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# B Physics and CP Violation*

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## Abstract

In these three lectures the basic ideas and of the subject and some current issues are presented, but no attempt is made to teach calculational techniques and methods. (There is a significant overlap in the material here with my earlier lectures presented at the 2002 European Summer School.)

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# 1 LECTURE 1: Introduction and background

The flavor sector is that part of the Standard Model which arises from the interplay of quark weak gauge couplings and quark-Higgs couplings. The matrix of quark weak couplings (in the mass eigenstate basis) encodes these effects in four parameters, one of which is a CP violating phase. From a theorist's perspective, the aim of the game in flavor physics today is to search for Standard Model predictions that cleanly relate measurements to these parameters. Such relationships always have some theoretical uncertainty, a clean relationship is one where that uncertainty is well-defined and small. $B$ physics provides multiple channels that can be used to measure various combinations of Standard Model parameters. Overdetermination of the parameters obtained by different measurements provides a test of the theory, or alternatively a probe for physics beyond the Standard Model. To be interesting, any inconsistency with Standard Model relationships must be large compared to both experimental and theoretical uncertainties.

Interesting tests also arise in cases where the Standard Model predicts that a particular effect is either zero, or very small. Often these predictions can be dramatically altered in extensions of the Standard Model containing new particles. Searches for such effects thus provide another set of tests of the theory.

Why do we look for physics beyond the Standard Model? The chief reason is that it is surely not a complete theory. Just a few of the many reasons for this statement: that it does not comprehend gravity; it includes no dark matter candidate particles; and it fails to account for the matter/antimatter asymmetry of the Universe. There are some reasons to suspect that the scale of new physics is low enough to be observable via its impacts in $B$ decays, perhaps even before it new particles are directly observable via their production (for example at LHC). This is what motivates us to pursue measurements that, were the Standard Model the full story, would simply be redundant measurements of Standard Model parameters.

When searching for new physics we are of course constrained by what we already know. The Standard Model generation structure was invented to give a gauge theory of weak interactions that includes a $Z$ boson with no flavor changing neutral coupling. Such effects are not seen, and their size is strongly constrained by the small mass difference between the neutral kaon mass eigenstates. Likewise any CP-violating and thus, via CPT, $T$-violating effects in extensions of the Standard Model are strongly constrained by the small upper bound on the neutron electric dipole moment. The match between theory and experiment for the quantity ($g$-2) of both the electron and the muon also provides constraints. All these effects put lower bounds on the masses of particular additional particles, and hence, in general, on the size of amplitude contributions for $B$ decays involving such particles. New tests, the focus of much of these lectures, will provide further constraints on model building, or, if we are lucky, a glimpse of new physics.

These lectures begin with some basic background on the nature of CP violation in field theories. I review the particular case of CP violation in the quark sector of the Standard Model and how (and why) it is embedded in the matrix of quark-weak couplings known as the CKM matrix. I then treat the special case of neutral but flavored mesons which can exhibit two types of CP violation effects not seen in other processes. I emphasize the relationships between measurements and the underlying theory parameters, stressing the issues of the theoretical uncertainties that arise in such relationships.

There are some excellent textbooks available covering $B$ physics and CP Violation in much more detail that these three lectures can. I refer students to these texts for further study. [1] Another useful reference is the BaBar Physics Book [2], which summarizes a year long study effort planning the experimental program for that detector. As well as some introductory chapters and appendices that cover general issues in the theory, this report discusses in detail both the theoretical and the experimental issues for a number of interesting $B$ decay modes. While these references are a few years old they present a lot of the basics of the field in great detail and hence are worth the attention of anyone seriously seeking to learn this area. For somewhat more recent review including many experimental results see the Y. Nir.[3] Note also that I do not give many experimental numbers here, for that you need to look at the rest of the papers at this conference, and (now it is some months later) at more recent conferences.

## 1.1 Scales and expansion parameters

Where can we make incisive tests of the Standard Model? Why are heavy quarks so interesting in this regard? There are few measurements, such as that of the CP-violating asymmetry in $B \rightarrow J\psi K_s$ where hadronic physics does not enter the relationship between a measurement and a parameter in the Standard Model. Then there are cases where we can use symmetries, such as isospin invariance, to relate multiple measurements and extract a parameter without any calculation of hadronic physics effects. Finally there are cases where we can make a reliable expansion that controls the uncertainties in our calculation of hadronic physics.

To see when we can do this it is useful to ask what is the physical meaning of $\Lambda_{QCD}$? The formal definition as the scale where perturbation theory gives an infinite coupling constant for the strong gauge interactions is clearly not physical; no measurement can give infinity as its result, and perturbation theory breaks down well before the coupling grows so large. A better definition is to say that $\Lambda_{QCD}$ is the scale that defines the running of the strong coupling constant that should be seen in high energy jet physics, and in the binding of massive "onium" type states. Indeed these are the measurements used to determine it.

This is still a rather esoteric definition. What actually occurs at this scale, what quantity in low-energy physics depends on it? One needs to understand this to see the role that $\Lambda_{QCD}$ plays in $B$-decay physics. The answer is that this scale sets the size of hadrons and thus also gives the scale of the kinetic energy of quarks confined within these hadrons. For hadrons built solely of light quarks (quarks whose mass is small compared to the scale $\Lambda_{QCD}$) it thus also gives the scale of hadronic masses. (With this definition the up and down quarks are light quarks, but the strange quark is a borderline case.)

Conversely, quarks with masses large compared to $\Lambda_{QCD}$ are heavy quarks. There are two consequences of being heavy: the first and most obvious one is that the quark mass dominates the mass of any hadron containing that quark, and thus such quarks are effectively static components of hadrons (mass large compared to kinetic energy); the second is that the strong interaction coupling at the scale of that quark mass is small. Thus there are two small parameters for heavy quark physics $\Lambda_{QCD}/m_q$ and $\alpha_s(m_q)$. Expansions in both of these parameters are useful in calculating the impact of hadronic physics on weak decay processes of heavy quarks and thus we have better control over these effects for hadrons constraining heavy quarks than in the case of light hadrons. With this definition the $b$ quark is a heavy quark and the $c$ quark is a borderline case.

The top quark is in fact so heavy that the issues generally called heavy quark physics do not even enter in its decays. Weak interactions of hadrons are rare compared to hadron formation time if the masses of the $W$ and $Z$ bosons are large compared to the hadron masses. However the top quark has a mass greater than the $W$. The weak decays of the top quark occur so rapidly that it decays before it ever has time to form a hadron.

Thus when we talk of the heavy quark limit for hadronic physics, we take that limit while ignoring weak decays. One can ask how hadronic wave-functions and other hadronic properties scale as the quark mass goes to infinity, without considering the fact that any such hadron is never formed because of weak decays. The rigorous scaling properties derivable in this limit can then be used to constrain models and to inform predictions about the behavior of hadrons in the interesting heavy quark mass range—namely around the mass of the $b$-quark, which is conveniently large compared to $\Lambda_{QCD}$ while still small compared to $M_W$.

It would be very convenient for the study of flavor physics if there were more than one quark in this mass range. In fact there is a second quark that is almost so, the charm quark. The ratio $\Lambda_{QCD}/m_c$ is about 0.3, small, but not quite small enough. The QCD corrections to weak decay patterns, and the leading order $\Lambda_{QCD}/m_c$ effects are both quite large for charm quark states, which limits our ability to make clean predictions about charm decays. We can, and do, use heavy quark theory to simplify the analysis of $b \to c$ decays, but must take care to allow for the leading order $\Lambda_{QCD}/m_c$ corrections to these predictions. Cases where this leading correction is absent are particularly interesting.

At the opposite extreme, when quark masses are light compared to $\Lambda_{QCD}$ we can derive symmetries of the light hadrons. Since both $m_u$ and $m_d$ are small on this scale, both isospin symmetry, which is broken by their difference, and chiral symmetry, which is exact in the zero mass limit, can provide useful inputs for the study of hadronic physics. The additional symmetries that involve the strange quark as well—the full SU(3) flavor symmetry or its SU(2) subgroups $U$-spin (the symmetry of interchange of $s$ and $d$ quarks) and $V$-spin ($s$ and $u$ quarks)—have larger symmetry breaking effects. These are scaled by $m_s/\Lambda_{QCD}$, which is again borderline as a small parameter. The question of how to quantify corrections to the symmetry limit dominates the discussion of theoretical uncertainties for the relationship between measurement and Standard Model test in many cases. But that is always a gain over trying to quantify uncertainties in a case with no good limit known.

## 1.2 The Universe and its matter antimatter asymmetry

*"... I would like to have the asymmetry between positive and negative electricity in the laws of nature (it does not satisfy me to shift the empirically established asymmetry to one of initial conditions)"*

*Wolfgang Pauli, in a letter to Heisenberg, June 1933.*

This remarkable quote from Pauli shows he felt that matter-antimatter asymmetry in the equations, the asymmetry we now know as CP violation, is preferable to an initial condition for understanding the matter-antimatter asymmetry of the Universe. Pauli aside, it seems that most physicists accepted a complete symmetry in the laws of physics between those for matter and those for antimatter as a natural condition of their theories until the empirical discovery that this could not be true—the observation of the two pion decay of the long-lived neutral kaon (the supposed odd-CP eigenstate).

Now, almost forty years later, not only do we have a theory that accommodates CP violation, namely the three-generation Standard Model, but also we have observations of new CP-violating effects in $B$ meson decays, and the expectation that further such effects will soon be observed. This puts us in the exciting position of being able to test whether the observed patterns of CP-violation fit the tightly-constrained predictions of the Standard Model.

The puzzle of how and when the matter-antimatter imbalance in the Universe arose is still unanswered. Indeed it provides one of the strong motivations for studying CP symmetry violation effects. Sakharov took Pauli's statement a two steps further, when he summarized the three conditions required for such an imbalance to arise (rather than to persist from an initial condition), namely baryon number changing processes, CP violation (as remarked by Pauli), and a non-equilibrium situation.[4]

Now that we know that neutrinos have mass there are two places in the theory where CP violation occurs, namely the quark and the lepton weak coupling matrices. (We will later study the quark case in some detail.) Thus there are two classes of scenarios for the matter-antimatter imbalance: one that it developed first from the neutrino sector and was later thermalized to baryon number via standard model processes that change both baryon number and lepton number but not the difference between them, or alternately that it developed due to CP violations in the Higgs-quark couplings at the time of the phase transition in which the Higgs vacuum expectation value became non-zero and thus the particles became massive.

