

# Marangoni Convection Induced Ripple on Grazing Incidence Liquid Metal Mirror (GILMM) Used for Laser Inertial Fusion Energy

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# Marangoni convection induced ripple on grazing incidence liquid metal mirror (GILMM) used for laser inertial fusion energy

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## Abstract

A spatial variation of temperature in the sodium film on the surface of the grazing incidence liquid metal mirror (GILMM) will give rise to convection due to the temperature dependent variation in surface tension. This is called thermal capillary convection or the Marangoni effect and causes the surface to have ripples or waves. This note estimates the magnitude of this effect and finds, with care, design parameters can be chosen to make the resulting ripples sufficiently small so that a laser beam can be focused on a target of 1/4 mm spot size at 30 m distance, for example. Smaller spot sizes are discussed.

## Introduction

An inertial fusion energy (IFE) power plant that uses lasers will have final optics exposed directly to the microexplosion. Grazing incidence metal mirrors have been proposed for the final optics. Even more robust would be liquid metal mirrors because liquids can recover after each burst of neutrons, x-rays and debris coming from the exploding target. The question then becomes, "can the liquid surface be maintained sufficiently smooth to allow an adequately small spot size?" The spot size is made up of laser pointing or jitter considered here as well as diffraction spreading, optics aberrations and other effects. The objective of this note is to address that question. The grazing incidence liquid metal mirror (GILMM) arrangement is shown in Fig. 1 and described in Ref. 1.

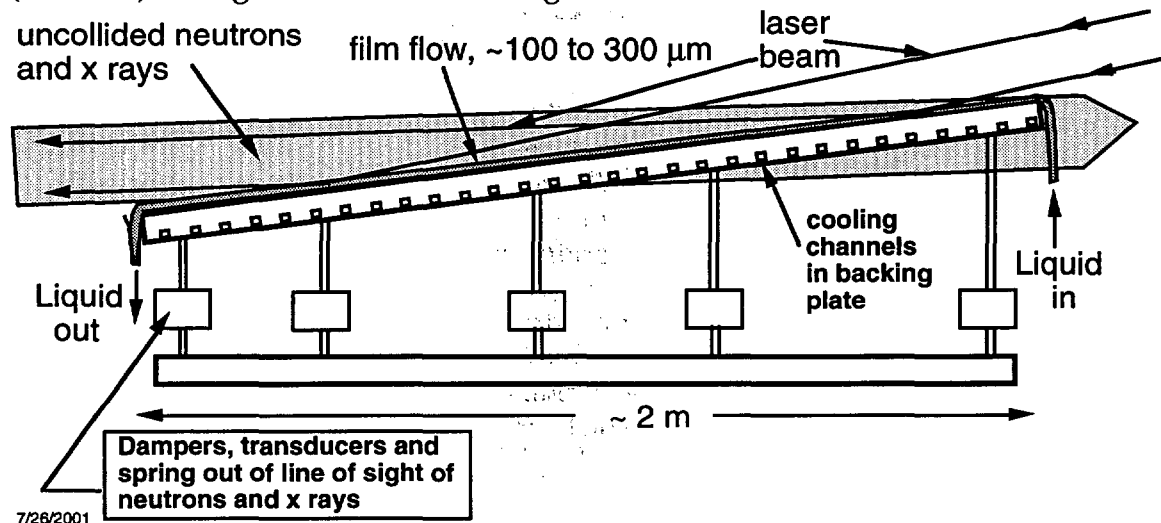


Fig. 1. Grazing incidence liquid metal mirror.

Ideally, the thin film of liquid flowing down the inclined plane will be stable and perfectly smooth forming a good reflecting surface. Viscous effects damp out disturbances for a sufficiently thin film. However, disturbances can be caused by a number of effects resulting in a ripple shown in Fig. 2.

