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Muon Acceleration

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September 30, 2003

Abstract

One of the major motivations driving recent interest in FFAGs is their use for the cost-effective acceleration of muons. This paper summarizes the progress in this area that was achieved leading up to and at the FFAG workshop at KEK from July 7–12, 2003. Much of the relevant background and references are also given here, to give a context to the progress we have made.

1 Introduction

Much of the recent interest in FFAGs has been centered on their use in accelerating muons for a neutrino factory. In a neutrino factory, one generally wants to accelerate to around 20 GeV. Since the muons are decaying, that acceleration must be rapid, generally corresponding to over 1 MV/m on average. The accelerator must also have sufficient acceptance for the input beam. In recent studies, the transverse acceptance under consideration has been 30 mm normalized (transverse normalized acceptance is $A^2mc/(\beta p)$, where p is the particle momentum or reference momentum, β is the beta function, A is the maximum beam size, m is the particle mass, and c is the speed of light).

The longitudinal acceptance, however, varies widely depending on what scenario is under consideration. In the US, the scheme that has been given the most consideration to this point involves taking initial distribution which has a very large longitudinal phase space area and creating a 201.25 MHz bunch train in the front end. In this scheme, the longitudinal acceptance for each bunch is around 50 meV-s. In contrast, the scheme considered in Japan does not have such a bunching scheme, and the longitudinal acceptance required is much higher.

These requirements will determine the design of any FFAG for muon acceleration.

2 Basic Lattice Properties

An FFAG is a lattice that uses alternating gradient focusing and has a very wide energy acceptance. This is achieved by having a highly symmetric lattice, where every cell is identical or nearly so, and insuring that in that single cell, one does not cross any linear resonances over the desired wide energy range. There have been two successful approaches to achieving this: they are referred to as scaling and non-scaling FFAGs.

2.1 Scaling FFAGs

The scaling FFAG is the original notion of an FFAG [1]. It is the only kind of FFAG which has actually been built [2, 3, 4].

Scaling FFAGs use highly nonlinear magnets to create a cell whose tune is independent of energy. Such a lattice can be created by constructing a vertical magnetic field which is described in a cylindrical coordinate system by

$$B_y(r,\theta) = B_{y0}(\theta)(r/r_0)^k. \qquad (1)$$

Then the momentum dependent closed orbits $R(\theta,p)$ are related to one another by

$$R(\theta,p) = R(\theta,p_0)(p/p_0)^{1/(k+1)}, \qquad (2)$$

and the closed orbit lengths $L(p)$ are related to one another by

$$L(p) = L(p_0)(p/p_0)^{1/(k+1)}. \tag{3}$$

The resulting lattice will have tunes which are independent of energy [1]. The fact that $R(\theta, p)$ is a θ-independent function times a momentum-independent function of θ is why this lattice is called "scaling."

In addition to the constant tunes, this type of lattice is distinguished by the fact that the closed orbit length is a monotonically increasing function of momentum. This will become important when one considers longitudinal dynamics.

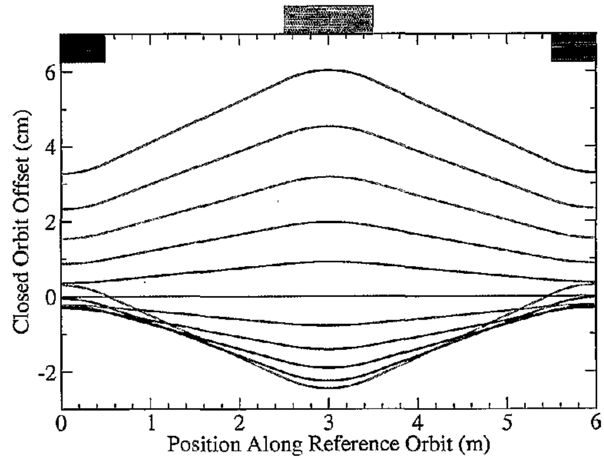

Figure 1: Energy-dependent closed orbits in a non-scaling FODO FFAG. Magnet positions are shown at the top.

2.2 Non-Scaling FFAGs

Non-scaling FFAGs [5] are an attempt to achieve the wide energy acceptance of an FFAG by using highly linear magnets instead of the nonlinear magnets that are found in the scaling FFAG. One can hope to achieve a larger transverse dynamic aperture in non-scaling FFAGs than in scaling FFAGs. It seems, however, that the advantages of a non-scaling FFAG over a scaling FFAG are likely to lie elsewhere.

The constant tune of the scaling FFAG gives it a substantial advantage during acceleration. One can choose a working point that is far from nonlinear and imperfection resonances, and the momentum-independent tune means that you remain at that working point throughout the acceleration cycle. In a non-scaling FFAG, the tune is no longer constant, and thus one passes through all of these resonances during acceleration. This could lead to substantial beam loss and/or emittance growth. It is hoped that such a machine is still useful for acceleration of muons, however, since it is necessary to accelerate muons very rapidly to avoid decays. One should pass through any resonances very quickly, which would hopefully lead to at most only a small emittance growth. A non-scaling FFAG is likely to be be problematic if one were accelerating slowly, however.

