LS-304

Effect of Emittance and rms Phase Error on Angular Flux Density and Pinhole Flux—A Simulation Study of Two Undulators at 10.5 mm Gap Including Very High Harmonics

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Introduction

There is a trade-off between how much effort should go into the tuning of insertion devices to reduce their rms phase errors and the actual benefits achieved in spectral quality when the real APS beam emittance and beam energy spread are taken into account. In the magnetic measurement laboratory, the measured magnetic fields are analyzed in terms of the rms phase error and the angular flux density, which is calculated from the measured fields for an ideal electron beam, i.e., a zero-emittance beam. In this study, we go beyond the case of an ideal beam to study the effect of the APS beam emittance and beam energy spread on the angular flux density and the pinhole flux (for a typical pinhole size that covers most of the central cone of the radiation) for real-field insertion devices to get an estimate of how low an rms phase error is reasonable to attain. The results presented here are directly applicable to the APS "canted" undulators of type A (planar permanent-magnet hybrid insertion devices 2.1 m long and 3.3 cm period length) but also to the standard undulators A (which have a similar design with the same period length but are 0.3 m longer) unless otherwise noted, e.g., the asymptotic ratios of the real-to-ideal intensity at very high harmonic numbers differ.

To obtain the sensitivity to the rms phase error on the spectra, two devices were studied—one that exemplifies a device with smaller than average rms phase error (3.7° at 10.5 mm gap; henceforth labeled the "low-phase-error device") and one that represents a device with larger than average rms phase error (6.8° at 10.5 mm gap; henceforth labeled the "high-phase-error device"). It should be noted that, although this device has a relative large rms phase error, it is by no means performing poorly and is well within the APS tolerance specification (of 8° rms phase error at 11.5 mm gap). The spectra were calculated up to 100 keV to study degradation of very high harmonics due to magnetic field errors, an important consideration for medium-energy storage rings in particular, where use of the higher harmonics is commonplace or commonly proposed. The results should be used as guidance only for such facilities since they depend on the specifics of the beam parameters.

Device and Emittance

A series of calculations was performed with the code UR[1] to study the effect of the APS emittance and beam energy spread on the angular flux density and pinhole flux for two devices that show a large difference in the rms phase error at the same gap setting (10.5 mm): undulators type A, 1) a 2.1-m-long “canted” insertion device with period length 3.3 cm and 3.7° rms phase error, and 2) a similar device with 6.8° with rms phase error.

Two emittances are compared at 7.0 GeV beam energy and 100 mA current: i) the routinely-used low-emittance lattice with emittance 2.5 nm-rad, coupling 2.9%, and beam energy spread 9.6×10^{-4} (beam parameters from a typical run from May 30, 2003 were used), ii) the high-emittance lattice[2] with emittance 7.7 nm-rad, coupling 1.0%, and beam energy spread 9.6×10^{-4} (same as for the low-emittance lattice).

Results

The results are summarized in five tables and five figures below. Table 1 shows the performance ratios for the angular flux density and table 2 the pinhole flux ratios at 10.5 mm gap. Table 3 is a summary table for the lower odd harmonics that shows the author’s estimate of the expected performance for both quantities, and table 4 shows additional calculations at the larger gap of 18.5 mm. The very high harmonics (up to and including harmonic 33) are examined in table 5 for energies up to 100 keV. The figures are complementary to the tables and show calculated spectra (figs. 1 – 4) and predicted intensity degradation due to field errors and emittance (fig. 5).

The listed intensities are the peak values. The ratio for first harmonic is always close to 100% and was omitted (except in table 5). The calculated values of the ratios in this study are accurate to about ±1% for the angular flux density and ±2% for the pinhole flux for the specific set of measured field files being used. For another set of measured field files (or for an assembly average of measured field files) with approximately the same rms phase errors, one may get ratios that differ from the results here by twice the values given above—it is an estimate only as a statistical study was not done.

[1] R.J. Dejus and A. Luccio, Nucl. Instrum. Methods, A**347**, 61 (1994).

[2] R.J. Dejus, I.B. Vasserman, S. Sasaki, and E.R. Moog, Argonne National Laboratory Report, ANL/APS/TB-45, May 2002, p. 31.

Table 1. Comparison of calculated angular flux density at 10.5 mm gap for two insertion devices and three emittances for harmonics 3, 5, and 7. The values in parentheses are for the high-phase-error device (6.8° rms phase error), and the plain values for the low-phase-error device (3.7° rms phase error).