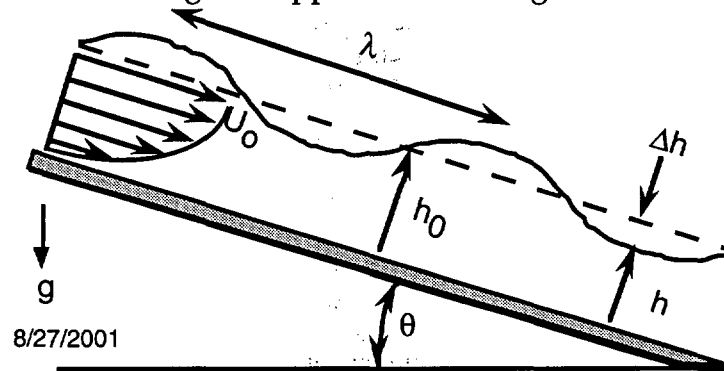


Fig. 2. Ripple shown on the film due to disturbances of any kind.

In reality there are disturbances induced by prior laser pulses incident on the liquid surface. These disturbances may be incompletely damped in the inter-pulse time of  $\sim 0.1$  s. The backing plate surface will not be perfectly smooth and its temperature will have small variations due to variations in the intensity of the prior laser pulses or due to the periodicity of the cooling channels near the back of the plate as shown in Fig. 1. We will leave most of these effects to future studies and treat the effect of the periodicity of the cooling channel.

A tractable problem is to estimate the ripple due to temperature variations in a thin film on a horizontal surface as shown in Fig. 3. The idea is to assume phenomena much the same will be superimposed on the flow down a plane.

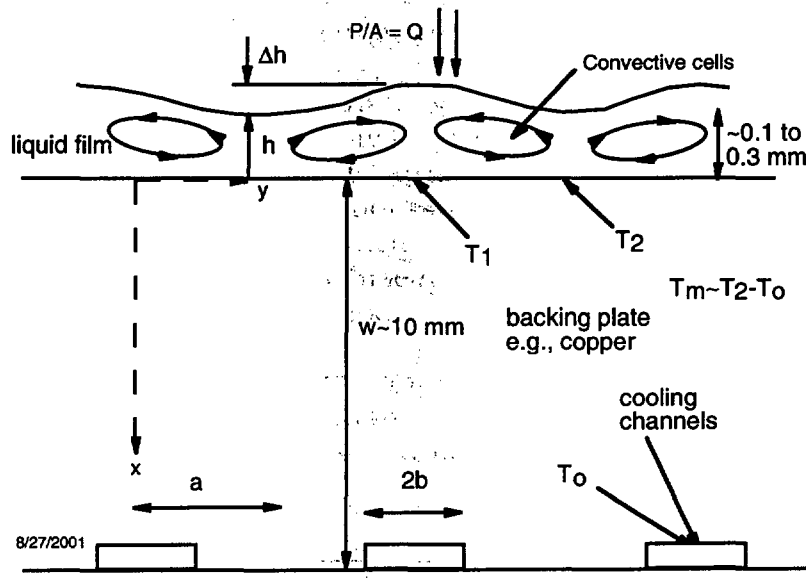


Fig. 3. Geometry of the backing plate for thermal management to minimize the Marangoni convection for GILMM. The liquid film is shown enlarged to emphasize the ripple and convective cells.

## Estimate of convection induced surface ripple

Steady flows are set up by variation in surface tension in a thin liquid film. The surface tension pulls the surface layer towards the colder regions. This convection drags the bulk by viscous drag until the liquid piles up so much that gravity driven convection just balances this flow. The thickness,  $h$ , is given in Ref. 2 for a thin film with small amplitude disturbances.

$$h^2 = h_0^2 \left( \frac{\rho_0}{\rho} \right)^{\frac{3}{2}} + \frac{3(\sigma - \sigma_0)}{\rho g} \quad (1)$$

Where  $\Delta h$  = variation in thickness,  $h$  when  $\sigma$  the surface tension and  $\rho$  the density are varied from there reference values.

For  $\Delta h \ll h$  and  $h \ll a$ .