The energy-dependent closed orbits in the non-scaling FFAG are no longer geometrically similar to one another as they were in the scaling FFAG. Figure 1 shows how this can be advantageous for the non-scaling FFAG. First of all, notice that in the defocusing quadrupole, the orbits are more tightly packed, as expected. Furthermore, the orbits at different energies cross each other are the low-energy end. In a scaling FFAG, the orbits are nearly just as widely spaced in the defocusing quadrupole as in the focusing quadrupole, and they never cross. For comparable designs, the scaling and non-scaling FFAGs have comparable orbit spacing in the focusing quadrupole; thus, the magnet size in the defocusing quadrupole can be substantially smaller in the non-scaling FFAG when compared to the scaling FFAG.

A further potential advantage of the non-scaling FFAG over the scaling FFAG lies in its time-of-flight behavior. When trying to accelerate rapidly, a large amount of RF voltage must be installed, requiring a large amount of stored energy in the RF cavities. It becomes expensive to change the RF frequency in such a machine. Since making many passes through the RF is necessary, the fact that the time-of-flight depends on energy can prevent the bunch from staying on-crest and being accelerated.

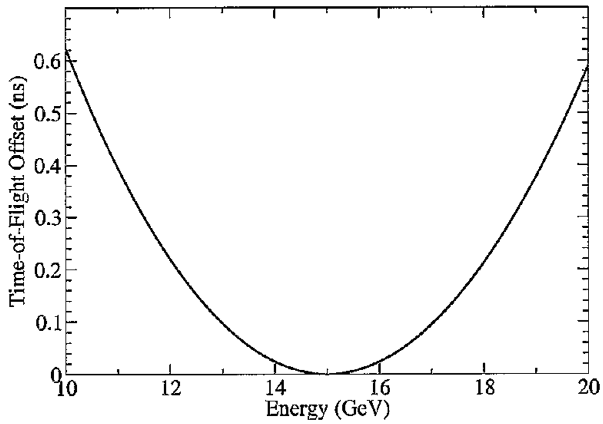

Figure 2: Time-of-flight as a function of energy in a non-scaling FFAG.

Solutions to this difficulty are discussed in the next section, but in general, the range in time-of-flight must be kept as small as possible. Figure 2 shows the time-of-flight as a function of energy in a non-scaling FFAG. Note that in contrast to the scaling FFAG, the time-of-flight in the non-scaling FFAG does not necessarily vary monotonically with energy. In fact, for a given maximum slope (which is characteristic of the lattice), the time-of-flight range will be substantially (about a factor of 2) lower for the parabolic-like behavior in the non-scaling lattice when compared to the scaling lattice.

3 Longitudinal Dynamics

Longitudinal dynamics is often one of the driving factors in the design of an FFAG. If one is not careful, it will not be possible even to accelerate from the minimum to the maximum energy. This is because of two factors: first, that the time-of-flight in an FFAG depends on the energy, and secondly that the requirement for rapid acceleration leads to large stored RF energies, making it prohibitively expensive to change the RF phase and/or frequency to match this changing time-of-flight. The result is a tradeoff between time-of-flight and RF voltage: the larger the time-of-flight range, the more RF voltage must be installed to accelerate over the desired energy range.

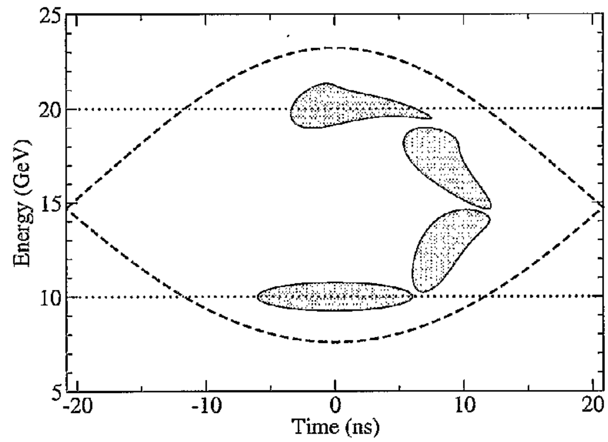

Figure 3: Acceleration in the RF bucket in a scaling FFAG. To accelerate, the bucket must extend from at least the minimum energy to the maximum energy (shown with horizontal dotted lines). If the RF voltage is too low (or the momentum compaction too high), the width of the bucket will be less than the energy range over which you wish to accelerate. The bunch starts at the bottom of the figure, and undergoes half a synchrotron oscillation to the top of the figure.

Lowering the time-of-flight range almost always increases the cost of the magnetic lattice.

3.1 Scaling FFAGs

For scaling FFAGs, the momentum compaction is independent of energy, and is $1/(k+1)$. If the energy is high enough such that one is above transition over the entire energy range, the time of flight will be a monotonically increasing function of energy. In this case, there is an RF bucket that is very similar to that found in synchrotrons, except that is has a large energy width. Acceleration in such an FFAG occurs by starting the bunch at the bottom of the bucket, and allowing the bunch to undergo half of a synchrotron oscillation, as shown in Fig. 3.

