V_{us} Calculation from Lattice QCD

Huey-Wen Lin*

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

(Dated: June, 2007)

I review recent progress in calculating $|V_{us}|$ from lattice QCD from koan and hyperon system. A preliminary result from the first dynamical calculation in the hyperon channel is also included.

I. INTRODUCTION

The Standard Model has successfully used quantum fields to describe most of the strong interactions in the past, where universe is composed by the the quarks with three flavors, which interact via glunoic gauge boson. Although the electromagnetism and strong interactions are not affected by quark flavor, the weak interaction, however, may change the flavor of the quarks. This amount of overlap between the various generations is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

In 1963, Cabibbo first introduced a 2×2 quark mixing matrix to explain semileptonic decay in baryons; Kobayashi and Maskawa later extended the matrix to include the then-undiscovered bottom quark sector. This becomes the CKM matrix that we are familiar with today:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1}$$

The Standard Model requires this matrix to be unitary. This gives six unitarity constraints, each of which may be graphically depicted as a unitarity triangle; one such constraint is

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \tag{2}$$

*Electronic address: hwlin@jlab.org

From the latest PDG 2006, V_{ud} in this unitarity equation is both the dominant term and also very well determined from proton beta decay, 0.97377(27); V_{ub} is very small, $4.21(30)\times 10^{-3}$. This leaves the third matrix element V_{us} as the weak link in deciding whether or not this unitarity equation holds. From the latest PDG number, $V_{us} = 0.2257(21)$ is determined to only 0.1%. In this proceeding, I will describe how this number can be obtained using calculations from lattice QCD. Note that although currently V_{us} seems to indicate that this unitarity equation holds within error bars, the number has been shifting around a lot during the past few years. In 2003, it was 0.2195(23), which is more than 10 standard deviations away from its current best value.

In quantum chromodynamics (QCD), physical observables are calculated from the path integral. For calculations where the coupling is weak, one can perform the integral by hand. However, for long distances the perturbative QCD series no longer converges. Thus, to calculate from first principles, one needs help from lattice gauge theory. Lattice QCD discretizes space-time such that the path integral over field strengths (especially at strong coupling) can be calculated numerically. Since the real world is continuous and infinitely large, by the end of the day we will have to take the lattice spacing $a \to 0$ and the volume $V \to \infty$ limits to connect to the physical world. However, to simulate at the real pion mass (while at the same time keeping the lattice box big enough to avoid finite-volume effects) would require much faster supercomputers that have not yet been born. Thus, we normally calculate with a few unrealistically large values of the pion mass, and then use chiral extrapolation to get back to the physical pion mass.

Here, we quickly mention a few choices of fermion action that have been commonly used in lattice QCD calculations. Each has its own pros and cons. They differ primarily by how they maintain symmetry, their calculation cost and their discretization error. (Improved) Staggered fermions (asqtad)[1–3] are relatively cheap for dynamical fermions, but it introduces mixing among parities and flavours or "tastes", which make its baryonic operators a nightmare to deal with. The $O(a)$-improved Wilson (Clover) fermion action[4] is moderate in cost and free of the disadvantages of the staggered actions. However, chiral symmetry is badly broken at non-zero lattice spacing which causes operator mixing issues. Chiral fermions (e.g. Domain-Wall or Overlap)[5–8] are free of all the above problems. They are automatically $O(a)$ improved, suitable for spin physics and weak matrix elements. These benefits comes at a great cost in computation power. Last but not least are mixed actions,

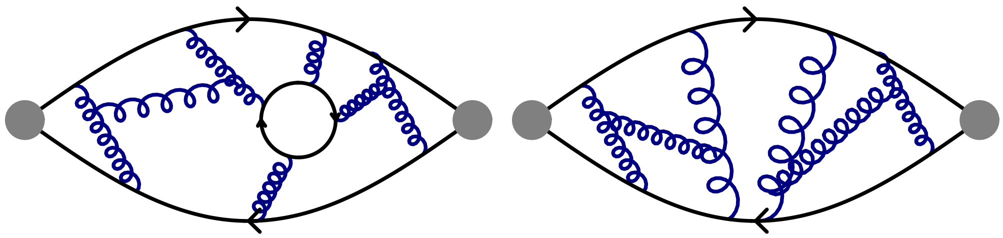

FIG. 1: A depiction of the difference between full QCD and the quenched approximation in the example of a two-point Green function.

where one chooses difference valence and sea quark discretizations. Later in this work, we present work done using staggered sea quarks (cheap) with domain-wall valence quarks (chiral); we match the sea Goldstone pion mass to the DWF pion.