Angular Flux Density (ph/s/mrad^2/0.1%bw) at 100 mA for Harmonic Number									
	3			5			7		
	Zero	Low	High	Zero	Low	High	Zero	Low	High
Real	9.33×10^{17} (8.92×10^{17})	3.84×10^{17} (3.69×10^{17})	2.23×10^{17} (2.16×10^{17})	1.08×10^{18} (8.40×10^{17})	2.63×10^{17} (2.30×10^{17})	1.48×10^{17} (1.31×10^{17})	1.01×10^{18} (6.10×10^{17})	1.64×10^{17} (1.31×10^{17})	9.14×10^{16} (7.46×10^{16})
Ideal	1.03×10^{18} (1.06×10^{18})	4.16×10^{17} (4.23×10^{17})	2.43×10^{17} (2.45×10^{17})	1.24×10^{18} (1.26×10^{18})	2.99×10^{17} (2.99×10^{17})	1.68×10^{17} (1.67×10^{17})	1.33×10^{18} (1.34×10^{18})	2.06×10^{17} (2.02×10^{17})	1.13×10^{17} (1.11×10^{17})
Ratio	91% (84%)	92% (87%)	92% (88%)	87% (66%)	88% (77%)	88% (78%)	75% (46%)	80% (65%)	81% (67%)

Notes and observations:

1. The "Ratio" is defined as the quotient of "Real" (real-field) to "Ideal" (ideal field) intensity. The "Low" and "High" in the table refers to the low- and high-emittance lattice, respectively. (A low-emittance lattice, with emittance 2.5 nm-rad, coupling 2.9%, and beam energy spread 9.6×10^{-4} was used, and a high-emittance lattice, with emittance 7.7 nm-rad, coupling 1.0%, and beam energy spread 9.6×10^{-4} was used.) The harmonic energies are not given as they are not important here; they differ slightly because of slightly different K values ($K = 2.798$ for the low-phase-error device and $K = 2.721$ for the high-phase-error device). The different K values are also the reason why the ideal intensities differ slightly.

2. The ideal sinusoidal field was generated for 60 periods with 3.3 cm period length. Comparison was also made with the code MA[3] for the 3rd and 5th harmonics for the zero-emittance case. Excellent agreement was found (differed at most by 1 percentage point). The real fields are from Hall probe magnetic measurements at 10.5 mm gap.

3. When the beam emittance and beam energy spread are introduced, the ratio improves in general in comparison with the zero-emittance case. But the effect is only marginal for the low-phase-error device (significant change is only seen for the 7th harmonic). For the high-phase-error device, the ratio is improved for all harmonics studied here. The low-phase-error device always shows a higher ratio (better performance) than the high-phase-error device. There is only a small improvement on the ratios for the high-emittance lattice, i.e., the low- and high-emittance lattices show approximately the same intensity ratios.

[3] Computer code MA for Magnetic Analysis of measured magnetic fields, unpublished. Roger J. Dejus.

Table 2. Comparison of calculated pinhole flux (2.5 x 1.0 mm at 30 m) at 10.5 mm gap for two insertion devices and two emittances for harmonics 3, 5, and 7. The values in parentheses are for the high-phase-error device (6.8° rms phase error), and the plain values for the low-phase-error device (3.7° rms phase error).

Pinhole Flux (ph/s/0.1%bw) at 100 mA for Harmonic Number									
	3			5			7		
	Zero	Low	High	Zero	Low	High	Zero	Low	High
Real	-	3.06×10^{14} (2.98×10^{14})	2.87×10^{14} (2.79×10^{14})	-	1.82×10^{14} (1.66×10^{14})	1.73×10^{14} (1.59×10^{14})	-	1.07×10^{14} (9.51×10^{13})	1.07×10^{14} (9.42×10^{13})
Ideal	-	3.31×10^{14} (3.26×10^{14})	3.07×10^{14} (3.06×10^{14})	-	2.03×10^{14} (1.97×10^{14})	1.92×10^{14} (1.89×10^{14})	-	1.26×10^{14} (1.20×10^{14})	1.28×10^{14} (1.20×10^{14})
Ratio	-	93% (91%)	93% (91%)	-	90% (84%)	90% (84%)	-	85% (80%)	84% (79%)

Notes and observations:

1. The "Ratio" is defined as the quotient of "Real" (real-field) to "Ideal" (ideal field) intensity. There is no comparison with zero-emittance beam here.

2. The low-emittance and high-emittance lattices show the same ratios (within margin of errors).

3. The low-phase-error device always performs better than the high-phase-error device, but the difference has become less noticeable in comparison with the ratios for the angular flux density (c.f., table 1).

4. All ratios have increased in comparison with the ratios for the angular flux density (c.f., table 1).

5. The angular extent of the pinhole is 2 – 3 times the APS beam divergence; therefore a considerably broadening of the spectra is expected, acting analogously to a very large emittance. Thus an improvement of the ratios is indeed expected.

Table 3. Comparison of estimated *rounded* ratios of real-to-ideal performance for harmonics 3, 5, and 7 of two insertion devices at 10.5 mm gap for the APS beam emittance (low- or high-emittance) and beam energy spread (values rounded to lie within 1-3%). The values in parentheses are for the high-phase-error device (6.8° rms phase error), and the plain values for the low-phase-error device (3.7° rms phase error).

Harmonic Number	Ratio: Angular Flux Density	Ratio: Pinhole Flux (2.5x1.0 mm @ 30 m)
3	90% (85%)	95% (90%)
5	85% (75%)	90% (85%)
7	80% (65%)	85% (80%)

Figure 1 shows an example of the calculated angular flux density. Peak intensities from this graph and similar graphs of other harmonics were used to generate the data shown in the tables.

