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Mass and Density, Criticality Relationships

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Mass and Density, Criticality Relationships

Background

Here I present some well known relationships that allow the variation in critical mass versus density to be written in a simple analytical form; these relationships have appeared extensively in the open literature for over 50 years, but seem to be periodically forgotten. These relationships are exact for bare, homogeneous systems, and approximate [but reasonably accurate] for reflected systems. With these relationships anyone can quickly estimate the critical mass corresponding to any given density, using nothing more complicated than a hand calculator.

Bare Systems

For ANY spherical, homogeneous, bare system we have the EXACT relationship,

K-eff = C*rho*R

K-eff = K effective of the system; a constant in this relationship
C = a constant to be determined
rho = density of the material (grams/cc)
R = radius of the sphere (cm)

We can derive this relationship directly from the Boltzmann equation without any approximations, simply by changing to dimensionless variables. WARNING - this relationship is NOT intended for interpolation between different values of K-eff. It is only intended to be used for fixed values of K-eff to allow us to define the relationship between rho and R. What this relationship says is that all systems that have the same product, rho*R, will have the same K-eff. In other words for constant K-eff, such as K-eff = 1, the product rho*R is a constant.

The mass of a spherical system is,

Mass = $(4*pi/3)*rho*R^3$

Mass*rho^2 = $(4*pi/3)*[rho*R]^3$ Mass/R^2 = (4*pi/3)*[rho*R]

Since we know that the product rho*R is a constant, the entire right hand side of this equation is a constant, and we find the very simple relationship between density and mass,

Mass = A/rho^2 Mass = B*R^2

Therefore all we need do is define the mass at one density [to define the constant A], and we can then define the mass at any other density.

Applications to Bare PuO2 Systems

Consider two PuO2 systems with the following compositions in atom fractions,

Case	Pu239	Pu240	Pu241	Pu242	O16
#1	0.3116	0.0200	0.0017	0.0000	0.6667
#2	0.2100	0.0756	0.0420	0.0057	0.6667

By running TART calculations for a variety of densities between 2 and 12 grams/cc I was able to determine that for these systems the relationships between density (grams/cc) and mass (kilograms) is,

Case #1: Mass = $3729/\text{rho}^2$ rho = 2, Mass = 932: rho = 12, Mass = 25.9

Case #2: Mass = $4488/\text{rho}^2$ rho = 2, Mass = 1122: rho = 12, Mass = 31.2

For any given density the mass of Case#2 is roughly 120% (i.e., 4488/3729) that of Case#1.

Note, because we have this very simple relationship between density and critical mass, if you plot the data and use log scaling for both x and y axii, you will find straight lines across the entire density range; see the example plot below.

