LBNL-56404
UCB-PTH-04/25
hep-ph/0410001
September 2004

# Quadratic Divergences in Effective Supergravity from the Heterotic Superstring[1 2]

Mary K. Gaillard

*Department of Physics, University of California*
*and*
*Theoretical Physics Group, 50A-5101, Lawrence Berkeley National Laboratory*
*Berkeley, CA 94720, USA*

**Abstract**

Results from studies of effective Lagrangians for gaugino condensation are summarized and re-examined with an eye to previously neglected one-loop quadratically divergent corrections.

[1]Talk presented at the Nathfest, Northeastern University, Boston, MA, August 14–15, 2004, to be published in the proceedings of PASCOS04.

[2]This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-0098840.

## Disclaimer

# 1 Introduction

The subject of my talk is closely related to the pioneering work [1] of Arnowitt, Chamseddine and Nath on locally supersymmetric Grand Unified Theories, with the difference that the theory above the scale of unification is a string theory rather than a field theory. Specifically, I will consider effective supergravity theories obtained from compactification of the weakly coupled heterotic string. In the next section I summarize results [2, 3] from the study of modular (T-duality) invariant effective Lagrangians for gaugino condensation. These are characterized in particular by

- Dilaton dominated supersymmetry breaking. The auxiliary fields of the T-moduli (or Kähler moduli) have vanishing vacuum values (*vev*'s): $\left\langle F^T \right\rangle = 0$, thus avoiding a potentially dangerous source of flavor changing neutral currents (FCNC).

- The constraint of vanishing (or nearly so) vacuum energy leads to a variety of mass hierarchies that involve the $\beta$-function coefficient of the condensing gauge group.

These results were obtained at tree level in the effective supergravity theory for gaugino condensation, which includes the quantum corrections in the strongly coupled gauge sector whose elementary degrees of freedom have been integrated out, as well as the four dimensional Green-Schwarz (GS) terms needed at the quantum level to cancel field theory anomalies. In addition, the logarithmically divergent and finite ("anomaly mediated" [4]) one-loop corrections to soft supersymmetry-breaking parameters have been extensively studied [5, 6, 7]. These analyses did not include quadratically divergent loop corrections that are for the most part corrections to terms in the tree Lagrangian, and are suppressed by the loop expansion parameter

$$\epsilon = 1/16\pi^2. \tag{1.1}$$

However, since some of these terms have coefficients proportional to the number of fields in the effective supergravity theory, it has been argued that they may not be negligible. In particular, their contributions to the cosmological constant [8] and to flavor changing neutral currents [9] have been emphasized. Both are important for the phenomenology of the above condensation models; thus we need to revisit [10] their effects.

# 2 Modular invariant gaugino condensation

One starts above the (reduced) Planck scale $m_P$ with the heterotic string theory in 10 dimensions. Just below the string scale $\mu_s = g_s m_P$, where $g_s$ is the gauge coupling at the string scale, physics is described by $N = 1$ modular invariant supergravity in four dimensions, where here modular invariance refers to T-duality under which the Kähler moduli $T$ transform as

$$T \to \frac{aT - ibT}{icT + d}, \qquad ad - bc = 1, \qquad a, b, d, c \in \mathcal{Z}. \tag{2.2}$$

Modular invariance – and in many compactifications [11] a $U(1)$ gauge group factor called $U(1)_X$ – is broken by anomalies at the quantum level of the effective field theory, and the symmetry is restored by an appropriate combination of threshold effects [12] and four dimensional GS term(s) [13, 14]. The precise form of these loop effects in the Yang-Mills sector of the effective supergravity theory have been determined by matching the string and field theory amplitudes at the quantum level [15].

If an anomalous $U(1)$ is present, the corresponding GS term leads to a Fayet-Illiopoulos (FI) D-term in the effective Lagrangian [14] and some $U(1)$-charged scalars $\phi^A$ acquire *vev*'s at a scale $\mu_D$ one or two orders of magnitude below the Planck scale such that the overall D-terms vanish:

$$\left\langle \frac{1}{\ell(s,\bar{s})} \sum_A q_A^a (t+\bar{t})^{n_A} |\phi^A|^2 \right\rangle = \frac{1}{2}\delta_X \delta_{Xa}, \tag{2.3}$$

where $\delta_X \ell$ is the coefficient of the FI term, $n^A$ is the modular weight of $\phi^A$, $q_A^a$ is its charge under the gauge group factor $U(1)_a$, and $t, s$ are the scalar components of the Kähler moduli and dilaton chiral superfields $T, S$. The function $\ell(s+\bar{s})$ is the dilaton field in the dual, linear supermultiplet formulation; in the classical limit $\ell = (s+\bar{s})^{-1}$. The combination of fields that gets a vacuum value is modular invariant. Thus modular invariance, as well as local supersymmetry, is unbroken at this scale, and the moduli fields $s, t$ remain undetermined [16]. The $\phi^A$ vacuum is generically characterized by a high degree of further degeneracy [17, 18] that may lead to problems for cosmology.