Before we discuss the details of the lattice calculation, it is important to mention the "quenched" approximation or 0-flavor calculations. There are a couple of calculations mentioned later which use this approximation. In the path integral, the correct way to carry out the fermionic part of integral is to integrate out the quark/antiquark fields first. This leaves the remaining integral as a function only of the gauge links and introduces a fermionic determinant. The "quenched" approximation fixes the fermionic determinant as a constant. This means that when we calculate, say, a two-point Green function, as depicted in Figure 1, such as a meson correlator, internal fermion loops have been omitted; thus, there are no internal fermion loops.

The quenched approximation was a product of the old days when the computers were slow and algorithms were not yet sped up to today's standards. Quenching very greatly reduces the cost of a lattice calculation by eliminating the fermionic determinant. Of course, modern calculations have moved on to focus on *un*quenched calculations. This is a lot better, since when one ignores the fermion loops, it is very difficult to estimate how much this will affect the final numbers; the size of the effect tends to vary greatly between different physical quantities. However, since the calculations using the quenched approximation can be done very fast, one can test new methodologies and ideas using this approximation before performing unquenched (dynamical) calculations. The first lattice calculations in K_{l3} and hyperon semileptonic decay calculations were demonstrated in this approximation.

In the following, I will review the latest V_{us} calculations. There has been a series of works devoted to K_{l3} decays but only quenched calculations in the hyperon channel so far. In the second half of this work, I will show the first lattice dynamical calculation of hyperon decays using mixed action. In the final part, I will summarize the current standing of V_{us} from lattice QCD and give some future outlook.

II. LATTICE V_{us} CALCULATIONS

In this work, we will concentrate on three determinations of V_{us}: leptonic decay ratios (mainly for completion), K_{l3} decays and hyperon decays. We will compare all of them to the number listed in PDG 2006. So far, the number from K_{l3} decays has the smallest errorbar for V_{us}.

A. Leptonic Decays

If one looks at the decays $K_{\mu 2}$ and $\pi_{\mu 2}$, their branching ratios can be written in terms of V_{us}/V_{ud} and the ratio of the kaon to pion decay constant:

$$\left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 = \left[\left(\frac{f_K}{f_\pi}\right)^2 \frac{M_K\left(1-m_\mu^2/M_K^2\right)^2}{M_\pi\left(1-m_\mu^2/M_\pi^2\right)^2}\left(1+\frac{\alpha}{\pi}\left(C_K-C_\pi\right)\right)\right]^{-1}\frac{\Gamma(K\to\mu\bar{\nu}_\mu)}{\Gamma(\pi\to\mu\bar{\nu}_\mu)}, \qquad (3)$$

where C_K and C_π are the radiative-inclusive electroweak corrections, and the rest of the numbers can be obtained from experimental measurements. William J. Marciano[9] used decay constant ratios $f_K/f_\pi = 1.210(4)(13)$ from a 2+1f staggered fermion calculation of f_π and f_K in 2004 done by MILC collaboration and found $V_{us} = 0.2219(25)$. Of course, there have been other full-QCD calculations since 2004. For example, RBC/UKQCD use dynamical chrial fermions (DWF) to obtain the ratio 1.24(2)[10]. However, none of the other collaborations have come out with a number with competitive errorbar yet In 2006, MILC updated their own calculation, $1.208(2)(^{7}_{14})$[11]; this yields $V_{us} = 0.2223(^{26}_{14})$.

