Nuclear Energy Research Initiative (NERI) Program
DE-FG03-99SF21922

Topical Report
Phase 1
August 15, 1999 - August 14, 2000

An Innovative Reactor Analysis Methodology Based on a Quasidiffusion Nodal Core Model

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1 Task 1

1.1 Introduction

Research efforts in Task 1 were concentrated on studies of the problem in one-dimensional geometry in order to develop basic ideas of functionalization and homogenization procedures. To model reactor physics phenomena of interest, first, we developed a code for solving 1D one-group eigenvalue neutron transport problems based on the quasidiffusion (QD) method [1]. The low-order QD equations are approximated by means of finite-volume (integro-interpolation) approach [2]. The transport equation is discretized by the characteristic method. Then we developed a code for solving 1D multigroup transport problems by means of the QD method [3, 4] that is based on the same discretization schemes.

To model assembly level calculations of MOX and uranium assemblies, we formulated a set of 1D test problems. The definitions of the tests are based on cross section data corresponding to realistic fuel assemblies. By means of these model problems we studied details of behavior of the transport solution and the QD functionals that are specific for reactor physics phenomena. We used these tests to investigate and develop procedures for homogenization of assemblies parameters.

We have formulated certain approaches for spatial assembly homegenezation that are based on consistent discretization spatially averaged QD low-order equations and their fine-mesh discretization. Several variants were considered and studied.

1.2 Homogenization Procedure

In this section the development of spatial homogenization procedure is considered. Such procedure must preserve the averaged reaction rates, surface-averaged group currents, and eigenvalue [6]. The homogenization fits naturally into the framework of the QD method [1, 3] that is based on the idea of successive averaging of the transport equation over angular and energy variable. The averaging over spatial variable is the next logical step [3]. The

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coarse-mesh discretization of the QD low-order equations that is exactly consistent with a fine-mesh discretization is described. These results will be presented on Winter ANS Meeting 2000 in Washington D.C. and published in its transactions [5].

1.2.1 The Quasidiffusion Method

We consider the k-eigenvalue single-group transport problem for 1D slab geometry ($a \leq x \leq b$, $-1 \leq \mu \leq 1$)

$$\mu \frac{\partial}{\partial x}\psi(x,\mu) + \Sigma_t \psi(x,\mu) = \frac{1}{2}\left(\Sigma_s + \frac{1}{k}\nu\Sigma_f\right)\phi(x)\,, \quad \psi(a,\mu)|_{\mu>0} = 0\,, \quad \psi(b,\mu)|_{\mu<0} = 0\,, \tag{1}$$

The low-order QD equations [1] for the scalar flux ϕ and the current J are

$$\frac{d}{dx}J(x) + \left(\Sigma_a - \frac{1}{k}\nu\Sigma_f\right)\phi(x) = 0\,, \tag{2}$$

$$\frac{d}{dx}\left(E(x)\phi(x)\right) + \Sigma_t J(x) = 0\,, \tag{3}$$

$$J(a) = C_a\phi(a)\,, \quad J(b) = C_b\phi(b)\,, \tag{4}$$

where the functionals

$$E(x) = \int_{-1}^{1} \mu^2\psi(x,\mu)d\mu \Big/ \int_{-1}^{1} \psi(x,\mu)d\mu\,, \tag{5}$$

$$C_a = \int_{-1}^{0} \mu\psi(a,\mu)d\mu \Big/ \int_{-1}^{0} \psi(a,\mu)d\mu\,, \quad C_b = \int_{0}^{1} \mu\psi(b,\mu)d\mu \Big/ \int_{0}^{1} \psi(b,\mu)d\mu \tag{6}$$

are calculated by the transport solution.