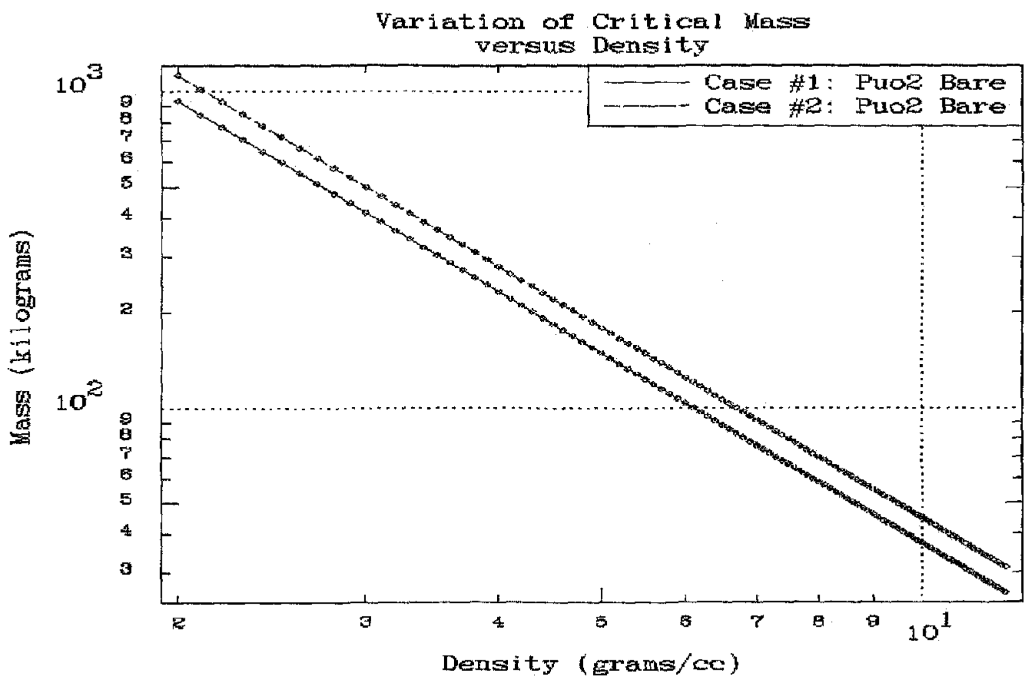

Fig. 1: Critical Mass vs. Density for Bare PuO2 Systems

Applications to Bare UO2 and U3O8 Systems

Consider two systems, UO2 and U3O8, with the following compositions in atom fractions,

Case	U234	U235	U238	O16
#3	0.0033	0.3100	0.0200	0.6667
#4	0.0027	0.2536	0.0164	0.7273

As in the above PuO2 case, by running TART calculations for a variety of densities between 2 and 12 grams/cc I was able to determine that for these systems the relationship between density (grams/cc) and mass (kilograms) is,

Case #3: Mass = $13664/\text{rho}^2$ rho = 2, Mass = 3416: rho = 12, Mass = 94.9
Case #4: Mass = $13030/\text{rho}^2$ rho = 2, Mass = 3257: rho = 12, Mass = 90.5

For any given density the mass of Case#4 (U3O8) is roughly 95% (i.e., 13030/13664) that of Case#3 (UO2).