If the full energy width of the bucket is less than $E_{\max} - E_{\min}$, then it is not possible to remain inside the bucket and accelerate from $E_{\min}$ to $E_{\max}$ by undergoing half of a syn-

$p_{\min}$ (GeV/c)	0.3	0.3	1	1	3	10
$p_{\max}$ (GeV/c)	1	1	3	3	10	20
Lattice	DFD	DFD	DFD	DFD	FD	FD
$B_{\max}$(F) (T)	1.8	2.8	1.8	3.6	6.04	6.05
$B_{\max}$(D) (T)	1.8	2.8	1.8	3.6	5.69	5.55
Cells	32	16	64	32	100	120
k	50	15	190	63	350	450
Ring radius (m)	21	10	80	30	55	120
Orbit excursion (cm)	50	77	46	52	18	18
Drift (m)	2.06	212	4.325	3.229	1.25	2.09
Cell phase advance (H)	120°	131°	132°	154°	123.1°	120.2°
Cell phase advance (V)	61°	103°	33°	46°	57.2°	61.9°
Bucket height (GeV/c)	1.64	0.72	4.96	2.2	8.82	14.8
Transition momentum (GeV/c)	0.75	0.41	1.46	0.84	1.98	2.24

Table 1: Proposed parameters for non-scaling muon FFAGs [6, 7]. RF frequencies are about 24 MHz, and average voltage gradients are in the 0.75–1 MV/m range.

chrotron oscillation. Furthermore, if for some energy between $E_{\min}$ and $E_{\max}$, $\gamma^2 = k + 1$, then the longitudinal dynamics will be different from this. Table 1 gives the bucket height and momentum at transition for a sequence rings that are proposed as designs for a muon acceleration scheme using scaling FFAGs. Notice that especially for the lower energy rings, there are serious problems because the transition momentum is within the momentum range of the ring, and because the bucket height is not much larger than the desired momentum range. Furthermore, the bucket height calculations are not taking into account effects from the transition energy being within the energy range of acceleration.

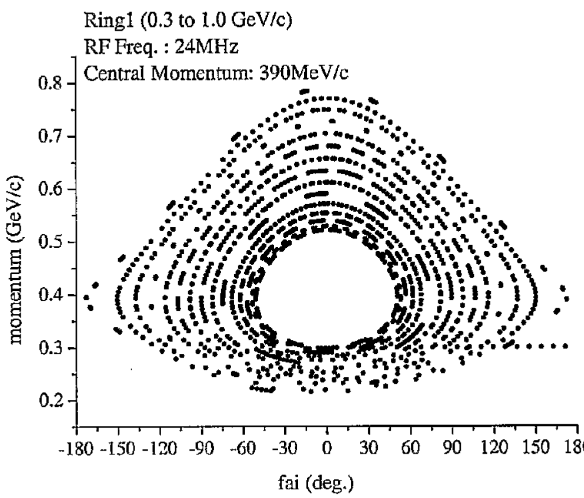

Figure 4: Tracking in a low-energy scaling FFAG with 24 MHz RF [7].

Figure 4 shows what tracking in a scaling FFAG can look like when the transition energy is within the range over which one is accelerating. The particles are clearly not capable of accelerating from 0.3 GeV/c to 1.0 GeV/c as required. Lowering the RF frequency, as shown in Fig. 5, clearly improves the situation.

To avoid the difficulties with longitudinal dynamics in low-energy lattices, one can avoid transition by keeping k either very low (but this leads to large orbit swings, and so is probably undesirable), or by making k very high. If needed, the bucket area can be increased by lowering the RF frequency, but it may be more challenging to achieve the required RF gradients in this case. Operating with the transition energy within the acceleration range is not completely out of the question, but requires a more complicated dynamical analysis. The next subsection describes a mode of operation which may be useful in that case.

Finally, there is another possible acceleration scheme (applicable in principle to both non-scaling and scaling lattices) wherein one

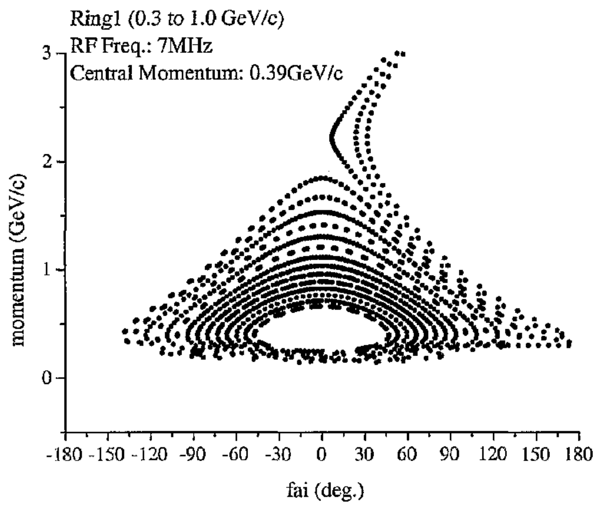

Figure 5: System in Fig. 4, but with 7 MHz RF instead [7].

uses two separate RF systems set up so as to have two overlapping RF buckets, one directly above the other, such that as the bunch is accelerated, the bunch is passed from one bucket to the next [6].