Values of surface tension,  $\sigma$ , is given from Ref. 3 for sodium, whose melt temperature is 98 °C. The density,  $\rho$ , of sodium is 970 kg/m<sup>3</sup>.

$$\begin{aligned} \sigma &= 0.2064 \text{ N/m}; \quad T = 100^\circ \text{C} \\ &= 0.1995 \text{ N/m}; \quad T = 250^\circ \text{C} \end{aligned} \quad \begin{array}{l} \text{Ref. 3, Liquid metals handbook} \\ \end{array} \quad (2)$$

$$\sigma = 0.2064 \cdot (1 - \alpha \Delta T) \text{ N/m} \quad (3)$$

$$\Delta T = T - 100^\circ \text{C} \quad (4)$$

$$\alpha = 0.000223$$

From Ref. 3 more detailed data, see Fig. 4.

$$\alpha = 0.000506 \quad (5)$$

We will use the larger value of  $\alpha$ .

$$\Delta h = h - h_0 \quad (6)$$

$$\Delta h = \frac{3\sigma_0 \alpha \Delta T}{2h\rho g} = \frac{3 \cdot 0.2064 \cdot 0.000506 \cdot \Delta T}{2h \cdot 970 \cdot 9.8} = 1.65 \cdot 10^{-8} \cdot \frac{\Delta T}{h} \quad (7)$$

A typical film might be 0.1 to 0.3 mm thick. We will use the lower value for our estimates. Then

$$\Delta h = 1.65 \cdot 10^{-4} \Delta T \quad (8)$$

The film thickness variation depends on temperature but not on distance directly. In a distance of  $1/4$  wavelength ( $\lambda=2a$ ), the temperature varies by  $\pm\Delta T$  and  $\Delta h = \pm 1.65 \times 10^{-4} \Delta T$  in the example above.

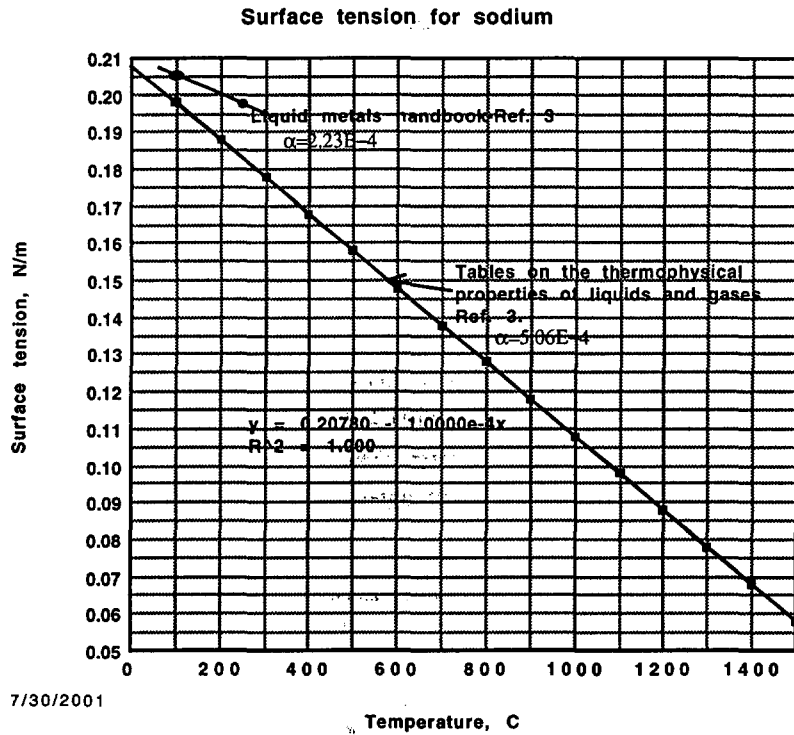


Fig. 4. Data on surface tension for sodium

### Allowed surface ripple

In Ref. 1 we show a sinusoidal surface ripple of  $\pm\Delta h$  over a wavelength,  $\lambda$ , will cause the surface to ripple in angle by  $\pm 2\pi\Delta h/\lambda$ , which will deflect the laser beam by twice this amount. An allowed angle might be  $\pm 0.25 \text{ mm}/30 \text{ m} = \pm 0.83 \times 10^{-5}$  radians, based on targets typical dimensions being  $\sim 2.5 \text{ mm}$ .

