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The Contribution from Neutrino Yukawa Couplings to Lepton Electric Dipole Moments

Yasaman Farzan[1,2,3] and Michael E. Peskin[1,3]

[1] *Stanford Linear Accelerator Center*
2575 Sand Hill Road, Menlo Park, California 94025 USA
[2] *Scuola Internazionale di Studi Avanzati (SISSA)*
via Beirut 4, I-34014, Trieste, ITALY

ABSTRACT

To explain the observed neutrino masses through the seesaw mechanism, a supersymmetric generalization of the Standard Model should include heavy right-handed neutrino supermultiplets. Then the neutrino Yukawa couplings can induce CP violation in the lepton sector. In this paper, we compute the contribution of these CP violating terms to lepton electric dipole moments. We introduce a new formalism that makes use of supersymmetry to expose the GIM cancellations. In the region of small $\tan\beta$, we find a different result from that given previously by Ellis, Hisano, Raidal, and Shimizu. We confirm the structure found by this group, but with a much smaller overall coefficient. In the region of large $\tan\beta$, we recompute the leading term that has been identified by Masina and confirm her result up to minor factors. We discuss the implications of these results for constraints on the Y_ν.

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Contents

1 Introduction

The discovery of neutrino mass has not only required revision of the Standard Model of particle physics but also of theories that go beyond the Standard Model. A compelling idea for the origin of the observed small neutrino masses is the seesaw mechanism. This requires the introduction of heavy singlet leptons, that is, right-handed neutrinos. In the context of supersymmetric theories, these singlet leptons belong to new chiral supermultiplets N_i, one for each fermion generation. The Yukawa couplings and soft supersymmetry breaking terms associated with these right-handed neutrino supermultiplets can play important roles in lepton flavor violating processes [1] and in the production of the baryon number of the universe through leptogenesis [2].

A particularly important aspect of this model is the appearance of new sources of CP violation. In addition to new CP violating parameters generic to new physics—in supersymmetry, for example, the phases of μ and the A—new phases are possible in the neutrino Yukawa couplings and in the neutrino B term ($BM\tilde{N}\tilde{N}$). Complex Yukawa couplings can lead to observable CP violation in neutrino oscillations, and all of these parameters can be the source of the CP violation that generated a fermion-antifermion asymmetry in the early universe [2,3].

To test whether the observed matter-antimatter asymmetry indeed arose from leptogenesis, it is necessary to determine the CP violating phases from microscopic measurements. There has been much analysis of CP violating observables in neutrino mixing. In principle, it is possible to determine all seesaw parameters studying neutrino and sneutrino mass matrices but, in practice, it will be quite challenging, if possible at all, to extract all of these parameters in the near future [4]. Another possible experimental approach to test CP violation in the lepton sector is to measure the electric dipole moments (EDMs) of charged leptons [5]. There are in fact many possible ways that underlying CP violating couplings could give rise to lepton EDMs. Thus, it is important to classify these effects and, if possible, to learn how to separate them from one another.

If CP violation is provided by phases of soft supersymmetry breaking parameters, it is straightforward to generate a contribution to lepton electric dipole moments in one-loop order. This possibility has been explored by many authors [6]. However, it is also possible to generate lepton EDMs in models in which the soft supersymmetry breaking terms are CP conserving, by making use of phases in the neutrino Yukawa couplings. A particularly simple context to study this effect is to consider models in which the soft supersymmetry breaking scalar masses are exactly flavor-universal and the A terms are exactly proportional to the Yukawa couplings. Such models arise in the simplest paradigms for gravity-mediated supersymmetry breaking [7]. The idea of 'gaugino mediation' provides an attractive way to realize this scheme in the context of a complete unified or superstring model [8].

In this class of models with universal soft supersymmetry breaking interactions, the flavor and CP violation due to the neutrino Yukawa couplings is computed by integrating out the right-handed neutrino N_i superfields. This particular model has been studied in a number of papers, beginning with [9]. In particular, the contribution of neutrino Yukawa couplings to lepton EDMs has been studied in this context by Ellis, Hisano, Raidal, and Shimizu (EHRS) [10], and by Masina [11]. These authors found that the analysis is complicated by GIM cancellations, so that the first nonzero contribution to the EDMs arises in two-loop order and has the form of a commutator of different combinations of the Yukawa matrices.

The work of [10] and [11] used a renormalization group approach to evaluate the leading logarithmic contributions to the lepton EDMs. We thought that it might be valuable to extend these calculations by evaluating the complete contribution to the lepton EDMs without making leading-log approximations. In this paper, we present a new accounting method for the CP violating effects of the right-handed neutrino sector that makes this calculation straightforward. Our results, however, differ from those of [10] even at the leading-log level. We confirm the general structure of the answers found by this groups—in particular, the commutator structure noted in the previous paragraph. However, we claim that there are further cancellations not found in their papers that one must resolve to obtain the correct detailed formulae. We confirm Masina's result for the large $\tan\beta$ region, up to some minor factors, using a method that is much more transparent.