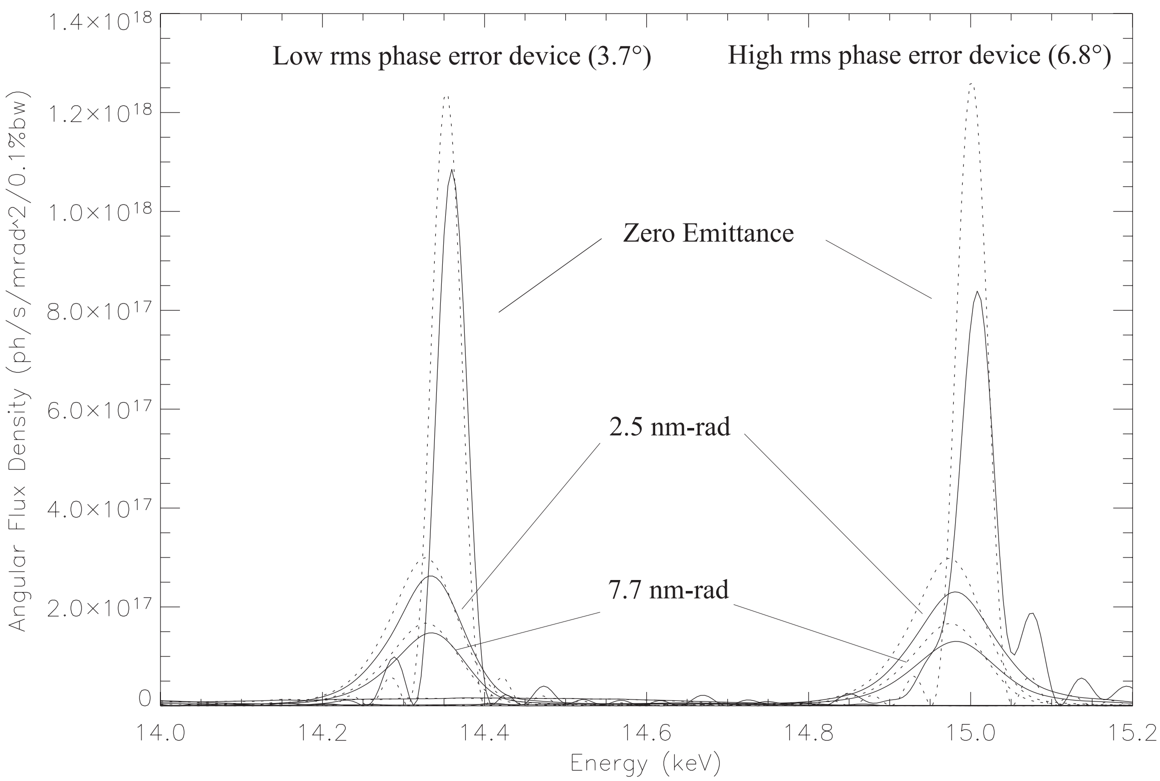

Figure 1. Example of calculated angular flux density of the 5th harmonic for the two devices at 10.5 mm gap at 7.0 GeV energy and 100 mA current—zero-emittance calculations and emittance calculations with the beam energy spread included. Calculations for ideal magnetic fields are indicated by the dotted lines and for measured magnetic fields by the solid lines. The location of the peaks differs by 0.65 keV because the *K* values are slightly different.

Gap Dependence

There is a gap dependence on the ratio of the real-to-ideal performance. The emittance-calculated intensity ratios tend to improve with increased gap (smaller *K* value) for a given period length and fixed rms phase error. In reality, the rms phase error becomes smaller (on average) with increased gap, thus improving the ratio even further. Hence, the 10.5-mm-gap study represents the worst case, i.e., the largest loss of intensity. Therefore, to get an estimate of the relative improvement of the ratios, the angular flux density was calculated at an intermediate gap setting of 18.5 mm (table 4). As can be seen by comparing table 4 and table 1, the effect is not large, but it confirms the general trend. For this example, we used the high-phase-error device that has a measured rms phase error of only 3.0° at 18.5 mm gap (c.f., 6.8° at 10.5 mm gap).

Table 4. Comparison of calculated angular flux density at 18.5 mm gap for the high-phase-error device (rms phase error 3.0°) and two emittances for harmonics 3, 5, and 7.

Angular Flux Density (ph/s/mrad^2/0.1%bw) at 100 mA for Harmonic Number									
	3			5			7		
	Zero	Low	High	Zero	Low	High	Zero	Low	High
Real	7.95×10^{17}	1.85×10^{17}	-	3.44×10^{17}	4.68×10^{16}	-	1.18×10^{17}	1.10×10^{16}	-
Ideal	8.59×10^{17}	1.95×10^{17}	-	4.01×10^{17}	5.20×10^{16}	-	1.64×10^{17}	1.34×10^{16}	-
Ratio	93%	94%	-	86%	90%	-	72% *)	82%	-

Notes and observations:

*) Zero-emittance ratio is 3 percentage points smaller than the corresponding case at 10.5 mm gap (table 1) for approximately the same rms phase error. It reflects the statistical fluctuations in the results arising from the actual phase-error distribution.