$$\frac{\pm 4\pi\Delta h}{\lambda} = 0.83 \times 10^{-5} \quad (9)$$

As an example we assume:

$$\lambda = 10 \text{ mm}$$

$$\Delta h = \pm 6.6 \text{ nm} \quad (10)$$

$$\Delta T = \pm 4.0 \times 10^{-5} \text{ } ^\circ\text{C}$$

In a distance of 10 mm between cooling channels ( $\lambda = 10 \text{ mm} = 2a$ ) we can tolerate  $\Delta h = \pm 6.6 \text{ nm}$ . This requires  $\Delta T = \pm 4.0 \times 10^{-5} \text{ } ^\circ\text{C}$ . In a distance of 100 mm the temperature difference can be  $\pm 4.0 \times 10^{-4} \text{ } ^\circ\text{C}$  and over 1 m it can be  $\pm 4.0 \times 10^{-3} \text{ } ^\circ\text{C}$ .

The flow speed down the plane is typically 10 mm/s, so that in 1/6 s the liquid has moved ~1 mm between shots. This will have only modest temperature smoothing effects.

To achieve the above small temperature variations along the front of the plate will require good thermal management. Can parameters in Fig. 3 be chosen to obtain these small temperature variation? The spacing of the cooling channels will have to be small,  $a/w < 1$  and  $b/a$  approaches 1. The cooling system design requires a standard heat transfer analysis, which is covered in the next section.

A way to achieve smaller contribution to the spot size would be to go to shorter focal distance than the 30 m example and films flowing down only slightly inclined planes so the film could be several times thicker than the example in Eq. 8. These combined effects might allow a pointing jitter of  $\pm 25 \mu\text{m}$ .

### Heat transfer analysis

The temperature of the film will be near the temperature of the surface of the backing plate at the time of the next shot. The distance a heat pulse travels,  $l$ , or diffuses is given as

$$l^2 = \alpha \tau \quad (11)$$

where  $\tau$  is the interpulse time and the thermal diffusivity,  $\alpha$ , is

$$\alpha_{Na} = \frac{k}{\rho C} = \frac{89.7 \text{ W/mK}}{927 \text{ kg/m}^3 \cdot 1386 \text{ J/kgK}} = 6.98 \times 10^{-5} \text{ m}^2/\text{s} \quad (12)$$

where  $k$  is thermal conductivity,  $\rho$  is density and  $C$  is heat capacity.

$$l = 3.4 \text{ mm for } \tau = 1/6 \text{ s.} \quad (13)$$

During a 10 ns pulse the heat has diffused only  $0.8 \mu\text{m}$  but in 1/6 s the film should be at the temperature of the front of the plate because the thermal diffusion distance is far larger than the film thickness. The temperature of the film will equilibrate with the temperature of the plate after a shot.

For the copper backing plate the thermal diffusivity is

$$\alpha_{Cu} = \frac{k}{\rho C} = \frac{380 \text{ W/mK}}{8900 \text{ kg/m}^3 \cdot 385.2 \text{ J/kgK}} = 1.1 \times 10^{-4} \text{ m}^2/\text{s} \quad (14)$$

$$l = \sqrt{1.8474 \times 10^{-5}} = 4.3 \text{ mm for } \tau = 1/6 \text{ s.} \quad (15)$$

The backing plate is likely to be 10 to 20 mm thick, which is much larger than the 4 mm thermal diffusion distance in copper. This means the temperature at the front of the plate is due to the energy deposited by many laser pulses and

justifies the assumption of steady heat flow to determine the temperature of the film. Also the variation from shot to shot is smoothed out as well. Other backing plate materials such as aluminum or stainless steel might be advantageous. Stainless steel would have a longer thermal diffusivity distance and therefore smooth out the pulses for past laser shots more. Aluminum would be lighter.