1.2.2 Fine-Mesh Discretization

We discretize spatially the low-order QD equations using a second-order difference scheme [2]. A spatial mesh is defined so that $x_{i+1/2}$ ($1 \leq i \leq N$) correspond to the mesh edges, where $x_{1/2} = a$ and $x_{N+1/2} = b$. The mesh widths are given by $h_i = x_{i+1/2} - x_{i-1/2}$. Integer $\pm\frac{1}{2}$ subscripts refer to cell-edge quantities, and integer subscripts refer to cell-average quantities. The difference scheme for the low-order equations of the QD method is defined by ($i = 1, ..., N$)

$$J_{i+1/2} - J_{i-1/2} + \left(\Sigma_{a,i} - \frac{1}{k}\nu\Sigma_{f,i}\right)h_i\phi_i = 0\,, \tag{7}$$

$$E_i\phi_i - E_{i-1/2}\phi_{i-1/2} + \frac{1}{2}\Sigma_{t,i}h_i J_{i-1/2} = 0\,, \tag{8}$$

$$E_{i+1/2}\phi_{i+1/2} - E_i\phi_i + \frac{1}{2}\Sigma_{t,i}h_i J_{i+1/2} = 0\,, \tag{9}$$

$$J_{1/2} = C_a\phi_{1/2}\,, \quad J_{N+1/2} = C_b\phi_{N+1/2}\,. \tag{10}$$

Hereafter we assume that the angular flux is found by means of some transport differencing method, and hence, all necessary functionals (E_i, $E_{i+1/2}$, C_a, and C_b) are calculated.

1.2.3 Consistent Coarse-Mesh Discretization

Let us consider a two-zone problem: $a \le x \le g$ (α-zone) , $g \le x \le b$ (β-zone). We introduce $x_{M+1/2} = g$, two sets of indices $I_\alpha = \{i : 1 \le i \le M\}$ and $I_\beta = \{i : M+1 \le i \le N\}$, and a coarse mesh consisting of just one cell per zone. The coarse-mesh solution is defined by $\overline{\Phi}_\xi$, Φ^R_ξ, Φ^L_ξ, J^R_ξ, and J^L_ξ, where ξ is the index of a zone and the corresponding coarse cell ($\xi = \alpha, \beta$), $\overline{\Phi}_\xi$ is the cell-average scalar flux for the ξ-cell (zone), and the superscripts L and R indicate respectively the left and right cell-edge values in the ξ-th coarse cell. The coarse-mesh discretization of the QD low-order equations consistent with the difference scheme (7)-(10) is given by the set of equations for $\overline{\Phi}_\xi$, Φ^R_ξ, Φ^L_ξ, J^R_ξ, and J^L_ξ:

$$J^R_\xi - J^L_\xi + \left(\overline{\Sigma}_{a,\xi} - \frac{1}{k}\overline{\nu\Sigma}_{f,\xi}\right) H_\xi \overline{\Phi}_\xi = 0 \,, \quad \xi = \alpha, \beta \,, \tag{11}$$

$$\widetilde{E}^L_\xi \overline{\Phi}_\xi - E^L_\xi \Phi^L_\xi + \frac{1}{2}\Sigma^L_{t,\xi} h^L_\xi J^L_\xi = 0 \,, \tag{12}$$

$$E^R_\xi \Phi^R_\xi - \widetilde{E}^R_\xi \overline{\Phi}_\xi + \frac{1}{2}\Sigma^R_{t,\xi} h^R_\xi J^R_\xi = 0 \,, \tag{13}$$

$$J^R_\alpha = J^L_\beta \,, \quad \Phi^R_\alpha = \Phi^L_\beta \,, \tag{14}$$

$$J^L_\alpha = C_a \Phi^L_\alpha \,, \quad J^R_\beta = C_b \Phi^R_\beta \,, \tag{15}$$

where the coefficients of these equations are defined by fine-mesh functions in the following way:

$$E^L_\alpha = E_{1/2} \,, \quad E^R_\alpha = E^L_\beta = E_{M+1/2} \,, \quad E^R_\beta = E_{N+1/2} \,, \tag{16}$$