3.2 Non-Scaling FFAGs

Dynamics in non-scaling FFAGs are significantly different from that in scaling FFAGs. Non-scaling lattices have a non-constant momentum compaction, leading to a time-of-flight that varies roughly parabolically with energy. To reduce the variation in time-of-flight, the parabola is generally chosen so that its minimum (i.e., the point of isochronicity) is near the center of the energy range [8].

Examining Fig. 6, the phase space for this time-of-flight variation gives two ways to accelerate: making half a synchrotron oscillation within the buckets (this is probably what is happening in Figs. 4 and 5), and going between the buckets. Figure 6 shows that for a given RF voltage, one can accelerate by a larger amount if one goes between the buckets.

For a parabolic time-of-flight variation with energy, the problem can be transformed to scaled variables [10]. The longitudinal equations of motion when RF cavities are dis-

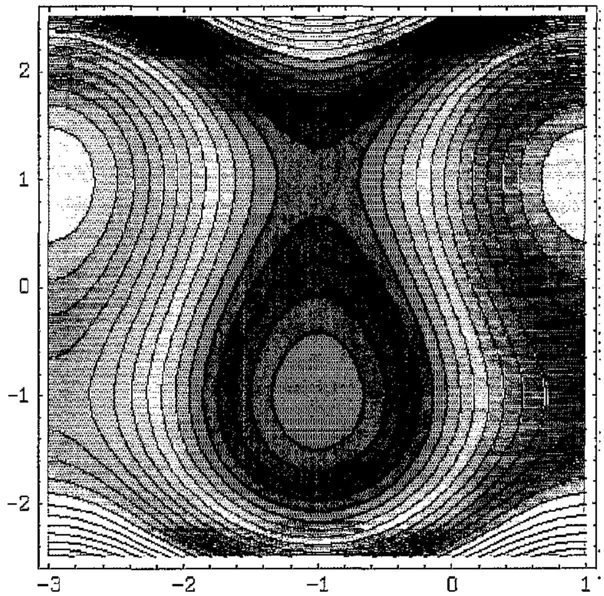

Figure 6: Phase space for FFAG acceleration with parabolic time-of-flight variation with energy [9].

tributed smoothly around the ring and all RF cavities have the same frequency and phase are

$$\frac{dx}{du} = (2p-1)^2 - z \qquad \frac{dp}{du} = w\cos(x), \tag{4}$$

where

$$x = \omega\tau \qquad p = \frac{E - E_{\min}}{\Delta E} \tag{5}$$

$$u = \frac{\omega \Delta T s}{L} \tag{6}$$

$$z = \frac{T_0}{\Delta T} \qquad w = \frac{V}{\omega \Delta T \Delta E}, \tag{7}$$

s is the arc length along the reference orbit, E is the particle energy, τ is the time minus the the time of the RF crest, ω is the angular frequency of the RF, and V is the amount of RF voltage in the length L. We desire to accelerate particles from $E_{\min}$ to $E_{\min} + \Delta E$. The change in τ over the length L depends on particle energy: it is a parabola whose minimum is at $E_{\min} + \Delta E/2$, and has a value there of $-T_0$; its value at $E_{\min}$ and $E_{\min} + \Delta E$ is $\Delta T - T_0$.

Figure 7 shows the phase space where the bunch can be accelerated in the normalized phase space. The normalized phase space acceptance increases with increasing w, but the

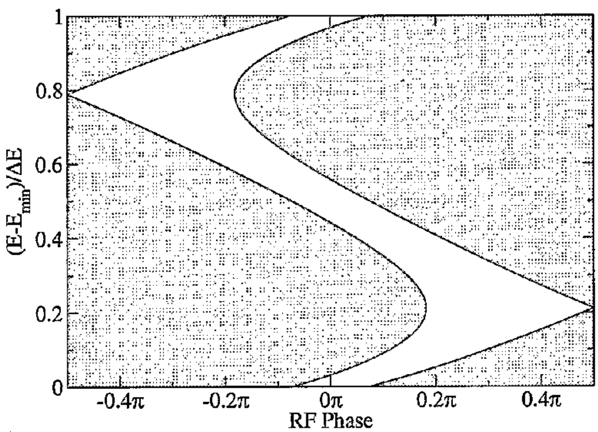

Figure 7: Transmitted (white) and blocked (dotted pattern) regions in normalized phase space [11].

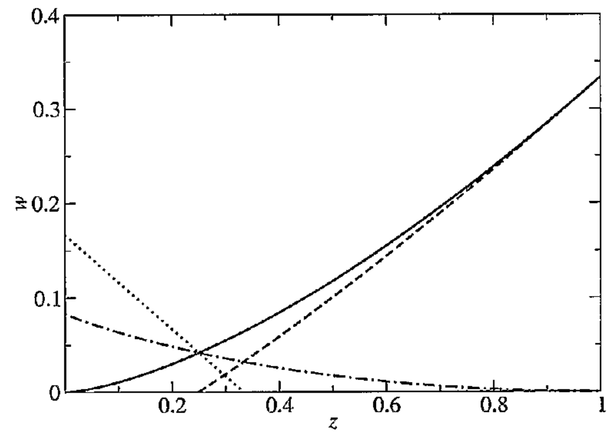

Figure 8: Relevant boundaries in parameter space. Above the solid line, there is a path through the center of the normalized phase space (0,0.5); above the dashed line, the right-hand separatrix crosses $p = 0$; above the dashed-dotted line, the left-hand separatrix crosses $p = 0$; above the dotted line, a line through the center of the phase space also crosses $p = 0$ [11].