The reflectivity of the sodium film for  $1/3 \mu\text{m}$  light is calculated to be  $99.5\%^1$ . For a copper plate 1 cm thick and a 6 Hz pulse rate with  $5 \text{ J/cm}^2$  laser and  $0.025 \text{ J/cm}^2$  absorbed, we get a surface temperature rise of the front of the plate over that of the back of the plate,  $T_m$ . Again,  $T_m$  is the average temperature difference from the front to the back of the plate. The plate thickness is  $w$ .

$$\frac{P}{A} = Q = \frac{0.025 \text{ J/cm}^2}{\frac{1}{6} \text{ s}} = 0.15 \text{ W/cm}^2 \quad (16)$$

where  $P/A$  is the absorbed power per unit area on the film

$$k = 3.8 \frac{\text{W}}{\text{cmK}} = \text{thermal conductivity for copper}$$

$$\frac{P}{A} = k \frac{T_m}{w} = Q \quad (17)$$

$$T_m = \frac{1 \text{ cm} \cdot 0.025 \text{ J/cm}^2}{\frac{1}{6} \text{ s} \cdot 3.8 \frac{\text{W}}{\text{cmK}}} = 0.0395 \text{ }^\circ\text{C} \quad (18)$$

The temperature across a  $100 \mu\text{m}$  sodium film on average will be  $1.7 \times 10^{-3} \text{ }^\circ\text{C}$ , for this example. If the film happens to be thicker at the time of the next shot, then its temperature will increase proportionally and Marangoni convection will tend to reduce this thickness. This stabilizing effect is left for future analysis.

### Temperature variation due to variations in laser power

Since the allowed temperature variation along the surface transverse to the channel direction ( $4.0 \times 10^{-5} \text{ }^\circ\text{C}$  for 10 mm spacing) is very small compared to the temperature difference from the film to the back of the plate ( $0.0395 \text{ }^\circ\text{C}$ ), any spatial variation in incident power or plate thickness will be important. For example, if the laser power varies by 0.1% over a distance of 10 mm, then there will be a  $\Delta T$  of  $4 \times 10^{-5} \text{ }^\circ\text{C}$ , which is just the allowed value (see Eq. 10). Over a distance of 100 mm, the laser can vary in power by 1% and by 10% over 1 m. In order to hit a spot size ten times smaller,  $\pm 25 \mu\text{m}$ , will require laser power variations ten times smaller than those of our example

### Temperature variation along channel due to coolant heat up



There will be a temperature variation along the cooling channels due to the coolant warming up from the laser heating. Assume the coolant is sodium in cooling channels equivalent to 4 mm thick layer flowing at 1 m/s. The temperature rise of the coolant and therefore of the front of the plate is:

$$\frac{dT}{dt} = \frac{Q\tau^{-1}}{\delta \rho C} = \frac{0.025 J/cm^2 \cdot 10^4 cm^2/m^2 \cdot 6 Hz}{0.004 m \cdot 927 kg/m^3 \cdot 1386 J/kgK} = 0.29 ^\circ C/s \quad (19)$$

$$\frac{dT}{dz} = \frac{dT}{dz} \frac{dz}{dt} = 0.29 ^\circ C/s \cdot 1 m/s = 0.29 ^\circ C/m \quad (20)$$

This temperature rise is 70 times that of  $\pm 4.0 \times 10^{-3} ^\circ C$  quoted above for 1 m. If the temperature rise is perfectly uniform along the direction of flow, the convective cell will give rise to a layer thickening uniformly along the flow. It is the non-uniformity along the cooling direction that would give rise to a ripple. Clearly, many aspects of thermal management will be important.

#### Fourier analysis of surface temperature variation due to cooling channels

Since small variations of temperature on the film will cause convection generated ripples it is important to minimize this temperature variation. Therefore we will solve the heat conduction problem by Fourier analysis following a similar problem solved by VanSant (Ref. 4). The geometry is shown in Fig. 3. We assume the depth of the cooling channels in the x direction are negligible compared to w, the backing plate thickness. The simplified geometry is shown in Fig. 5 and the boundary conditions are given below:

$$1. \partial T(x,0)/\partial y = 0 \quad (21)$$

$$2. \partial T(x,a)/\partial y = 0 \quad (22)$$

$$3. k \partial T(0,y)/\partial x = -Q = -P/A \quad (23)$$

$$4. T(w,y) = T_0 \quad (0 < y < b) \quad (24)$$

$$\partial T(w,y)/\partial x = 0 \quad (b < y < a)$$

Beginning with Laplace's equation in two dimensions ( $\nabla^2 T(x,y) = 0$ ) and using the separation of variable technique, the following can be obtained:

$$T(x,y) = [C_1 \cosh(\lambda x) + C_2 \sinh(\lambda x)] \cdot [C_3 \cos(\lambda y) + C_4 \sin(\lambda y)] \quad (25)$$