At a lower scale $\mu_c$, a gauge group $\mathcal{G}_c$ in the hidden sector becomes strongly coupled, and gauginos as well $\mathcal{G}_c$-charged matter condense. The potential generated for the moduli is T-duality invariant and the Kähler moduli $T$ are stabilized at self-dual points with $\left\langle F^T \right\rangle = 0$, while $\left\langle F^S \right\rangle \neq 0$, so that, in the absence of an anomalous $U(1)$, supersymmetry breaking is dilaton mediated [2]. In the presence of an anomalous $U(1)$, *vev*'s of D-terms are generically generated as well and tend to dominate supersymmetry breaking; these may be problematic for phenomenology. On the plus side, at least some of the degeneracy of the $\phi^A$ vacuum is lifted by $\phi^A$ couplings to the condensates [3].

To briefly summarize the phenomenology of these models, the condition of vanishing vacuum energy introduces the $\beta$-function coefficient of the condensing gauge group $\mathcal{G}_c$ into the supersymmetry breaking parameters in such a way as to generate a variety of mass hierarchies. Defining

$$b_c = \frac{1}{16\pi^2}\left(3C^c - C_M^c\right), \tag{2.4}$$

where $C^c(C_M^c)$ is the adjoint (matter) quadratic Casimir for $\mathcal{G}_c$, in the absence of an anomalous $U(1)$ one has at the condensation scale [2] (one can also have $m_0 \sim m_T \gg m_{\frac{3}{2}}$ if gauge-charged matter couples to the GS term)

$$\begin{aligned} m_0 &= m_{\frac{3}{2}}, \qquad m_{\frac{1}{2}}^a = \frac{4b_c^2}{9} g_a(\mu_c) m_{\frac{3}{2}}, \\ m_T &\approx \frac{b}{b_c} m_{\frac{3}{2}}, \qquad m_S \sim b_c^{-2} m_{\frac{3}{2}}, \qquad m_a = 0. \end{aligned} \tag{2.5}$$

where $m_{0,\frac{1}{2},\frac{3}{2}}$ refer to observable sector scalars and gauginos, and the gravitino, respectively; $m_{T,S,a}$ are the Kähler moduli, dilaton and universal axion masses. The expression for $m_T$ assumes $b \gg b_c$, where $b$ is the $\beta$-function coefficient appearing in the modular invariance restoring GS term [13]. For example in the absence of Wilson lines, $b = b_{E8} \approx .57$, and viable scenarios for electroweak symmetry breaking [19] and for neutralinos as dark matter [20] require $b_c \approx .05 - .06$. These numbers give desirably large moduli and dilaton masses, while the scalar/gaugino mass ratio is perhaps uncomfortably large, but no worse than in many other models.

When Wilson lines are present the condition $b \gg b_c$ may not hold; for example $b_c = b$ in a $Z_3$ compactification [21] with an $SO(10)$ hidden sector gauge group; this would give vanishing T-moduli masses in the above class of models. However when an anomalous $U(1)$ is present, the T-moduli couplings to the condensates are modified, giving additional contributions to their masses, and a hierarchy with respect to the gravitino mass can still be maintained [3]. In this scenario the gaugino, dilaton and axion masses are determined only by the dilaton potential, as before. A stable vacuum with a positive metric for the dilaton is most easily achieved in a "minimal" class of models in which the number of Standard Model (SM) gauge singlets that get *vev*'s at the scale $\mu_D$ is equal to the number $m$ of broken $U(1)$'s (in which case there are no massless "D-moduli" [18] associated with the degeneracy of the $U(1)$-charged $\phi^A$ vacuum), or $N$ replicas of these with identical $U(1)$ charges [yielding $(N-1)m$ D-moduli]. In this case the gaugino and dilaton masses are unchanged from (2.5) (the axion mass always vanishes). The most significant change from the above scenario is a D-term contribution to scalar squared masses $m_0^2$ that is proportional to their $U(1)$ charges. At weak coupling, and neglecting nonperturbative effects, this term dominates the one in (2.5) by a factor $b_c^{-2} \gg 1$, and is not positive semi-definite. Thus unless SM particles are uncharged under the broken $U(1)$'s (or have charges that, in a well-defined sense [3], are orthogonal to those of the $\phi^A$ with large *vev*'s), these models are seriously challenged by the SM data: a very high scalar/gaugino mass ratio for positive $m_0^2$, and the danger of color and electromagnetic charge breaking if $m_0^2 < 0$.