B. K_{l3} decay

Another way to determine V_{us} is to look at the K_{l3} decay. When one integrates out the short-distance dependence, one is left with a low energy non-perturbative matrix element for K to π, which can be calculated directly in lattice QCD. Using Lorentz invariance, we can decompose the matrix element into two form factors (f_+ and f_-) with differing momentum

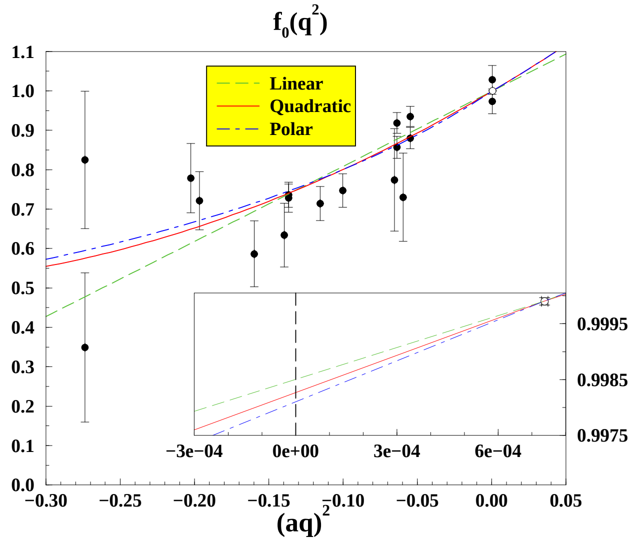

FIG. 2: Momentum extrapolation from Ref. [13]

dependence:

$$\langle \pi(p')|V_\mu|K(p)\rangle = (p_\mu + p'_\mu)f_+(q^2) + (p_\mu - p'_\mu)f_-(q^2). \tag{4}$$

The so-called "double ratio" technique:

$$\frac{\langle \pi|\bar{s}\gamma_0 u|K\rangle\langle K|\bar{u}\gamma_0 s|\pi\rangle}{\langle K|\bar{s}\gamma_0 s|K\rangle\langle \pi|\bar{u}\gamma_0 u|\pi\rangle} = |f_0(q^2_{\max})|^2\frac{(m_K+m_\pi)^2}{4m_K m_\pi}, \tag{5}$$

which has been used to look at B-to-D decays[12], can also be applied to the K to π decay. The result is a form factor that only depends on q^2, the momentum transfer between the initial and final states, and some kinetic factors. This f_0 can be connected to $f_\pm$ through

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2}f_-(q^2). \tag{6}$$

When we extrapolate to $q^2 = 0$, $f_0 = f_+$. We sometimes study different extrapolation forms to estimate the systematic error caused by such an extrapolation:

$$f_0(q^2)^{\text{Linear}} = f_0(0)(1+\lambda_0 q^2) \tag{7}$$

$$f_0(q^2)^{\text{Quadratic}} = f_0(0)(1+\lambda_0 q^2 + cq^4) \tag{8}$$

$$f_0(q^2)^{\text{Polar}} = f_0(0)/(1-\lambda_0 q^2), \tag{9}$$

which is shown in Figure 2 by Becirevic et al. [13]

To remove uncertainty due to the momentum extrapolation, we can calculate the matrix element directly at $q^2 = 0$. The trick to this technique is to include help from "twisted" boundary conditions on the fermions as

$$\psi(x+e_jL)=e^{2\pi i\theta_j}\psi(x). \tag{10}$$

The discretized momenta on the lattice will reflect the choice of this θ_j as $p_j=\theta_j\frac{2\pi}{L}+n_j\frac{2\pi}{L}$ with n_j integer. We can select θ wisely to cancel out the mass difference, so that we can obtain $f_+(0)$ directly. Guadagnoli et al.[14] first demonstrated the advantage in the quenched approximation. Later, UKQCD made an exploratory study on a 2+1-flavor DWF calculation[15]. They showed that with 50% more statistics, one can get number competitive with conventional extrapolation calculations. The advantage of this method over the conventional one is smaller or no systematic error due to q^2 extrapolation. Thus, total error on the calculation is reduced.