$$\Sigma^L_{t,\alpha} = \Sigma_{t,1} \,, \quad \Sigma^R_{t,\alpha} = \Sigma_{t,M} \,, \quad \Sigma^L_{t,\beta} = \Sigma_{t,M+1} \,, \quad \Sigma^R_{t,\beta} = \Sigma_{t,N} \,, \tag{17}$$

$$H_\alpha = g - a \,, \quad H_\beta = b - g \,, \quad h^L_\alpha = h_1 \,, \quad h^R_\alpha = h_M \,, \quad h^L_\beta = h_{M+1} \,, \quad h^R_\beta = h_N \,, \tag{18}$$

$$\overline{\Sigma}_{a,\xi} = \sum_{i \in I_\xi} \Sigma_{a,i} \phi_i h_i \Big/ \sum_{i \in I_\xi} \phi_i h_i \,, \quad \overline{\nu\Sigma}_{f,\xi} = \sum_{i \in I_\xi} \nu\Sigma_{f,i} \phi_i h_i \Big/ \sum_{i \in I_\xi} \phi_i h_i \,, \quad \xi = \alpha, \beta \,, \tag{19}$$

$$\widetilde{E}^L_\alpha = F^L_\alpha E_1 \,, \quad \widetilde{E}^R_\alpha = F^R_\alpha E_M \,, \quad \widetilde{E}^L_\beta = F^L_\beta E_{M+1} \,, \quad \widetilde{E}^R_\beta = F^R_\beta E_N \,, \tag{20}$$

$$F^L_\alpha = \phi_1 H_\alpha \Big/ \sum_{i \in I_\alpha} \phi_i h_i \,, \quad F^R_\alpha = \phi_M H_\alpha \Big/ \sum_{i \in I_\alpha} \phi_i h_i \,, \tag{21}$$

$$F^L_\beta = \phi_{M+1} H_\beta \Big/ \sum_{i \in I_\beta} \phi_i h_i \,, \quad F^R_\beta = \phi_N H_\beta \Big/ \sum_{i \in I_\beta} \phi_i h_i \,. \tag{22}$$

The coarse-cell balance equations (11) were obtained by spatial averaging of the fine-mesh equations (7) over each zone and introducing the averaged cross sections (19). To get Eqs. (12) and (13), in each of coarse cells we approximated the first-moment QD equation (3) relating cell-edge and cell-average values of the coarse-mesh solution in a way that is similar to one used to obtain the difference equations (8) and (9). To accomplish this equivalently, we derived the definition of cell-average functional E [Eqs. (20)] which is based on special

factors F_ξ^L and F_ξ^R [Eqs. (21) and (22)]. We note that these quantities are similar to discontinuity factors [6]. However, the proposed consistent coarse-mesh discretization preserves the continuity of the scalar flux and the current on cell interfaces [Eqs. (14)].

Theorem

The coarse-mesh discrete QD low-order equations (11)-(22) are exactly consistent with the fine-mesh discrete QD low-order equations (7)-(10) and the coarse-mesh solution has the following form in terms of the fine-mesh solution of Eqs. (7)-(10):

$$\overline{\Phi}_\xi = \sum_{i \in I_\xi} \phi_i h_i \Big/ \sum_{i \in I_\xi} h_i \ , \quad \xi = \alpha, \beta \ , \tag{23}$$

$$J_\alpha^L = J_{1/2} \ , \quad J_\alpha^R = J_\beta^L = J_{M+1/2} \ , \quad J_\beta^R = J_{N+1/2} \ , \tag{24}$$

$$\Phi_\alpha^L = \phi_{1/2} \ , \quad \Phi_\alpha^R = \Phi_\beta^L = \phi_{M+1/2} \ , \quad \Phi_\beta^R = \phi_{N+1/2} \ , \tag{25}$$

and k-eigenvalue is the same.