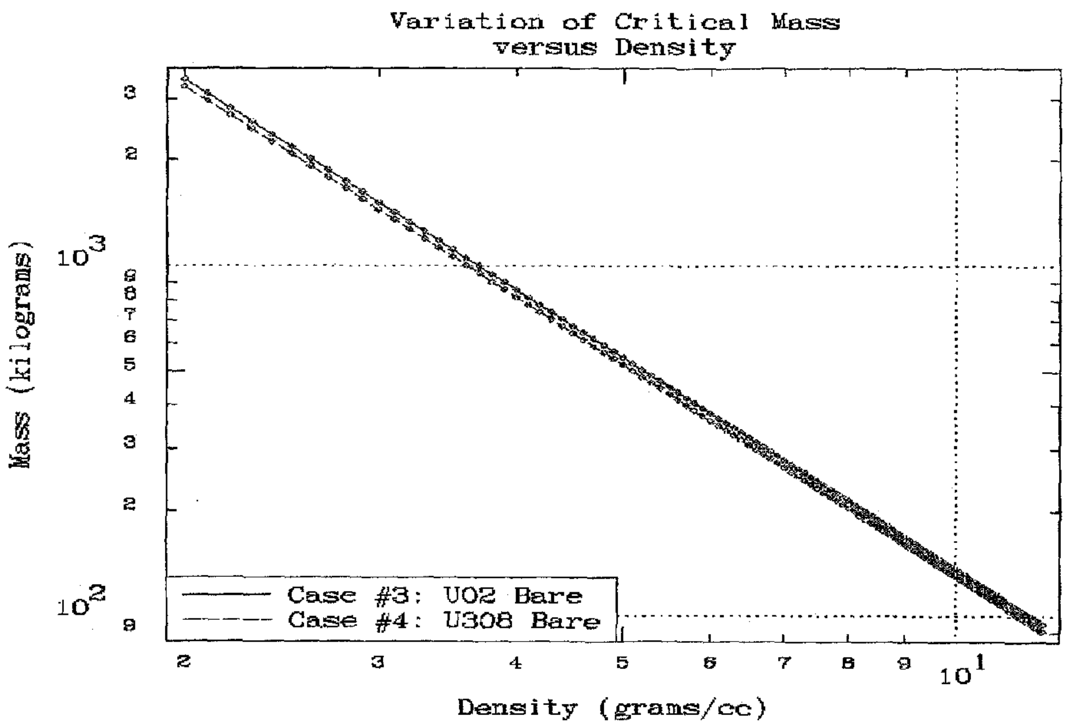

Fig. 2: Critical Mass vs. Density for Bare UO2 and U3O8 Systems

Reflected Systems

Unfortunately there is no simple EXACT extension from homogeneous, bare systems, to reflected systems. But there are some simple empirical relationships that can be traced back at least as far as a 1945 paper by Fermi, who references earlier work by Oppenheimer [1], as the source of an empirical relationship between density and critical mass for totally reflected systems, in the form,

$\text{Mass} = A/\text{rho}^{1.5}$

So there is an indication that the power of rho varies from 2 for bare system to 1.5 for totally reflected systems. For a water reflector, totally reflected mean about 30 cm thickness of water. For a smaller thickness of water, we expect significant amounts of leakage, that could change this relationship. Below I present results for the same composition as the above Cases #1 through #4, in this case they are reflected by 10 cm of water.

In each case I have found that the critical mass can be written in the form,

$\text{Mass} = A/\text{rho}^{x}$

where x is close to 1.5; x is called the "core density exponent". Here are the results,

Case #1: $\text{Mass} = 467/\text{rho}^{x}$ x=1.46, rho = 2, Mass = 170.7: rho = 12, Mass = 12.48

Case #2: $\text{Mass} = 619/\text{rho}^{x}$ x=1.50, rho = 2, Mass = 219.7: rho = 12, Mass = 14.96

Case #3: $\text{Mass} = 1661/\text{rho}^{x}$ x=1.48, rho = 2, Mass = 604.2: rho = 12, Mass = 41.11

Case #4: $\text{Mass} = 1770/\text{rho}^{x}$ x=1.51, rho = 2, Mass = 621.5: rho = 12, Mass = 41.53

rho (grams/cc)	Case#1	Case#2	Case#3	Case#4
2	170.7	219.7	604.2	621.5
4	61.62	76.6	210.5	218.2
6	34.23	41.8	117.9	118.9
8	22.65	27.66	76.75	76.83
10	16.35	19.72	54.46	54.41
12	12.48	14.96	41.11	41.53

Table 1: Critical Mass (kilograms) vs. Density for 4 cases

Note, that compared to bare results, where the critical mass of Case #4 is about 95% of the critical mass of Case #3, here the results are that the critical mass of Case #4 is slightly larger than that of Case #3.

[1] This reference was supplied by Dave Heinrichs, Criticality Safety Group. "Critical Mass Measurements for a 25 Sphere in Tu and WC Tampers", E. Fermi, et. al., LA-442, October 30, 1945.

In order to illustrate the simple relationship between critical mass and density for these water reflected systems, I used TART to define the critical mass for densities of 2, 4, 6, 8, 10, and 12 grams/cc, and below I show a comparison for the four cases between the TART calculated values and the simple relationships. Note, that in all cases the simple relationship [continuous line] passes within 1% of the TART calculated values [discrete points].