The outline of this paper is then as follows: In Section 2, we specify the model in which we are working. In Section 3, we describe our procedure for integrating out the N_i supermultiplets and identifying CP violating contributions. In Section 4, we carry out this procedure for the leading CP violating contribution proportional to Y_ν^4, where Y_ν is the neutrino Yukawa coupling. We find a result that is parametrically smaller than that of EHRS by one power of a large logarithm. In Section 5, we reconsider the analysis of EHRS and show how that logarithm cancels out using their method. In Section 6, we give a formula for the lepton EDMs that arises from this contribution.

In [11], Masina pointed out that, for large values of $\tan\beta$, a different contribution can dominate the evaluation of the lepton EDMs. This new term arises at one higher loop order, at order $Y_\nu^4 Y_\ell^2$, where Y_ℓ is the charged lepton Yukawa coupling. In Section 7, we evaluate this contribution, which requires a nontrivial two-loop diagram calculation.

In Section 8, we make numerical estimates of the electron EDM from our new formulae and compare these to the results of other models of lepton CP violation.

Our calculations in this paper look specifically at the terms resulting from integrating out the right-handed neutrino sector. We work from an initial assumption that the soft supersymmetry breaking terms are universal and flavor-independent. In a model with renormalizable interactions that violate the flavor and CP symmetries, this initial condition is not technically natural. Thus, there will in general be other CP violating contributions, for example, from the thresholds at the grand unification scale $M_{\rm GUT}$, that should be added to the formulae we present here. Because, in all of our formulae, the leading logarithmic behavior cancels due to a GIM cancellation, our terms are not parametrically enhanced over those from the GUT threshold. In specific models, the GUT scale terms can be numerically smaller than the terms from the right-handed neutrino scale; the authors of [9], for example, argue this for their $SU(5)$ GUT model. In any event, our formulae are computed precisely for the effective theory of Section 2 with minimal subtraction (in the $\overline{DR}$ scheme) at $M_{\rm GUT}$. By noting this prescription, it should be straightforward to add GUT threshold corrections to our results when these are computed in a particular GUT model.

2 The model

We consider the supersymmetric Standard Model coupled to three chiral supermultiplets N_i which contain the heavy right-handed neutrinos associated with the seesaw mechanism. The superpotential of the model contains the following terms involving lepton supermultiplets:

$$W = Y_\ell^{ik}\epsilon_{\alpha\beta}H_{1\alpha}E_iL_{j\beta} - Y_\nu^{ij}\epsilon_{\alpha\beta}H_{2\alpha}N_iL_{j\beta} - \mu\epsilon_{\alpha\beta}H_{1\alpha}H_{2\beta} + \frac{1}{2}M_{ij}N_iN_j \ . \qquad (1)$$

In this equation, $L_{j\beta}$ is the supermultiplet containing the left-handed lepton fields $(\nu_{jL}, \ell^-_{jL})_\beta$, E_i is the superfield whose left-handed fermion is ℓ^+_{iL}, and N_i is the superfield whose left-handed fermion is $\overline{\nu}_{iL}$. The N_i are singlets of $SU(2)\times U(1)$. We introduce the right-handed neutrino masses M_{ij} by hand, and we do not assume any *a priori* relation of these parameters to the other couplings in (1).

Without loss of generality, we can choose the basis and phases of L, E, and N such that M_{ij} and Y_ℓ^{ij} are real and diagonal. We will refer to the diagonal elements of these matrices as M_i, $Y_{\ell i}$. These choices exhaust the freedom to redefine fields, and so the matrix Y_ν^{ij} is in general off-diagonal and complex. The mass matrix of light neutrinos is given by

$$(m_\nu)_{ij} = \sum_k \frac{Y_\nu^{ki}Y_\nu^{kj}}{M_k}\left\langle H_2^0\right\rangle^2 \ . \qquad (2)$$

If the neutrino Yukawa couplings Y_ν^{ik} are of order 1, the requirement of small neutrino masses ($m_\nu \sim 0.1$ eV) leads to large values of the M_k, of the order of 10^{14} GeV.

To the Lagrangian generated by (1), we must add appropriate soft supersymmetry breaking interactions. In this paper, we will assume that slepton masses are universal at the messenger scale (of the order of $M_{\rm GUT}$) and that A terms are strictly proportional to the corresponding Yukawa couplings, with a real constant of proportionality. We will assume that the phases of μ and of the gaugino masses are zero. If these conditions are not met, it is possible to generate EDMs from 1-loop diagrams, a possibility that has been exhaustively explored in the literature [6].

This restriction to universal, CP invariant, flavor invariant soft supersymmetry breaking terms is not a natural restriction of the model in the technical sense. It is violated by loop corrections due to the neutrino Yukawa couplings. In fact, our analysis in this paper is to calculate the CP violation induced by these corrections. Consequently, the effects we find can be cut-off dependent. As we have explained in the introduction, we will impose the universality and flavor symmetry of the soft supersymmetry breaking interactions as an initial condition, defined by minimal subtraction in the $\overline{DR}$ scheme at $M_{\rm GUT}$.