exact dependency is not clear. There is likely to be a dependence on z as well. What is clear is that as ΔE decreases (considering lower energy stages), the acceptance in the normalized phase space will decrease for a given w; thus w must be increased as one goes to lower energy stages. This requires increasing the RF voltage or decreasing the time-of-flight range (ΔT). Figure 8 shows the (z, w) parameter space, showing the region of parameter space where particles can be accelerated over the desired energy range (assuming that one wishes to accelerate in the region between the buckets). It is not currently known how to precisely characterize the relationship between the desired normalized phase space acceptance and the point where one should operate in this parameter space. It does appear that $z = 1/4$ is the optimal operating point, since it gives the lowest minimum value for w (of 1/24), but this has not been definitively proven in the case where one wants to transmit a finite phase volume.

The discussion so far has assumed that the cavities all have the same phase (in the sense that for M uniformly distributed cavities, the phase of cavity number k is $2\pi kK/M$ for some integer K). One can instead allow all cavities to have different initial phases. There are several things one can do to optimize the system performance in this case; one choice [12] is to adjust the cavity phases and frequency (same for all cavities) to minimize the quantity

$$\sum_{i=1}^{N}\sum_{j=1}^{M}(\phi_{ij} - \bar{\phi}_j)^2, \tag{8}$$

where

$$\bar{\phi}_j = \frac{1}{N}\sum_{i=1}^{N}\phi_{ij} \tag{9}$$

and ϕ_{ij} is the phase for which the bunch arrives at cavity j on turn i.

One of the things that makes defining a strict criterion for phase space acceptance difficult is that what the term "acceptance" really means is unclear. Generally, one wants to transmit a volume of phase space without "significant" distortion. One potential way to substantially decrease phase space distortion is to add higher harmonic RF to the ring [12, 13]. Figure 9 shows the longitudinal phase space after acceleration with and without the third harmonic RF, showing significant reduction in the phase space distortion with higher

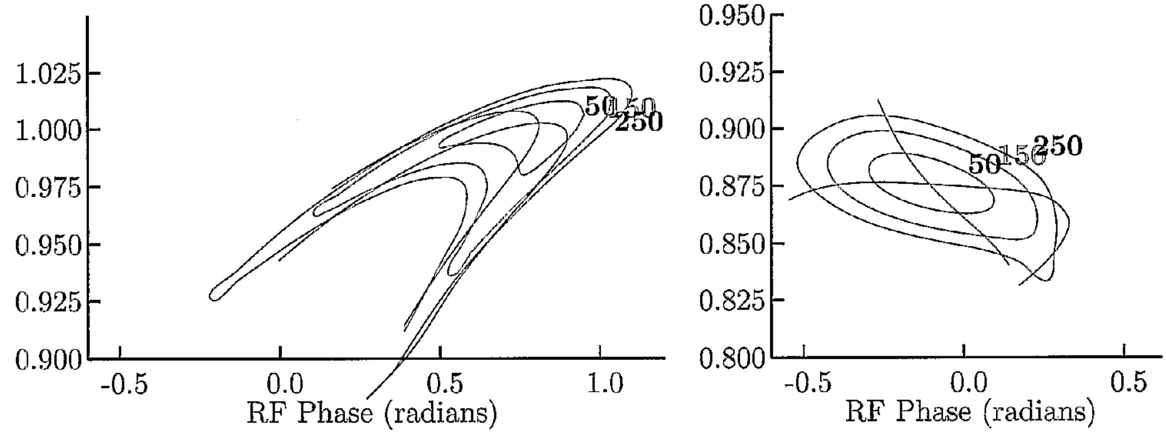

Figure 9: Phase space after acceleration, without (left) and with (right) third harmonic cavities [13].

harmonic RF. Higher harmonics do not significantly increase the size of the region of phase space where particles are accelerated; the effect seems to be mostly in increase in the linearity of the central part of that region.

4 Lattice Design

FFAG lattices generally consist entirely of a repetition of a relatively simple cell. Symmetry breaking is avoided as much as possible. Under these conditions, only the linear and nonlinear resonances in a single cell (as opposed to the entire ring or superperiod) need to be considered in the lattice design.

One must completely avoid the single-cell linear resonances, since they lead to extremely rapidly particle loss (within a couple of cells). There are essentially two ways to do this: keeping the tunes constant (scaling FFAGs), and keeping the tunes below 0.5 over the entire energy range (non-scaling FFAGs). Other non-scaling FFAG designs have been attempted where the tunes were not constant, but kept between 0.5 and 1.0 using sextupoles [14]. These designs have essentially been abandoned due to their poor dynamic aperture which resulted from the strong sextupoles required for chromaticity correction.

Nonlinear resonances present a more challenging problem. The scaling FFAG lattice addresses them by fixing the tunes at a good operating point, as far from significant nonlinear resonances as possible. This comes at a cost of significant nonlinearity in the magnets, which can limit the transverse dynamic aperture (due to both an increased tune footprint and an increase in the strength of nonlinear resonances). The non-scaling lattice deals with the nonlinear resonances by accelerating through them sufficiently rapidly that any emittance growth due to them is minimized. The non-scaling lattices are therefore not appropriate for very slow acceleration; but for muons, it is desirable to accelerate rapidly anyhow.