After we obtain $f_0(0)$, the next step is: how do we extrapolate the pion mass to the physical one? We can get some help from the Ademollo-Gatto (AG) theorem[16, 17]. We know that the $SU(3)$ symmetry-breaking Hamiltonian is:

$$H=\frac{1}{\sqrt{3}}\left(m_s-\frac{m_d+m_u}{2}\right)\overline{q}\lambda^8q \tag{11}$$

The AG theorem tells us that there is no first-order correction due to $SU(3)$-breaking; thus, the correction starts at second order

$$f_0(0)=f_0(0)^{SU(3)}+O(H^2). \tag{12}$$

What would be a good measure for $SU(3)$ breaking? The most natural candidate would be the mass splitting between the kaon and pion. So, we expect the remaining correction should be small; thus, one would expect the "corrected" lattice f_0 (after subtracting the chiral log), f', should differ from $f_0^{SU(3)}$ by only a small amount. If we construct a ratio

$$R(m_K,m_\pi)=\frac{f^{SU(3)}-|f'(0)|}{a^4(m_K^2-m_\pi^2)^2}, \tag{13}$$

where $f^{SU(3)}$ is the $SU(3)$-limit value; in this case, it is 1. We expect the remaining mass dependence in Eq. 13 should be relatively small. We then extrapolate the remaining mass dependence to the physical sum of the pion and kaon masses-square

$$R(m_K,m_\pi)=c_0+c_1a^2(m_K^2+m_\pi^2). \tag{14}$$

Thus, V_{us} can then be obtained from

$$\Gamma(K_{l3})=\frac{G_F^2M_K^5}{128\pi^3}|V_{us}|^2S_{\mathrm{EW}}|f_+(0)|^2C_K^2I_K^l(\lambda_i)(1+\delta^K_{SU(2)}+\delta^{Kl}_{\mathrm{EM}}). \tag{15}$$

Group	N_{f}	S_{f}	M_π (GeV)	# conf	$f_+(0)$
SPQcdR[20]	0	Wilson	0.500–1.000	230	0.961(09)
JLQCD[21]	2	Clover	0.440–0.960*	N/A	0.967(06)
RBC[22]	2	DWF	0.475–0.700	94	0.955(12)
HPQCD[23]	2+1	Staggered	0.500–0.700	N/A	0.962(11)
RBC/UKQCD[19]	2+1	DWF	0.390–0.700	150	0.961(05)

TABLE I: Summary of existing published f_+ calculation from K_{l3} decay

The decay width, $\Gamma(K_{l3})$, is taken from experiment, while the phase-space integral I^l, isospin breaking $\Delta_{SU(2)}$, long-distance electromagnetic corrections Δ_{EM} and short-distance radiative corrections S_{EW} are taken from perturbative calculations.

Table I summarizes the results from various lattice QCD groups: quenched, partially quenched and full QCD, fermion action variety and the range of the pion mass. Figure 3 f_+ is taken from individual calculations; combined with the latest PDG 2006 number for $|f_+V_{us}| = 0.2169(9)$. Note the V_{us} number may be very be different from the ones given in the original papers, due to the updated progress in experimental measurements. The grey band in the graph is the range allowed assuming unitarity holds. All the lattice calculations so far have agreed with the old estimation from Leutwyler-Roos in 1984[18]. However, not every paper has complete estimations of the systematic errors due to lattice artifacts. The work done by RBC/UKQCD[19] is one of the exceptions, and thus I quote their number as representative of the V_{us} from K_{l3} decay channel: 0.2257(14).

C. Hyperon decays

Hyperon decays provide us with an additional independent channel for determining V_{us}. We start by looking at the low-energy contribution of the transition matrix elements for hyperon beta decay, $B_1 \to B_2 e^- \overline{\nu}$; in low-energy effective theory this can be written as

$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O_\alpha^{\mathrm{V}} + O_\alpha^{\mathrm{A}}) u_{B_1} \overline{u}_e \gamma^\alpha (1+\gamma_5) v_\nu. \tag{16}$$