With this prescription, we will take the soft supersymmetry breaking terms for the lepton sector to be

$$\text{Ł}_{SSB} = -m_0^2 \sum_f \widetilde{f}^* \widetilde{f} - m_a \overline{\lambda}_a \lambda_a - a_0 \left(Y_{\ell i} \epsilon_{\alpha\beta} H_{1\alpha} \tilde{E}_i \tilde{L}_{i\beta} - Y_\nu^{ij} \epsilon_{\alpha\beta} H_{2\alpha} \tilde{N}_i \tilde{L}_{j\beta} \right)$$
$$-(\frac{1}{2} B_\nu M_i (\tilde{N}_i)^2 + h.c.) - (\frac{1}{2} b_H \mu H_1 H_2 + h.c.) \qquad (3)$$

where $\tilde{f}$ collectively represents sfermions, and we assume that a_0, b_H and B_ν are real parameters. The parameters m_0, m_a, a_0, and b_H all have the dimensions of mass and are of order $M_{\rm SUSY} \sim 100$ GeV $-$ 1 TeV. CP violating phases arise both from the neutrino Yukawa couplings and from the neutrino A term, but, in this model, they are controlled by the same parameters. We should note that if any of the parameters a_0, b_H or B_ν has an imaginary part, the corresponding term can give a large contribution to lepton EDMs. This point is discussed in some detail elsewhere [6]. The specific effects of the B_ν term have been analyzed in [12]

In computing the effects of the N_i supermultiplets, it is convenient to work in components, keeping the auxiliary fields (the F fields) as independent fields. We use two-component notation for the fermion fields. With the effects of the Majorana mass term included, the propagators for the component fields of the N_i take the form

$$\left\langle \tilde{N}_j(q) \tilde{N}_k^*(-q) \right\rangle = \frac{i}{q^2 - M_i^2} \delta_{jk} \qquad \left\langle \tilde{N}_j(q) F_{N_k}(-q) \right\rangle = \frac{-iM_j}{q^2 - M_i^2} \delta_{jk}$$

$$\left\langle N_j(q)N_k^\dagger(-q)\right\rangle = \frac{i\sigma\cdot q}{q^2-M_i^2}\delta_{jk} \qquad \left\langle N_j(q)(N^k(-q))^T\right\rangle = \frac{-iM_jc}{q^2-M_i^2}\delta_{jk}$$
$$\left\langle F_{N_j}(q)F_{N_k}^*(-q)\right\rangle \;=\; \frac{iq^2}{q^2-M_i^2}\delta_{jk} \tag{4}$$

where $\sigma^\mu = (1,\vec{\sigma})^\mu$ and $c = -i\sigma^2$ are 2×2 components of the Dirac matrices and the charge conjugation matrix.

3 Radiative corrections due to Y_ν

As it is well known, radiative corrections will distort the form of Eq. (3) and break the exact mass degeneracy between the sfermions. In this section, we will focus on those radiative corrections to the parameters of Eq. (3) that can induce CP-violating phase and EDMs, in particular, the effects of diagrams involving the neutrino Yukawa and A terms. We will discuss the form of the effective Lagrangian at scales just below the right-handed neutrino mass scale. When we compute the induced EDMs in Section 6 and 7, we will need to take into account some additional effects that come from renormalization group running down to the electroweak scale. In our analysis, we will always assume that the right-handed neutrino masses M_k are much larger than the supersymmetry breaking mass terms, of order $M_{\rm SUSY}$, so that any contribution suppressed by $M_{\rm SUSY}/M_k$ can be neglected. In this limit, the calculation that integrates out the right-handed neutrino sector divides neatly into a part that corrects the supersymmetric Lagrangian and a part that corrects the supersymmetry breaking perturbations.

First, we consider the radiative corrections to the supersymmetric part of the Lagrangian. We begin by noting that, to a good approximation, we can neglect diagrams that include vertices from the supersymmetry breaking terms. Except for the μ term, all coefficients in the supersymmetric Lagrangian are dimensionless, while all supersymmetry breaking terms have coefficients with mass parameters of order 1 TeV or smaller. Therefore, corrections to the dimensionless coefficients from the supersymmetry breaking terms are at most of order of $M_{\rm SUSY}/M_k$, completely negligible. Corrections to the μ term are at most of the order of $\mu b_0 a_0/M_k^2$, again, a negligible correction.

The radiative corrections within the supersymmetric theory are strongly restricted by the constraints of supersymmetry. All component fields within supermultiplet receive the same radiative corrections. By the non-renormalization theorem [13], the superpotential receives no corrections. The result of this theorem constrains only the leading term in a Taylor series in external momenta, but, since these diagrams are evaluated at external momenta of order $M_{\rm SUSY}$, terms that depend on external momenta are suppressed by powers of $M_{\rm SUSY}/M_k$ and can be ignored. Then the most general effective Lagrangian obtained by integrating out the N_k multiplets will have the form

$$Ł_{\rm eff} = \int d^4\theta\,\overline{L}_i(1+\delta Z_L)^{ij}L_j + \int d^4\theta\,\overline{E}_i(1+\delta Z_E)^{ij}E_j + \int d^2\theta\,W + H.c. \qquad (5)$$

Since the Lagrangian is real-valued, the matrices $(\delta Z_L)^{ij}$ and $(\delta Z_E)^{ij}$ must be Hermitian to all orders in perturbation theory. Note that while $(\delta Z_L)^{ij}$ receives off-diagonal corrections at the 1-loop level, $(\delta Z_E)^{ij}$ receives off-diagonal elements only at the two-loop level because E does not have any flavor number violating coupling.