The answer that Pauli did not like, that it is an initial condition protected by a conservation law is a viable third alternative but unattractive today. It is unattractive in principle, as Pauli declared, and in practice as well, because the required asymmetry the early Universe is approximately 1 in 10 billion, which seems to require a very fine-tuned initial state. (However, while small for an initial condition, this ratio is too big to be compatible with the idea that the asymmetry arose as a quantum fluctuation in the early Universe.) Further, for an initial condition to be relevant, it must be protected by a conservation law, for baryon number, lepton number (or their difference). Modern theories (GUTS, etc.) rarely have such a law applicable in the early Universe.

The quark scenario for baryogenesis does not work in the Standard Model. Given current limits on the Higgs mass we find that the phase transition when the Higgs vacuum expectation value appeared is not first order. Even if it were, the amount of mater-antimatter that the known CP violation would give in this scenario is too small. Extensions to the Standard Model can fix both problems. In general such extensions would have observable consequences in $B$ physics. That is one reason why we find $B$ decays a very interesting place to search for violations of Standard Model predictions.

## 1.3 Background: How and when do theories exhibit CP violation

For any field theory Lagrangian, three discrete transformations can be defined for all fields. These are: $C$, charge conjugation, which interchanges particle and antiparticle; $P$, parity, which reverses all spatial co-ordinates; and $T$, time-reversal, which interchanges in-states and out-states. The product of these three operations, CPT, is an exact symmetry in any local Lagrangian field theory. It follows from the locality, Lorentz Invariance, and hermiticity of the Lagrangian. This means that any two rates which are related to one another by the operation of CPT must be equal in any field theory. Tests for CPT violation are thus testing for physics which lies outside the realm of local Lagrangian field theory.

The combination CP, and thus $T$, is also an automatic symmetry in pure gauge theories, and hence, in particular in QED and QCD. These theories also have separate $C$ and $P$ conservation, as long as all quarks are massless. In this same limit weak interactions maximally violate $C$ and $P$ but conserve CP. The pattern of automatic CP conservation applies for many theories beyond QED, and indeed was the only one known to theorists prior to the experimental discovery of CP violation [5], which explains why this discovery was such a shock to the physics community.

However we now understand that CP violation, unlike CPT violation, is readily accommodated in field theories. CP violation can arise when there is a phase difference between two couplings in the theory that cannot be removed by any set of phase redefinitions of the fields. I will explain this point further below.

First let us understand why complex couplings give CP violation. Two amplitudes contributing to the same process interfere with one another when they have relative phases. Any coupling constant phase occurs with opposite sign for a decay and the CP-conjugate decay, and this can lead to CP-violating rate differences. The rate difference can be seen by the following simple algebra. Let the amplitude for the decay of interest be

$$A = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2} \tag{1}$$

where the $A_i$ are real amplitudes, the $\phi_i$ are the coupling constant phases (known as weak phases), and the $\delta_i$ are the phases from absorptive parts in the amplitude (known as strong phases). Imagine that the two terms come from two different sub-processes that contribute to the same transition, for example two different Feynman diagrams for the quark weak decay. Now the CP conjugate amplitude is given by

$$\overline{A} = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2} \ . \tag{2}$$

The weak phases reverse sign between $A$ and $\overline{A}$ because the CP-conjugate rate is governed by the complex conjugate couplings. The strong phases, however, are the same in the two cases, because whatever absorptive parts contribute to the first process are matched by the CP-conjugate absorptive parts in the second. The difference in rates is

$$|A|^2 - |\overline{A}|^2 = 2A_1A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2) \ . \tag{3}$$

Note that a CP-violating rate difference of this type requires that both the weak and strong phases are different for the two terms in the amplitude. This is called direct CP violation, or CP violation in the decay, and is characterized by $|\overline{A}/A| \neq 1$.

Any two terms that contribute to the same rate must correspond to the same overall set of quark fields (valence quarks) for the external particles of the process. A phase redefinition of any subset of fields will change both terms in the same way. Thus the difference $\phi_1 - \phi_2$ is phase convention invariant, and can have physical consequences, while the individual values of the phases are convention dependent and hence have no physical meaning.

Now let us explore what theories can have such phase differences of couplings. For many years physicists forgot Pauli's critique of matter-antimatter symmetry and wrote theories that, like Dirac's equation have exact CP symmetry. Why? I think chiefly because many theories automatically have this property. Indeed it turns out one needs a theory with many different particle types before one can introduce any CP violating couplings.

A Lagrangian must be Hermitian. That alone makes the gauge coupling constants real. In QED one could add a complex fermion mass, but that can be made real by a chiral rotation, which is simply a phase redefinition of the fermion field.† Once we add the left-handed gauge coupling of the weak interaction we cannot even add a fermion mass term, but again Hermiticity is enough to ensure that all the gauge couplings are real.

Once one adds a scalar field to a multi-fermion theory things get more interesting. Scalar couplings can take the form

$$Y_{ij}\phi\overline{\psi}_i\psi_j + Y_{ij}^*\phi^*\overline{\psi}_j\psi_i \ . \tag{4}$$

Here, I wrote the Hermitian conjugate coupling explicitly. You can see that Hermiticity does automatically force the coupling $Y_{ij}$ to be real, because the role of the flavors $i$ and $j$ is reversed in the second term. However I can readily make any one such coupling real by a phase redefinition of either of the fermion fields (and if the scalar field $\phi$ is complex I can redefine its phase too.)

As additional copies of each fermion type are added to the theory the number of possible couplings of the form in Eq. (4) grows more rapidly than the number of fields. Eventually, with enough independent fields in the theory, we reach the point where, starting with all complex couplings allowed by the symmetries of the Lagrangian, there is not enough rephasing freedom to make all couplings real. Such a theory has CP violating effects.

In the Standard Model the quark-Higgs Yukawa couplings allow the possibility of CP violation. Quark masses come from the Higgs field vacuum expectation value via the Yukawa couplings of the quarks to the Higgs field. These couplings thus define what combination of quark weak eigenstates (states paired to a given up quark in weak decays) form a definite mass down-type quark. Thus, in the quark mass-eigenstate basis, these couplings are the source of the quark mixing-matrix parameters, which define the strength of the various $W$-emission transitions. Thus the possible CP violations of quark processes are encoded within this matrix.

## 1.4 The CKM matrix

In a two generation Standard Model, all couplings can be made real by field redefinitions, starting from the most general complex but Hermitian Lagrangian with the symmetries imposed. For three generations of quarks in the Standard Model, after field redefinitions have removed as many phases as possible, and the constraints due to unitarity of the theory have been imposed, the 3×3 quark-$W$ coupling matrix, known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [6] contains four independent parameters, one of which is a complex phase that causes CP violation. The aim of $B$ physics experiments is to over determine the CKM parameters by many independent sets of measurement and to search for effects that are not consistent with the predictions of this theory.

†For QCD the situation is a little more complicated, if there are no massless quarks then chiral rotations that make all quark masses real introduce a CP violating term of the form $\epsilon_{\mu\nu\eta\sigma}F^{\mu\nu}F^{\eta\sigma}$.

The CKM matrix [6] elements $V_{ij}$ denote the transition between an up-type quark $i$ and a down-type quark $j$ by W-emission or absorption. The magnitudes of these matrix elements are physically measurable quantities. I will discuss ways in which they are determined in the next lecture.

One commonly used convention for the parameters of the CKM matrix was suggested by Wolfenstein [7], namely

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)\ . \end{aligned} \tag{5}$$

Empirically, we know that the parameter $\lambda \equiv V_{us}$ is a small number, of order 0.2. The higher powers of this parameter that appear in the more off-diagonal matrix elements $V_{cb}$ and $V_{ub}$ have no theoretical basis; they are simply a way of denoting the empirical fact that these matrix elements are successively smaller. The powers of $\lambda$ are chosen so that the parameters $A$, defined by $|V_{cb}|$ and $\rho^2+\eta^2$, defined by $|V_{ub}|$, are of order 1. The matrix elements $V_{cd}$, $V_{td}$ and $V_{ts}$ are then fixed by unitarity to take the form given here, up to corrections of order $\lambda^4$. The Wolfenstein parametrization is also a choice of phase convention for this matrix. The parameter $\eta$ is a CP-violating parameter; couplings in which appears are complex.

The weak interaction gauge symmetry requires that the CKM matrix is unitary, unless there are additional quark types beyond the three generations of the Standard Model. The unitarity constraints, which have been built into this parameterization, take the form

$$\sum_{i=u,c,t} V_{ij}V^*_{ik} = \delta_{jk} \tag{6}$$

and likewise

$$\sum_{j=d,s,b} V_{ij}V^*_{kj} = \delta_{ik}\ . \tag{7}$$

The off-diagonal relationships in Eq. (6) or (7) (a sum of three complex numbers is equal to zero) can be represented as a closed triangle of vectors in the complex plane. These are called the Unitarity triangles. Notice that the angles in any of these triangles, that is the relative phases of the terms in any one of the sum relationships, cannot be changed by any set of phase redefinitions of the quark fields; they are rephasing invariant quantities. In fact, all these triangles are related; the true rephasing invariant statement that there is only one independent CP-violating parameter in the matrix is the condition that all these triangles have the same area, $J/2$, where $J$ is called the Jarlskog invariant, for Cecilia Jarlskog who first proved this fact.[8] $J$ is proportional to the area of the unitarity triangles. Obviously $J$ is zero if all couplings are relatively real.

While all the triangles have the same area they come in three distinct types. Consider the case for Eq. (6) with $i = d$ and $j = s$. Then two of the terms are of order $\lambda$ and the third is of order $\lambda^5$. The area of this triangle is thus of order $\lambda^6$, however its small angle is of order $\lambda^4$. Asymmetries proportional to such a small parameter are extremely unlikely to be measured. For the case $i = s$ and $j = b$ one finds two sides of order $\lambda^2$ and one of order $\lambda^4$, again an area of order $\lambda^6$. Here the small angle of order $\lambda^2$, difficult but perhaps not impossible to measure. Finally for the case $i = d$ and $j = b$ we find all three sides are of order $\lambda^3$ and thus all angles are of order 1.

This last is the interesting case for CP violation studies, which measure quantities directly proportional to the angles of the triangle. While the overall effect is order $\lambda^6$ here we have CP asymmetries of order 1 in rare processes, as compared to the first case where the CP asymmetries are of order $\lambda^4$ but could occur in leading weak decay rates. In $B$ physics, when people talk of "the unitarity triangle" they mean this last triangle. Conventionally it is drawn with the sides rescaled by the quantity $V_{cd}V_{cb}^*$ so the base is real and of unit length and the apex of the triangle is the point $\overline{\rho}, \overline{\eta}$ in the complex plane where $\overline{\rho} = \rho(1 - \lambda^2/2)$, and $\overline{\eta} = \eta(1 - \overline{\lambda}^2/2)$.