# 3 Quadratically divergent corrections

When local supersymmetry is broken, there is a quadratically divergent one-loop contribution to the vacuum energy [22]

$$\langle V_{1-\text{loop}} \rangle \ni \frac{\Lambda^2}{32\pi^2} \left\langle \mathrm{STr}\mathcal{M}^2 \right\rangle , \tag{3.6}$$

where $\mathcal{M}$ is the field-dependent mass matrix, and the gravitino contribution is gauge dependent. For example in minimal supergravity [1] with $N_\chi$ chiral and $N_G$ Yang-Mills superfields, one obtains, using the gravitino gauge fixing procedure of Ref [23].

$$\langle \delta V_{1-\text{loop}} \rangle \ni \frac{\Lambda^2}{16\pi^2} \left( N_\chi m_0^2 - N_G m_{\frac{1}{2}}^2 + 2m_{\frac{3}{2}}^2 \right) . \tag{3.7}$$

In the MSSM we have $N_\chi = 49$ and $N_G = 12$. The much larger field content of a typical $Z_3$ orbifold compactification [24, 11] of the $E_8 \otimes E_8$ heterotic string has $N_\chi \gtrsim 300$ and $N_G \lesssim 65$, suggesting [8] that this contribution to the vacuum energy is always positive.

However, in order to maintain manifest supersymmetry, a supersymmetric regularization of ultraviolet divergences must be used. Pauli-Villars (PV) regularization [25] meets this criterion. The regulation of quadratic divergences requires *a priori* two subtractions; in the context of PV regularization, the number $S$ of subtractions is the number of PV fields for each light field. Once the divergences are regulated (*i.e.* eliminated), we are left with the replacement

$$\Lambda^2 \mathrm{STr}\mathcal{M}^2 \to \mathrm{STr}\mu^2\mathcal{M}^2\ln(\mu^2)\eta_S, \qquad \eta_S = \sum_{q=1}^{S}\eta_q\lambda_q\ln\lambda_q, \tag{3.8}$$

where $\mu$ represents the scale of new physics, and the parameter $\eta_S$ reflects the uncertainty in the threshold for the onset of this new physics. The squared PV mass of the chiral supermultiplet $\Phi^q$ is $\lambda_q\mu^2$ (so $\lambda_q > 0$), and $\eta_q = \pm 1$ is the corresponding PV signature. The sign of the effective cut-off is determined by the sign of $\eta_S$, which is positive definite only[1] if $S \leq 3$. Cancellation of all the ultraviolet divergences of a general supergravity theory requires [27] at least 5 PV chiral multiplets for every light chiral multiplet and even more PV supermultiplets to regulate gauge loops. Therefore one cannot assume that the effective cut-offs are all positive.

Including the full (cut-off) one-loop quadratically divergent contribution gives the effective bosonic Lagrangian

$$\begin{aligned} e^{-1}\mathcal{L}(\Lambda) &= e^{-1}\mathcal{L}_{\rm tree} + \epsilon\Lambda^2\frac{r}{4}\left(N_\chi - 7 - N_G\right) \\ &\quad + \epsilon\Lambda^2\left(5V + m_{\frac{3}{2}}^2 - 2K_{i\bar m}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar z^{\bar m} - \mathcal{D}\right) \\ &\quad - \frac{\epsilon\Lambda^2}{\mathrm{Re}s}\mathcal{D}_a D_i(T^a z)^i + \epsilon\Lambda^2 R_{i\bar m}\left(F^i\bar F^{\bar m} + \mathcal{D}_\nu z^i\mathcal{D}^\mu\bar z^{\bar m}\right) \\ &\quad - \epsilon\Lambda^2 N_\chi\left(V + m_{\frac{3}{2}}^2 - \mathcal{D}\right) + \epsilon\Lambda^2 N_G\left(\frac{\partial_\mu s\partial^\mu\bar s}{(s+\bar s)^2} + m_{\frac{1}{2}}^2\right), \end{aligned} \tag{3.9}$$

where $m_{\frac{3}{2}}, m_{\frac{1}{2}}$ are now understood to be field-dependent,

$$\mathcal{D}_a = K_i(T^a z)^i, \qquad F^i = -e^{K/2}K^{i\bar m}\bar W_{\bar m}, \tag{3.10}$$

are the usual auxiliary fields, the tree level potential is

$$V_{\rm tree} = V = D + K_{i\bar m}F^i\bar F^{\bar m} - 3m_{\frac{3}{2}}^2, \qquad D = \frac{D^a\mathcal{D}_a}{s+\bar s}, \tag{3.11}$$

and $R_{i\bar m}$ is the Kähler Ricci tensor. After a Weyl transformation to restore the Einstein term to canonical form, we obtain

$$e^{-1}\mathcal{L}(\Lambda) \;=\; e^{-1}\mathcal{L}_{\rm tree}(g_R) + \epsilon\Lambda^2\left(m_{\frac{3}{2}}^2 - 2V + \frac{3}{2}K_{i\bar m}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar z^{\bar m} - \mathcal{D}\right)$$