1. The “Ratio” is defined as the quotient of “Real” (real-field) to “Ideal” (ideal field) intensity. The values are for the high-phase-error device with an rms phase error of 3.0° at 18.5 mm gap and a *K* value of 1.208 (still labeled the “high-phase-error” device although the rms phase error is relative small at this gap setting).

2. Only the low-emittance lattice was calculated, since no significant difference was found on the ratios of the two emittances.

3. The effect of the emittance on the ratios is more substantiated now (favorable) in comparison with the ratios obtained at the larger *K* value at 10.5 mm gap (c.f., table 1, the low-phase-error device with 3.7° rms phase error).

4. The real field was taken from Hall probe magnetic measurements at 18.5 mm gap.

5. The ratios for the low-emittance lattice for the 9th and 11th harmonics are 76% and 70%, respectively (not in the table but harmonics are shown in figure 4).

Very High Harmonics

There is an increasing interest to use the very high harmonics of undulator radiation to reach x-ray energies as high as 10 keV with appreciable intensity in newly constructed storage rings operating at medium beam energy of 2 – 3 GeV. To get an estimate of the effect of the magnetic field errors on the spectrum at very high harmonics, the spectra were calculated up to 100 keV for the APS low-emittance lattice at 7.0 GeV. Figures 2 and 3 compare the calculated angular flux densities from the measured magnetic field with the ideal field at 10.5 mm gap for the low- and high-phase-error devices, respectively. Similarly, in figure 4, a comparison is made for the high-phase-error device at 18.5 mm gap to illustrate the overall quick drop in intensity with increasing harmonic number for a smaller *K* value. It should be emphasized that, although both devices approach the same ratio at high harmonics (35%) at 10.5 mm gap, the actual level reached depends on many factors, e.g., length of the device and emittance (see discussion in relation to figure 5 that succinctly displays the different cases).