Most designs under consideration require that some drift space be placed in each cell to allow for RF cavities and potentially other hardware. For most non-scaling designs, superconducting 201.25 MHz RF is being considered. When one is bunching the beam, from both the point of view of longitudinal acceptance in the acceleration as well as minimizing the length of the initial phase rotation before (or during) bunching, it is desirable to have the lowest possible RF frequency where one still has a good ratio of surface field to accelerating field. Due to practical size considerations, that frequency is around 200 MHz. Superconducting RF is desirable since it is less costly than room-temperature RF, due to its lower peak power requirement. The length of the cavity cell and the requirement that the magnets be kept sufficiently far away from the superconducting cavities to keep the magnetic fields below about 0.1 T [15] lead to a requirement that there be a drift in the cell that is at least about 2 m long. This provides one of the critical constraints in non-scaling designs; similar constraints are required for scaling designs.

4.1 Lattice Structure

There are some basic constraints on the lattice that are generally related to the hardware that is required for the machine. The primary one is that there must be at least one drift per cell of sufficient length to hold an RF cavity; that length will depend on the choice of RF technology. There must be sufficient distance between adjacent magnets to accommodate coils and any other necessary hardware (diagnostics, for example). Pole tip fields may be limited (or at least will have a strong effect on cost).

For a given set of physical constraints, one can choose different types of lattices: some examples might be a FODO lattice, a doublet lattices, or triplet lattices. The triplet lattice may be arranged with the horizontally defocusing quadrupole in the middle (FDF) or the horizontally focusing quadrupole in the middle (DFD).

In the original design for muon acceleration using a scaling FFAG [16], DFD triplets were used. The motivation for using this configuration over an FDF configuration is that the vertical beam size will be smaller. The magnets are required to be wide in any case due to the horizontal orbit excursion, so keeping the vertical beta functions low seemed wise since it would at least reduce the vertical aperture. More recent designs [6] have switched to a doublet configuration for higher energy lattices (see parameters in Tab. 1). This has allowed for both a smaller circumference and a higher k, the latter leading to reduced orbit excursions and larger bucket heights for a given RF voltage.

The original non-scaling FFAG lattices were FODO lattices [5]. There has been interest in switching to a triplet lattice coming from two fronts. First of all, ΔT is approximately proportional to the cell length for a given design [17]. The drift required for the RF is relatively long, and a FODO lattice requires two of them. One could shorten the cell by choosing a lattice which requires only a single drift, and a triplet is a good choice [18]. The triplet is also the logical extension of the FFAGs based on a minimum-emittance lattice [14, 19]. The earlier lattices had tunes over 0.5, and therefore needed strong sextupoles to reduce their chromaticity (to avoid the linear resonances) and give a wide energy acceptance. To reduce the tune below 0.5, elements needed to be removed, leading to a triplet lattice.

If one uses the triplet, there is a question of whether to use a DFD arrangement of the magnets, or an FDF arrangement. For the non-scaling lattice, one can show that in a bending magnet, the normalized dispersion changes by an amount $\theta\sqrt{\beta_x}$ in a bending magnet which bends by an angle θ with a horizontal beta function of β_x [20]. In the FDF arrangement, β_x is lower where there is the most bending compared to the DFD arrangement, thus leading to a lower dispersion function and therefore a lower ΔT. Similarly, in a scaling lattice, the horizontal tune is lower and the vertical tune is higher in a FDF lattice than in a DFD lattice with the same F to D ratio and k value [7]. Since the horizontal tune is higher than the vertical tune in these scaling lattices, and the horizontal tune crossing the half integer is one thing that prevents k from being raised (tunes increase with increasing k since the quadrupole strengths increase), one can achieve a higher k in the FDF lattice than in the DFD lattice [6]. Larger k leads to a smaller orbit excursion and a either a lower RF voltage requirement or better longitudinal acceptance. Furthermore, injection and extraction for a scaling FFAG is significantly easier in a FDF lattice than in a

Minimum energy (GeV)	10	
Maximum energy (GeV)	20	
Cells	150	59
D-F drift (cm)	50	17
Cavity drift (cm)	200	206
D quad center (mm)	8.07	3.79
D quad radius (cm)	7.97	9.41
D quad length (cm)	83.4	150
D quad dipole (T)	3.07	5.81
D quad gradient (T/m)	-54.8	-35.7
D pole tip field (T)	7.0	9.0
F quad center (mm)	1.36	-2.16
F quad radius (cm)	5.95	10.70
F quad length (cm)	29.4	50
F quad dipole (T)	-1.58	-3.41
F quad gradient (T/m)	93.2	67.0
F pole tip field (T)	7.0	10.7
w	1/12	1/12
Circumference (m)	663.2	288.8
RF Voltage (MV)	995	607
Voltage/cell (MV)	6.6	10.3
$c\Delta T$	28 cm	17 cm

Table 2: Parameters for 10–20 GeV FDF triplet FFAGs. The left are for an optimization where the pole tip fields, D-F drift, and average RF gradient were fixed [11]; the second is a design trying to achieve a reduced circumference and ΔT [22].