To generate a lepton electric dipole moment, we require a flavor-diagonal matrix element of an electromagnetic form factor to have an imaginary part [14]. However, the radiative corrections from the supersymmetric Lagrangian, treated to first order, will be proportional to the matrices δZ_L and δZ_E. Since the diagonal elements of a Hermitian matrix are real, none of these corrections, acting alone, can induce a lepton electric dipole moment. This is an important constraint, which we will continue to follow through our analysis.

The soft supersymmetry breaking part of the Lagrangian receives corrections proportional to the supersymmetry breaking parameters. However, the form is still quite constrained. The most general effective Lagrangian has the form

$$\begin{aligned} Ł_{SSB} = &-(m_0^2+\delta m^2_{\tilde{L}})_{ij}\widetilde{L}_i^\dagger\widetilde{L}_j - (m_0^2+\delta m^2_{\tilde{E}})_{ij}\widetilde{E}_i^\dagger\widetilde{E}_j \\ &-(a_0 Y_{\ell i}\delta_{ij}+\delta\mathcal{A}^{ij})\epsilon_{\alpha\beta}H_{1\alpha}\tilde{E}_i\tilde{L}_{j\beta} + H.c. \end{aligned} \qquad (6)$$

Since $Ł_{SSB}$ is Hermitian, $(\delta m^2_{\tilde{E}})_{ij}$ and $(\delta m^2_{\tilde{L}})_{ij}$ must be Hermitian matrices to all orders in perturbation theory. The A term can in general receive non-Hermitian contribution. However, we will show in Appendix A that, up to order Y_ν^4, the corrections to the A term have the form

$$\delta\mathcal{A}^{ij} = a_0 Y_{\ell i}\delta Z_A^{ij}\ , \qquad (7)$$

where δZ_A is Hermitian. Here again, the form of the radiative corrections as Hermitian matrices limits their ability to contribute to electric dipole moments.

To work with the effective Lagrangian written in (5) and (6), it is useful to bring the lepton and slepton fields into a canonical normalization by rescaling by $(1+\delta Z)^{-1/2}$. Then the superpotential becomes

$$W = -[(1+\delta Z_E)^{-1/2}]^{ki}Y_{\ell i}[(1+\delta Z_L)^{-1/2}]^{ij}\epsilon_{\alpha\beta}H_{1\alpha}E_kL_j \qquad (8)$$

and the soft terms become

$$\begin{aligned} Ł_{SSB\rm eff} = &-[(1+\delta Z_L)^{-1/2}(m_0^2+\delta m^2_{\tilde{L}})(1+\delta Z_L)^{-1/2}]^{ij}\widetilde{L}_i^\dagger\widetilde{L}_j \\ &-[(1+\delta Z_E)^{-1/2}(m_0^2+\delta m^2_{\tilde{E}})(1+\delta Z_E)^{-1/2}]^{ij}\widetilde{E}_i^\dagger\widetilde{E}_j \end{aligned}$$

$$-a_0[(1+\delta Z_E)^{-1/2}Y_\ell(1+\delta Z_A)(1+\delta Z_L)^{-1/2}]^{ij}\epsilon_{\alpha\beta}H_{1\alpha}\tilde{E}_i\tilde{L}_{j\beta}+H.c. \tag{9}$$

One more step is needed. To identify the mass basis for leptons, we need to re-diagonalize the lepton Yukawa coupling. Decompose the coefficient of (8) into a product of a unitary matrix, a real positive diagonal matrix, and another unitary matrix:

$$[(1+\delta Z_E)^{-1/2}Y_\ell(1+\delta Z_L)^{-1/2}]_{ij} = [(1+\delta V)^T\mathcal{Y}_\ell(1+\delta U)]_{ij} \tag{10}$$

Then $(1+\delta V)^T$ can be absorbed into the superfields E and $(1+\delta U)$ can be absorbed into the superfields L. The soft supersymmetry breaking terms now take a form similar to (6):

$$\begin{aligned}Ł_{SSB} = & -(m_0^2+\Delta m^2_{\tilde{L}})_{ij}\widetilde{L}_i^\dagger\widetilde{L}_j - (m_0^2+\Delta m^2_{\tilde{E}})_{ij}\widetilde{E}_i^\dagger\widetilde{E}_j \\ & -a_0\mathcal{Y}_{li}(\delta_{ij}+\Delta Z_A^{ij})\epsilon_{\alpha\beta}H_{1\alpha}\tilde{E}_i\tilde{L}_{i\beta}+H.c.\end{aligned} \tag{11}$$

where

$$\begin{aligned}(m_0^2+\Delta m^2_{\tilde{L}}) &= [(1+\delta U)(1+\delta Z_L)^{-1/2}(m_0^2+\delta m^2_{\tilde{L}})(1+\delta Z_L)^{-1/2}(1+\delta U)^{-1}] \\ (m_0^2+\Delta m^2_{\tilde{E}}) &= [(1+\delta V)(1+\delta Z_E)^{-1/2}(m_0^2+\delta m^2_{\tilde{E}})(1+\delta Z_E)^{-1/2}(1+\delta V)^{-1}] \\ a_0\mathcal{Y}(1+\Delta Z_A) &= a_0\mathcal{Y}_\ell(1+\delta U)(1+\delta Z_L)^{1/2}(1+\delta Z_A)(1+\delta Z_L)^{-1/2}(1+\delta U)^{-1}\ .\end{aligned} \tag{12}$$

At this point, the only signs of CP-violation from the neutrino Yukawa couplings occur in the coefficient functions listed in (12). It is still true that the first two coefficient functions are Hermitian matrices with real diagonal elements, and that the diagonal elements of the A term coefficient are real through two-loop order (order Y_ν^4). For the mass matrices, this result is obvious. For the A term an additional slightly technical argument is needed, which we give in Appendix B.

