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Linear Collider Collaboration Tech Notes

Planar Undulator Considerations

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rev. 2: 7/03/02

References:

1. H. Wiedemann, **Particle Accelerator Physics II**, 1995
2. V.V. Balandin, *et al*., *Conceptual Design of a Positron Injector for the TESLA Linear Collider,* Moscow-Hamburg 2000, DESY-TESLA-00-12.
3. **TESLA TDR II Accelerators**, chapters 4 and 5.

Abstract

This note consists of informal working notes that document an effort to understand the TESLA baseline, unpolarized, undulator based positron source. This is the first step in the design process of an undulator based positron system for the NLC. The expressions and methodologies developed herein are used in subsequent memos that reference this note. In regards to the TESLA design, it is found that a 135 m long (versus 100 m length) undulator is consistent with the performance descriptions in the TDR text. And while operation of the TESLA system with a 250 GeV drive beam energy provides a safety margin of a factor of ≈ 2 in e+ yield, operation of the same system at 160 GeV results in a yield of about 0.6.

Question: How long is the undulator? Start with how many positrons are required, N_p, and a multiplicative safety margin, f>1:

$$fN_p = N_e \, {}_eY_{h\nu} L_u \, {}_{h\nu}Y_p V_c \tag{1}$$

where N_e = number of electrons, ${}_eY_{h\nu}$ = yield of electrons to photons per meter of undulator, L_u = the length of the undulator in meters, ${}_{h\nu}Y_p$ = yield of photons to positrons leaving the target, and V_c = the capture efficiency. Note that ${}_eY_{h\nu}$, ${}_{h\nu}Y_p$, and V_c all depend in detail of the undulator parameters, the incident electron energy, the spot size on the target, and details of the collection and capture system. L_u is extracted from (1) to give:

$$L_u = \frac{fN_p}{N_e \, {}_eY_{h\nu} \, {}_{h\nu}Y_p V_c} \tag{2}$$

For the TESLA design, $Np=Ne$ and f =2 for E_e = 250 GeV, the electron energy. For the TESLA undulator, B_0 = 7.5 kG and λ_p = 1.42 cm (page II-122)[3]. From Wiedemann[1],

$$K = 9.344 B_0(kG)\, \lambda_p(m) = 0.995 \tag{3}$$

The total radiated energy per electron per meter is given as *DE*:

$$\Delta E(eV) = 725\frac{E_e^2(GeV)K^2}{l_p^2(cm)} = 2.23\times 10^7 eV\ /m = 22.3MeV\ /\ m \qquad (4)$$

The TESLA design report says that on average 3 GeV is lost, this implies an undulator length of $\frac{3GeV}{22.3MeV\ /\ m} = 134.8m$. This is at odds with the text of the report which indicates that the undulator is 100 m long. Similarly, the report indicates that 135 kW of power is generated by the wiggler. From (4) the average power output is given as P_{avg}:

$$P_{avg} = qN_e\Delta E(eV)\ /\ m\times L_u = 135kW \qquad (5)$$

Plugging in the numbers, $q = 1.602\times 10^{-19}\frac{J}{eV}$, and $N_e = 2.8\times 10^{14}\ /\ s$ and DE=22.3 MeV/m, then gives a value for L_u of

$$L_u = \frac{135kW}{qN_e\Delta E(eV)\ /\ m} = 135.2m \qquad (6)$$

This is at least consistent with the 3 GeV average loss. Next a quick check of the cutoff energy of the undulator first harmonic, E_{c10}:

$$E_{c10} = \hbar w_{10} = \hbar\frac{4pg^2\, c/l_p}{\left(1+K^2/2\right)} \qquad (7)$$

For λ_p = 1.42 cm , E_{c10} = 28 MeV (27.96 MeV , actually) which is essentially the same as in Table 4.3.2 (II-120).