When the first boundary condition is applied to Eq. (25), the result is

$$C_4 = 0 \quad (26)$$

If the second boundary condition is next applied to Eq. (25), one obtains

$$\sin(\lambda a) = 0, \quad \lambda_n = n\pi/a, n=0,1,2, \dots \quad (27)$$

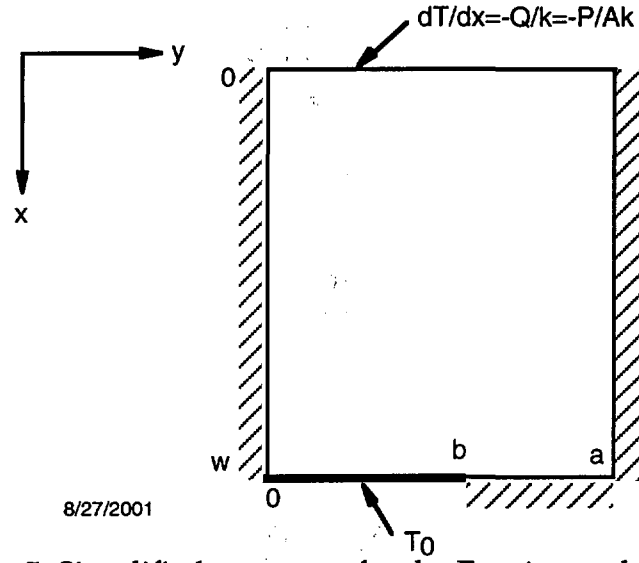


Fig. 5. Simplified geometry for the Fourier analysis.

The equation for temperature can now be rewritten as

$$T(x, y) = \sum_{n=0}^{\infty} [A_n \cosh(n\pi x / a) + B_n \sinh(n\pi x / a)] \cos(n\pi y / a) \quad (28)$$

The third boundary condition requires that

$$k \frac{\partial T(0, y)}{\partial x} = -Q \quad (29)$$

$$k \sum_{n=0}^{\infty} \left[ A_n \frac{n\pi}{a} \sinh(n\pi x / a) + \frac{n\pi}{a} B_n \cosh(n\pi x / a) \right] \cos(n\pi y / a) = -Q \quad (30)$$

$$k \sum_{n=0}^{\infty} \left[ \frac{n\pi}{a} B_n \right] \cos(n\pi y / a) = -Q \quad (31)$$

$$\int_0^a k \frac{n\pi}{a} B_n \cos^2(n\pi y / a) dy = -Q \int_0^a \cos(n\pi y / a) dy \quad (32)$$

$$k \frac{n\pi}{a} B_n \frac{a}{2} = -Q \frac{a}{n\pi} [\sin(n\pi a) - \sin(0)] = 0 \quad (33)$$

$$B_n = 0, \quad n > 0 \quad (34)$$

The integral in Eq. 25 is now done for  $n=0$ .

$$k \left[ \frac{n\pi}{a} B_n \right] a = -Qa, \quad n = 0 \quad (35)$$

$$B_n = \frac{-Qa}{n\pi k}, \quad n = 0 \quad (36)$$

The second term is evaluated by taking the limit as  $n$  approaches zero.

$$B_n \sinh(n\pi x / a) = -Q \frac{x}{k}, \quad n = 0 \quad (37)$$

Eq. 28 is now rewritten.

$$T(x, y) = \sum_{n=0}^{\infty} [A_n \cosh(n\pi x / a) \cdot \cos(n\pi y / a)] - \frac{Qx}{k} \quad (38)$$