The angles of this triangle have, unfortunately, two conventionally used sets of names, they are either $\alpha, \beta, \gamma$ or $\phi_2, \phi_1, \phi_3$, where the first named is at the apex and the order is clockwise around the triangle. One can of course determine this triangle by measuring the lengths of its sides, all of which are CP-conserving quantities. The match between the angles determined by measuring sides and those found by measuring CP-violating quantities is a test of the Standard Model. In the next lecture we will discuss how, and how well, these various quantities are measured.

## 1.5 Quantum states of neutral flavored mesons

There are a number of pairs of neutral but flavored pseudoscalar mesons, $K^0, \overline{K}^0$; $D^0, \overline{D}^0$; $B^0, \overline{B}^0$; and $B_s, \overline{B}_s$. In each case there are two distinguishable quark flavor eigenstates. Let us use the notation $P^0$, and $\overline{P}^0$ to denote any such pair of particles, with the phase convention chosen so that CP $P^0 = \overline{P}^0$. However these flavor eigenstates are not mass eigenstates . In general the two mass eigenstates can be written as

$$P_{H(L)} = pP^0 \pm q\overline{P} \tag{8}$$

with the constraint $p^2 + q^2 = 1$. The subscripts $H$ and $L$ denote mass, $H$ for the heavier and $L$ for the lighter, with mass difference $\Delta M$.

The mixing of a $P^0$ and $\overline{P}^0$ that determines these mass eigenstates arises from two-$W$ quark box diagrams, as seen in Fig. 1. If the quark-$W$ couplings were all relatively real this would give mass and width differences but no CP violation effects. If CP were an exact symmetry then the two CP-eigenstates $(P^0 \pm \overline{P}^0)/\sqrt{2}$ would have to be the mass eigenstates, and thus $|q/p| = 1$. Phase differences between the couplings that contribute for different intermediate quarks in the box diagrams can

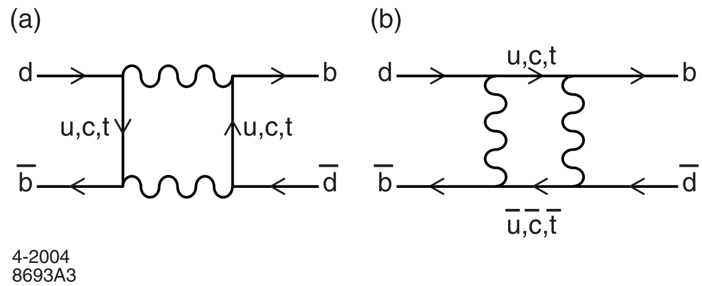


Figure 1: Diagrams for $B^0 \leftrightarrow \overline{B}^{\,0}$ mixing.

lead to mass eigenstates for neutral but flavored mesons that are not CP-eigenstates (and cannot be made so by any set of phase redefinitions). The decay of the $K_L$ state to two pions showed that this is the case for the neutral $K$ system. ($K_L$ is one of the two mass eigenstates; in the $K$ case it is conventional to label the two eigenstates $S$ and $L$ for short- and long-lived as the lifetime difference dominates the mass difference in their phenomenology.) This decay was the first a clear indication of CP violation. Any value of $|q/p| \neq 1$ gives CP violation in the mixing.

These two meson systems have interesting quantum mechanics. Production, decay and interactions with matter are flavor dependant, but the mass eigenstates are not flavor eigenstates. The effects has been well explored in the case of the kaons. For D mesons the effects of mixing are as yet undetected, as these mesons decay so rapidly on the scale of their mass differences that the initial coherent superposition of the two mass eigenstates scarcely has time to evolve. For the $B_d$ mesons the width difference is expected to be small compared to the mass difference; most treatments of $B_d$ decays ignore width difference effects. For $B_s$ mesons the two effects are expected to be of a similar magnitude.

## 1.6 Three Types of CP violation

We can define three types of CP violation. (We have already mentioned two of them.) The first type is a CP-violating difference in the magnitude of the amplitude $A$ for any process and the amplitude $\overline{A}$ for the CP-conjugate process (and thus a difference in the rates), as described above in Eq. (2). This is generally known as direct CP violation, though a better description is CP violation in the decay. This can occur for both charged and neutral particle decays.

Direct CP violation has historically been a topic of considerable theoretical and experimental attention, chiefly because it distinguishes between a class of theories called "superweak" that predict no such effect, and all others. Its magnitude depends on strong interaction phases, which are notoriously difficult to calculate. Thus, it is generally very difficult to use an observation of direct CP violation to pin down theoretical parameters or otherwise test the Standard Model theory. One exception is obvious: those cases where the Standard Model predicts no, or very small, direct CP violation effects. In such cases observation of significant direct CP violation would be a clean signature that some new physics effect is contributing to the amplitudes. Hence such channels are important to identify and to study.

The remaining two types of CP-violating effect are peculiar to the neutral flavored meson systems discussed above. The second type is called CP violation in the mixing, $|q/p| \neq 1$. In that case the mass eigenstates cannot be the CP eigenstates.

The third type of CP violation can occur even when $|\overline{A}/A| = 1$ and $|q/p| = 1$. It occurs for decays of neutral pseudoscalar mesons to a CP eigenstate final state $f$ when such a state is accessible both from decay of $P^0$ (with amplitude $A_f$) and that of $\overline{P}^0$ (with amplitude $\overline{A}_f = \eta_f \overline{A}_{\overline{f}}$ where $CP \mid f\rangle = \left|\overline{f}\right\rangle = \eta_f \mid f\rangle$). The CP quantum number $\eta_f = \pm 1$ depends on the particular state f under study. A particle that is $P^0$ at time $t = 0$ can decay to f either directly, or by first mixing to $\overline{P}^0$ and then decaying to the final state. These two paths interfere to give a time-dependent CP-violating effect a difference between the rate for an initial $P^0$ and that for an initial $\overline{P}^0$ to produce the state $f$. Let us define

$$\lambda_f = \frac{q\overline{A}_f}{pA} \ . \tag{9}$$

One finds a contribution to the rate difference that is proportional to $\sin(\Delta M t)\mathrm{Im}\lambda_f$. This third type of CP violation occurs whenever the weak phase of the decay amplitude is different from the weak phase of the mixing amplitude. In addition there is a possible direct CP violation contribution to this rate difference, proportional to $\cos(\Delta M t)(1 - |\lambda_f|^2)$. The quantity $\lambda_f$ is one we see over and again in discussions of CP violation in neutral $B$ decays, so it worthwhile remembering its definition.

This third type of CP violation is particulary interesting in the case where the other two are not present, $|\lambda_f| = 1$. Then the imaginary part of $\lambda_f$ directly measures the phase difference between the mixing and the decay amplitudes, a quantity that is cleanly predicted in the Standard Model. In this case the magnitudes and strong phases of the decay amplitudes do not enter the asymmetry result (the difference of rates divided by the sum), so there are no hadronic physics uncertainties in extracting CKM phases from such a measurement.

The case $\psi K_S$ (where $\psi$ here stands for any $c\overline{c}$ resonance, including the $\eta_c$ type) is an example of this type; in this case the asymmetry is proportional to $\sin 2\beta$ where $\beta$ (or $\phi_1$) is the bottom right-hand corner of the $b$-decay unitarity triangle, the angle between $V_{cb}V_{cd}^*$ and $V_{td}V_{td}^*$. (We will shortly examine generic $B$ decays to see why this is so.)

All three types of CP violation have been observed. In $K$ decays the quantity $\epsilon'$ is the first type—direct CP violation or $|\overline{A}/A| \neq 1$. The quantity $\mathrm{Re}(\epsilon)$ measured in the decays $K_L \to \pi\pi$ is the second type, CP violation in the mixing or $|q/p| \neq 1$. The asymmetry in $B$ decays to $\psi K_S$ (and $\psi K_L$) is of the third type, asymmetry due to interference between decay with and without mixing, or $\mathrm{Im}\lambda_f \neq 0$.

## 1.7 Quark-level b decay processes and B meson decays

We next study the generic processes that contribute to $B$ decays. We need to look at the processes that occur for decay of a $b$-quark. We include both leading order (tree) and one-loop (penguin) quark decay diagrams.

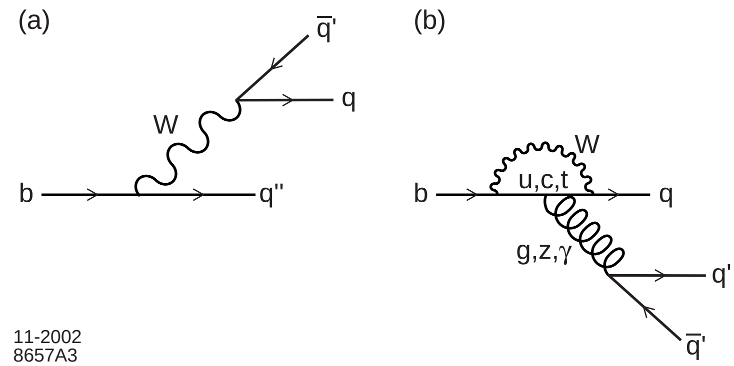


Figure 2: Quark level tree (a) and penguin (b) graphs for weak decay.

These two types of quark-level process are the underlying structure for all possible decay amplitude contributions. One then draws all ways of getting from the initial quarks within the $B$ meson to the valence quarks of the final meson. Note however that the diagrams one obtains in this way are not Feynman diagrams. The many soft gluons, and even possible hard gluon exchanges, are not drawn. One must therefore be somewhat wary of language about $B$ decay processes that uses these diagrams as a way of specifying the various contributions and their relative sizes. This language is imprecise and even counts the same process in more than one way. The diagrams and names for the various diagrammatic topologies are a useful way to categorize the processes and to compare various channels, not a tool for precision calculation. In the third lecture I will talk a little about the formalism that is being developed to move from a schematic diagrammatic description to a well-controlled calculation, at least of the leading terms in a $\Lambda_{QCD}/M_q$ expansion.

Two major factors govern the general size of an amplitude contribution where the second quark of the $B$ meson becomes a valance quark of one of the final mesons. One is the size of the CKM coefficient that appears in it, the more factors of $\lambda$ there are the smaller the contribution. The second is whether it is a tree or a loop diagrams. Loop diagrams have an additional factor of order $\alpha_S(m_q)/4\pi$, due to the gluon emission. Thus, for heavy flavor decays they are suppressed relative to trees.

Topologies where the second quark in the $B$ meson participates in the becomes one of the legs of the quark diagrams in Fig. 2 are generally considered to be suppressed contributions, because they require the two quarks in the $B$ meson to be a short distance apart on the scale of $\Lambda_{QCD}$. These are called exchange or annihilation contributions.