Proof

The coarse-mesh equations (11)-(22) were derived by means of equivalent manipulations. No approximations were made. If one substitutes the solution (23)-(25) into (11)-(22), then it easy to see that it meets these coarse-mesh discretized equations. ■

To develop a coarse-mesh solution that contains more details about the solution inside a zone, one can define in each zone a mesh consisted of three cells: two original fine-mesh boundary cells in the given zone, and one central cell that covers the rest area in the zone. Then, the described procedure of coarse-mesh discretization is applied to the central cell. We note that the proposed methodology can be extended to arbitrary number of zones without any changes.

1.2.4 Spatial Decomposition

It is possible to reformulate equivalently the derived above two-cell problem (11)-(15) in the form of two one-cell problems each of which will reproduce the corresponding part of the original solution and give the same k-eigenvalue. To perform such decomposition, we replace two interface equations (14) by special boundary conditions. These conditions can be presented in different forms [7]. We propose the albedo form:

$$J_\alpha^R = \frac{1 - \lambda_1^\alpha}{1 + \lambda_0^\alpha} C_g^\alpha \Phi_\alpha^R \ , \quad J_\beta^L = \frac{1 - \lambda_1^\beta}{1 + \lambda_0^\beta} C_g^\beta \Phi_\beta^L \ , \tag{26}$$

$$\lambda_n^\xi = \int_0^{\mu_\xi^*} \mu^n \gamma(-\mu) \psi(g, \mu) d\mu \Big/ \int_0^{\mu_\xi^*} \mu^n \psi(g, \mu) d\mu \ , \quad C_g^\xi = \int_0^{\mu_\xi^*} \mu \psi(g, \mu) d\mu \Big/ \int_0^{\mu_\xi^*} \psi(g, \mu) d\mu \ , \tag{27}$$

where $n = 0, 1$, $\xi = \alpha, \beta$, $\mu_\alpha^* = 1$, $\mu_\beta^* = -1$, and $\gamma(\mu)$ is the albedo. The albedo and any of these functionals [Eq. (27)] can be generated by the known fine-mesh transport solution. Note that integrals in Eq. (27) should be calculated on the angular mesh that is used to solve the transport equation and with quadrature sets that are identical to those utilized

to compute QD functionals. The availability of such transformation of the original problem creates a theoretical basis for generating tables of group constants for each zone separately by means of which one can reproduce the solution in the whole domain exactly.

1.2.5 Discussion

We have developed an approach for exact spatial averaging the discretized QD low-order equations and generating a coarse-mesh discretization that is consistent with the given fine-mesh discretization. The presented technique can be applied to any number of spatial zones. This is a rigorous mathematical result. The proposed method uses the quantities that are similar by their definitions to discontinuity factors, however, the resulting solution preserves continuity of both the scalar flux and the current on interfaces. The procedure of spatial decomposition based on albedo boundary conditions was formulated. The proposed methodology creates a theoretical background for homogenization of spatial regions. The presented approach of consistent coarse-mesh discretization can be extended to multigroup and multidimensional problems, as well as to different kind of discretization methods.

1.3 Functionalization Procedure

We proceed working on methodology of functionalization of group data with respect to albedo boundary conditions. We have built basic software that is necessary to perform analysis of the developed approach. Some modifications of the functionalization procedure have been made. We will report the results of these studies in our future reports.

2 Task 2

During the past year, the Oregon State University (OSU) team engaged in several activities related to Task 2. The goal of the first year of our research was to develop a nodal discretization technique for the QD low-order equations, and then to investigate efficient solvers for this discretized system. In Task 2, we are focusing on the solution of the *few-group* problem; however, because of the equivalence procedures we employ, the solution of this system will yield multigroup transport results.

The initiation of the research in Task 2 was delayed for approximately 3.5 months because of the unavailability of graduate student support. Razvan Nes, the OSU graduate student currently working on this project, joined the research team in January, 2000.