So next figure out yields for this particular undulator. The number of undulator photons is found by equating the integral of "universal" undulator spectrum, *hsum(ww10)*[note: see z:/positrons/polarized positrons/1149df.m and teslsa150_250wrkspc.m], to the total energy radiated to find the normalization constant which in turn is used with the integral of the photon number spectrum. So here goes (Matlab is used through out for the numerical results).

$$a\int dw\, hsum = a\,dw\sum hsum = \frac{P_{avg}}{N_e} \qquad (8)$$

which gives

$$a = \frac{P_{avg}/N_e}{dw\sum hsum}(J\ /\ e^-) \qquad (9)$$

Similar thinking leads to the ratio of the number of photons radiated to the number of electrons, ${}_eY_{hn}$:

$$ {}_eY_{hu} = \frac{adw}{L_u} \sum hsum./w = \frac{P_{avg}(W) \big/ N_e(e^- / s)}{qE_{c10}(eV)\, L_u(m)} \times \frac{\sum hsum./ww10}{\sum hsum} \qquad (10) $$

Note, a plot of the photon energy spectrum, hsum(ww10), is shown below; ww10 is the photon energy (frequency) normalized by the cutoff energy of the first harmonic, E_{c10}:

$$ ww10 = \frac{w}{w_{10}} \qquad (10a) $$

wherein $w_{10} = E_{c10} / \hbar$.

Also the quantity $\frac{\sum hsum./ww10}{\sum hsum}$ depends only on K. For K=1, and summing over the first 4 harmonics

$$ \frac{\sum hsum./ww10}{\sum hsum} = 1.2648 \qquad (11) $$

So, for the L_u = 135 m, K=1, λ_p = 1.42 cm, and E_{c10} = 28 MeV ,

$$ {}_eY_{hu} = \frac{135.7}{135} = 1.005(photons/meter/electron) \qquad (12) $$

One can also figure out the average photon energy, ${}_{hu}E_{avg}$,

$$ {}_{hu}E_{avg} = E_{c10} \times \frac{\sum hsum}{\sum hsum./ww10} \qquad (13) $$

For the case at hand,

$$ {}_{hu}E_{avg} = 22.1385 MeV \qquad (14) $$

And the product,

$$ {}_eY_{hu} \times L_u \times {}_{hu}E_{avg} = 3.0045 GeV \qquad (15) $$

as it should.

In order to determine the yield of photons to positrons leaving the target, $_{hn}Y_{p}$? one needs to use EGS4. A version of EGS4 user code (ucRTZspc.mortran, ucRTZ.data, pncum_k1_120.out, ww10c_k1_120.out, and Xegs4run*) has been written (with significant help and guidance from R. Nelson) which generates positrons from an incident photon beam of arbitrary spectrum. The code also allows one to specify an incident rms gaussian beam size in x and y. The programs all reside in

/afs/slac.stanford.edu/u/ad/jcs/egsruns/yieldcalc/phtnspctrm in Unix land.

Figures 3-8 follow the text. Figures 3 and 4 are scatter plots of the x-px phase space of positrons coming out of the back surface of the conversion target. It should be noted that the emittance of the emitted positrons are only a factor of 2-3 time smaller than the phase space of conventionally produced positrons. This is in large part due to the fact that the beam size of the emitted positrons is large because of the 1.4 cm thickness of the 0.4 r.l. length Ti-alloy target. Figure 5 illustrates the energy spectrum of emitted positrons for the conditions noted in the figure. Figures 6-8 show the spectrum and transverse profiles of the incident photon beam. Figures 3-8 are made from files written by EGS4.