This implies that, through order Y_ν^4, we cannot obtain a contribution to the lepton electric dipole moments from any individual term in (11). However, we can obtain a matrix with an imaginary part by taking the product of two different matrices from (11). For example,

$$C_i = \mathrm{Im}\left[\Delta Z_A\Delta m^2_{\tilde{L}}\right]_{ii} \tag{13}$$

can have nonzero diagonal elements. Since both matrices are Hermitian, this quantity can be written more illustratively as

$$C_i = \frac{1}{2i}\left([\ \Delta Z_A\ ,\ \Delta m^2_{\tilde{L}}\ ]\right)_{ii}\ . \tag{14}$$

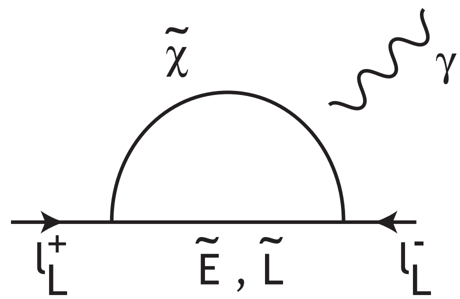

Figure 1: The general form of the diagrams contributing to the EDM of a charged lepton ℓ. The photon line should be attached at all possible positions in the diagram.

Note that to compute C_i to order of Y_ν^4, it suffices to calculate ΔZ_A and $\Delta m^2_{\tilde{L}}$ to the one-loop level. Through two-loop order, this is the only structure in the theory that can contribute to a lepton electric dipole moment.

At three-loop order, products of $\Delta m^2_{\tilde{E}}$ with the other matrices in (11) can give additional contributions of a new structure. A specific CP-violating quantity that will be important to us is

$$D_i = \mathrm{Im}\left((\Delta m^2_{\tilde{E}})^T\, m_\ell\; \Delta m^2_{\tilde{L}}\right)_{ii} \tag{15}$$

This quantity also has a commutator structure, as we will see in Section 7. It is smaller than (14) by a factor of $Y_\ell^2/4\pi$. Nevertheless, as we will see in Section 7, this term can give the dominant contribution to lepton electric dipole moments in models with large $\tan\beta$. To obtain the contribution from this structure of order $Y_\nu^4 Y_\ell^2$, it suffices to calculate $\Delta m^2_{\tilde{E}}$ to two-loop order and $\Delta m^2_{\tilde{L}}$ to one-loop order.

We can be somewhat more concrete about how the structures C_i and D_i arise from Feynman diagrams. Contributions to the lepton EDM's come from diagrams of the general form of Fig. 1, in which a right-handed lepton and is converted to a left-handed lepton through a photon vertex diagram. A lepton line runs through the diagram, and the matrices (12) appear as insertions on this line. By the arguments just given, we need to consider contributions with two separate insertions. The product (14) comes uniquely from diagrams of the form of Fig. 2(a), with the photon inserted in all possible positions on the lepton line. The product (15) comes from diagrams of the form of Fig. 2(b). In the latter diagram, the left-right mixing contributes the factor of m_ℓ. We will evaluate these diagrams in Sections 6 and 7, respectively.

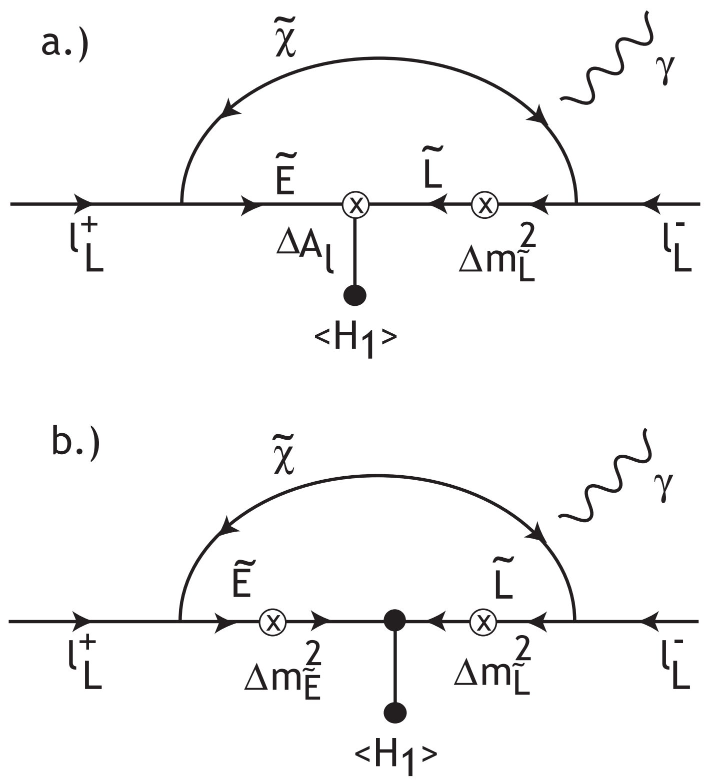

Figure 2: Diagrams giving the dominant contribution to EDM of charged lepton ℓ (a) for small $\tan\beta$, (b) for large $\tan\beta$.