[1] See appendix C of [26]. and the discussion in [10].

$$
\begin{aligned}
&-\frac{\epsilon\Lambda^2}{\mathrm{Re}s}\mathcal{D}_a D_i(T^a z)^i + \epsilon\Lambda^2 R_{i\bar{m}}\left(F^i\bar{F}^{\bar{m}} + \mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\right)\\
&-\epsilon\Lambda^2 N_\chi\left(m^2_{\frac{3}{2}} - \mathcal{D} + \frac{1}{2}K_{i\bar{m}}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\right)\\
&+\epsilon\Lambda^2 N_G\left(\frac{\partial_\mu s\partial^\mu\bar{s}}{(s+\bar{s})^2} + m^2_{\frac{1}{2}} - V + \frac{1}{2}K_{i\bar{m}}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\right),
\end{aligned}
\tag{3.12}
$$

where $g_R$ is the one-loop renormalized metric. The Lagrangian (3.12) does not respect supersymmetry. With a supersymmetric PV regularization, PV masses arise from quadratic couplings in the superpotential

$$
W_{PV} \ni \mu_{IJ}(Z^k)Z^I_{PV}Z^J_{PV}, \qquad Z^k\Big| = z^k. \tag{3.13}
$$

Then the squared cut-off in (3.12) is replaced by suitably weighted linear combinations of PV squared masses

$$
\Lambda^2 \to (M^2)^I_J = e^K K^{I\bar{K}}(z)K^{\bar{L}M}(z)\bar{\mu}_{\bar{K}\bar{L}}(\bar{z})\mu_{MJ}(z) \tag{3.14}
$$

that are generally field-dependent. Moreover, the couplings (3.13) induce additional terms proportional to $M^2$ that cannot be obtained by a straight cut-off procedure. The resulting effective Lagrangian takes the form [25]

$$
\mathcal{L}^1_{eff} = \mathcal{L}_{\mathrm{tree}}(g,K) + \mathcal{L}_{1-\mathrm{loop}} = \mathcal{L}_{\mathrm{tree}}(g_R,K_R) + O(\epsilon\ln\Lambda^2_{eff}) + O(\epsilon^2), \tag{3.15}
$$

where

$$
K_R = K + \Delta K \tag{3.16}
$$

is the renormalized superpotential. The action obtained in this way is only perturbatively supersymmetric:

$$
\delta S^1_{eff} = \int d^4x\delta\mathcal{L}^1_{eff} = O(\epsilon^2). \tag{3.17}
$$

Writing

$$
\Delta K = \frac{\epsilon}{2}\left[N\Lambda^2_\chi - 4N_G\Lambda^2_G + O(1)\Lambda^2_{\mathrm{grav}}\right] + O(\epsilon\ln\Lambda^2_{eff}) + O(\epsilon^2), \tag{3.18}
$$

where $\Lambda_{\chi,G,\mathrm{grav}}$ are the effective cut-offs for chiral, gauge and gravity loops, and $\Lambda_{eff}$ is a generic effective cutoff, if $N_\chi, N_G \sim \epsilon^{-1}$, we must retain the full effective Lagrangian as derived from $K_R$. This amounts to resuming the leading terms in $\epsilon N\Lambda^2_{eff}$, with the result, as dictated by supersymmetry, just a correction to the Kähler potential. I will discuss the consequences of this correction in the following sections.

# 4 The vacuum energy

Consider first the possibility that we can choose the $Z^k$-dependence of the PV Kähler potential and superpotential such that the effective cutoffs are constant. For example, one needs

PV superfields $Z^I_{PV}$ with the same Kähler metric as the light superfields $Z^i$: $K^Z_{I\bar{M}} = K_{i\bar{m}}$. If we introduce superfields $Y_I$ with Kähler metric: $K_Y^{I\bar{M}} = e^{-K}K^{-1}_{i\bar{m}} = e^{-K}K^{i\bar{m}}$, the superpotential coupling

$$W_{PV} = \mu Z^I Y_I \tag{4.19}$$

yields a constant squared mass $M^2 = \mu^2$ if $\mu$ is constant, and the quantum corrected potential just reads

$$V_{eff} = \mathcal{D} + e^{\Delta K}\left(F^i K_{i\bar{m}} F^{\bar{m}} - 3m^2_{\frac{3}{2}}\right)_{\rm tree} + O(\epsilon \ln \Lambda^2_{eff}). \tag{4.20}$$

If supersymmetry breaking is F-term induced: $\langle \mathcal{D}\rangle = 0$, the tree level condition $\left\langle F^i K_{i\bar{m}} F^{\bar{m}} = 3m^2_{\frac{3}{2}}\right\rangle$ for vanishing vacuum energy is unmodified by these quantum corrections.