One strategy to search for new physics in $B$ decays to look for channels that are forbidden at the tree level, or have only CKM-suppressed tree contributions. Since any new physics process is, by definition, mediated by heavy particles, it is unlikely to compete with unsuppressed tree-level Standard Model processes. However such effects could be large compared to a rare or forbidden Standard Model contribution. Hadronic corrections introduce uncertainty in the size of the Standard Model contribution, but for such channels the discrepancy arising from new physics effects could be large even compared to this uncertainty. In $B$-decays processes that occur only via loop diagrams are $B \to s\gamma$, and $b \to s\bar{s}s$ (such as $B \to \phi K_S$). Since these are rare decays both the rate and the size of the time-dependent CP violation in any such channel are interesting places to search for new physics.

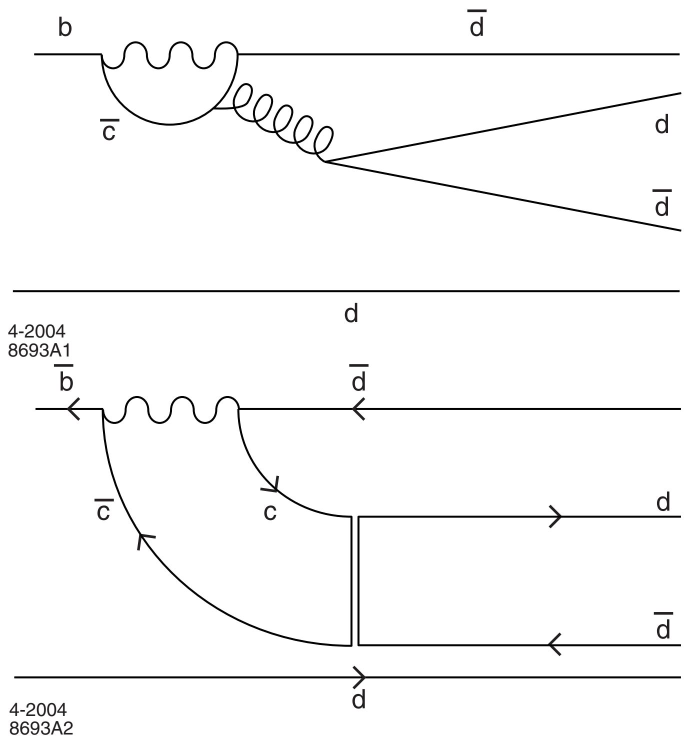


Figure 3: Two ways of thinking about the same thing. (a)penguin diagram (b)tree diagram plus meson exchange.

None of the diagrams for $B$ decay to few meson final states are Feynman diagrams, as they all involve a lot of hidden physics of the strong interactions. Even the distinction between a “tree” and a “penguin” process is not unique, as shown by comparing the two diagrams in Fig. 3. In the first, a gluon is explicitly shown (and thus implicitly assumed to be hard), whereas in the second, the same quark lines appear with no hard gluon. Here the gluon is implicit, inside the exchanged meson, and thus implicitly a soft gluon. However there is no sharp way to distinguish these two contributions until a whole formalism for explicitly separating the hard and soft gluon effects is applied. Similarly some so-called annihilation contributions can equally well arise as flavored-meson exchanges. Such "rescattering" effects are probably somewhat suppressed, but it is not easy to calculate by how much. All this is a warning to take arguments based on different magnitudes of diagrams as rough rules of thumb, rather than firm predictions.

## 1.8 Consequences of unitarity

The Unitarity relationships are the residual effect of the initial gauge symmetry. Thus they protect the theory form divergent contributions. Let us look at this for the case of a penguin graph contribution to $B$ decay, as shown in Fig. 2b, say for the process $b \to c\bar{c}s$.

The internal quarks in the loop in Fig. 2 can be any one of the three up-type quarks. The sum of all such diagrams is given by

$$P_{c\bar{c}s} = \Sigma_{i=u,c,t} V_{is} V_{id}^* f(m_i) \ . \tag{10}$$

Here the quantity $f(m_i)$ represents the Feynman integral over loop momenta for the diagram containing the $i$-th up-type quark in the internal lines of the loop. Notice that this is a divergent integral, so in fact $f$ is not well-defined unless I introduce some regularization prescriptions. If all three up-type quarks had equal mass, then the Unitarity condition (Eq. (6)) would say this amplitude vanishes. The divergent term in the integral does not depend on the quark mass. Thus the Unitarity relationship guarantees that the divergences cancel. One way to make this explicit is to use the unitarity relationship to eliminate one set of CKM coefficients, say $V_{ts}V_{td}^*$. This gives

$$P_{c\bar{c}s} = V_{cb}V_{cs}^*[f(m_c) - f(m_t)] + V_{ub}V_{us}^*[f(m_u) - f(m_t)] \ . \tag{11}$$

The divergence cancellation is now explicit in the differences of the $f(m_i)$. Furthermore we have now arranged this amplitude so that we see that the first term has the same CKM coefficient (the same weak phase) as the larger tree graph contribution. The second term is suppressed by an additional two powers of the small parameter $\lambda$. Thus we can see that the amplitude for a $b \to c\bar{c}s$ decay is very strongly dominated by a single weak phase and hence, for any CP eigenstate f $|\overline{A}_f/A_f| = 1$ up to corrections that are at most a few percent.

This same unitarity pattern is used over and over again in the Standard Model to combine terms from similar diagrams with three different internal quarks to give two manifestly finite contributions. In addition to demonstrating divergence cancellations it is a useful way to group terms both to display the CKM phase structure and discuss the relative size of the two terms. There is no fixed rule on which term to eliminate, so that choice may be made differently in different papers. Shorthand notation often labels the entire contribution that includes both tree and penguin parts as the "tree piece" and the term with a different weak phase as the "penguin". This is fine if you are just keeping track of weak phases, but if you want to use other processes to determine the magnitude of a particular contribution you need to keep track of all the pieces explicitly. That is why diagrammatic analyses include many more than two terms, even though only two distinct weak phases appear.

## 1.9 Time-dependent $B$ decay formalism

Now we turn to the general formalism that is needed to discuss experiments to measure time-dependent CP violating effects in $B$ decays. Let us define $M = (M_H + M_L)/2$, and $\Delta M = M_H - M_L$, and similarly for $\Gamma$ and $\Delta\Gamma$, where the subscripts $H$ and $L$ denote the heavier and lighter mass eigenstates respectively. (Another warning about conventions is needed here; there are, unfortunately, two of them floating around. With the convention defined above $\Delta M$ is obviously positive, however the sign of $\mathrm{Re}\, q/p$ is a physical quantity to be explored. The other convention labels the two states as 1 and 2 and defines $q/p$ to be positive; 1 is the state with $+q$, and 2 has $-q$ in the superposition Eq. (8). In this alternate convention the sign of $\Delta M$ is *a priori* undefined. I use the first of these two conventions.)

We define the states $B^0(t)$ ($\overline{B}^{\,0}(t)$) as the time-dependent superposition of a $B^0$ and a $\overline{B}^{\,0}$, (or, equivalently, of a $B_H$ and a $B_L$) which at time $t = 0$ was, or will be, a pure $B^0$ (or $\overline{B}^{\,0}$ respectively).

$$\begin{aligned} B^0(t) &= g_+(t)\, B^0 + \frac{q}{p}\, g_-(t)\overline{B}^{\,0} \\ \overline{B}^{\,0}(t) &= \left(\frac{p}{q}\right) g_-(t)\, B^0 + g_+(t)\overline{B}^{\,0}\ . \end{aligned} \tag{12}$$

The functions $g_\pm(t)$ can readily be found by writing the state $B^0(t=0)$ as a superposition of $B_H$ and $B_L$ and allowing that state to time evolve. A little algebra gives

$$\begin{aligned} g_+(t) &= \frac{1}{2}\, e^{-(\Gamma t/2)} e^{iMt} \left\{ \cos\frac{\Delta M t}{2} \left(e^{-(\Delta\Gamma t/4)} + e^{+(\Delta\Gamma t/4)}\right) \right. \\ &\qquad \left. + i \sin\frac{\Delta M t}{2} \left(e^{-(\Delta\Gamma t/4)} - e^{+(\Delta\Gamma t/4)}\right) \right\} \\ &\to e^{-(\Gamma t/2)} e^{iMt} \cos\frac{\Delta M t}{2} \quad \text{for} \quad \frac{\Delta\Gamma}{\Gamma} \to 0 \end{aligned} \tag{13}$$

and

$$\begin{aligned} g_-(t) &= \frac{1}{2}\, e^{-(\Gamma t/2)} e^{iMt} \left\{ i \sin\frac{\Delta M t}{2} \left(e^{-(\Delta\Gamma t/4)} + e^{+(\Delta\Gamma t/4)}\right) \right. \\ &\qquad \left. + \cos\frac{\Delta M t}{2} \left(e^{-(\Delta\Gamma t/4)} - e^{+(\Delta\Gamma |t|/4)}\right) \right\} \\ &\to i e^{-(\Gamma t/2)} e^{iMt} \sin\frac{\Delta M t}{2} \quad \text{for} \quad \frac{\Delta\Gamma}{\Gamma} \to 0\ . \end{aligned} \tag{14}$$

From here on we use the approximation $\frac{\Delta\Gamma}{\Gamma} \to 0$ to simplify our equations, for $B_d$ it is a good approximation. Note that the states $B(t)$ are perfectly well-defined for $t < 0$. What this means physically is that superposition of $B^0$ and $\overline{B}^0$ which, if it does not decay, would evolve to be pure $B^0$ (or pure $\overline{B}^0$) at time $t = 0$. (The state so defined has norm greater than 1 at $t < 0$ in order to arrive at time $t = 0$ with norm 1. This is a bit artificial, in any real case we normalize the state for any particle at production time and then evolve that state with a decaying exponential.)