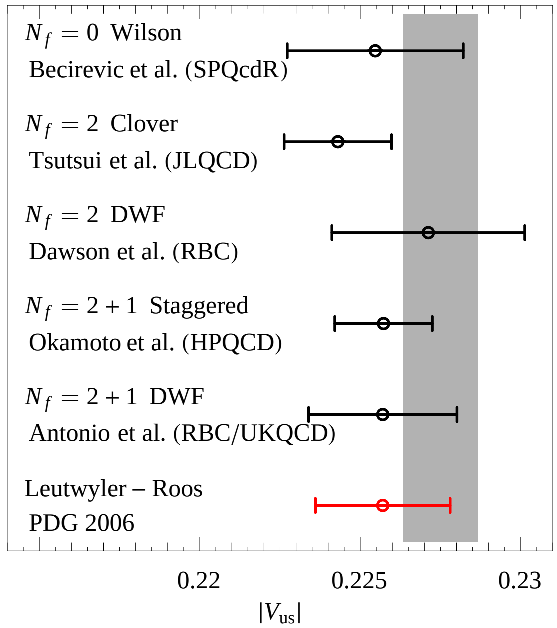

FIG. 3: Lattice $|V_{us}|$ summary with unified experimental numbers from PDG

From Lorentz symmetry, we expect the matrix element composed of any two spin-1/2 nucleon states, B_1 and B_2, to have the general form

$$O_\alpha^V = f_1(q^2)\gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_{B_1}}q_\alpha \tag{17}$$

$$O_\alpha^A = \left(g_1(q^2)\gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_{B_1}}q_\alpha\right)\gamma_5 \tag{18}$$

with transfer momentum $q = p_{B_2} - p_{B_1}$ and V, A indicating the vector and axial currents respectively.

The vector form factor is connected to V_{us} via

$$\Gamma = G_F^2\,|V_{us}|^2\,\frac{\Delta m^5}{60\pi^3}\,(1+\delta_{\mathrm{rad}}) \tag{19}$$

$$\times\left[\left(1-\frac{3}{2}\beta\right)\left(|f_1|^2+|g_1|^2\right)+\frac{6}{7}\beta^2\left(|f_1|^2+2|g_1|^2+\mathrm{Re}(f_1f_2^*)+\frac{2}{3}\,|f_2^2|\right)+\delta_{q^2}\right], \tag{20}$$

with $\Delta m = m_{B_1} - m_{B_2}$, $\beta = \Delta m/m_{B_1}$, the radiative corrections δ_{rad}, and $\delta_{q^2}(f_1, g_1)$ taking into account the transfer-momentum dependence of f_1 and g_1 [24]. Generally, the ratios of g_1/f_1 from experiment and f_2/f_1 in the $SU(3)$ limit are used to get V_{us} from hyperon decays.

In 2003, Cabibbo et al.[25] used f_2/f_1 and f_1 in the $SU(3)$ limit, combined with experimental decay width (or rate) and g_1/f_1, to obtain V_{us} from various channels of hyperons

Channel	$f_1^{SU(3)}$	$\|f_1 V_{us}\|$	$(g_1/f_1)^{SU(3)}$	$(g_1/f_1)^{\text{exp}}$
$n \to p$	1	n/a	$F+D$	1.2670(30)
$\Lambda \to p$	$-\sqrt{3/2}$	0.2221(33)	$F+D/3$	0.718(15)
$\Sigma^- \to n$	-1	0.2274(49)	$F-D$	$-0.340(17)$
$\Xi^- \to \Lambda$	$\sqrt{3/2}$	0.2367(97)	$F-D/3$	0.25(5)
$\Xi^- \to \Sigma^0$	$\sqrt{1/2}$	n/a	$F+D$	n/a
$\Xi^0 \to \Sigma^+$	1	0.216(33)	$F+D$	1.32(22)

TABLE II: Summary of a few hyperon numbers

decay, as shown in Table II. It is not hard to see that if lattice calculations can provide better estimates of g_1/f_1, we can improve the precision of V_{us} from hyperon decays and possibly get a better estimation than the K_{l3} channel ones.