However not all PV masses can be chosen to be constant because of the anomaly associated with Kähler transformations $K(Z,\bar{Z}) \to K(Z,\bar{Z}) + F(Z) + \bar{F}(\bar{Z})$ that leave the classical Lagrangian invariant. In the presence, for example, of an anomalous $U(1)_X$, with generator $T_X$, there is a quadratically divergent term proportional to ${\rm Tr}T_X\Lambda^2$ that cannot be canceled by $U(1)_X$-invariant PV mass terms, since the contribution to ${\rm Tr}T_X$ from each pair in the invariant superpotential cancels. As a consequence, there must be some PV masses $\propto e^{aV_X}$, where $V_X$ is the $U(1)_X$ vector superfield. Similarly, in the presence of a Kähler anomaly there is a term

$$\mathcal{L}_{1-\rm loop} \ni c\epsilon K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \Lambda^2, \tag{4.21}$$

that cannot be canceled unless some PV superfields have masses $M^2_{PV} \propto e^{\alpha K}$. In addition, PV regulation of the gauge + dilaton sector requires some PV masses proportional to the field-dependent string-scale gauge coupling constant: $M^2_{PV} \propto g^s(s,\bar{s}) = 2(s+\bar{s})^{-1}$.

What are the effects of this field-dependence on the condensation models described above? In order to implement the correct Bianchi identity for the gaugino condensate composite superfield – as well as the GS anomaly cancellation mechanism – the linear multiplet formulation for the dilaton was used in the construction of the effective Lagrangians for these models. The results have been recast [7] in the more familiar language of the chiral multiplet formalism, so that the effective tree-level potential below the scale of condensation takes the standard form (3.11) with

$$m_{\frac{3}{2}} = \frac{3}{2} b_c u, \qquad F^S = -\frac{1}{4} K^{-1}_{S\bar{S}} \left(1 - \frac{2}{3} b_c K_S\right) \bar{u}, \tag{4.22}$$

where $u$ is the vacuum value of the condensate. The modular invariance of these models assures that the moduli $T$ are stabilized at self-dual points with vanishing auxiliary fields: $\left\langle F^T \right\rangle = 0$. Supersymmetry breaking is dilaton-dominated and the condition for vanishing vacuum energy at tree level in the effective theory is

$$\langle V_{eff}\rangle = \left\langle K_{S\bar{S}} |F^S|^2 - 3m^2_{\frac{3}{2}} \right\rangle = 0, \qquad \left\langle K^{-1}_{S\bar{S}} \right\rangle = \frac{4b_c^2}{3(1 - \frac{2}{3}\langle K_S\rangle b_c)^2}, \tag{4.23}$$

Classically,

$$-2\langle K_S\rangle = 2\left\langle K^{\frac{1}{2}}_{S\bar{S}} \right\rangle = 2\left\langle (s+\bar{s})^{-1}\right\rangle = g_s^2 \approx \frac{1}{2}. \tag{4.24}$$

with the approximate value of $g_s$ inferred from low energy data. The model is phenomenologically [19] and cosmologically [20] viable if $b_c \approx .05$–$.06$, so it is clear that (4.23) cannot be satisfied without a modification of the Kähler potential for the dilaton; one approach [28] has been to invoke nonperturbative string [29] and/or QFT [30] corrections to the dilaton Kähler potential. Specifically we require

$$K_{s\bar{s}}^{-1} \ll K_{s\bar{s}}^{-1}\Big|_{\text{classical}} . \tag{4.25}$$

Avoiding dangerously large D-term contributions to scalar masses in the presence of an anomalous $U(1)$ may further require [3]

$$-K_s \gg -K_s|_{\text{classical}} , \tag{4.26}$$

suggesting that weak coupling may not be viable [31].