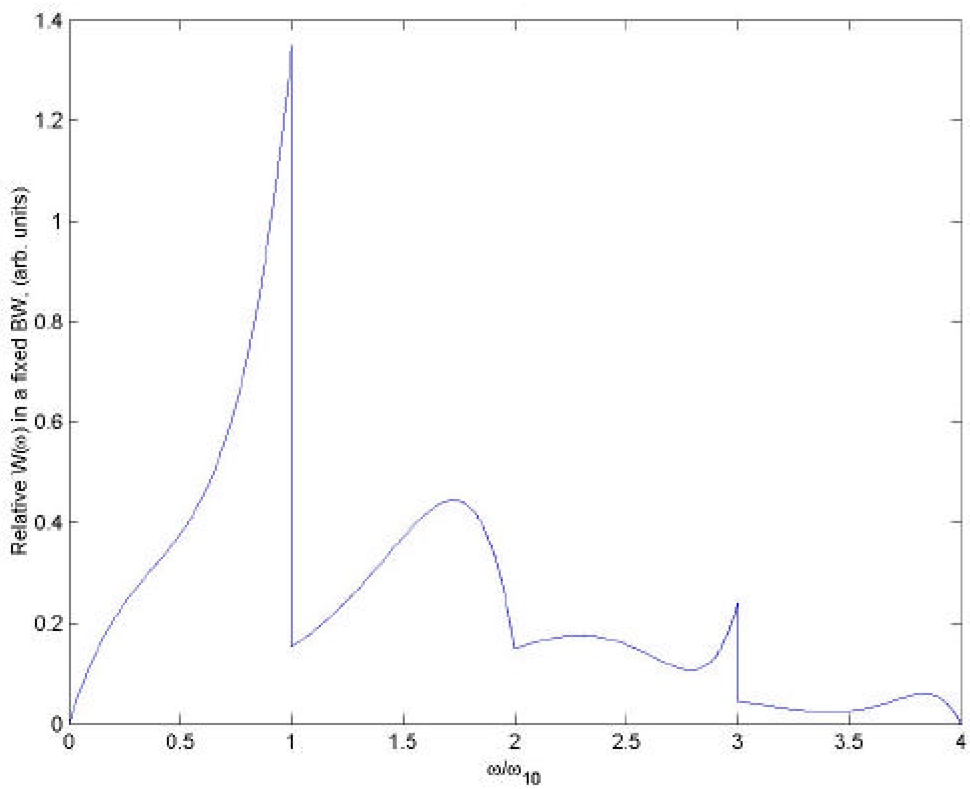

Figure 1: Universal undulator energy spectrum, hsum.