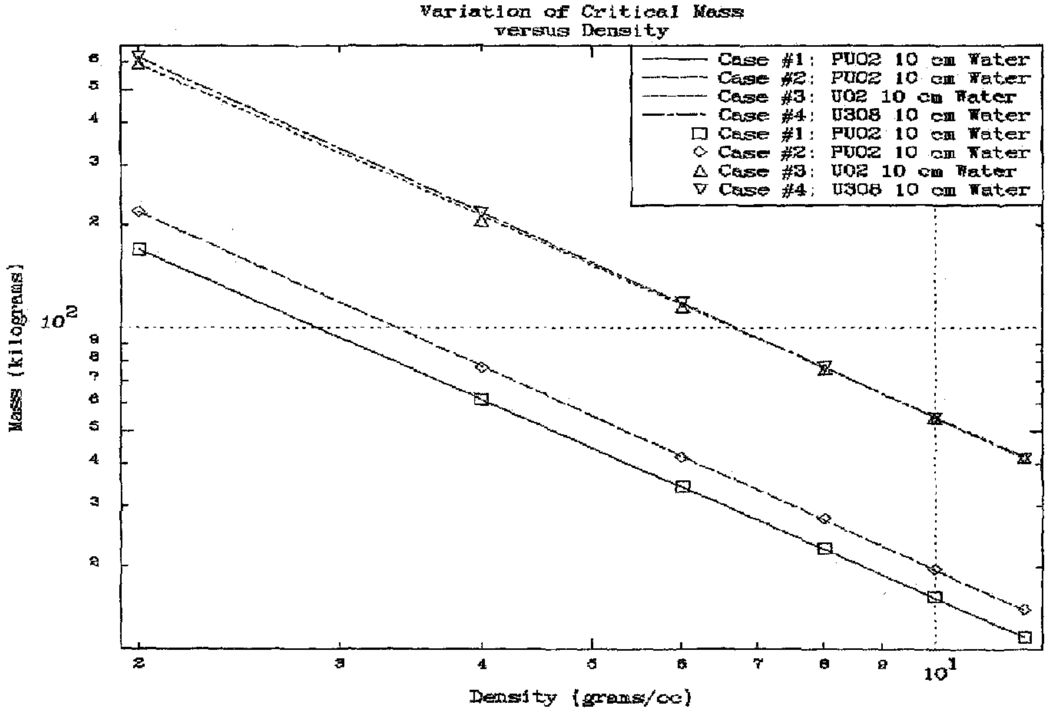

Fig. 2#: Critical Mass vs. Density for Water Reflected Systems

Summary

In interpreting the above results it is important to consider their accuracy; this can be summarized as follows,

1) The TART calculations were run to a high degree of accuracy, and should not introduce any additional uncertainty.
2) I estimate that the nuclear data introduces an uncertainty of about 3 to 5 %.
3) The results are a function of temperature. Here I've based all calculations on room temperature. Anyone who has ever held a ball of plutonium in their hand knows that due to alpha decay, it isn't at room temperature. However, reasonably small variations from room temperature (as with plutonium), will have a very small effect on the results.
4) Here I modeled each system as completely isolated, surrounded only by vacuum, which means no reflection. In any real situation the surroundings can lead to significant reflection. I estimate that "room return" introduces an uncertainty of about 10 %.
5) Here I modeled the reflectors as pure water. The critical mass will be very sensitive to any impurities in the reflector, that can absorb neutrons. The uncertainty introduced by this assumption is hard to quantify, because of the wide variety of available reflector materials and their impurities.
6) Isotopics is a major source of uncertainty. If you are interested ONLY in EXACTLY the compositions considered here, there is no source of uncertainty. However, if you are really interested in the critical masses of real systems that you may encounter, you should be aware that the critical mass is very strongly dependent on additional scatterers, as can be seen above for UO2 versus U3O8, as well as how much neutron poison is included in the composition: for Pu: Pu240, Pu242, and for U: U234, U236, nd U238. Increasing the amount of neutron poison will increase the critical mass, and deceasing it will decrease the critical mass. For the limit of no neutron poisons, below I include a comparison of the above results for the bare Case#1 through Case#4 to PuO2 containing only Pu239 and O16, and UO2 containing only U235 and O16,

Only Pu239:	Mass = $3569/\text{rho}^2$	rho = 2, Mass = 892: rho = 12, Mass = 24.8
Case #1:	Mass = $3729/\text{rho}^2$	rho = 2, Mass = 932: rho = 12, Mass = 25.9
Case #2:	Mass = $4488/\text{rho}^2$	rho = 2, Mass = 1122: rho = 12, Mass = 31.2
Only U235:	Mass = $12295/\text{rho}^2$	rho = 2, Mass = 3017: rho = 12, Mass = 85.4
Case #3:	Mass = $13664/\text{rho}^2$	rho = 2, Mass = 3416: rho = 12, Mass = 94.9
Case #4:	Mass = $13030/\text{rho}^2$	rho = 2, Mass = 3257: rho = 12, Mass = 90.5