So far, there are only two quenched lattice calculations of hyperon beta decay, and they are in different channels, $\Sigma \to n$ and $\Xi^0 \to \Sigma^+$. Guadagnoli *et al.*[26] extrapolate the matrix element $\Sigma \to n$ via an AG ratio, similar to the discussion in the K_{l3} decay case, but using a dipole form to extrapolate to the zero-transfer momentum point. All of the pion masses are larger than 700 MeV; their final numbers are $f_1 = -0.988(29)_{\text{stat}}$ and $V_{us} = 0.230(5)_{\text{exp}}(7)_{\text{lat}}$. Sasaki *et al.*[27] use lighter pion masses 530–650 MeV and DWF to look at the Ξ^0 decay channel. They extrapolate the vector form factor f_1 via the variable $\delta = (m_{B_2} - m_{B_1})/m_{B_2}$. The Ademollo-Gatto theorem suggests the leading-order effect should be δ^2, and thus one can fit $f_1(0)$ to the form $c_0 + c_1\delta^2$. Their final numbers are $f_1 = 0.953(24)_{\text{stat}}$ and $V_{us} = 0.219(27)_{\text{exp}}(5)_{\text{lat}}$. Unfortunately, the experimental determination of the decay rate is lousy; despite f_1 in Ξ decay channel being compatible within errors, V_{us} is not well-determined. This may further improve in the future with updates from Fermilab KTeV and CERN NA48 collaborations. One important thing to note is that neither of the calculations has systematic error estimate from quenching effects, which we expect might be significant.

We have taken data looking at both hyperon channels with a dynamical lattice calculation for the first time. We use a mixed action, meaning that the sea (staggered) and valence (DWF) fermions have different discretization. Our pion masses are relatively lighter than the quenched calculations. We only simulate one strange quark mass, which unfortunately does not reproduce the correct strange-strange Goldstone mesons. We find a box size around

Label	m_π (MeV)	m_K (MeV)	$\Sigma^- \to n$ conf.
m010	358(2)	605(2)	600
m020	503(2)	653(2)	420
m030	599(1)	688(2)	561
m040	689(2)	730(2)	306

TABLE III: Configuration information

2.6 fm and list a few other important parameters in Table III. In this work, we report on our preliminary calculation in the $\Sigma^- \to n$ channel. We use a projection operator $T = (1-\gamma_5\gamma_3)(1+\gamma_4)/2$ in both two- and three-point Green functions and construct a ratio

$$\begin{aligned} R_{j_\mu} &= \frac{Z_V \Gamma^{\Sigma N}_{\mu,GG}(t_i,t,t_f,\vec{p}_i,\vec{p}_f\,;\,T)}{\Gamma^{NN}_{GG}(t_i,t_f,\vec{p}_f\,;\,T)} \sqrt{\frac{\Gamma^{\Sigma\Sigma}_{PG}(t,t_f,\vec{p}_i\,;\,T)}{\Gamma^{NN}_{PG}(t,t_f,\vec{p}_f\,;\,T)}} \\ &\times \sqrt{\frac{\Gamma^{NN}_{GG}(t_i,t,\vec{p}_f\,;\,T)}{\Gamma^{\Sigma\Sigma}_{GG}(t_i,t,\vec{p}_i\,;\,T)}} \sqrt{\frac{\Gamma^{NN}_{PG}(t_i,t_f,\vec{p}_f\,;\,T)}{\Gamma^{\Sigma\Sigma}_{PG}(t_i,t_f,\vec{p}_i\,;\,T)}}, \end{aligned} \tag{21}$$

to cancel out kinetic and overlap Z factors. With multiple intertions of the momentum, we can solve for the individual form factors in Eq. 17.

We need to extrapolate to zero momentum. We use a dipole form, as has been used in momentum extrapolation for many baryons' momentum dependence. For the mass extrapolation, a similar approach to the K_{l3} case can be applied here with the help of Ademollo-Gatto theorem. We first construct a ratio and then extrapolate the mass dependence according to Eq. 13, as shown in Figure 4. Since our heaviest pion mass is much closer to the strange Goldstone meson mass, due to the mass difference in the denominator of the ratio, the magnitude of this point, both central value and especially the errorbar size, increases a lot. Thus, it provides only very weak constraint to the fit; hence this is not an ideal solution for our data.

Alternatively, we can combine the given two-step process into one, by performing a two-dimensional fit to the sum and difference of kaon and pion masses, as shown in Figure 5. But this fit is not ideal either since the constraints from the data points are not strong: only four points to define a two-dimensional surface.