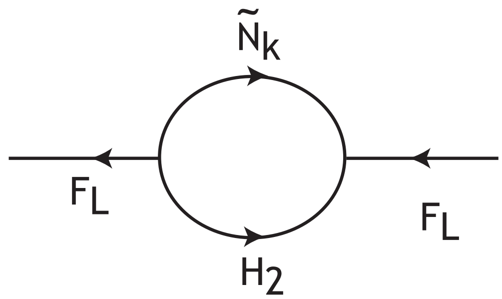

Figure 3: Diagram giving the field strength renormalization of the supermultiplet L_i. In this and the next few figures, we treat F components as independent fields; the F terms of N_k multiplets have the propagators (4).

4 One-loop corrections

To estimate the lepton electric dipole moments at order Y_ν^4, we should next compute ΔZ_A and $\Delta m_{\tilde{L}}^2$. According to the arguments of the previous section, only the leading-order contributions are needed. To this order

$$\Delta m_{\tilde{L}}^2 = \delta m_{\tilde{L}}^2 - m_0^2 \delta Z_L \qquad \Delta Z_A = \delta Z_A \tag{16}$$

The factor δZ_L is most easily computed as the one-loop correction to the F_L field strength. There is only one diagram, shown in Fig. 3; its value is

$$(\delta Z_L)^{ij} = (Y_\nu^{ki})^* Y_\nu^{kj} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{p_E^2 (p_E^2 + M_k^2)} \ , \tag{17}$$

where p_E is a Euclidean momentum after Wick rotation.

The factor δZ_A arises from the diagram shown in Fig. 4. The vertex marked with a heavy dot is an A_ν vertex. The value of the diagram is

$$a_0 Y_{\ell i} (\delta Z_A)^{ij} = -a_0 Y_{\ell i} (Y_\nu^{ki})^* Y_\nu^{kj} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{p_E^2 (p_E^2 + M_k^2)} \ . \tag{18}$$

The tensor structure is exactly the same as in (17). This fact is used in Appendix B.

The matrix δm_L^2 arises from the four diagrams shown in Fig. 5. The first diagram has two A_ν vertices; the other three have supersymmetry breaking mass insertions. It should be noted that there is a contribution in which m_0^2 in inserted into the F_N propagator, which results from the mixing of F_N with $\widetilde{N}$ through the Majorana mass term. The final result is

$$(\delta m_{\tilde{L}}^2)^{ij} = -(Y_\nu^{ki})^* Y_\nu^{kj} \int \frac{d^4 p_E}{(2\pi)^4} \left[\frac{m_0^2 + a_0^2}{p_E^2 (p_E^2 + M_k^2)} + \frac{m_0^2}{(p_E^2 + M_k^2)^2} - \frac{m_0^2 M_k^2}{p_E^2 (p_E^2 + M_k^2)^2} \right] , \tag{19}$$

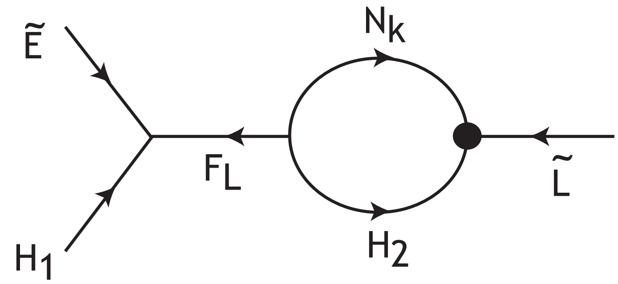

Figure 4: Diagram giving the one-loop radiative correction to the vertex A_ℓ. The heavy dot is an A_ν vertex.

which is quite similar to (17) and (18), except that some terms appear with two massive propagators. The small difference in structure between (19) and the earlier equations will be significant.

As we have explained in Section 2, we regularize these diagrams by dimensional regularization and minimal subtraction at the scale $M_{\rm GUT}$. This gives

$$\begin{aligned}
\delta Z_L^{ij} &= \frac{1}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left[\log\frac{M_{\rm GUT}^2}{M_k^2}+1\right] \\
\delta Z_A^{ij} &= -\frac{1}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left[\log\frac{M_{\rm GUT}^2}{M_k^2}+1\right] \\
(\delta m_{\tilde{L}}^2)^{ij} &= -\frac{2m_0^2}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left[\log\frac{M_{\rm GUT}^2}{M_k^2}\right] - \frac{a_0^2}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left[\log\frac{M_{\rm GUT}^2}{M_k^2}+1\right] . \quad (20)
\end{aligned}$$

so that

$$\Delta m_{\tilde{L}}^{2\,ij} = -\frac{1}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left(m_0^2[3\log\frac{M_{\rm GUT}^2}{M_k^2}+1] + a_0^2[\log\frac{M_{\rm GUT}^2}{M_k^2}+1]\right) . \quad (21)$$

and $\Delta Z_A = \delta Z_A$.