In an $e^+e^-$ $B$ factory the initial system is produced in a coherent state which remains exactly $B^0\overline{B}^0$ until such time as one of the particles decays. Better said, both particles oscillate, but they do so coherently, so that the probability of finding two $B^0$ particles or two $\overline{B}^0$ particles vanishes at all times, as long as both are present. However once one particle decays the other continues to oscillate until such time as it decays. If one $B$ decays to a flavor-tagging mode and the other decays to a CP-study mode we have an event that can be used to reconstruct the time dependence of the asymmetry. We find the rate for the production of such events is given by

$$\begin{aligned} R(t_{\rm tag}, t_f) \;\propto\; & e^{-\Gamma(t_{\rm tag}+t_f)/2} \left|\overline{A}_{\rm tag}\right|^2 |A_f|^2 \\ & \times \left\{ \frac{1+|\lambda_f|^2}{2} \mp \cos\, \Delta m(t_f - t_{\rm tag}) \left(\frac{1-|\lambda_f|^2)}{2}\right) \right. \\ & \left. \pm\; \sin\, \Delta m(t_f - t_{\rm tag})\, {\rm Im}\, \lambda_f \right\} . \end{aligned} \tag{15}$$

The CP asymmetry for a final state $f$ is thus

$$\begin{aligned} a_f &= \frac{R(\overline{B}_{\rm tag}) - R(B_{\rm tag})}{R(\overline{B}_{\rm tag}) + R(B_{\rm tag})} \\ &= [\cos(\Delta M t)(1-|\lambda_f|^2) - 2\sin(\Delta M t){\rm Im}\lambda f]/(1+|\lambda_f|^2) . \end{aligned} \tag{16}$$

In this last equation we have set $t = t_f - t_{\rm tag}$. In an asymmetric $B$ factory we can measure this time from the physical separation of the two $B$-decay vertices, since the pair is produced with known large momentum in the direction of the higher-energy beam. (This time difference is negative when the tagging decay occurs later than the CP-eigenstate decay.)

Eventually we will also have $B$ physics results from hadron colliders. The two types of experiments have different advantages and disadvantages; both are needed to carry out the full program of $B$ physics measurements. One can produce many more $B$-mesons per hour in a hadron collider, but along with them one also produces many other hadrons, and many more events that do not contain b-hadrons. For each decay mode one must devise a way to trigger on the events of interest, separate the particles produced from the $B$ decay from other hadrons, and from backgrounds that fake a $B$ event, and tag the initial flavor of the $B$ meson. Each of these steps is somewhat more difficult, and less efficient, in the hadronic environment. How much more difficult depends on the mode in question. However since one is starting with a much higher production rate, lower efficiencies can be acceptable. In addition, all types of $b$-hadrons are produced, so a hadron collider can study processes that are inaccessible at an $e^+e^-$ collider-based $B$ factory, which makes only $B_d$ type mesons when running at the $\Upsilon_{4s}$ resonance. $B_s$ mesons are not accessible to the current $B$-factories; their decays are as interesting for testing the Standard Model as those of $B_d$ mesons. Conversely, the mode $B \to \pi^0\pi^0$ is important and cannot be readily studied except at the $e^+e^-$ $B$ factories. [These are examples to show why both approaches are needed; they are not the only cases.] In these lectures I focus chiefly on the electron-positron colliders, because those are the currently active experiments.

Note that, in the case of a coherent $B^0\overline{B}^0$ state a single CP-violating term survives if $|\lambda_f| = 1$. It is proportional to an odd function of time, and hence would vanish if one were to integrate over all times. This quantity is particularly interesting because it gives us a result that directly measures the difference of weak phases of the mixing and decay terms, and thus the relative phases of certain CKM matrix elements, with no uncertainties from hadronic physics effects.

There are many channels where the pattern of tree plus penguin amplitudes leaves gives $|\lambda_f| \neq 1$ and the relationship between $\mathrm{Im}\lambda_f$ and any CKM parameter is more complicated. In some cases we can use further inputs, measured in other related channels, to determine or constrain these shifts. My third lecture will focus on such examples.

## 1.10 $B \to J/\psi K_S$

An example of a decay with $|\lambda_f| = 1$ to very high accuracy is the decay $f = \psi K_S$. At the quark level this is a $b \to c\bar{c}s$ decay, which I showed above is dominated by a single CKM matrix element and has $|\lambda_f| = 1$ up to corrections at the few percent level. Thus the third type of CP violation, interference between decay with and without mixing is the only one that can play a role here. For the two decay paths to interfere we need both $B$ mixing and $K$ mixing. The phase measured here is

$$
\begin{aligned}
\frac{\arg(q/p)_B \arg(\overline{A}_(\psi K_S))}{A_(\psi K_S) \arg(q/p)_K} &= 2arg V_{tb}^* V_{tb} \arg(V_{cb}V_{cs}^*) \arg(V_{cs}V_{cd}^*) \\
&= 2 \arg V_{tb}^* V_{tb} V_{cb} V_{cd}^* = 2\beta \ . \qquad (17)
\end{aligned}
$$

This decay is clean both theoretically and experimentally, a rare situation! The channel is readily recognized, for example by the two-lepton decay of the $\psi$-type resonance and the two-charged-pion decay of the $K_S$.

The results from both BaBar and Belle now show a clear CP violation in this channel.[9] The extracted value for $\sin(2\beta)$ is in good agreement with the value given by measuring the sides of the triangle. Figure 4 shows these results, and that from CP violation in $K$ decays, as well as the allowed regions for the apex given by measuring the sides. The figure is taken from the CKM Fitter website: ckmfitter.in2p3.fr/ .[10] The website describes in detail what measurements have been used for each quantity in this figure and how they are combined to give the allowed region. You can see that there is a common region for the apex that is consistent with all these measurements. The Standard Model has survived yet another test!

It will take some years more work on $B$ decay physics to complete the next set of tests to a comparable level of accuracy. There are many interesting channels to study. Few of them have sufficient statistical accuracy as yet to give refined tests of the theory.

Modes where a similar analysis predicts no CP violation because the decay weak phase cancels the mixing weak phase provide a good test of the theory. This is the case for example, up to few percent corrections, for the channel $B_s \to \psi\phi$. A large observed CP violation this channel would be a clear indication of physics beyond the Standard Model. However the existing $B$ factories cannot study it, so this and other $B_s$ results await a good $B$-physics detector at a hadron collider.

## 1.11 Other sin(2$\beta$) modes

There are a number of other modes that should have the same asymmetry as the $J/\psi K_S$ mode in the Standard Model. Any CP eigenstate mode dominated by the quark level decay process $b \to s\bar{s}s$ falls into this class; the precision with which this statement applies must be estimated separately for each mode.

The quark transition $B \to s\bar{s}s$ is pure penguin, and can, like the $b \to c\bar{c}s$ transition amplitude, be written as a sum of a term proportional to $V_{cb}V_{cs}^*$ and a $\lambda^2$-suppressed term proportional to $V_{ub}V_{us}^*$. In the $c\bar{c}s$ case the first of these terms was further enhanced by the presence of a tree graph contribution, whereas in the $s\bar{s}s$ case the two coefficients multiply similar penguin type contributions, differing only by replacing the mass of the c quark by the mass of the u-quark in the integrand of the loop diagram. Thus the impact of the second term is slightly larger here, naively the ratio of the two terms is about 5%. Thus, to be conservative, we can say that the mode $\phi K_S$ should have the same asymmetry as that for $J/\psi K_S$ with less than 10% theoretical uncertainty in the Standard Model.

The current experimental situation is tantalizing. Both BaBar and Belle have measured the asymmetry in this mode. The most recent numbers for the coefficient of the sin $\Delta Mt$ term are [9, 11, 12]

$$\begin{aligned} \mathrm{Im}\lambda_{\psi K_S} &= 0.736 \pm 0.049 \\ \mathrm{Im}\lambda_{\phi K_S} &= -0.96 \pm 0.50^{+0.09}_{-0.11} \quad \text{Belle} \\ \mathrm{Im}\lambda_{\phi K_S} &= 0.47 \pm 0.34^{+0.08}_{-.06} \quad \text{BaBar}\ . \end{aligned} \tag{18}$$

The Belle result shows a significant discrepancy from the Standard Model. If this were the only measurement one could get quite excited at this. However the BaBar

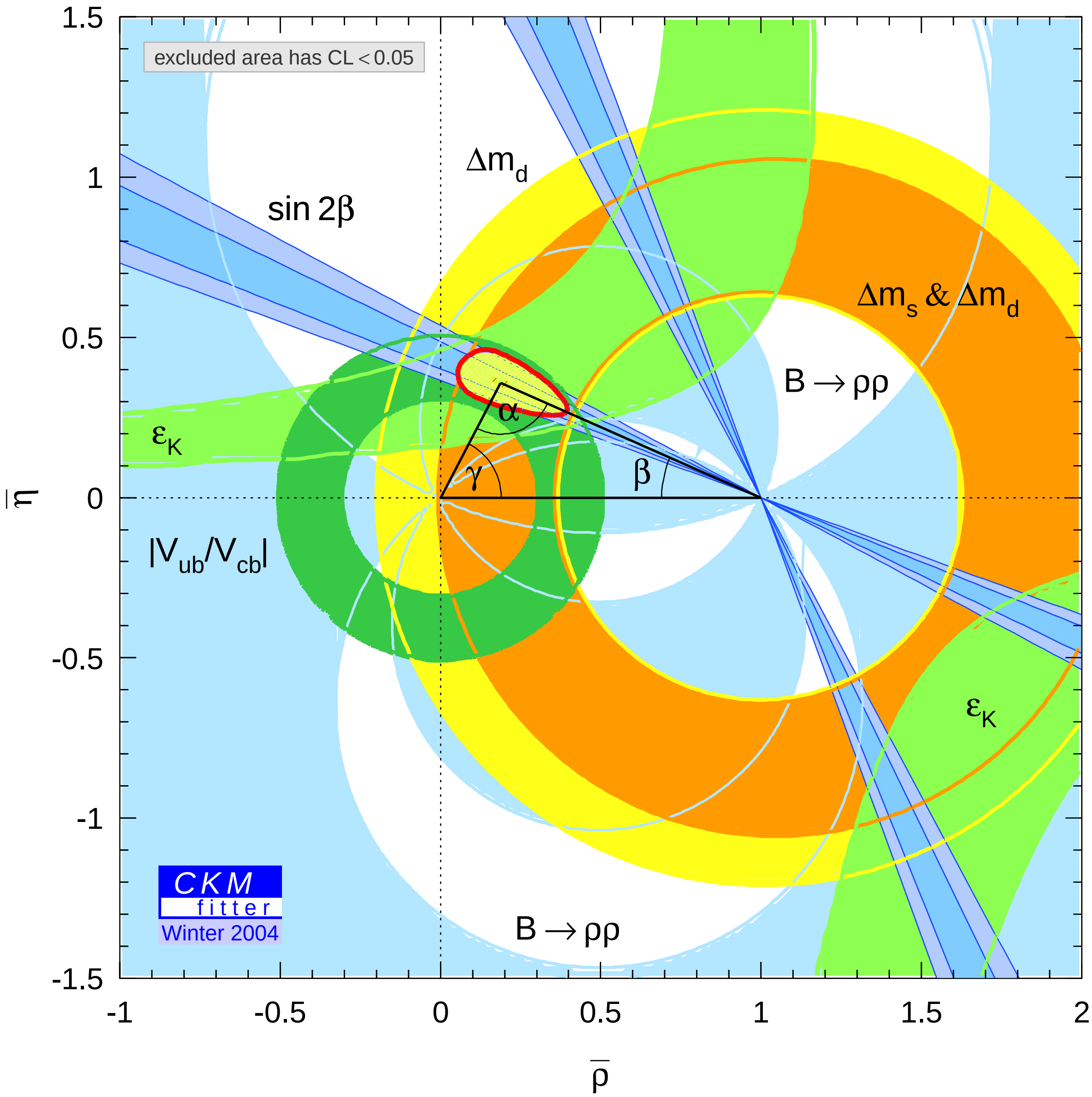


Figure 4: This plot, taken from the CKM fitter site, shows that, to date, all measurements are compatible with a single choice of CKM parameters.