A better constrained fit can be composed by combining the momentum and mass depen-

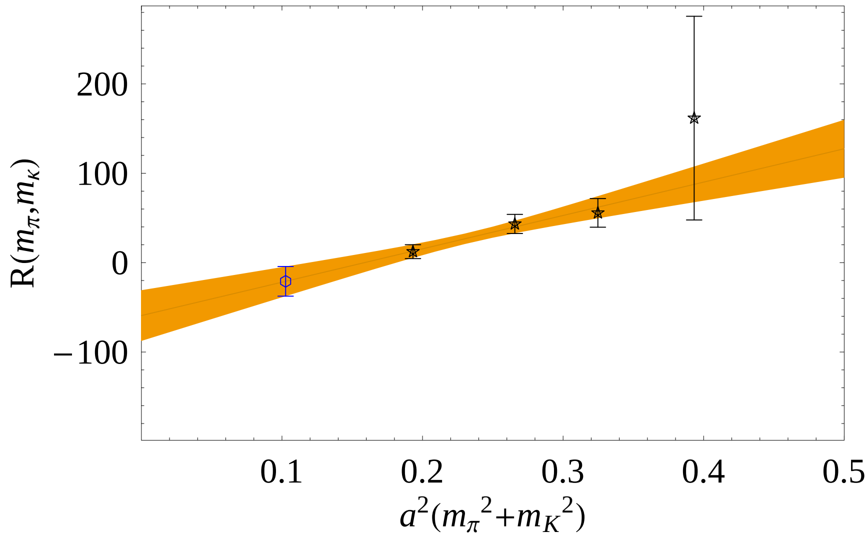

FIG. 4: AG ratio extrapolation to physical $m_K^2 + m_\pi^2$

dence into a single simultaneous fit:

$$f_1(q^2) = \frac{1 + (M_K^2 - M_\pi^2)^2 \left(A_1 + A_2 \left(M_K^2 + M_\pi^2\right)\right)}{\left(1 - \frac{q^2}{M_0 + M_1\left(M_K^2 + M_\pi^2\right)}\right)^2}. \tag{22}$$

Figure 6 shows the result from simultaneously fitting over all q^2 and mass combinations. The z-direction indicates f_1, while the x- and y-axes indicate mass and transfer momentum. The surface is the fit using Eq. 22 with color to indicate the different mass. The columns are the data and the momentum points from different pion masses line up in bands. Our preliminary result for f_1 is $-0.88(15)$. This leads us to a V_{us} somewhat larger in central value than the other calculations but still agrees with them due to the large errorbar. The statistics will be greatly improved at the lightest pion mass data in the near future.

III. CONCLUSION AND OUTLOOK

To summarize, there are various ways that lattice QCD calculations can help to determine V_{us} in the CKM matrix. (Similar approaches can be applied to the rest of the elements with effective lattice fermion actions.) Firstly, we can use the lattice input from the kaon and pion decay constant ratios. Currently, MILC has best determined ones, resulting in $V_{us} = 0.2226(^{26}_{15})$. Secondly, we can use the form factor from K_{l3} decay matrix elements: here we use the number from RBC/UKQCD, 0.2257(14), in which a sound study and proper