Now some significant simplifications appear. First, in evaluating (14), we can drop any terms in $\Delta m_{\tilde{L}}^2$ that are proportional to the tensor structure of δZ_A. Thus, we can replace

$$\Delta m_{\tilde{L}}^{2\,ij} \to -\frac{1}{(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}(-2m_0^2) . \quad (22)$$

Second, after making this simplification, we can drop any terms in δZ_A^{ij} that are proportional to the structure $(Y_\nu^{ki})^* Y_\nu^{kj}$. In particular, we can change $M_{\rm GUT}$ inside the logarithm to any other value that is independent of k. We then find

$$C_i = \frac{m_0^2}{(4\pi)^4}\frac{(\,[\mathbf{Y}_0,\mathbf{Y}_1]\,)_{ii}}{i} , \quad (23)$$

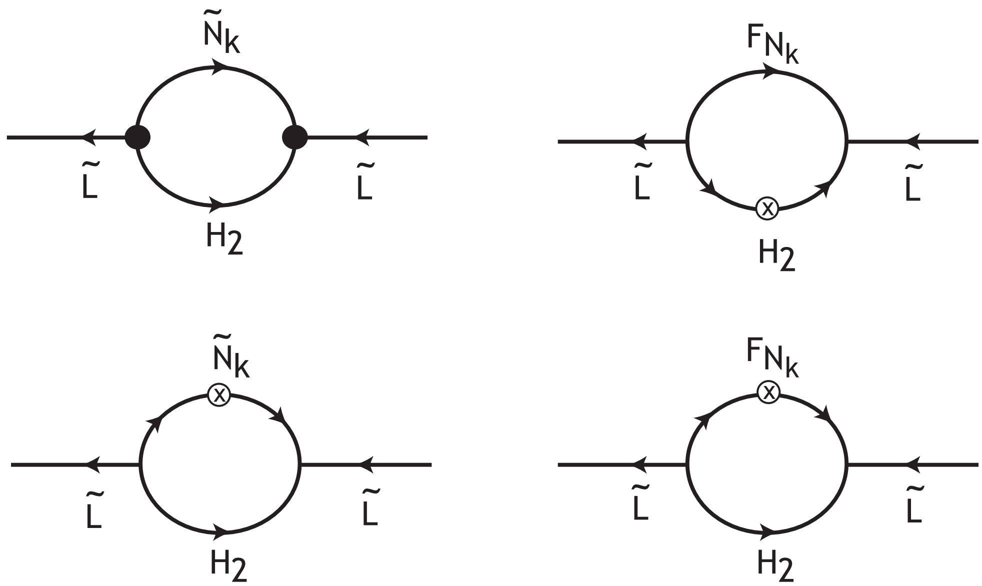

Figure 5: Diagrams giving the one-loop corrections to the supersymmetry breaking $\tilde{L}$ mass term. The heavy dot is an A_ν vertex; the marked insertion is a soft mass term m_0^2.

where

$$(\mathbf{Y}_0)^{ij} = (Y_\nu^{ki})^* Y_\nu^{kj} \qquad (\mathbf{Y}_1)^{ij} = (Y_\nu^{ki})^* Y_\nu^{kj} \log \frac{M_N^2}{M_k^2} \; . \tag{24}$$

As is explained just above, the expression for C_i actually does not depend on the parameter M_N. It is convenient to choose M_N to be the geometric mean of the M_k to minimize the individual logarithms that appear in (24).

Our final result for C_i is simple and cutoff-independent. However, we remind the reader that this result is derived in the simple picture in which we ignore threshold effects at the GUT scale and regulate diagrams using the $\overline{DR}$ scheme. Because of the major cancellations that occurred in the simplification of C_i, these threshold corrections, which depend in a model-dependent way on GUT-scale physics, can be of the same order of magnitude as (23).

5 Comparison to the RGE approach

It is remarkable that, to order Y_ν^4, the only contribution to the lepton EDM comes from the invariant C_i and that there is no contribution from $\mathrm{Im}[A_\ell]$. This conflicts with previous results on lepton EDMs given by EHRS [10] and Masina [11]. In this section, we will compute the leading logarithmic contributions to $\mathrm{Im}[A_\ell]$ using the renormalization group method and demonstrate explicitly that they cancel. At the end of the section, we will compare our analysis to that of [10] and [11].

To carry out the renormalization group analysis, we must integrate the RGEs from an initial condition at M_{GUT} to the heaviest N mass, M_3, then from M_3 to M_2, then from M_2 to M_1. This procedure is valid only if $M_1 \ll M_2 \ll M_3$. Let us define

$$t(Q) = \frac{1}{(4\pi)^2} \log Q \ . \tag{25}$$

and

$$t_3 = t(M_{\mathrm{GUT}}) - t(M_3) \ , \qquad t_2 = t(M_3) - t(M_2) \ , \qquad t_1 = t(M_2) - t(M_1) \ . \tag{26}$$

For a hierarchical spectrum of masses, we expect this procedure to reproduce the results of two-loop calculations up to the order $Y_\nu^4 t^2$.