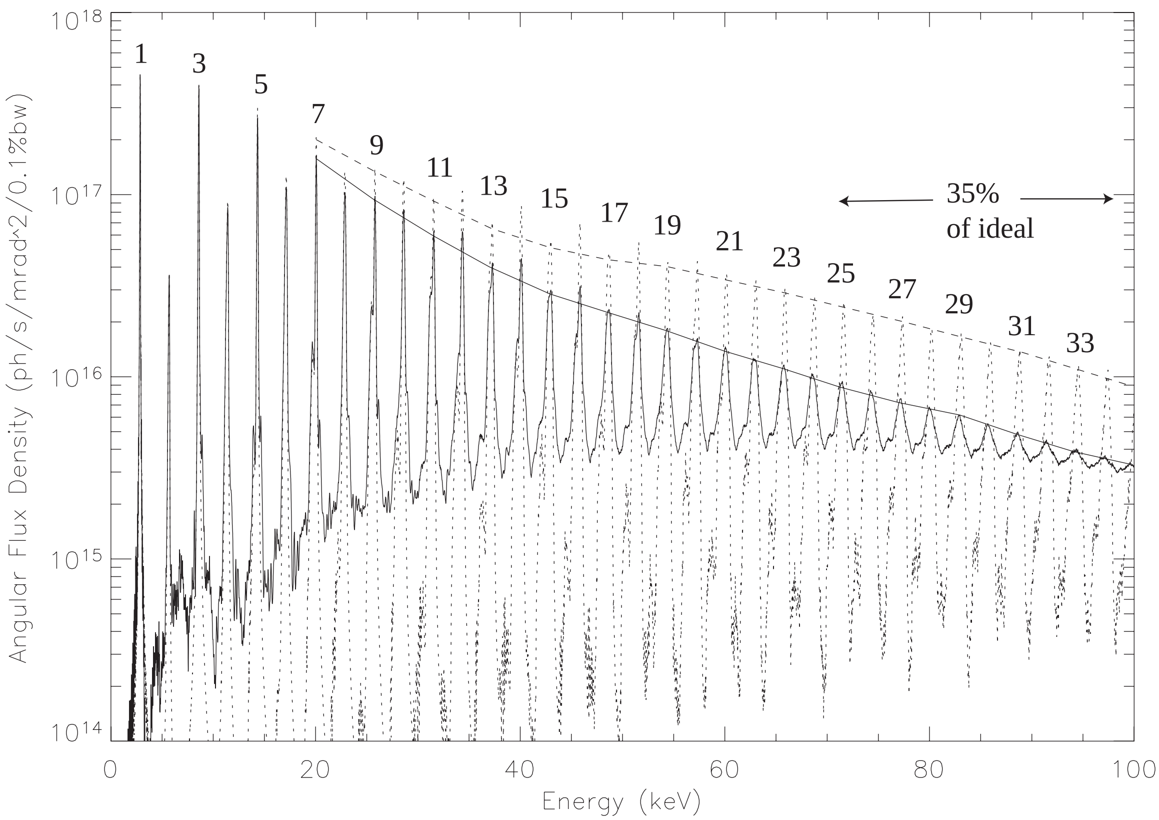

Figure 2. Calculated angular flux density for all harmonics up to 100 keV at 10.5 mm gap for the low-phase-error device (K = 2.798, E_1 = 2.87 keV; rms phase error 3.7°) for the 2.5 nm-rad low-emittance lattice at 7.0 GeV energy, 100 mA current, and beam energy spread of 9.6x10^{-4}. Calculation for the ideal magnetic field is indicated by the dotted line and for the measured magnetic field by the solid line. The odd harmonic numbers are labeled, and the odd harmonic peaks have been connected to emphasize the reduction in intensity due to magnetic field errors alone. Above harmonic 25, the ratio of the real-to-ideal performance is constant at 35%, a value that depends on the length of the device and the emittance. The reduction of each odd harmonic is plotted in figure 5 and listed in table 5.

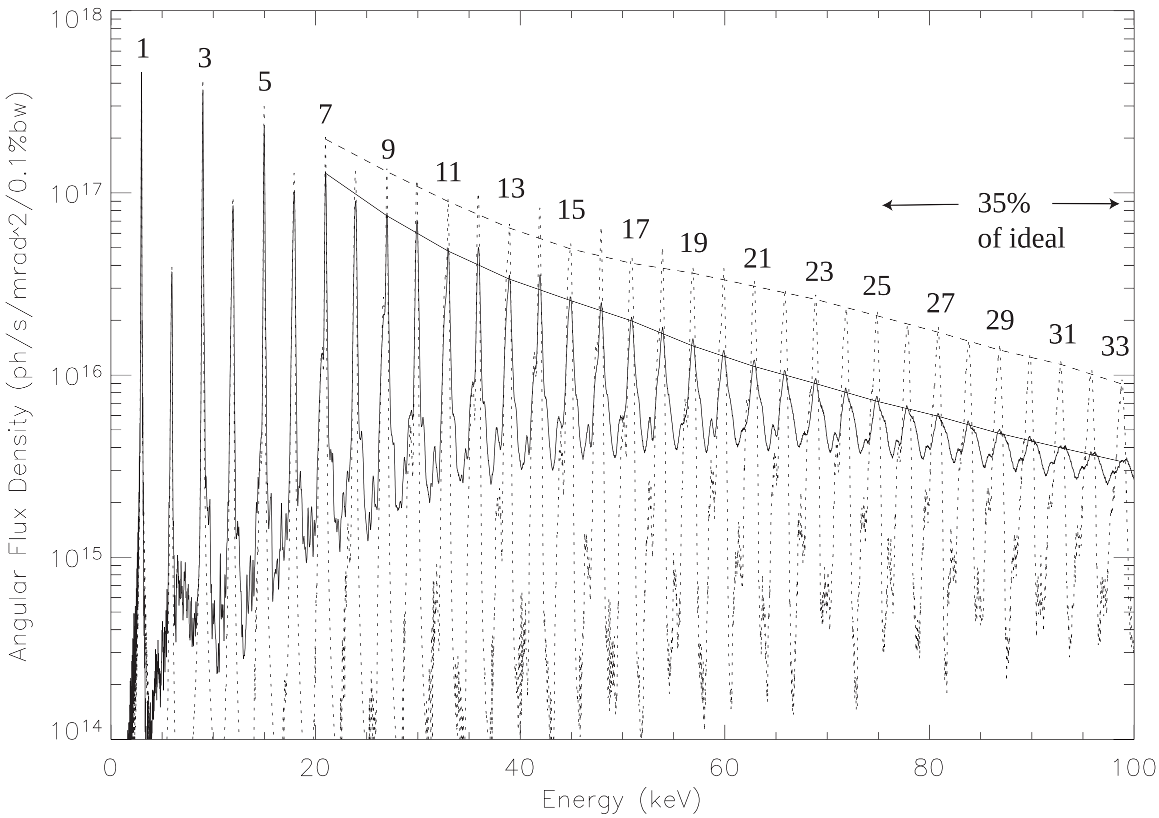

Figure 3. Calculated angular flux density for all harmonics up to 100 keV at 10.5 mm gap for the high-phase-error device (K = 2.721, E_1 = 3.00 keV; rms phase error 6.8°) for the 2.5 nm-rad low-emittance lattice at 7.0 GeV energy, 100 mA current, and beam energy spread of 9.6×10^{-4}. Calculation for the ideal magnetic field is indicated by the dotted line and for the measured magnetic field by the solid line. The odd harmonic numbers are labeled, and the odd harmonic peaks have been connected to emphasize the reduction in intensity due to magnetic field errors alone. Above harmonic 25, the ratio of the real-to-ideal performance is constant at 35% (same value as for the low-phase-error device and depends on the length of the device and the emittance). The reduction of each odd harmonic is plotted in figure 5 and listed in table 5.