We subsequently began the project by familiarizing ourselves with the details of the QPANDA nodal discretization technique and the QD system of equations. Our initial approach to the QD nodal discretization consisted of applying a standard nodal discretization technique (QPANDA) to the QD system of equations by forcing the left-hand side of the QD equations to look like standard diffusion, and moving the remainder of the QD operator to the right-hand side. After applying a standard transverse integration procedure to generate a coupled system of one-dimensional equations, several significant issues remain to be considered. The most influential is how to approximate the spatial dependence of the QD tensor, $\mathbf{E}$, a quantity which is influenced by the nature of the transport physics. Specifically, the approximation made to the spatial dependence of $\mathbf{E}$ will depend on the multigroup QD low-order equations from which our few-group QD low-order equations are constructed. Near strong discontinuities in angular flux, the exact $\mathbf{E}$ can have a substantial spatial variation.

We began the investigation of the convergence behavior of candidate iterative schemes for the solution of these coupled one-dimensional nodal QD low-order equations. We spent even more time dissecting the QPANDA scheme; in particular, the quadratic transverse leakage approximation and the relationship between the coefficients in this representation and the polynomial flux coefficients. As the end of July drew near, we were actively developing the code to perform the Fourier analysis of this system.

In early August, 2000 all the active participants in this NERI research project met in Idaho Falls, Idaho. Those in attendance were Kord Smith, Dmitriy Anistratov, Marvin Adams, Todd Palmer, Razvan Nes and Scott Palmtag. After many detailed technical discussions, the approach for Task 2 was significantly modified. The most important motivation for changing the methodology used to formulate nodal QD equations is the need for advanced nodal techniques when solving MOX-UOX systems. In his Ph.D. thesis at the Massachusetts Institute of Technology Scott Palmtag developed, computationally implemented and benchmarked several improvements upon state-of-the-art nodal methods for the simulation of current generation nuclear reactors [10]. These improvements were aimed at MOX-UOX systems and include the use of analytic solution functions as basis functions in the thermal group solution, the elimination of the transverse leakage approximation in favor of a traditional finite-element type approach, the use of non-separable expansion functions, a dynamic homogenization technique which computes cross-sections based on the *actual* flux shape found in the reactor, the development of pin-power reconstructions technique which is completely consistent with this new homogenization procedure, and the introduction of empirical correlations to alleviate inadequacies in the modeling of spectral interactions.

Task 2 will now focus on the adaptation of these models to the solution of the low-order QD equations. Since the August meeting, we have completed the derivation of the advanced QD nodal equations. We briefly summarize this derivation here.

The equations have been derived starting from the two-group, fast and thermal, low-order QD equations in two-dimensional Cartesian geometry with constant cross-sections written for rectangular nodes i, j:

$$\vec{\nabla} \cdot \vec{J}_1(x,y) + \Sigma_{r,1}^{i,j}\phi_1(x,y) = \frac{1}{k}\left[\nu\Sigma_{f,1}^{i,j}\phi_1(x,y) + \nu\Sigma_{f,2}^{i,j}\phi_2(x,y)\right], \tag{28}$$

$$\vec{\nabla} \cdot \vec{J}_2(x,y) + \Sigma_{a,2}^{i,j}\phi_2(x,y) = \Sigma_{s,12}^{i,j}\phi_1(x,y), \tag{29}$$

$$J_{x,g}(x,y) = -\frac{1}{\Sigma_{t,g}^{i,j}}\left(\frac{\partial}{\partial x}\left(\mathbf{E}_{xx,g}^{i,j}(x,y)\phi_g(x,y)\right) + \frac{\partial}{\partial y}\left(\mathbf{E}_{xy,g}^{i,j}(x,y)\phi_g(x,y)\right)\right), \tag{30}$$

$$J_{y,g}(x,y) = -\frac{1}{\Sigma_{t,g}^{i,j}}\left(\frac{\partial}{\partial x}\left(\mathbf{E}_{yx,g}^{i,j}(x,y)\phi_g(x,y)\right) + \frac{\partial}{\partial y}\left(\mathbf{E}_{yy,g}^{i,j}(x,y)\phi_g(x,y)\right)\right), \tag{31}$$