The fourth boundary condition requires that

$$T(w, y) = T_0, \quad 0 < y < b \quad (39)$$

and

$$k \frac{\partial T(w, y)}{\partial x} = 0, \quad b < y < a \quad (40)$$

These boundary conditions are mixed value and derivative conditions, which gives us trouble. We know that the average heat flux by conservation of energy is  $Qa/b$  between 0 and  $b$ . If, we take the value as constant rather than the average then the boundary condition is a derivative condition all along this boundary and this altered problem becomes tractable.

$$k \frac{\partial T(w, y)}{\partial x} = Qa/b, \quad 0 < y < b \quad (41)$$

$$\int_0^a \left[ A_n \frac{n\pi}{a} \sinh(n\pi w / a) \cdot \cos^2(n\pi y / a) \right] dy - \int_0^a \frac{Q}{k} \cos(n\pi y / a) dy = - \int_0^b \frac{Q}{k} \frac{a}{b} \cos(n\pi y / a) dy \quad (42)$$

$$A_n = \frac{-\frac{Qa}{kb} \int_0^b \cos(n\pi y / a) dy}{\frac{n\pi}{a} \sinh(n\pi w / a) \int_0^a [\cos^2(n\pi y / a)] dy} \quad (43)$$

$$A_n = -2 \frac{Qa^2}{kb} \left( \frac{1}{n\pi} \right)^2 \frac{\sin(n\pi \frac{b}{a})}{\sinh(n\pi w / a)} \quad (44)$$

Eq. (31) now becomes

$$T(x, y) = A_0 + \sum_{n=1}^{\infty} [A_n \cosh(n\pi x / a) \cdot \cos(n\pi y / a)] - \frac{Qx}{k} \quad (45)$$

When  $x=w$ ,  $a=b$ , then  $A_0=0$  and  $T=T_0$ .

$$T_0 = A_0 - \frac{Qw}{k} \quad (46)$$

It is not clear what the value of temperature is along the boundary,  $x=w$ ,  $0 < y < b$ . However, with this condition, the temperature then becomes

$$T(x, y) = T_0 + \frac{Q(w-x)}{k} + \sum_{n=1}^{\infty} [A_n \cosh(n\pi x / a) \cdot \cos(n\pi y / a)] \quad (47)$$

The temperature variation in the film is given as

$$\Delta T = T(0, a) - T(0, 0) = \sum_{n=1}^{\infty} [A_n \cdot (-1)^n] - \sum_{n=1}^{\infty} [A_n] = 2 \sum_{n>0, odd}^{\infty} A_n \quad (48)$$

The fractional temperature variation is given as

$$\frac{\Delta T}{T_m} = \frac{T(0, a) - T(0, 0)}{Qw/k} = \frac{4a^2}{bw} \sum_{n>0, odd}^{\infty} \frac{1}{(n\pi)^2} \frac{\sin(n\pi b / a)}{\sinh(n\pi w / a)} \quad (49)$$

where  $T_m$  is nominally the temperature difference from front to back of the plate. The results in the above two equations under estimate the temperature variation somewhat due to the altered boundary condition of Eq. 41. The fractional temperature variation,  $\Delta T/T_m$ , is plotted in Fig. 6 for typical cooling system parameters defined in Fig. 3 and 5.

To check the accuracy of the Fourier analysis, Laplace's equation was solved numerically using the relaxation method (see Ref. 5) for the conditions shown in Fig. 5. with the boundary conditions of Eq. 17. The results were within a few percent of the values given in Fig. 6