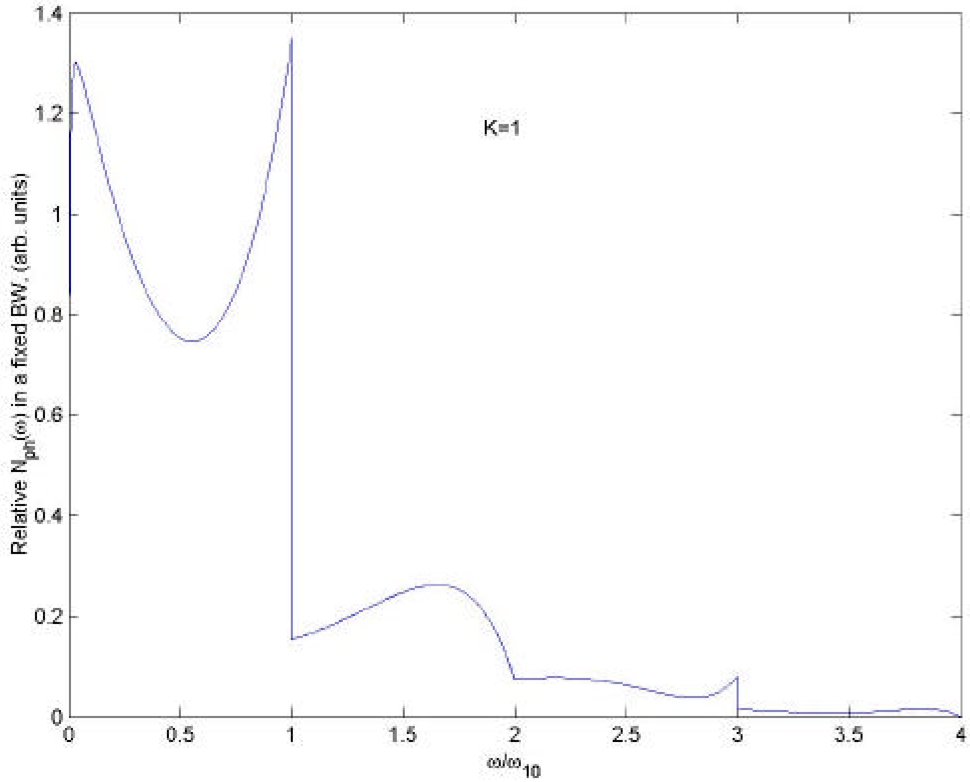

Figure 2: The universal photon number spectrum, hsum/(ww10)

At 250 GeV through the undulator, E_{c10} = 28 MeV , $s_x = s_y = 0.7mm$, 100,000 photons into 0.4 R.L. Ti target results in 5404 emitted positrons from the exit face, a yield of ${}_{hn}Y_p$ = 5.4%. Similarly, at 160 GeV through the undulator, E_{c10} = 11.45 MeV, $s_x = s_y = 1.09mm$, 100,000 photons into 0.4 R.L. Ti target results in 1750 emitted positrons from the exit face, a yield of ${}_{hn}Y_p$ = 1.75%.

The INR report[2] states that the capture efficiency into $\varepsilon_x = \varepsilon_y = 0.036$ m is $V_c \approx 25\%$. So, putting this all together, I find

$$f(E_e = 250GeV) = \frac{N_e}{N_p} {}_eY_{hu} L_u {}_{hu}Y_p V_c = 1 \times 1.005 \times 135 \times 0.054 \times 0.25 = 1.83$$

At 160 GeV,

$$f(E_e = 160GeV) = \frac{N_e}{N_p} {}_eY_{hu} L_u {}_{hu}Y_p V_c = 1 \times 1.005 \times 135 \times 0.0175 \times 0.25 = 0.59$$

The ratio

$$f(160GeV)/f(250GeV) = 0.59/1.83 = 0.32\,.$$

This is similar in value to simply counting the number of positrons within the 5-D phase space cuts which are shown in the emitted positron figures. From the figures,

$$N_p(160)/N_p(250) = 1750\times 0.75/5404\times 0.941\times 0.721 = 1312/3666 = 0.36.$$

On the face of this, it looks as if TESLA has real problems making positrons at 160 GeV. To get up to a capture yield of $f(160GeV) = 1$, the capture efficiency needs to increase by a factor of 1.7 from 0.25 to 0.42. Seems tough; $V_c \approx 25\%$ also feels optimistic and is quoted at a positron energy of 274 MeV [2], not through the full 5 GeV of acceleration and transport. [Note 11/19/01: Yuri Batygin indicated that $V_c \approx 25\%$ may be doable but is very sensitive to the relative phasing of the capture rf and also of the linac phases, at the several degree level.]

Time to do our own flux concentrator simulations. Also can figure out the energy cross over point.