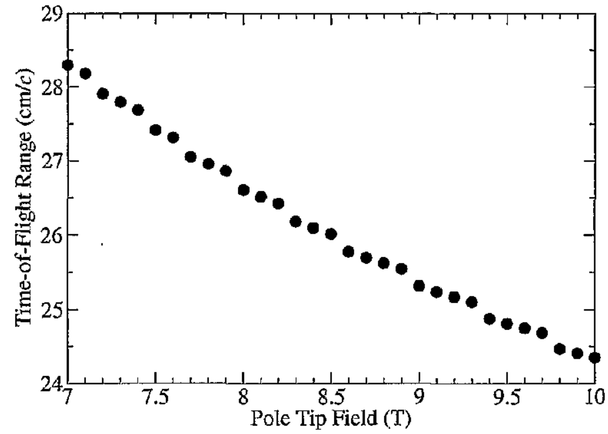

Figure 10: Dependence of ΔT on maximum pole tip field [11].

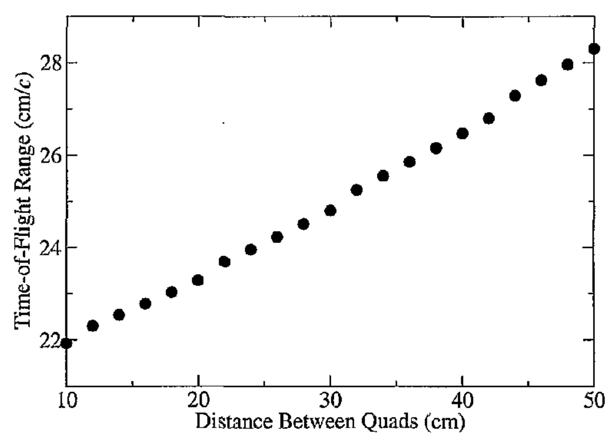

Figure 11: Dependence of ΔT on D-F spacing in triplet [11].

DFD lattice [21].

Optimization techniques can be used to produce designs that meet certain constraints and/or to minimize/maximize some quantity, such as machine cost. They also allow one to examine how lattice properties depend on constraints, such as pole tip fields or drift lengths. Certain dependencies have already been demonstrated, such as a linear dependence of ΔT on cell length, or the inverse dependence of ΔT on the number of of cells [17]. It is not optimal to simply increase the number of cells to increase ΔT, since that not only increases the arc cost, but it also either increases the RF voltage that one must install, or results in increased muon decays due to the lower average RF gradient. For the 10–20 GeV non-scaling FFAG, optimization techniques were used to produce an initial FDF triplet lattice for the workshop, with parameters as listed in Tab. 2. Later, a lattice was produced by hand which had a significantly smaller $c\Delta T$ (see Tab. 2).

The improvement can be ascribed (roughly equally) to three factors: that the drift lengths between the quadrupoles were reduced, that the pole tip fields in the magnets were increased, and that the RF gradient required to have a given longitudinal acceptance was increased. Figures 10 and 11 show the dependence of ΔT on the pole tip fields and the drift space between the magnets in the triplet. It therefore seems best to place the

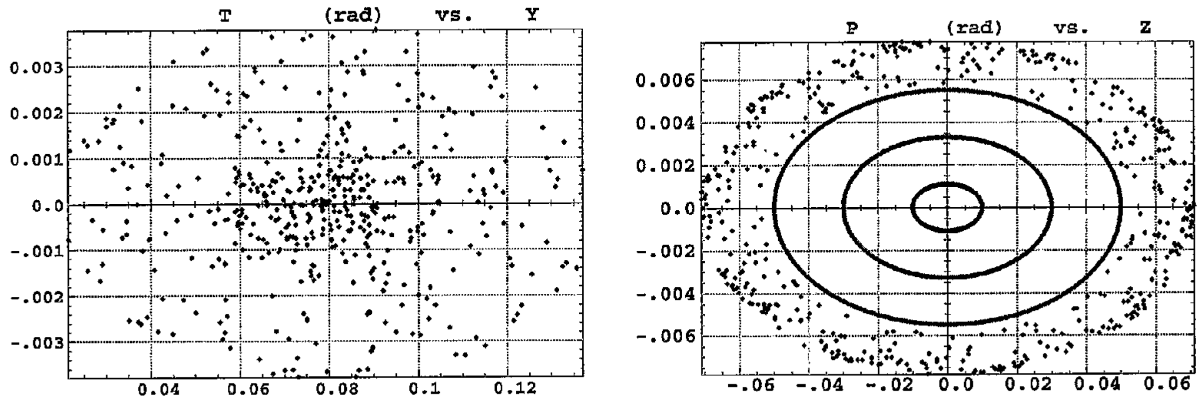

Figure 12: Particle tracking in an example non-scaling triplet FFAG lattice: particles are launched at nonzero amplitudes in the vertical phase space plane, as shown to the right; the motion of those particles in the horizontal plane are shown to the left. Particles are tracked at 20 GeV [26].

quadrupoles in the triplet as close as is practical. There are several constraints that prevent the reduction in the distance between the quadrupoles: the need for space for the coils to turn around (especially if one wishes to reduce higher-order multipoles), the need to put diagnostics (however, there is probably plenty of space elsewhere), the need for trim coils (which can hopefully be incorporated into the magnets themselves), and finally the fact that the adjacent magnets with opposite field arrangements will tend to cancel each other's fields.