result is consistent within errors with the Standard Model. The two experiments are marginally consistent with one another. This leaves a quandary, either someone is making a mistake, or, with more data the results will converge. We could eventually have a result that is quite interesting. Given the numbers, it will take at least three times the present data to clarify this situation. Theorists of course are actively exploring models that would explain such a discrepancy, see for example.[13]

The modes $\eta' K_S$ and $\eta K_S$ also have significant $s\bar{s}s$ contribution, but they also get contributions from $b \to d\bar{d}s$ and $b \to u\bar{u}s$. This last introduces tree terms proportional $V_{ub}V_{us}^*$. Thus the CKM-suppressed term is somewhat enhanced by the tree contribution in these decays and the uncertainty in the expected match of the asymmetry to that for $J/\psi K_S$ is larger. However these modes are interesting to study, as any new physics that affects $\phi K_S$ will also play a role here.

# 2 LECTURE 2: DETERMINING CKM MATRIX ELEMENTS

## 2.1 Magnitudes from $B$ physics

The Wolfenstein parametrization encodes some of what is known about the magnitudes of CKM matrix elements in terms of powers of the Cabibbo quantity $V_{us} = \lambda$, which is well measured. The remaining parameters $A$, $\rho$, and $\eta$ (or equivalently $\overline{\rho} = \rho(1 - \lambda^2/2)$, and $\overline{\eta} = 1 - \overline{\lambda}^2/2)$ enter into $B$-physics. These are less well known. This lecture is about how, and how well, one can determine them that is, determine the magnitudes of the CKM matrix elements in which they enter. An excellent review of this subject is included in the particle data book.[14]

In almost all cases the largest uncertainty comes from the theory. Theory uncertainties have two unfortunate features, first they are difficult to quantify, and second they are generally not statistical, so it is not clear that one treats them correctly by adding them in quadrature. (These two statements apply also to many types of systematic errors in experiments.) This is why results are typically quoted nowadays with three terms in the error: first the experimental statistical uncertainty; second the systematic error that is intrinsic to the experimental measurement; and third the systematic error that arises when one tries to interpret the result as a measurement of some parameter in the theory. The difference between the second and third terms is not always well-defined. For example one measures a particular branching fraction but must impose some cuts to reduce backgrounds. The cuts introduce an uncertainty already at the level of a branching fraction result because one must use some theoretical input to determine the impact of the cut on the branching fraction. Often a Monte Carlo model is used, but that model is built using some theory, as well as data inputs. This kind of uncertainty is usually called an experimental systematic uncertainty. When one makes the step of using a measured branching fraction to extract the value of a CKM parameter another set of theoretical inputs are needed. One must calculate the ratio of the branching fraction (or other measured quantity) to the desired parameter. The uncertainty in this last step is what is usually called the theoretical uncertainty.

## 2.2 $|V_{cb}|$

The transition $b \to cl\nu$ is proportional to the $b \to cW$ coupling and thus the mixing matrix element $V_{cb}$. There are two ways to try to extract this quantity, inclusive measurements and exclusive decays to particular channels. As a first guess you might think it obvious that the inclusive hadronic semileptonic decay rate measures the quark level semileptonic decay rate; after all, if the quark decays then the hadron must. This idea is called quark-hadron duality, or to be more precise local quark-hadron duality (as opposed to the more rigorously defendable "global quark hadron duality" that applies for example for the energy-averaged cross-section for $e^+e^- \to$hadrons).[15]

Certain effects can be seen quite clearly, for example the fact that the initial and final states are mesons not free quarks alters the phase space available. The question is then how much does the environment affect the decay rate for a $b$-quark in a hadron? The naive guess—not much—seems a good one. One would not expect the energetics of the decay to be greatly changed by the confinement of an additional light quark around the heavy $b$-quark (certainly no exclusion principle forbids the decay, as occurs for neutrons in stable nuclei). This argument is compelling but not rigorous. Its biggest flaw is that it does not tell us how to calculate the size of the error we are making in carrying out an approximate treatment of the effect of the environment. How big is "not much"?

The formal treatment that gives some improvement over the naive statements is the operator product expansion. The physical idea behind this approach is that we can separate the hard (or short-distance) physics and its time scale from the soft (long-distance effects). This is a weak form of the quark-hadron duality assumption, it assumes that the environment is properly accounted for in the matrix elements, and does not affect the short-distance or hard part of the physics. The hard physics gives us a set of local operators (products of fields and their derivatives) that are generated by the quark-level weak decay vertex and the hard QCD corrections to that vertex. At each order in an expansion in $\alpha_S(m_q)$ and $\Lambda_{QCD}/m_q$ further operators can appear. The coefficients of these operators are calculated from the weak decay and hard QCD corrections to it; these are quark and gluon level diagrams. The soft physics gets lumped into the matrix elements of the operators. These are not perturbatively calculable. However, there are a finite number of such matrix elements that appear in inclusive $B$ decays at each order in $\Lambda_{QCD}/m_q$. The hope is that these can be determined by combining several sets of measurements that, according to this theory, depend on the same set of matrix elements. In some cases the needed matrix elements can be calculated using lattice QCD calculations.

To take the path of using other measurements, note that the same quantities that determine the spectrum of the meson weak decay rate, weighted integrals over quark distributions in the $B$ meson, also govern the charmless hadronic spectrum seen in the decay $B \rightarrow X_s\gamma$ where $X_s$ denotes any final states containing non-cancelled strangeness. Here we know the relevant coupling constant. So eventually, with enough data for both processes, and assuming the expansions in $\Lambda_{QCD}/m_q$ and $\alpha_S(m_b)$ converge well enough, we should be able to fit for the leading set of unknown quantities and $V_{cb}$ simultaneously. (The method of choice is to use moments of the spectra in both experiments and to parameterize these in terms of heavy quark expansion parameters). The impact of terms that we have dropped via our heavy-quark and QCD expansions can be estimated on the basis of the sizes of the terms that we have kept. Indeed, with sufficient data, the parameters of the leading and next-leading terms can be over constrained, so the basic assumptions of the method can be tested by checking for self-consistent results.

In defining the operator matrix elements and their coefficients we introduced an artificial scale, the division between what we call a hard gluon and what we call a soft one. Both the operator coefficients and the matrix elements depend on this scale. If we could treat both exactly there should be no dependence on it in the final result. The calculations also depend on a second scale, which is the scale at which we choose to renormalize the strong coupling constant. Again this scale should not enter into the final result if the calculation is done consistently, but can do so when approximations are made. Both scales appear in a similar form in the results; they are usually chosen to be equal. Scale dependence appears both because we truncate the perturbative calculation of operator coefficients at some low order, and because we do not have exact methods to determine the matrix elements of the operators.

Additional scale dependence appears because of the dependence of the rate on another unphysical parameter, the mass of the $b$-quark. Quark masses cannot be directly measured. There are a number of different perturbatively-defined prescriptions for what we mean by the quark mass. These prescriptions introduce some further scale dependence. Care is needed to ensure that a consistent definition is used for all parts of an analysis of data.

If you recall the calculation for muon decay, semileptonic decay rates scale as the mass of the decaying particle to the fifth power. The situation is not quite that bad, as three of the five powers of mass are actually phase-space factors, which scale as the mass difference $m_b - m_c$. This is more readily determined; it is given by the difference of $B$- and $D$-type meson masses up to corrections suppressed by $\Lambda_{QCD}/m_c$.

The true answer for any physical parameter should not depend on the artificially-introduced scales at all. However, all calculations yield results that do have some scale dependence because approximations are made that do not correctly treat these details. What scale should we choose to define the answer? What theoretical uncertainty arises because of this choice? The usual prescription in $b$-decay physics is to say the right scale is the mass of the $b$ quark, since this sets the physical scale of energy release in the problem. The uncertainty due to scale-dependence is typically estimated from the amount the answer changes as the scale is varied from $m_b/2 \leq \mu \leq 2m_b$. Clearly this prescription is quite arbitrary! Fortunately, scale-dependence is much decreased when higher-order QCD effects are calculated, and when the choice of mass definition is sufficiently physical. Then there is a range of choices for the scale around $m_b$ over which the result is quite stable. It is generally assumed that this gives the correct scale-independent result with small uncertainty.

Recent experimental results include calculation of the higher order perturbative corrections and the use of moments of the hadronic spectral distribution to determine the non-perturbative parameters (the operator matrix elements that appear up to the first two powers of the $\Lambda_{QCD}/m_q$ expansion). The parameters so determined include $m_b$ (in whatever prescription is used). This approach shows promise of giving a very accurate value (less than 5% uncertainty) for $V_{cb}$. [16]. A recent BaBar paper using this approach quotes the result as $41.4 \pm 0.04 \pm 0.04 \pm 0.06$. [17] Here the first error is statistical, the second is the theory uncertainty due to higher order corrections to the heavy quark expansion and error, the third is an estimate of the remaining theory uncertainty.

The alternative determination of $V_{cb}$ comes from the exclusive decay to $D^*l\nu$. The fact that both the $b$ and $c$ quarks are massive on the scale of $\Lambda_{QCD}$ gives us a very nice situation. (For the moment let us put aside the concern that the charm quark is not really so very massive on this scale, and talk as if this is a good limit.) In this limit both the $B$ and the $D$ mesons can be pictured as a massive static quark around which the light quark is located in a distribution with a size (and thus a light-quark momentum) scaled by $\Lambda_{QCD}$. We don't know a lot about this distribution; so it is often referred to as "the brown muck". However we do know that QCD is flavor blind, so, up to terms of order $\Lambda_{QCD}/m_q$, the light-quark distribution is independent of which massive quark is at its core. Indeed, it is also independent of the spin orientation of the massive quark, so, in the heavy quark limit, it is the same for the $B$, $B^*$, $D$ and $D^*$ mesons.