where $\vec{J}_g(x,y) = [J_{x,g}(x,y), J_{y,g}(x,y)]^T$ is the net neutron current in group g, $\phi_g(x,y)$ is the scalar neutron flux in group g, k is the reactor eigenvalue, $\nu\Sigma_{f,g}^{i,j}(x,y)$ is the number of neutrons emitted per fission multiplied by the macroscopic fission cross-section in node i,j for group g, $\Sigma_{r,1}^{i,j}(x,y)$, $\Sigma_{a,2}^{i,j}(x,y)$, and $\Sigma_{s,12}^{i,j}(x,y)$ are the fast removal, thermal absorption and slowing-down macroscopic cross sections in node i,j, and $E_{ql,g}^{i,j}$ is the symmetric positive-definite QD tensor in node i,j.

The first two equations are neutron balance equations, and the third is an exact representation of the group current, where the QD tensor's components are defined by

$$E_{ql,g}^{i,j} = \int \frac{\Omega_q\Omega_l\psi_g^{i,j}(x,y,\hat{\Omega})d\hat{\Omega}}{\psi_g^{i,j}(x,y,\hat{\Omega})d\hat{\Omega}} \qquad g = 1,2, \qquad l = x,y \qquad q = x,y, \tag{32}$$

where $\psi_g^{i,j}(x,y,\hat{\Omega})$ is the neutron angular flux in group g.

The group structure is chosen such that thermal group fission spectrum is zero and there is no upscattering from the thermal group to the fast group. The group removal cross-section is the total cross-section minus the in-group scattering cross-section.

The relatively smooth fast flux in the interior of each node is accurately approximated by a two-dimensional, non-separable expansion of polynomial functions of the form:

$$\phi_1^{i,j}(x,y) = \sum_{m=0}^{4}\sum_{n=0}^{4} a_{m,n}^{i,j} f_m(u) f_n(v), \tag{33}$$

where

$$u = \frac{2x - x_{i+1} - x_i}{2h}, \tag{34}$$

$$v = \frac{2y - y_{j+1} - y_j}{2h}, \tag{35}$$

$$h = x_{i+1} - x_i = y_{j+1} - y_j. \tag{36}$$

The polynomial expansion functions are:

$$f_0(\xi) = 1, \tag{37}$$

$$f_1(\xi) = \xi, \tag{38}$$

$$f_2(\xi) = 3\xi^2 - \frac{1}{4}, \tag{39}$$

$$f_3(\xi) = 4\xi\left(\xi + \frac{1}{2}\right)\left(\xi - \frac{1}{2}\right), \tag{40}$$

$$f_4(\xi) = \left(\xi^2 - \frac{1}{20}\right)\left(\xi + \frac{1}{2}\right)\left(\xi - \frac{1}{2}\right). \tag{41}$$

The thermal flux in the interior of each node is approximated by a two-dimensional, non-separable expansion of polynomial and hyperbolic functions of the form:

$$\phi_2^{i,j}(x,y) = \phi_2^{P,i,j}(x,y) + \phi_2^{H,i,j}(x,y), \tag{42}$$

$$\phi_2^{P,i,j}(x,y) = \sum_{m=0}^{4}\sum_{n=0}^{4} b_{m,n}^{i,j} f_m(u) f_n(v), \tag{43}$$

$$\phi_2^{H,i,j}(x,y) = \sum_{m=0}^{4}\sum_{n=0}^{4} c_{m,n}^{i,j} g_m(u) g_n(v). \tag{44}$$

The hyperbolic expansion functions are:

$$g_0(\xi) = 1, \tag{45}$$

$$g_1(\xi) = cosh(\kappa^{i,j}\xi), \tag{46}$$

$$g_2(\xi) = sinh(\kappa^{i,j}\xi), \tag{47}$$

$$g_3(\xi) = cosh(\frac{\kappa^{i,j}\xi}{\sqrt{2}}), \tag{48}$$

$$g_4(\xi) = sinh(\frac{\kappa^{i,j}\xi}{\sqrt{2}}), \tag{49}$$

and κ^{ij} is defined as:

$$\kappa^{i,j} = \sqrt{\frac{h^2\Sigma_{a,2}^{i,j}}{D_2^{i,j}}}. \tag{50}$$

Note that this function depends on the diffusion coefficient in the thermal group, which should be defined by means of QD functionals. This is an outstanding issue which we are currently addressing.

