a constrained problem with binary decision variable with a Pareto front as shown in Figure 6.10. This problem was been solved for two objectives with 500 items and 750 items. The results reported here correspond to 750 items and two knapsacks. The results are compared with those obtained from the CMEA and a Binary Linear Programming (BLP) solver that was solved using CPLEX. The BLP solution represent the best available estimation of the Pareto front. Non-dominated solutions obtained using TMEA compare well with respect to the BLP solution, and perform relatively better than the solutions obtained using CMEA. The performance metrics compared in Tables 6.1, 6.2 and 6.3 show the superior performance of TMEA with respect to accuracy, spread, and coverage.

Table 6.1: Accuracy comparison of TMEA with CMEA, NSGA-II, SPEA, and PESA for different test problems. A larger value indicates a better performance

The MOEAs Compared ($MOEA_1$ vs $MOEA_2$)	Problem instance	(S_1, S_2) : (S factor for $MOEA_1$ data set, S factor for $MOEA_2$ data set)
(TMEA vs NSGA-II)	CTP2	(0.613, 0.6075)
(TMEA vs CMEA)	CTP2	(0.613, 0.6123)
(TMEA vs NSGA-II)	CTP3	(0.578, 0.5823)
(TMEA vs CMEA)	CTP3	(0.578, 0.5931)
(TMEA vs CMEA)	CTP6	(0.525, 0.560)
(TMEA vs NSGA-II)	CTP6	(0.525, 0.5663)
(TMEA vs NSGA-II)	CTP7	(0.512, 0.1543)
(TMEA vs CMEA)	CTP7	(0.512, 0.6563)
(TMEA vs NSGA-II)	Knapsack	(0.731, 0.623)
(TMEA vs CMEA)	Knapsack	(0.731, 0.707)
(TMEA vs PESA)	Knapsack	(0.731, 0.602)
(TMEA vs SPEA)	Knapsack	(0.731, 0.607)

Table 6.2: *Spread* comparison of TMEA with CMEA, NSGA-II, SPEA, and PESA for different test problems. A larger value indicates better performance

The MOEAs Compared ($MOEA_1$ vs $MOEA_2$)	Problem instance	($Z1$ spread for $MOEA_1$, $Z1$ spread for $MOEA_2$)	($Z2$ spread for $MOEA_1$, $Z2$ spread for $MOEA_2$)
(TMEA vs NSGA-II)	CTP2	(1.000, 1.000)	(0.997, 1.000)
(TMEA vs CMEA)	CTP2	(1.000, 1.000)	(0.997, 1.000)
(TMEA vs NSGA-II)	CTP3	(0.991, 0.996)	(1.000, 1.000)
(TMEA vs CMEA)	CTP3	(0.991, 0.997)	(1.000, 1.000)
(TMEA vs NSGA-II)	CTP6	(0.967, 0.974)	(0.934, 0.973)
(TMEA vs CMEA)	CTP6	(0.967, 0.999)	(0.934, 1.000)
(TMEA vs NSGA-II)	CTP7	(0.755 , 0.036)	(0.510 , 0.234)
(TMEA vs CMEA)	CTP7	(0.755, 0.989)	(0.510, 1.000)
(TMEA vs NSGA-II)	Knapsack	(1.000 , 0.269)	(0.925 , 0.269)
(TMEA vs CMEA)	Knapsack	(1.000 , 0.922)	(0.925 , 0.920)
(TMEA vs PESA)	Knapsack	(1.000 , 0.280)	(0.925 , 0.261)
(TMEA vs SPEA)	Knapsack	(1.000 , 0.342)	(0.925 , 0.341)

Table 6.3: Coverage comparison of TMEA with CMEA, NSGA-II, SPEA, and PESA for different test problems. A smaller value indicates better performance