We can now find by trial and error typical parameters that satisfy the small thermal convection induced ripple.  $T_m=0.0395$  °C from the example of Eq. (18) earlier. In a distance of 10 mm between cooling channels we can tolerate  $\Delta h=\pm 6.6$  nm, in order to have a spot size of  $\pm 0.25$  mm (see Eq. 9 & 10). This requires  $\Delta T=\pm 4.0 \times 10^{-5}$  °C. Thus the required  $\Delta T/T_m=\pm 4.0 \times 10^{-5}$  °C. /  $0.0395$  °C =  $\pm 1.01 \times 10^{-3}$ . The parameter  $a=10$  mm, for  $w=10$  mm,  $w/a=1$ . This gives  $b/a > 0.95$ , i.e., no solution. A better set of parameters might be  $w=10$  mm,  $a=5$  mm, the required  $\Delta T/T_m=\pm 2.0 \times 10^{-5}$  °C /  $0.0395$  °C =  $\pm 5.56 \times 10^{-4}$ ,  $b/a=0.8$ , which is close to acceptable. For  $w=15$  mm,  $a=6.5$  mm, the required  $\Delta T/T_m=\pm 2.6 \times 10^{-5}$  °C /  $0.059$

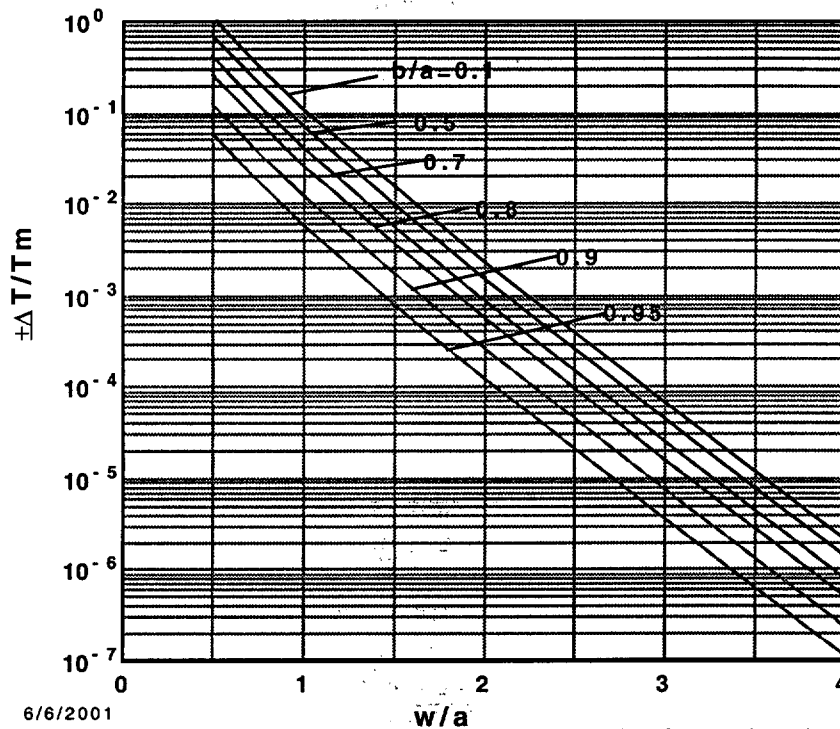


Fig. 6. Fractional temperature variation along the front divided by the temperature difference front to back.

$^{\circ}\text{C} = \pm 4.4 \times 10^{-4}$ ,  $b/a = 0.5$ , which seems like a practical set of cooling channel parameters. If we need to hit a spot size of  $\pm 25 \mu\text{m}$ , parameters from Fig. 6 can easily be chosen to achieve this result.

Apparently a sufficiently small temperature variation due to the discrete periodic cooling channels can be obtained; however the fact that we require less than a variation of  $\pm 4 \times 10^{-5} ^{\circ}\text{C}$  over distances of order 10 mm should put us on alert for effects not included.

## Conclusions

In general, thermal management will be an important subject for successful application of the GILMM concept. As an example of our results, for a  $100 \mu\text{m}$  sodium film we found small enough ripple for a backing plate 15 mm thick and cooling channels spaced 7 mm apart of 4 mm width. This allowed the jitter portion of spot size for hitting a target  $\pm 1/4 \text{ mm}$  at 30 m. If a smaller jitter is required, such as  $\pm 25 \mu\text{m}$ , the ripple due to all effects must be ten times smaller than the examples used in this report. The temperature variation due to the cooling channels can be made small enough by design choices, however, other requirements such as laser spatial uniformity might be difficult to achieve and they will need further study.

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