Now consider the weak $B \to D^*l\nu$ decay at the kinematic point where the $D^*$-type meson is at rest in the $B$ rest frame. The rate is the quark decay times the matrix element of the operator between the meson wave functions. But the heavy quark limit tells us that the meson wave functions are identical at this particular kinematic point! So we know the matrix element. The operator simply switches the core quark type (and perhaps also its spin) and the wave function overlap is 1. Of course there are corrections to this statement, for finite mass quarks. It turns out that for the transition $B \to D^*l\nu$ the corrections begin at order $(\Lambda_{QCD}/m_q)^2$, while for $Dl\nu$ the first correction is of order $\Lambda_{QCD}/m_q$. This makes the $D^*l\nu$ decay a particularly good way to fix the parameter $V_{cb}$, since even for the charm quark the second order correction is small. In addition there are calculable perturbative QCD corrections.

There is a catch however. The situation in which the leptons carry off all the energy of the $b \to c$ transition is clearly a kinematic endpoint. The cross section vanishes at this point, because of phase space factors going to zero! So, in actuality, one must measure at some distance from this end point and then extrapolate to it. This introduces some theoretical uncertainty in the relationship between the measurement and the parameter $V_{cb}$. We must postulate and fit how the wave-function overlap changes as we move away from the known end-point. The uncertainty from this fitting can be reduced if the measurement is made closer to the end-point, but of course the rate is smaller there. So there is an interplay here between theoretical uncertainty and statistical uncertainty. In such a case more data can shift the result to smaller uncertainty. Currently the accuracy obtainable by this method is also at the $5-10\%$ level, with the range depending on how one combines various non-statistical sources of error.[18]

## 2.3 Determinations of $V_{ub}$

The situation for the parameter $V_{ub}$ is in principle quite similar to that for $V_{cb}$; one can pursue either an inclusive or an exclusive semi-leptonic measurement to fix this quantity. Additional difficulties arise in both cases.

In the inclusive case the problem is to discriminate the rare $b \to u$ decays from the much more copious $b \to c$ decays (including the effects where the $c$-quark decays to a $d$-quark, so no strange particles flag its presence). This requires kinematic cuts to exclude any region reachable via a charm quark decay. Then one must determine what fraction of the $b \to u$ events is excluded by this cut. This determination depends on theoretical modelling of the spectrum. At the quark level the spectrum is readily calculated, but the hadron level spectrum has a different end-point. There are also noticeable effects from hadronic resonant states near the end-point. These do not appear in the quark-level calculation.

In addition, because of the unseen neutrino, there are several different choices for how to impose the cut to exclude charm decays. One can use the charged lepton momentum, or the hadronic invariant mass, or some combination of these two. As in the case of charm decays the assumption of quark-hadron duality can be more reasonably applied for a set of a few moments of the spectral function, than for the detailed spectrum itself. The calculation of $V_{ub}$ can be given in terms of such moments. The moments of the quark distribution in the $B$ meson obtained from $B \to X_s\gamma$ can also be used to fix some of the non-perturbative quantities that enter the extraction of $V_{ub}$.

Like the inclusive determination of $V_{cb}$, this method is based on the assumption that quark-hadron duality correctly gives the total rate and leading moments of the spectrum, though not the all details of the spectrum. It is difficult to quantify the residual theoretical uncertainty that comes from that assumption. One test is to check whether the result is stable as the choice of cuts is varied. Again this is currently a work in progress; it holds promise for accurate results.

For exclusive decays such as $B \to \rho\ell\nu$ or $B \to \pi\ell\nu$ one cannot use the heavy quark limit to constrain the transition matrix element. The heavy quark theory suggests that one could use comparison with the corresponding $D$ decays in matched kinematic regions for the transition matrix element, but the $\Lambda_{QCD}/m_c$ corrections can be large and, at least to date, are not well-controlled. Furthermore data on both the $B$ and $D$ decays is quite limited at present. The alternative approach is to fix the transition matrix element by a lattice calculation. At present such calculations have only been done in the "quenched" approximation, which means that the effect of internal light-quark loops is set to zero. Furthermore, the quark mass used for the light quarks is generally large compared to the physical value, so an extrapolation in that parameter is also needed. Both effects are sources of theoretical uncertainties. Both these issues can be clarified with sufficient computing time available. Methods to treat the quark-loop effects, and the computing power to calculate with lighter quark masses are beginning to appear, and certainly will be developed over the next few years. Perhaps by the time there is sufficient data to give a statistical accuracy of order of 5% for these decays there will also be sufficiently good lattice determinations of the transition matrix elements to give an overall 5% level accuracy for $V_{ub}$. However that day is at least a few years in the future.

## 2.4 The third side of the triangle

Like $V_{ub}$, the quantity $V_{td}$ is of order $\lambda^3$, so the prospect of measuring it directly in top-quark decays, where it must compete with the order 1 leading $t \to b$ decays is remote at best. Instead we must use loop effects that are dominated by top-quarks in the loop to fix the magnitude of $V_{td}$. The mass (and width) differences in the neutral $B$ meson systems, in other words the effects due to $B$-$\overline{B}$ mixing, provide the best option. These are mediated by diagrams like those of Fig. 1, with external $b$ and $d$ quarks for $B_d$ and $b$ and $s$ quarks for $B_s$, and internal up-type quarks.

We can look at any one quark line and write the contribution to the loop integrand for this line, summing over all three up-type quarks in the intermediate state. This gives

$$Q(k, m_t, m_c, m_u) = V_{tb}V_{ti}^* D(k, m_t) + V_{cb}V_{ci}^* D(k, m_c) + V_{ub}V_{ui}^* D(k, m_u) \tag{19}$$

where the functions $D(k, m)$ are the quark propagators. We use the unitarity relationship Eq. (6) to eliminate the term proportional to $V_{cb}V_{ci}^*$. This gives

$$\begin{aligned} Q(k, m_t, m_c, m_u) \quad &= \quad V_{tb}V_{ti}^*(m_t - m_c)D(k, m_t)D(k, m_c) \\ &\quad + V_{ub}V_{ui}^*(m_u - m_c)D(k, m_u)D(k, m_c) \ . \end{aligned} \tag{20}$$

Since the top quark mass is so much larger than the others and the typical loop momentum $k$ is large, the first term dominates (in the case $i = s$ the second term is also CKM suppressed). There are two such quark lines in each diagram, so the dominant contribution to the mixing amplitude is proportional to $V_{ti}^2$. This quantity multiplies a known coefficient times the matrix element of a local four-quark operator between the $B$ and $\overline{B}$ meson states.

For the $B_d$ system the mass-difference between the two mass-eigenstates is well-measured, so the dominant uncertainty in the extraction of $V_{td}$ comes from the uncertainty in the theoretical calculation of the operator matrix element. Lattice calculations for this quantity are steadily improving, but the resulting theoretical uncertainty is still quite large.

$V_{ts}$ is strongly constrained by unitarity hence a precise measurement of the $B_s$ system mass difference can give $V_{td}$ from the ratio of $B_d$ to $B_s$ mass differences. In this ratio much of the uncertainty in the matrix element cancels, since, up to the effect of the mass difference between an $s$ and a $d$ quark, the two matrix elements are the same. The largest difference in the two cases is that the non-leading terms in the box graph are CKM suppressed in the $B_s$ case, but not in the $B_d$ case; the uncertainty in the correction to the ratio due to these contributions thus affects this determination of $V_{td}$. There are also calculable perturbative QCD corrections; these are well understood. Further corrections arise from SU(3)-breaking effects, where the SU(3) in question is the flavor symmetry of the three light quarks. The uncertainty in these corrections gives the predominant uncertainty in this method to fix $V_{td}$. So far, only an upper limit on the mass difference for the $B_s$ system has been established.[19] The next round of experiments at the TeVatron should yield an actual value for the $B_s$ mass difference. Even the upper limit currently available significantly improves the constraints on $V_{td}$.

The three measurements $V_{cb}$, $V_{ub}$ and $V_{td}$ are in principle, sufficient to determine the unitarity triangle. The uncertainties in their values at present are quite large. In particular, if these measurements were all we had, it would be difficult to say with certainty that the CP-violating parameter is non-zero in the Standard Model. Of course, we know it is, because we observe CP violating effects in both $K$ and $B$ decays. The $K$ decay result gives a constraint on a combination of $\rho$ and $\eta$. The constraint has a large theoretical uncertainty, but excludes $\eta = 0$ which would give a vanishing rate for $K_L \to \pi\pi$. The theoretical uncertainty arises from the matrix element for the $K$-mixing operator, which is calculated on the lattice.

# 3 LECTURE 3: Some other two body modes

## 3.1 Why two body?

Given the large mass of the $B$ compared to charmed and, even more so, to charmless mesons it is relatively rare for the final state of $B$ decay to be a two-body or quasi two-body state (one with one or two unstable particles in the two body “final” state). So you may wonder why all the discussion of CP violations focuses on these relatively rare states. This is because we need to be sure we are studying a CP eigenstate. Even if the set of final particles is CP self-conjugate, multiparticle states are typically an unknown admixture of CP odd and CP even states. Since these contribute to the coefficient of $\sin(\Delta M t)$ with opposite signs, we cannot extract any information on CKM parameters unless we can determine the admixture. The cases where we can do this are two body states. If one of the two final particles has spin zero then there is a unique orbital angular momentum allowed so the state has definite CP. If both particles have non-zero spin we can often use an angular analysis of their decays to separate out the CP-odd and CP-even final states. (The CP state is angular momentum correlated because of the $(-1)^L$ factor for the parity of a state of orbital angular momentum $L$ between the two particles.)[20]

There is by now a large literature of suggestions of particular modes that can be analyzed to give constraints on Standard Model parameters. In the first lecture I talked about modes that measure $\sin(2\beta)$. Another theoretically clean set of modes are the modes that are pure tree that give gamma by interfering processes fed by $b \to c\overline{u}q$ with $b \to u\overline{c}q$, where q is either an $s$ or a $d$ quark. The modes are $B \to DK$ and $B \to D\pi$.[21] The interference can occur even though the mixing of $D^0$ to $\overline{D}^0$ is small; it occurs when one selects a CP eigenstate final state s for the $D$ decay that are common to the two $D$-type mesons, such as $\pi\pi$. Because there is not yet much data on these modes I will not discuss them further. They will eventually provide some interesting results. The detailed issues of limits to the accuracy of the analysis are different for the two cases.