To examine the effects of quadratically divergent perturbative corrections, I assume a form of the superpotential suggested by the discussion following (4.21):

$$\begin{aligned} K^R &= K + \frac{1}{2}\epsilon c_\chi f(T,\bar{T})e^K - \frac{4\epsilon c_G}{S + \bar{S} - V_{GS}} \\ &= K + \frac{1}{2}\epsilon c_\chi f(T,\bar{T})e^K - 2\epsilon c_G g_s^2(Z,\bar{Z}), \end{aligned} \tag{4.27}$$

where $V_{GS}(T+\bar{T})$ is the Green-Schwarz term (which slightly redefines the string-scale coupling $g_s$ at the quantum level), and $f(T,\bar{T})$ assures modular invariance of the second term. The loop-induced terms may not be negligible if $c_\chi, c_G \sim N_\chi, N_G$, respectively. Since the theory is still modular invariant we expect that the moduli are still stabilized at self-dual points, where the additional contributions to $F^T$ induced by these quantum corrections vanish. Setting the T-moduli at their *vev*'s and the matter fields to zero, and defining

$$K = k(2g_s^{-2}) + G(T+\bar{T}), \qquad \tilde{c}_\chi = \left\langle f(t,\bar{t})e^G \right\rangle c_\chi, \tag{4.28}$$

the renormalized Kähler potential and its $S$-derivatives read

$$\begin{aligned} K^R &= k + \frac{\epsilon}{2}\left(\tilde{c}_\chi e^k - 4c_G g_s^2\right), \qquad K_S^R = K_S\left(1 + \frac{\epsilon}{2}\tilde{c}_\chi e^k\right) + \epsilon c_G g_s^4, \\ K_{S\bar{S}}^R &= K_{S\bar{S}}\left(1 + \frac{\epsilon}{2}\tilde{c}_\chi e^k\right) + \frac{\epsilon}{2}\left(K_S^2\tilde{c}_\chi e^k - c_G g_s^8\right). \end{aligned} \tag{4.29}$$

The condition for vanishing vacuum energy is now given by (4.23) with $K \to K^R$, and the relevant parameter for phenomenology is now the *vev* of $1/K_{S\bar{S}}^R$, which remains strongly suppressed with respect to its classical value since $K_S^R$ is negative semi-definite.[2] Therefore the salient phenomenological features of the condensation models are essentially unaffected by these quantum corrections.

[2]The relation $\ell = -K_S^R$ holds at any given order in perturbation theory, where $\ell$ is the dilaton of the dual linear multiplet formalism.

However these corrections could lessen the need to invoke large nonperturbative effects. A large *negative* value of $c_G$ or a large *positive* value of $\tilde{c}_\chi$ would increase $-K_S^R$ and decrease $1/K_{S\bar{S}}^R$ for fixed $g_s^2 \approx 1/2$, which is the desired effect. One can reasonably assume that $|c_G| \leq N_G \leq 65 \sim .4\epsilon^{-1}$ in typical orbifold compactifications [11], so a significant effect cannot be obtained from the second term in (4.27). On the other hand $N\epsilon \sim 2$ for typical orbifolds. Quite generally the function $f(t,\bar{t})$ is of order one at a self-dual point, so if $\tilde{c}_\chi \sim N$, $e^k \sim 1$, it might be possible to reinterpret part of the needed modification of the dilaton Kähler potential in terms of perturbative quantum corrections.

# 5 Flavor Changing Neutral Currents

To address the question of what constraints are needed to avoid experimentally excluded FCNC effects, we first note that the tree potential of an effective supergravity theory includes a term

$$V_{\rm tree} \ni e^K K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} |W|^2. \tag{5.30}$$

The observed suppression of FCNC effects constrains the Kähler potential; to a high degree of accuracy we require that

$$K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} \not\ni \left\langle f(X,\bar{X}) \right\rangle \phi_f^A \bar{\phi}_{f'\neq f}^{\bar{A}}, \tag{5.31}$$

where $f, f'$ are flavor indices, $A$ is a gauge index, $\phi_f^A$ is any standard model squark or slepton, and $X$ is a singlet of the Standard Model gauge group. For example, in the no-scale models that characterize the untwisted sector of orbifold compactifications, we have

$$K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} = 3 + K_S K_{\bar{S}} K^{S\bar{S}}, \tag{5.32}$$

which is safe, since $K_S$ is a function only of the dilaton. The twisted sector Kähler potential is known only to quadratic order:

$$K_T = \sum_A (T+\bar{T})^{n_A} |\Phi_T^A|^2 + O(\Phi^3), \tag{5.33}$$

which is flavor diagonal and also safe. The higher order terms in (5.33) could be problematic if some $\phi^A = X^A$ have large *vev*'s (*i.e.* within a few orders of magnitude of the Planck scale). Thus phenomenology requires that we forbid couplings of the form $\phi_f^A \phi_{f'\neq f}^{\bar{A}} |\phi_{f''}^{A'}|^2 X^{B_1} \cdots X^{B_n}$, $n \leq N$, where $N$ is chosen sufficiently large to make the contribution $\left\langle X^{B_1} \cdots X^{B_n} \right\rangle$ to the scalar mass matrix negligible. The quadratically divergent one-loop corrections generate a term