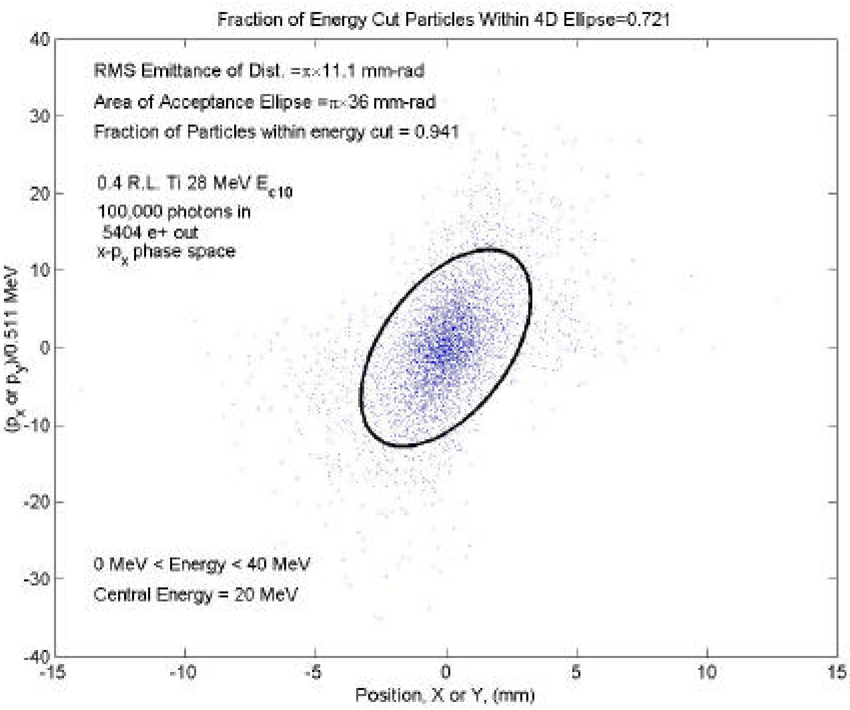

Figure 3: x-p_x phase space distribution of positrons exiting an 0.4 r.l. thick Ti-alloy conversion target. The incident photon spectrum is that of a K=1 undulator with $E_{c10} = 28\ MeV$ and $s_x = s_y = 0.7\ mm.$

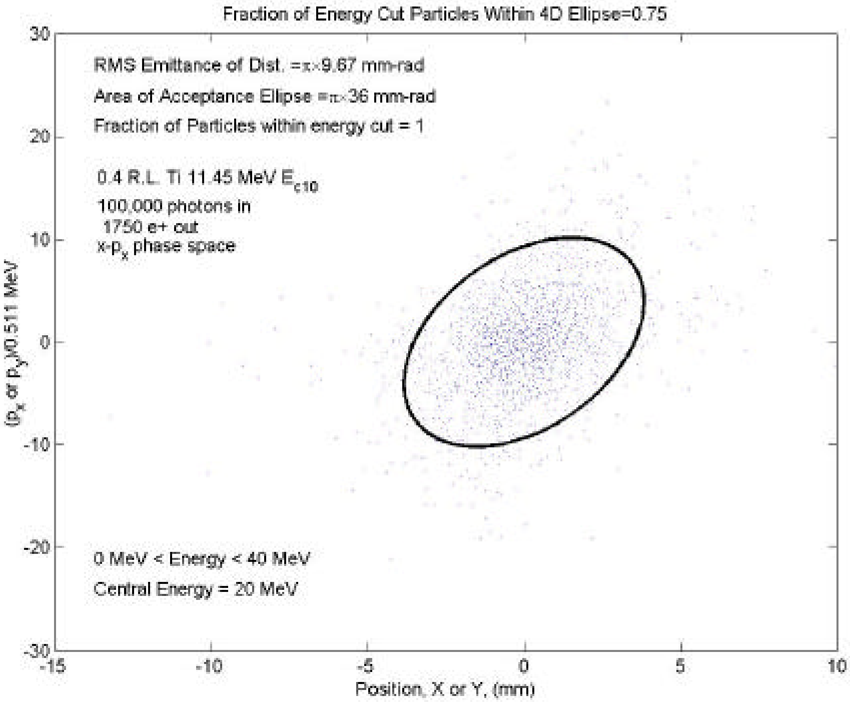

Figure 4: x-p_x phase space distribution of positrons exiting an 0.4 r.l. thick Ti-alloy conversion target. The incident photon spectrum is that of a K=1 undulator with $E_{c10} = 11.45\ MeV$ and $s_x = s_y = 1.09\ mm$.