Non-scaling lattices were produced for lower energy ranges [22, 23]. The cost per GeV of acceleration of the lower energy lattices is generally significantly larger than in higher energy lattices [13]. This comes about largely because the larger beam size requires larger apertures. Also, the value of the scaled voltage (w, above) required to transmit a given phase space area increases as the energy is reduced, due to the fact that the scaled energy p is really the energy divided by ΔE [11]. Thus, either greater voltages or lower ΔT are required at lower energies, increasing costs.

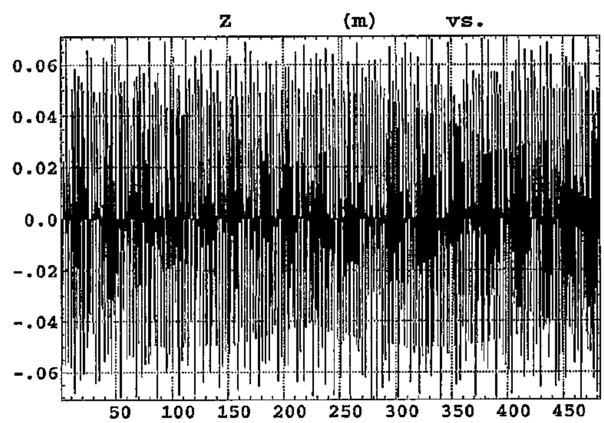

Figure 13: Vertical position versus cell number for particle at the outer amplitude in Fig. 12 [26].

5 Tracking Results

Single-particle tracking without acceleration was performed using the codes ICOOL [24] and ZGOUBI [25]. Tracking was performed both with and without fringe fields. There are significant effects of nonlinear coupling in both cases (Fig. 12), but there does not seem to be significant particle loss, even at the highest amplitudes (Fig. 13, where the amplitude shown is actually larger than the required acceptance). The fact that the particles do not follow perfect ellipses at large amplitudes may lead to an

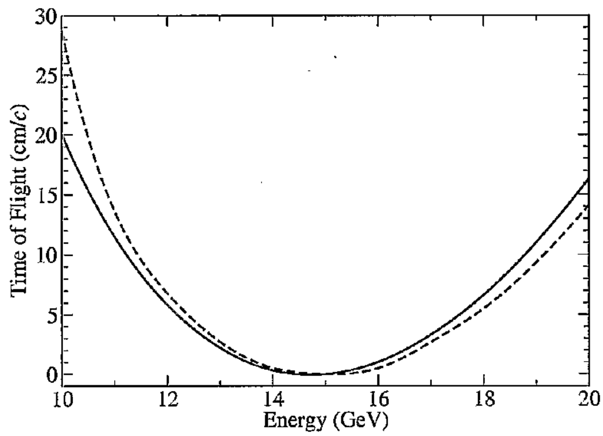

Figure 14: Time-of-flight as a function of energy for zero-amplitude particles (solid line) and particles the maximum amplitude (dashed line: they trace out an ellipse with normalized area 30π mm) [13].

increased aperture requirement beyond what is computed from a simple linear model, but that requirement appears to be modest [13, 26].

Another potentially significant observation from tracking is that the time-of-flight as a function of energy depends significantly on the transverse particle amplitude (Fig. 14). Since the longitudinal dynamics is highly dependent on this time-of-flight profile, and there is no phase focusing in the longitudinal direction, this could lead to high-amplitude particle loss if not taken into account. This would probably require a larger voltage to increase the longitudinal phase space acceptance, thus increasing costs. The precise amount of excess voltage needs to be quantified.

6 Conclusions

This paper summarizes the work of many participants in the FFAG03 workshop, both in giving presentations and in discussions that occurred during the workshop. They include Masamitsu Aiba, Carol Johnstone, Shinji Machida, Phil Meads, François Méot, Yoshiharu Mori, David Neuffer, Robert Palmer, Dejan Trbojevic, Masahiro Yoshimoto, and myself. Hopefully I have not forgotten anyone.

Both leading up to and during this workshop, we have made significant progress in our understanding of FFAG design. We have significantly improved our understanding of longitudinal dynamics in FFAGs. Designs for both non-scaling and scaling lattices have been improved, both in cost and performance. We have a better understanding of what lattice changes will give better performance. We have begun more detailed nonlinear tracking in these lattices, particularly the non-scaling lattices.

There are fruitful areas for future exploration. Full six-dimensional tracking still needs to be performed for the non-scaling lattices, and the results of this should be incorporated into lattice designs (for example, the effect of transverse amplitude on time-of-flight). Quantitative criteria should be found for determining appropriate parameters for longitudinal dynamics. We need to determine the optimal type of lattice to use for various design goals. We need to determine cost-optimized scenarios for a sequence of FFAGs for accelerating muons. Finally, there is interest in building an electron demonstration for a non-scaling FFAG, since such a device has never been built.

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