In all $b \to c\overline{c}d$ and $bu\overline{u}d$ cases there are two CKM coefficients of comparable magnitude and different weak phases contributing to the amplitude (after we use unitarity to remove one of the original three). This leads to $|\overline{A}_f/A_f| \neq 1$ and thus to $|\lambda_f| \neq 1$. If we could argue that penguin diagrams are insignificant compared to tree diagrams we could ignore the purely penguin term compared to the term which also has a tree contribution and then these modes would directly measure $\sin 2\beta$ and $\sin(2\alpha)$ respectively. However this is not the case. So then the question is how to constrain or remove the impact of the penguin term on the calculation of CKM parameters from the measured asymmetries in these modes. The remainder of this lecture discusses issues related to this problem.

## 3.2 Methods for extracting information when $|\lambda_f| \neq 1$

If we could reliably calculate the relative magnitudes and the relative strong phases of all the tree and penguin amplitude contributions the problem would be solved. Because of the soft hadronic physics contributions, that cannot be done without making further approximations and/or assumptions. Given any such approach we then must estimate the theoretical uncertainties that remain. When reading theory papers with suggestions for analyses it is important to ask when the paper was written, because early papers tend to be over optimistic about the accuracy obtained using what we now know are rather crude assumptions. More recent papers are more cautious, but it is well to review a variety of estimates rather than accepting any one at face value.

The technology of theoretical calculations has had approximately three stages, first naive factorization interpretation of diagrams, in which the spectator quark and the quarks from the b-decay become the valance quarks of the two final state particle, second two sets of attempts to develop systematic $\lambda_{QCD}/M_q$ and $\alpha_S(M_q)$ expansions (QCD factorization and perturbative QCD), and most recently the more detailed analyses of these expansions in the language of Soft Collinear Effective Theories (SCET). None of these analyses is completely predictive, all try to reduce the problem to a finite number of unknown matrix elements that appear in multiple processes. Eventually, if the number of measurements is greater than the number of unknowns, the system becomes predictive and can determine CKM parameters. In some cases matrix elements can be determined by lattice calculation.

Differences in predictions between the two intermediate methods, QCD factorization in perturbative QCD, came chiefly from different assumptions about the relative importance of some of these matrix elements. These issues are clarified by the more recent SCET analysis, but unfortunately not in a way that leads (as yet) to strong predictive power for the cases of interest here. However the work is ongoing and has clarified many of the issues that were confusing when two competing calculations apparently starting with the same tools led to quite different results. As discussed in the previous lecture it has also led to better control of uncertainties in extracting the parameters $V_{cb}$ and $V_{ub}$ from inclusive semileptonic decays.

Like the earlier approaches the SCET approach begins by organizing the calculation as a sum of operators, with coefficients defined in powers of $\lambda_{QCD}/M_b$ and $\alpha_S(M_b)$. However the operator matrix elements are then further broken down, explicitly including further QCD effects as collinear factors (impacts of gluons collinear to hard quarks, and soft factors (infra-red sensitive gluon effects). These latter must be carefully combined with appropriately-defined wave functions (or, more precisely, light cone quark distribution functions) to ensure cancellation of infra-red divergences. Both the collinear and the soft factors are universal in the sense that they do not depend on the flavor of the quark line or meson in question.

Two types of contribution appear, those where the spectator is not involved in the hard process, and those where it is. The first type involves a transition matrix element similar to that which enters a semileptonic decay. The second type can be written as a convolution of a hard six-quark kernel with wave functions for the initial and final mesons. Since wave functions are process independent, and the same transition matrix element appears in more than one process, there is a hope that this method can give some clean predictive relationships among multiple measurements. This hope is still being explored.

For the case of $B \to u\overline{u}d$ we have another possible tool to separate tree contributions from penguin contributions and that is the use of isospin. Isospin is a symmetry between up an down quarks if we neglect electromagnetic corrections (charges) and their mass differences, which are small compared to the scale $\Lambda_{QCD}$ (though not small compared to their average mass). Isospin is not a symmetry of weak interactions but we can classify amplitude contributions by the change in isospin in the weak decay $\Delta I$ and the isospin, $I$, of the four quark final state (this includes the spectator quark as well as those that result from the decay of the b-quark.)

For $B \to \pi\pi$ or $B \to \rho\rho$ this gives us a way to untangle the tree and penguin contributions. (There is no similar separation available for $b \to c\bar{c}d$ channels.) The tree amplitude has both $\Delta I = 1/2$ and $\Delta I = 3/2$ contribution, while, because a gluon is flavor blind, the penguin diagrams contribute only to $\Delta I = 1/2$. When we add in the spectator quark we can have, at first glance $I = 0$, 1 or 2 for the final state Bose statistics removes the $I = 1$ possibility when the two final particles are in a total $J = 0$ state, as they must be in a $B$ decay. (For the two $\rho$ case this is true because combined the space-spin state is always even parity for any $L$. Technically the result only applies for two identical $\rho$ particles, the impact of the broad $\rho$ resonance gives an experimentally distinguishable contribution that has $I = 1$ but vanishes when the mass of the two $\rho$ states seen are equal, in the rest of this discussion I ignore this complication.[23]) Thus we have three types of amplitude contributions, $\Delta I = 1/2, I_f = 0$ occurs for both CKM coefficients and $\Delta I = 3/2, I_f = 2$ only for tree coefficient $V_{ub}V_{ud}^*$. If we can effectively isolate the impact of this latter contribution we can measure $\alpha$ directly from the CP asymmetry in $B \to \pi^+\pi^-$ or $B \to rho^+\rho^-$ with angular analysis to separate the definite CP final states.

It turns out that one can in principle do this, up to a discrete set of ambiguities, if one also measures the isospin-related channels $B^- \to \pi^-\pi^0$ and $B^0 \to \pi^0\pi^0$ and their CP conjugates with sufficient precision.[24] Isospin tells us that the charged $B \to \pi\pi$ amplitude is pure $I = 2$, and so has no penguin contribution. It also tells us that there is a particular sum of the two neutral channel amplitudes that is equal to the charged amplitude. Thus the three amplitudes, appropriately summed form a closed triangle in the complex plane.

$$A_{+-}/\sqrt{2} + A_{00} = A_0 \ . \tag{21}$$

The three CP conjugate rates form a different triangle, but, up to an overall phase, the two triangles have a common side. The charged rates, being pure tree, must be equal in magnitude for $B^+$ and $B^-$. Figure 5 shows the triangles reoriented to a common base.

This figure, along with the measurement of the CP violating asymmetry parameters $C$ and $S$ (coefficients of $\cos(\Delta M t)$ and $\sin(\Delta M t)$ for the $\pi^+\pi^-$ channel, gives sufficient information to extract the value of alpha. $C$ gives us a measurement of $|\lambda_f|$ and

$$S = \frac{2|\lambda_f|}{1 + |\lambda_f|^2} \sin(2\alpha - \delta) \tag{22}$$

where $\delta$ is the angle so labelled in Fig. 5. Because the relative orientation of the

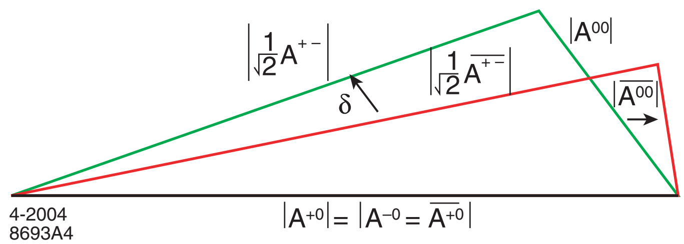


Figure 5: Isospin analysis triangles for two pion decays.

two triangles is not defined by measuring the magnitudes of all the sides, there is a fourfold ambiguity in the value of $\delta$.

The neutral rates $B^0 \to \pi^0\pi^0$ and $\overline{B}^0 \to \pi^0\pi^0$ are not yet separately measured, so this analysis cannot yet be carried out in full. However, given the sum of these two rates, one can set an upper bound on the shift of $\mathrm{Im}\lambda_f$ for $f = \pi^+\pi^-$ from $\sin(2\alpha)$.[26] Unfortunately this leaves a large uncertainty in $\alpha$. We will need the separated measurements, which requires about ten times the present data, to get a good value for $\alpha$ from $\pi\pi$ channels.

The same analysis can be used for $\rho\rho$, in conjunction with an angular analysis to separate out the states of definite CP. Angular analysis shows that both the $\rho^+\rho^-$ and the $\rho^+\rho^0$ final states are very strongly dominated by helicity zero. So far there is only an upper bound on the $B \to \rho^0\rho^0$ rate, but it is small enough to give a relatively good value for $\alpha$ once the CP asymmetry in $B \to \rho^+\rho^-$ is measured. (At the time of this lecture only CP-averaged rates were published, but a recent paper has changed that.[25])

Isospin-breaking effects introduce uncertainties in the results from this analysis, but it is still a significant improvement over the approximation of simply neglecting the penguin contributions to $B \to \pi\pi$ or $b \to \rho\rho$. The dominant isospin- breaking effect is $\pi^0$ mixing with $\eta$ states, (or $\rho\omega$ mixing for the $\rho$ case).[27] What this means is that the physical $\pi^0$ or $\rho^0$ states are not precisely the third member of the isospin triplet, so the possible contribution coming from the isospin singlet parts of these particles has to be considered. At present this effect is small compared to other uncertainties but with sufficient data it will be the factor that determines the ultimate accuracy of this approach. More precise studies of these issues, using the latest calculational methods, need to be done to get a more precise idea of just what that limit is. In the $\rho\rho$ case if one can also measure the asymmetries for $\rho^0\rho^0$, the relationship between these asymmetries and those for two charged $\rho$'s could put some constraints on isospin breaking effects. For two neutral pions the time-dependent asymmetry measurement is not feasible.

The full SU(3) symmetry of the $u$, $d$ and $s$ quarks, or its subgroup known as $u$-spin, which relates $d$ and $s$ quarks, can likewise provide useful information. It can be used, for example to calculate the penguin contribution to $\pi\pi$ by measuring $K\pi$ modes that are predominantly penguin driven. (In the calculations such as those referred to above as perturbative QCD, such relationships are used.) Again one must assign an error due to symmetry breaking effects. For SU(3) symmetry, in some cases, these are quite large. The classic example is the ratio of $f_\pi$ to $f_K$, where we see symmetry breaking of order 20%. Where appropriate this particular SU(3) breaking effect can be explicitly included in the calculation. Other SU(3) breaking contributions are not known and not well constrained. Thus it is not generally very effective to use SU(3) to constrain a dominant contribution. However, if the contribution being constrained by SU(3) is sub-leading, the impact of the uncertainties from SU(3) can be small enough in the final result that the constraint is useful.

None of my emphasis on theoretical uncertainties should be construed as saying one cannot test the Standard Model in heavy flavor decays. One can do so, but one must to it carefully.

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