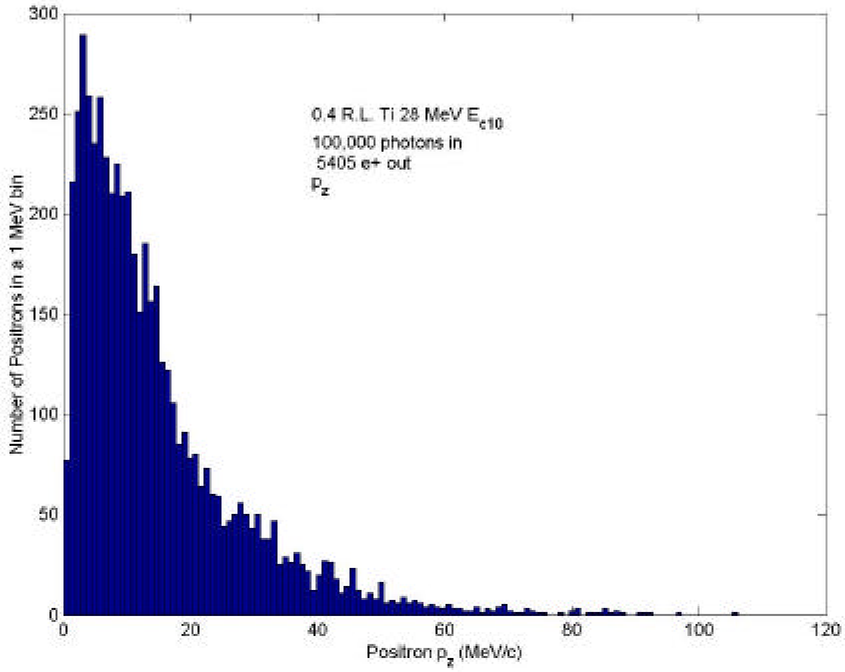

Figure 5.: Emitted positron energy spectrum for K=1 undulator with $E_{c10} = 28\ MeV$, 0.4 r.l. thick Ti-alloy target.

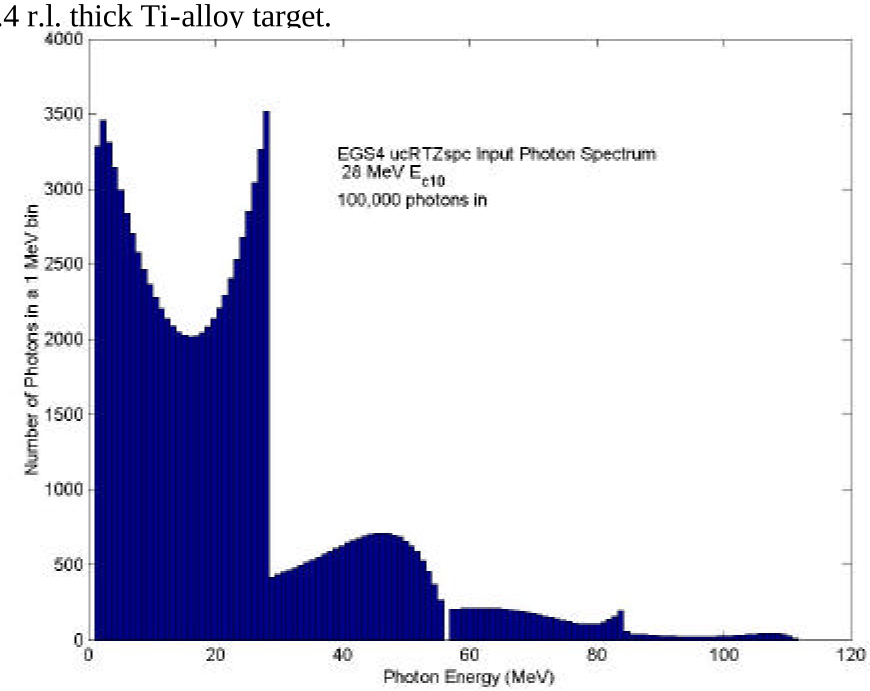

Figure 6.: Incident photon energy spectrum for K=1 undulator with $E_{c10} = 28\ MeV$.