$$V_{1-{\rm loop}} \ni e^K K_i K_{\bar{\jmath}} R^{i\bar{\jmath}} |W|^2, \qquad R^{i\bar{\jmath}} = K^{i\bar{k}} R_{\bar{k}l} K^{k\bar{\jmath}}. \tag{5.34}$$

where $R_{i\bar{\jmath}}$ is the Kähler Ricci tensor. The contribution (5.34) simply reflects the fact that the leading divergent contribution in a nonlinear sigma model is a correction to the Kähler metric proportional to the Ricci tensor (whence, *e.g.*, the requisite Ricci flatness of two dimensional conformal field theories). Since the Ricci tensor involves a sum of Kähler Riemann tensor elements over all chiral degrees of freedom, a large, order $N_\chi$, coefficient may be generated [9]. For example, for an untwisted sector $U$ with three untwisted moduli $T^n$ and Kähler potential

$$K^U = \sum_{n=1}^{3} K^n = -\sum_{n=1}^{3} \ln(T^n + \bar{T}^{\bar{n}} - \sum_{A=1}^{N_n} |\Phi_n^A|^2), \tag{5.35}$$

we get

$$R^n_{i\bar{\jmath}} = (N_n + 2) K^n_{i\bar{\jmath}}. \tag{5.36}$$

While this contribution is clearly safe, since the Ricci tensor is proportional to the Kähler potential, the condition that the tree potential be FCNC safe does not by itself ensure that (5.34) is safe in general. For this we require in addition the absence of Kähler potential terms of the form $\phi^A_f \bar{\phi}^{\bar{A}}_{f' \neq f} |\phi^{A'}_{f''}|^4 (X^B)^{n \leq N}$. On the other hand, if the Kähler metric is FCNC safe due to a *symmetry*, the same symmetry will protect the Ricci tensor from generating FCNC.

For example, the scalar metric $g_{ij}$ for the effective pion Lagrangian is dictated by chiral $SU(2)_L \otimes SU(2)_R$; there is a unique form of the two-derivative coupling:

$$g_{ij} = \delta_{ij} + \frac{\pi_i \pi_j}{v^2 - \pi^2}, \tag{5.37}$$

for a particular choice of field variables. Preservation of this symmetry at the one-loop level assures that $R_{ij} \propto g_{ij}$. Similarly, the kinetic term derived from the Kähler potential (5.35) possesses an $\prod_{n=1}^{3} SU(N_n + 1, 1)$ symmetry that is much larger than the $SL(2, R)$ (or possibly$[SL(2, R)]^3$) T-duality symmetry of the full Lagrangian, and we obtain the result (5.36). More generally, in effective supergravity from string compactifications there are a number of selection rules and/or symmetries that forbid superpotential couplings that are allowed by gauge invariance and the T-duality invariance group (see for example [21]). The Kähler potential has not been investigated in similar detail, but *a priori* one would expect an analogous pattern. In the absence of input from string theory one can work backwards and study [32] the constraints imposed by phenomenology.

Another handle on this issue is the requirement of the full cancellation of $U(1)_X$ and modular anomalies in the fully regulated effective supergravity theory. This requirement may restrict [27] the Kähler potential couplings of both the twisted sector and of the regulator PV fields that parameterize Planck scale physics.

# 6 Conclusions

I have outlined some of the promises as well as the problems of string phenomenology in models with supersymmetry broken by gaugino condensation. The issue of quadratic divergences in the effective supergravity theory was rephrased as a renormalization of the Kähler potential, with little impact on the phenomenology of these models, except for a possible reinterpretation of "string nonperturbative effects" in terms of perturbative contributions to the renormalized Kähler potential. I argued that FCNC bounds may constrain the twisted sector Kähler potential as well as that of PV fields. Such constraints would provide a "bottom up" probe of Planck scale physics. Conversely, the PV masses, that are determined in large part by the PV Kähler potential, play an important role in determining scalar masses if they are dominated by one-loop corrections [6]. Therefore any constraints on the Kähler potential from string theory calculations and/or the requirement of anomaly cancellation could provide a "top down" contribution to collider physics.

# Acknowledgments

I wish to thank my collaborators on the work described here. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grants PHY–0098840.

# References

[1] A. H. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** 970 (1982).

[2] P. Binétruy, M. K. Gaillard and Y.-Y. Wu, *Nucl. Phys.* **B493** 27 (1997) and *Phys. Lett.* **B412** 288 (1997).

[3] M. K. Gaillard, J. Giedt and A. L. Mints, hep-th/0312125 to be published in *Nucl. Phys.* **B**.

[4] L. Randall and R. Sundrum, *Nucl. Phys.* **B557** 79 (1999); G. Giudice, M. Luty, H. Murayama and R. Rattazzi, *JHEP* **9812** 027 (1998); J. Bagger, T. Moroi and E. Poppitz, *JHEP* **0004** 009 (2000).