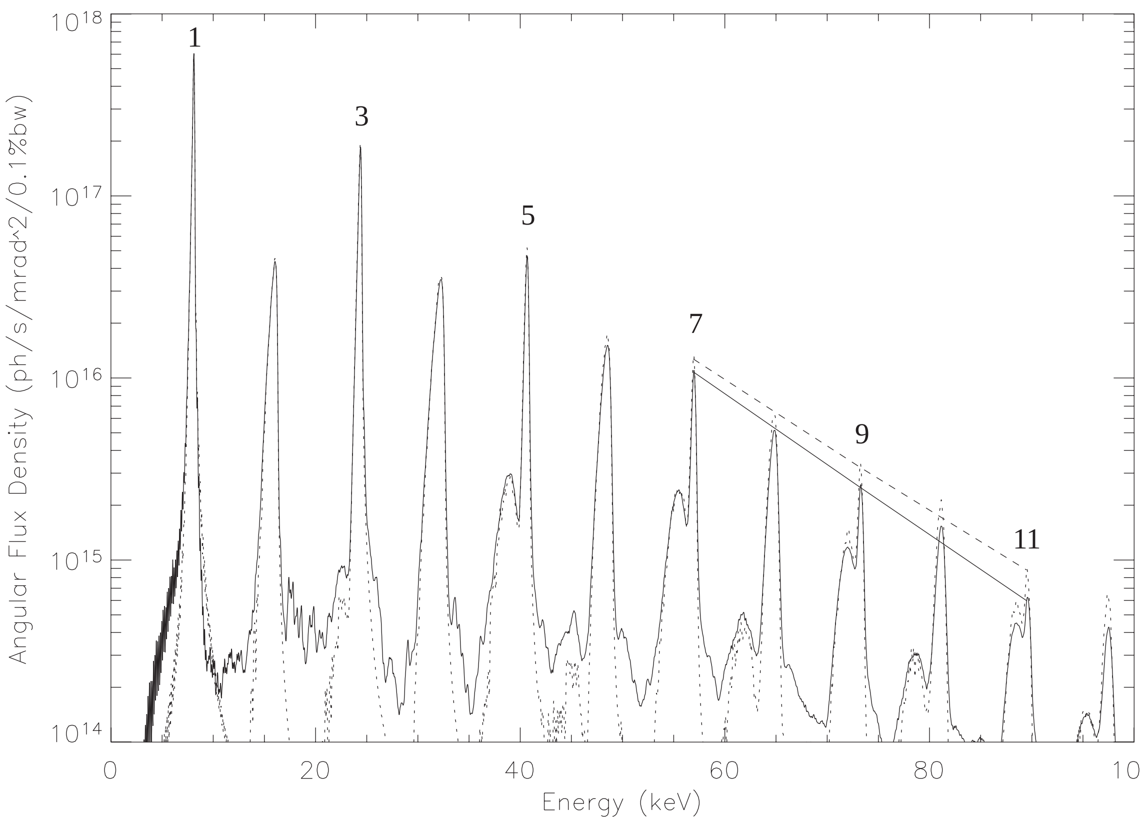

Figure 4. Calculated angular flux density for all harmonics up to 100 keV at 18.5 mm gap for the high-phase-error device (K = 1.208, E_1 = 8.15 keV; rms phase error 3.0°; still labeled the high-phase-error device because of the larger phase error at the small gap) for the 2.5 nm-rad low-emittance lattice at 7.0 GeV energy, 100 mA current, and beam energy spread of $9.6\text{x}10^{-4}$. Calculation for the ideal magnetic field is indicated by the dotted line and for the measured magnetic field by the solid line. The odd harmonic numbers are labeled, and the odd harmonic peaks have been connected to emphasize the reduction in intensity due to magnetic field errors alone. Ratio of real-to-ideal performance is for: 3^{rd} harmonic, 94%; 5^{th}, 90%; 7^{th}, 82%; 9^{th}, 76%; and 11^{th}, 70%.

The calculated reduction in intensity due to large and small magnetic field errors at 10.5 mm gap is summarized in figure 5. It shows the angular flux density for zero emittance and low emittance and the pinhole flux (low emittance only) to get a comprehensive understanding of the effect of field errors on the harmonic intensities. The accompanying data are listed in table 5.