The hyperbolic expansion functions added in the thermal flux expansion represent the attenuation of the thermal flux found in the analytic solution of the diffusion equation. They accurately describe the steep gradients that occur when a UO2 assembly is adjacent to a MOX assembly.

The most significant research issue in our current work is the determination of a proper treatment of the spatial dependence of the QD tensor. This is potentially a non-trivial deviation from Scott Palmtag's thesis work where the diffusion equation and constant diffusion coefficients within each node are considered. In our present derivation, QD tensor components have been approximated by two-dimensional non-separable expansions of polynomial functions:

$$E^{i,j}_{ql,g}(x,y) = \sum_{q=0}^{Q}\sum_{l=0}^{L} e^{i,j}_{ql,g} f_q(u) f_l(v). \tag{51}$$

We are currently investigating various expansions and/or analytic expressions for QD tensor components.

In order to obtain an unique solutions for flux expansion coefficients, one must apply linearly independent conditions to each node:

1. Weighted balance equations in the fast and thermal groups;
2. Surface continuity equations for surface-averaged flux and current in the fast and thermal groups;
3. Corner flux continuity equations in the fast and thermal groups.

The weight functions used in the derivations of weighted balance equations are:

$$w_0(x,y) = 1, \tag{52}$$

$$w_1(x,y) = f_1(u), \tag{53}$$

$$w_2(x,y) = f_1(v), \tag{54}$$

$$w_3(x,y) = 144 f_1(u) f_1(v), \tag{55}$$

$$w_4(x,y) = 4 f_2(u), \tag{56}$$

$$w_5(x,y) = 4 f_2(v), \tag{57}$$

$$w_4(x,y) = 16 f_2(u) f_2(v). \tag{58}$$

For instance, under these assumptions, the $w_p(x,y)$-weighted moment of the neutron balance equations written for node i,j is:

$$\langle w_p, \vec{\nabla}\cdot\vec{J}_1\rangle^{i,j} + \Sigma^{i,j}_{r,1}\langle w_p,\phi_1\rangle^{i,j} = \frac{1}{k}\left[\nu\Sigma^{i,j}_{f,1}\langle w_p,\phi_1\rangle^{i,j} + \nu\Sigma^{i,j}_{f,2}\langle w_p,\phi_2\rangle^{i,j}\right], \tag{59}$$

where

$$\langle w_p, F\rangle^{i,j} = \int_{-1/2}^{1/2} du \int_{-1/2}^{1/2} dv\; w_p(x,y) F(x,y). \tag{60}$$

At this time, we are writing a code which will solve these nodal equations in the thermal group only. This code is sufficiently flexible to allow for different spatial treatments of the QD tensor, should this be necessary. We will test this code initially on standard diffusion problems, since diffusion equation is an instance of the QD low-order equations, and then by means of QD functinals obtained from the transport calculations.

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Status Summary of NERI Tasks - Phase 1

Task 1

The development of the following methods in 1D slab geometry:

1. Homogenization and definition of discontinuity factors,
2. Group constants functionalization using assembly transport solution of multigroup eigenvalue problem with albedo boundary conditions,
3. Solving coarse-mesh effective few-group 1D QD moment equations using tables of data parametrized with respect to the ratio $\vec{n} \cdot \tilde{\vec{J}}^G / \tilde{\phi}^G$ on boundaries.

Planned completion date: August 14, 2000

Task 2

Development of a numerical method for solving the 2D few-group moment QD equations:

1. Development of a nodal discretization method for 2D moment QD equations,
2. Development of an efficient iteration method for solving the system of equations of the nodal discretization method for 2D moment QD equations.

Planned completion date: August 14, 2000