The MOEAs Compared ($MOEA_1$ vs $MOEA_2$)	Problem instance	($V1$ for $MOEA_1$, $V1$ for $MOEA_2$) (includes the known extreme points for each objective)	($V2$ for $MOEA_1$, $V2$ for $MOEA_2$) (excludes the known extreme points for each objective)
(TMEA vs NSGA-II)	CTP2	(0.074 , 0.350)	(0.262, 0.102)
(TMEA vs CMEA)	CTP2	(0.074 , 0.096)	(0.262, 0.096)
(TMEA vs NSGA-II)	CTP3	(0.142 , 0.360)	(0.165, 0.133)
(TMEA vs CMEA)	CTP3	(0.142 , 0.171)	(0.165, 0.161)
(TMEA vs NSGA-II)	CTP6	(0.053, 0.038)	(0.070, 0.035)
(TMEA vs CMEA)	CTP6	(0.053 , 0.152)	(0.070 , 0.091)
(TMEA vs NSGA-II)	CTP7	(0.222 , 1.590)	(0.999, 0.007)
(TMEA vs CMEA)	CTP7	(0.222 , 0.424)	(0.999, 0.4249)
(TMEA vs CMEA)	Knapsack	(0.082 , 0.089)	(0.082, 0.066)
(TMEA vs PESA)	Knapsack	(0.082 , 0.674)	(0.082, 0.010)
(TMEA vs SPEA)	Knapsack	(0.082 , 0.539)	(0.082, 0.022)

Chapter 7

Summary and Conclusions

This paper describes TMEA, a new EA-based multiobjective optimization procedure that builds upon the Tchebycheff method. Tchebycheff method has been established before and solved using mathematical programming procedures to estimate the non-inferior solutions for multiobjective optimization problems. The convergence of TMEA is enhanced by beneficial seeding of the population within the subiterations of the algorithm, a notion borrowed from mathematical programming-based MO analysis. Unlike other commonly reported MOEAs that attempt to converge the population of solutions simultaneously to the non-inferior set, TMEA attempts to converge first the population of solutions to an extreme non-inferior solution and then incrementally migrate the population to trace the non-inferior surface. As no new algorithm-specific operators or special encoding are needed, the structure of the algorithm enables easy integration with existing implementation of EAs for an optimization problem. This is important when analyzing large-scale realistic problems for which much effort is already spent on configuring and implementing the base evolutionary algorithms.

TMEA was evaluated by applying it to a number of test problems with different characteristics and levels of difficulty. This evaluation included problems involving continuous as well as combinatorial decision space, unconstrained as well as constrained optimization, real as well as binary variables, and concave as well as convex Pareto optimal sets. To characterize the performance of the algorithms, metrics quantifying accuracy, spread,

and coverage were used. Solutions were generated for several different random seeds, and TMEA showed robust behavior in generating the non-dominated solutions. Overall, TMEA performed relatively well with respect to these criteria and for the different problems tested.

By exploring specific regions of the objective space through different combinations of the γ weights in TMEA, the coverage and spread of the noninferior solutions are enforced explicitly and independent of noninferiority, resulting in excellent coverage while performing well with respect to noninferiority. While this approach can yield a relatively uniform coverage of the non-inferior region, depending on the shape of the Pareto front it is possible to converge to the same non-dominated solution for different γ weights, resulting in potential inefficiencies.

Further testing is needed to evaluate the effectiveness of applying TMEA to higher-order MO problems. As the number of objectives increases, the number of γ weight vectors increase, requiring increases in the combinatorial combinations of these weights. The computational as well as convergence performances associated with scale-up in the problem dimension need to be examined. TMEA is currently being applied to a real-world water resources management problem that requires MO analysis. The practicality of such realistic applications is being assessed and will be reported later.

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Appendix A

Computational Performance Evaluation of the TMEA

Table A.1: Number of function evaluations for solving the test problems

Problem	Num of Evaluations		
	First Iteration	Avg for subsequent iterations	Total
Schaffer	2550	1400	141150
ZDT3	6550	3653	368185
CTP2	9494	4041	409460
CTP3	9560	3960	401670
CTP6	9952	3750	381233
CTP7	8724	3839	388785