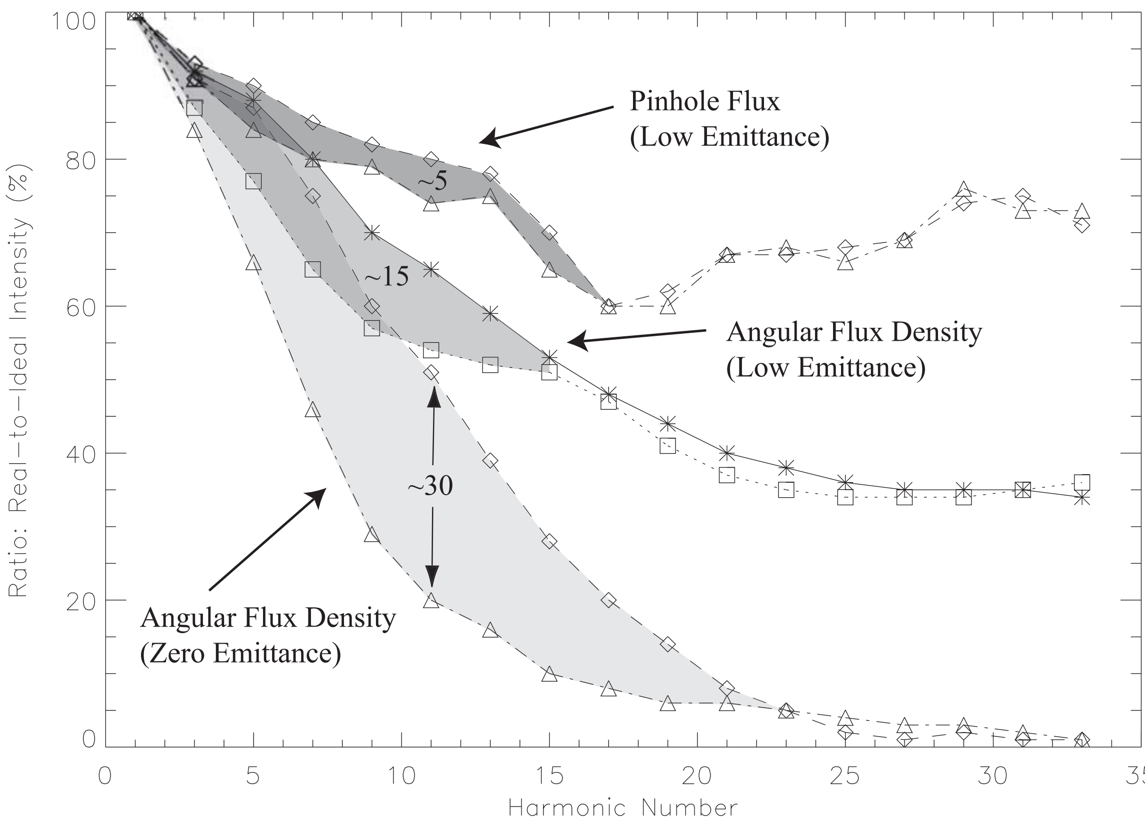

Figure 5. Comparison of calculated ratios of real-to-ideal field performance of the angular flux density and pinhole flux (2.5 x 1.0 mm at 30 m) for the two insertion devices at 10.5 mm gap for the 2.5 nm-rad low-emittance lattice and for zero emittance at 7.0 GeV for odd harmonics up to 100 keV (harmonic number 33). The upper boundaries for each of the three areas (stars and diamonds) are for the low-phase-error device (3.7° rms phase error), and the lower boundaries (squares and triangles) are for the high-phase-error device (6.8° rms phase error). The areas were shaded to elucidate the effect of the magnitude of the magnetic field error on the harmonic intensities. The maximum widths (difference) in percentage points are indicated for each area. For the practical case of a real beam (with emittance and beam energy spread taken into account), the maximum difference is about 15 percentage points for intermediate harmonic numbers (7 – 9) for the angular flux density but only about 5 percentage points for the pinhole flux, and, for both, smaller reductions are seen for the lower harmonics. Above a certain harmonic number, there is no difference between the high-phase-error and low-phase-error device. The asymptotic ratio depends on several factors including the length of the device (the longer the device, the smaller the asymptotic ratio) and the emittance (and the experimental setup, e.g., a pinhole smears the spectra in a way similar to the emittance). There are statistical fluctuations of the individual data points, but the overall shape of the areas is not expected to differ much for an assembly average of devices. The ratio for low-to-intermediate harmonics (3 – 9) depends on magnetic field quality and the emittance but not sensitively on the device length or period length.

Table 5: Tabulated values of the data plotted in figure 5.

Harmonic Number	Harmonic Energy (keV)	Ratio: Angular Flux Density (Zero Emittance)	Ratio: Angular Flux Density (Low Emittance)	Ratio: Pinhole Flux; 2.5 x 1.0 mm at 30 m (Low Emittance)
1	2.87 (3.0)	100% (100%)	100% (100%)	100% (100%)
3	8.61 (9.0)	91% (84%)	92% (87%)	93% (91%)
5	14.35 (15.0)	87% (66%)	88% (77%)	90% (84%)
7	20.09 (21.0)	75% (46%)	80% (65%)	85% (80%)
9	25.83 (27.0)	60% (29%)	70% (57%)	82% (79%)
11	31.57 (33.0)	51% (20%)	65% (54%)	80% (74%)
13	37.31 (39.0)	39% (16%)	59% (52%)	78% (75%)
15	43.05 (45.0)	28% (10%)	53% (51%)	70% (65%)
17	48.79 (51.0)	20% (8%)	48% (47%)	60% (60%)
19	54.53 (57.0)	14% (6%)	44% (41%)	62% (60%)
21	60.27 (63.0)	8% (6%)	40% (37%)	67% (67%)
23	66.01 (69.0)	5% (5%)	38% (35%)	67% (68%)
25	71.75 (75.0)	2% (4%)	36% (34%)	68% (66%)
27	77.49 (81.0)	1% (3%)	35% (34%)	69% (69%)
29	83.23 (87.0)	2% (3%)	35% (34%)	74% (76%)
31	88.97 (93.0)	1% (2%)	35% (35%)	75% (73%)
33	94.71 (99.0)	1% (1%)	34% (36%)	71% (73%)

Notes and observations:

1. The “Ratio” is defined as the quotient of “Real” (real-field) to “Ideal” (ideal field) intensity. All harmonics are from detailed analysis of plots similar to figures 2 and 3 (harmonics 3 – 7 were also listed in tables 1 and 2). The values in parentheses are for the high-phase-error device (6.8° rms phase error), and the plain values for the low-phase-error device (3.7° rms phase error). The ratios are accurate to about ±1% for the angular flux density and ±2% for the pinhole flux. (For another set of measured field files or for an assembly-average of devices, the uncertainties should be doubled.)