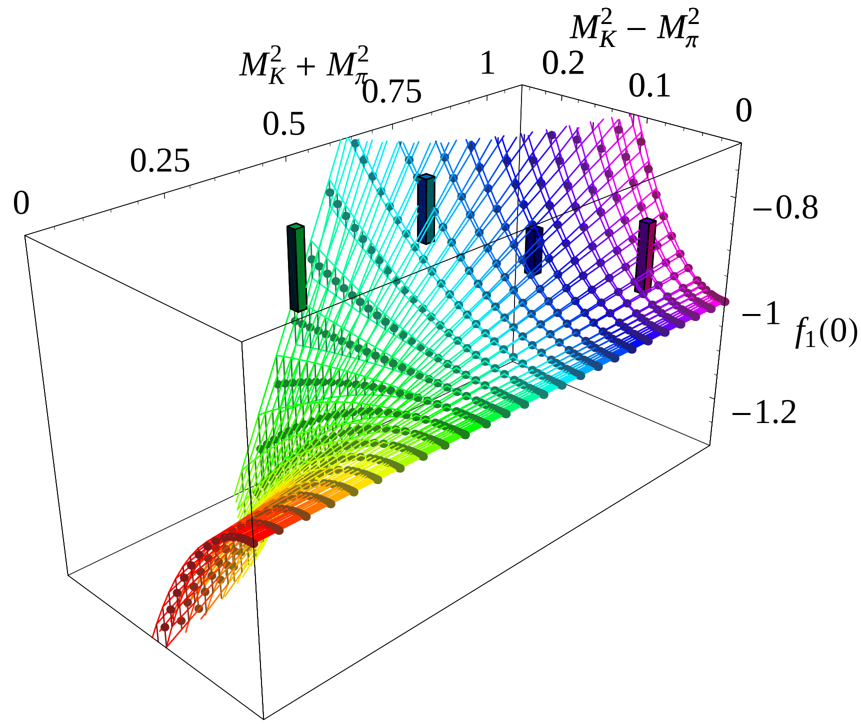

FIG. 5: Two-dimensional mass extrapolation after dipole extrapolation to zero-transfer point

systematics are included.

Finally, we can use the form factors from hyperon decays. We have started the first full-QCD 2+1-flavor dynamical calculation. Our preliminary results show consistency with previous calculations, but have larger errorbar due to the choice of lighter pion mass. The larger statistical error is partially compensated by the decrease in systematic error due to extrapolating the pion mass to the physical one. To improve the V_{us} value from the hyperon decays, reducing our statistical error on vector form factor and improving the accuracy on g_1/f_1 to replace the experimental one. Using these strategies, we can make our calculation of $|V_{us}|$ equivalent to or better than the one from the K_{l3} channel.

Acknowledgements

HWL thanks collaborator Kostas Orginos for useful discussions on hyperon decays. Computations were performed using the Chroma software suite[28] on clusters at Jefferson Lab-

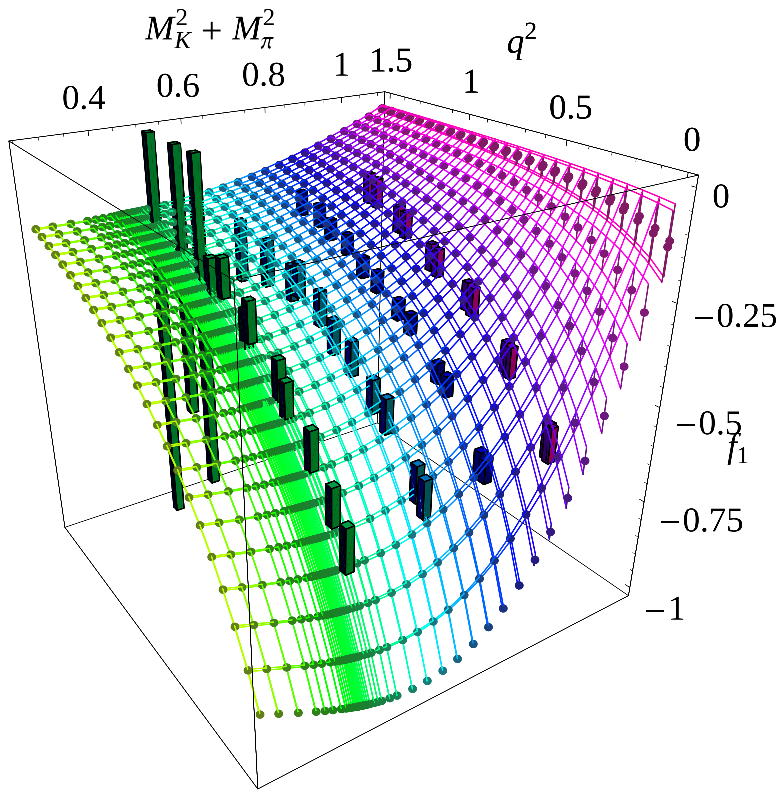

FIG. 6: Simultaneous extrapolation in q^2 and mass

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