[5] M. K. Gaillard, B. Nelson and Y.-Y. Wu, *Phys. Lett.* **B459** 549 (1999).

[6] M. K. Gaillard and B. Nelson, *Nucl. Phys.* **B588** 197 (2000).

[7] P. Binétruy, M. K. Gaillard and B. Nelson, *Nucl. Phys.* **B604** 32 (2001).

[8] K. Choi, J. E. Kim and H.-P. Nilles, *Phys. Rev. Lett.* **73** 1758 (1994).

[9] K. Choi, J. S. Lee and C. Muñoz, *Phys. Rev. Lett.* **80** 3686 (1998).

[10] M. K. Gaillard and B. D. Nelson, *Quadratic Divergences in Supergravity, Vacuum Energy and the Supersymmetric Flavor Problem*, paper in preparation.

[11] J. Giedt, *Ann. of Phys.* **289** 251 (2001) and **297** 67 (2002).

[12] L. J. Dixon, V. S. Kaplunovsky and J. Louis, *Nucl. Phys.* **B355** 649 (1991); I. Antoniadis, K. S. Narain and T. R. Taylor, *Phys. Lett.* **B267** 37 (1991).

[13] G. L. Cardoso and B. A. Ovrut, *Nucl. Phys.* **B369** 351 (1992); J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, *Nucl. Phys.* **B372** 145 (1992).

[14] M. Dine, N. Seiberg, E. Witten, *Nucl. Phys.* **B289** 585 (1987); J. J. Atick, L. Dixon, A. Sen, *Nucl. Phys.* **B292** 109 (1987); M. Dine, I. Ichinose, N. Seiberg, *Nucl. Phys.* **B293** 253 (1987).

[15] M. K. Gaillard and T. R. Taylor, *Nucl. Phys.* **B381** 577 (1992); V. S. Kaplunovsky and J. Louis, *Nucl. Phys.* **B444** 191 (1995).

[16] M. K. Gaillard, J. Giedt, *Nucl. Phys.* **B636** 365 (2002) and **B643** 201 (2002).

[17] F. Buccella, J.-P. Derendinger, S. Ferrara and C. A. Savoy, *Phys. Lett.* B 115 375 (1982);

[18] M. K. Gaillard and J. Giedt, *Phys. Lett.* **B479** 308 (2000).

[19] M. K. Gaillard, B. D. Nelson, *Nucl. Phys.* **B571** 3 (2000).

[20] A. Birkedal-Hansen, B. D. Nelson, *Phys. Rev.* **D64** 015008 (2001) and **D67** 095006 (2003).

[21] A. Font, L. E. Ibàñez, F. Quevedo and A. Sierra, *Nucl. Phys.* **B331** 421 (1990).

[22] B. Zumino, *Nucl. Phys* **B89** 535 (1975).

[23] M. K. Gaillard and V. Jain, *Phys. Rev.* **D49** 1951 (1994); M. K. Gaillard, V. Jain and K. Saririan, *Phys. Lett.* **B387** 520 (1996) and *Phys. Rev.* **D55** 883 (1997).

[24] J. A. Casas, M. Mondragon and C. Muñoz, *Phys. Lett.* **B230** 63 (1989).

[25] M. K. Gaillard, *Phys. Lett.* **B342** 125 (1995); *Phys. Rev.* **D58** 105027 (1998) and **D61** 084028 (2000).

[26] P. Binétruy and M. K. Gaillard, *Nucl. Phys.* **B312** 341 (1989).

[27] A. Birkedal, M. K. Gaillard, C. Park and M. Ransdorp, in progress.

[28] P. Binétruy, M. K. Gaillard and Y.-Y. Wu, *Nucl. Phys.* **B481** 109 (1996); J. A. Casas, *Phys. Lett.* **B384** 103 (1996).

[29] S. H. Shenker, in *Random Surfaces and Quantum Gravity*, Proceedings of the NATO Advanced Study Institute, Cargese, France, 1990, Ed. O. Alvarez, E. Marinari and P. Windey, NATO ASI Series, (Plenum, New York, 1990); E. Silverstein, *Phys. Lett.* **B396** 91 (1997).

[30] T. Banks and M. Dine, Phys. Rev. **D50** 7454 (1994).

[31] E. Witten, *Nucl. Phys.* **B471** 135 (1996); P. Hořava and E. Witten, *Nucl. Phys.* **B460** 506 (1996) and **B475** 94 (1996); T. Banks and M. Dine, *Nucl. Phys.* **B505** 445 (1997).

[32] M. K. Gaillard, P. Ko and B. D. Nelson, in progress.