2. The zero-emittance real-field calculated angular flux density showed a split peak starting at harmonic 25 for the low-phase-error device and harmonic 13 for the high-phase-error device. The peak value of the largest “side” peaks was recorded for each harmonic in those cases. For the pinhole flux calculations, there was substantial structure in the spectra beginning at harmonic 9—an average value was recorded.

3. The low-phase-error and high-phase-error devices approach the same ratio above a certain harmonic number—for the angular flux density (with the emittance included) the intensity drops to about 35% of the ideal value, whereas, for the pinhole flux, this ratio remains as high as 70%. The asymptotic ratio depends on the length of the device (2.1 m), the period length (3.3 cm), the experimental setup

(angular flux density versus pinhole flux), and the emittance; although the two emittances studied here gave the same results because the difference was small. (For the standard undulator A—2.4 m long, the asymptotic ratio was estimated at 28%.) Close to 100 keV, the high-phase-error device shows more structure in the spectrum than the low-phase-error device (c.f., figures 2 and 3), which is somewhat surprising; however, this, as well as the fluctuations in the derived ratios, reflect the statistical uncertainty in the data—the actual distribution of phase errors for the two devices affect different harmonics differently. For a fully smoothed spectrum, the ratio is about 30% and 70% of the ideal intensity near 100 keV for the angular flux density and pinhole flux, respectively. (The pinhole flux approaches a higher ratio because the pinhole itself smears out the ideal-field harmonics to a large extent. The opposite is true for a zero-emittance beam, where the asymptotic ratio is zero.)

4. It should be emphasized that both devices compared here are good-performing devices, and that the device labeled as a "high-phase-error" device is by no means performing poorly. Any device with an rms phase error less than 8° is a very good device. The true ideal-field performance can of course never be achieved in reality (as well as the zero-emittance beam, which is only used to quickly evaluate the quality of the undulator spectra and hence the magnetic field). In practice, an rms phase error of about 3° is as low as is reasonable practical (for the undulator type A devices). The ratios above should be used as *guidance only* for other devices at other facilities.

Summary

A zero-emittance calculation of the angular flux density from the measured magnetic fields *always* gives a conservative estimate of the *ratio* of the real-to-ideal field intensity, i.e., when the emittance is taken into account the ratio improves. Thus, it is legitimate to continue to evaluate the insertion devices in the magnetic measurement laboratory in terms of zero-emittance calculated ratios, which can be calculated quickly (in a few seconds). We found that the low- and high-emittance lattices gave approximately the same ratios of the real-to-ideal field intensity for a given rms phase error and that the ratio improved more at a larger undulator gap (smaller K value) than at a smaller gap (larger K value) in comparison with the zero-emittance calculated ratio. Further, the rms phase error typically increases with decreasing gap, making the 10.5 mm gap least sensitive to emittance-improved intensity ratios, and that is why the 10.5 mm gap was chosen in this study. The emittance-improved ratios was succinctly illustrated in figure 5, where the shaded areas became narrower and moved upwards (towards better performance) when the emittance was introduced (the pinhole acts in this regards analogous to a large emittance).

The original APS tolerance requirement of 8° rms phase error (at 11.5 mm gap) was set to ensure that the on-axis brilliance (and angular flux density) of the 3rd harmonic would be at least 70% of the ideal for a zero-emittance beam. At the APS, all insertion devices have been tuned to a value much less than that (device-assembly average of standard undulators A ~ 4.0° at 11.5 mm gap and slightly larger at 10.5 mm gap) to provide an even better performance.[4]

In this study, we found that, if a device will operate to generate high photon energies (and to use higher harmonics ~ 5 – 15) at relative small gaps (~ 10.5 mm), then, if effort and time permit, it is desirable to tune the insertion device to an rms phase error of 3 – 4° (rather than 6 – 7°), to get closer to ideal-field performance. The maximum improvement is expected for intermediate harmonic numbers (7 – 9) where one can gain approximately 15 percentage points in the angular flux density and 5 percentage points in the pinhole flux for today's APS typical emittance of 2.5 nm-rad (c.f., figure 5 and table 5, which summarize our results for the very high harmonics).

Acknowledgment

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[4] R.J. Dejus, I.B. Vasserman, S. Sasaki, and E.R. Moog, Argonne National Laboratory Report, ANL/APS/TB-45, May 2002, p. 38.