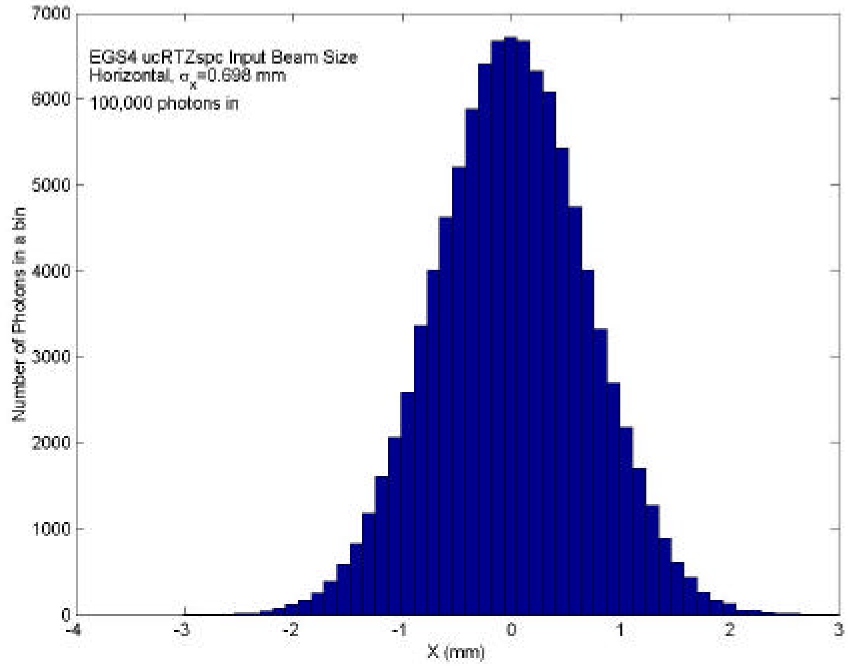

Figure 7.: Incident photon beam transverse distribution, horizontal plane.

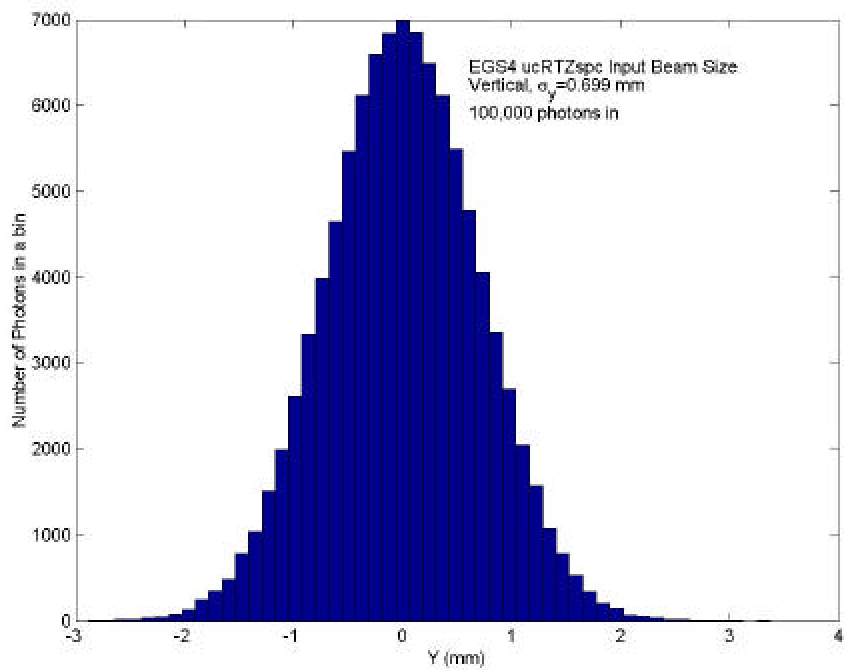

Figure 7.: Incident photon beam transverse distribution, vertical plane.