It is very important to write the RGEs in such a way that the right-handed neutrino thresholds are accounted correctly. There are two aspects to this. First, one should, at each stage of integration, project out those N_i's that have masses above the scale at which the RGE is being evaluated. To discuss this, it is useful to introduce projectors $P_3 = 1$, $P_2 = \mathrm{diag}(1,\ 1,\ 0)$, $P_1 = \mathrm{diag}(1,\ 0,\ 0)$, projecting onto the N mass eigenstates that are still active as we integrate through the various thresholds. Second, one should be careful to keep the matrices Y_ℓ and M diagonal, at least when heavy particles are integrated out.

We found it surprising that it is necessary to worry about off-diagonal terms in M, and so we would like to illustrate this with an example. In the appendix of [11], the RGE for the neutrino Yukawa coupling is given as

$$\frac{dY_\nu}{dt} = 3 Y_\nu Y_\nu^\dagger P_a Y_\nu + \cdots \tag{27}$$

The contribution on the right-hand side arises from the diagrams shown in Fig. 6. The projector eliminates contributions from the right-handed neutrinos with mass $M_k > Q$. Consider, in particular, integrating this equation down to a Q such that $M_2 < Q < M_3$. Let $t_Q = t(M_3) - t(Q)$. Then the integration gives

$$Y_\nu(Q) = Y_\nu(M_{\mathrm{GUT}}) + 3 Y_\nu Y_\nu^\dagger Y_\nu t_3 + 3 Y_\nu Y_\nu^\dagger P_2 Y_\nu t_Q \ . \tag{28}$$

However, direct calculation of the diagrams in Fig. 6 with Euclidean external momenta with $|Q^2| \ll M_3^2$ gives

$$Y_\nu(Q) = Y_\nu(M_{\mathrm{GUT}}) + 2 Y_\nu Y_\nu^\dagger P_2 Y_\nu (t_3 + t_Q) + Y_\nu Y_\nu^\dagger Y_\nu t_3 + Y_\nu Y_\nu^\dagger Y_\nu t_Q \ , \tag{29}$$

since in the first diagram the contribution from N_3 in the internal line labelled N_m has a propagator proportional to $1/(Q^2 + M^2)$ and so is suppressed for $Q^2 \ll M_3^2$.

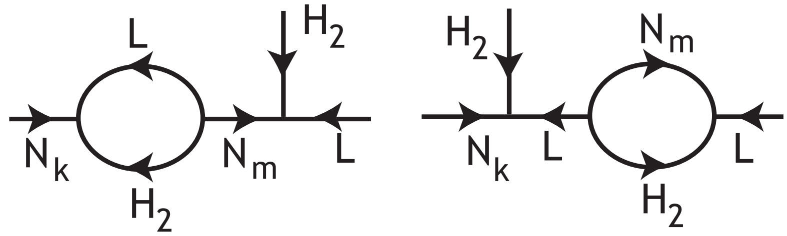

Figure 6: Diagrams giving the terms proportional to Y_ν^2 in the RGE evolution of the neutrino Yukawa coupling.

The direct calculation is correct. The problem is that, at this level, the application of the renormalization group method is incomplete. The neutrino mass matrix also acquires off-diagonal terms from the RGE. For $Q \gg M_3$

$$\frac{dM}{dt} = 2(Y_\nu Y_\nu^\dagger)M + 2M(Y_\nu Y_\nu^\dagger)^T + \cdots \ . \tag{30}$$

Thus, we should, first, integrate all of the RGEs down to a scale of the order of M_3, second, diagonalize the mass matrix M at this scale and rewrite the couplings in this new basis, third, integrate out its largest eigenvalue, and, finally, use the rotated couplings as the initial conditions for the stage of integration from M_3 to M_2.

It is not difficult to see that this prescription precisely eliminates the term that does not appear in (29) from (28). The first diagram in Fig. 6 modifies the neutrino Yukawa coupling by a field strength renormalization factor $Y_\nu \to Z_N^{-1/2} Y_\nu$. The equation (30) induces a similar modification in the mass matrix, $M \to Z_N^{-1/2} M (Z_N^{-1/2})^T$. The Z_N factors are the same in the two expressions due to the nonrenormalization theorem. When we now diagonalize M at the scale M_3, the change of basis cancels the off-diagonal 1–3 and 2–3 elements of $Z_N^{-1/2}$ that affect Y_ν.

To control this effect, one must integrate through all three thresholds by carefully solving the RGE for all couplings and mass terms. However, here we only wish to develop expressions for the effective couplings to order t^2, in order to check the results of the previous sections. For this, it is easier and more direct to use the following procedure: First, we integrate the renormalization group equations for the couplings. Then we identify terms that correspond to diagrams such as the first one in Fig. 6 with decoupling internal lines, and we remove these contributions by hand.

We should be careful also to remove diagrams with intermediate F_N lines, since F_N also decouples, as we see from the last line of (4). One-loop diagrams involving the supersymmetry breaking a_0 term can produce mixing of L and F_L or N and F_N, for example, as in the diagram shown in Fig. 4. In the RGE evolution of A_ν, we encounter a term in which the intermediate line is F_{N3}. In this contribution, the off-diagonal terms decouple and should be removed at the same time that we remove intermediate N_3 lines. Unless this is done, one cannot see the complete cancellation of $\mathrm{Im}[A_{\ell ii}]$ that we will present below. In an RGE analysis, this step requires treating F_N as a separate field in the Lagrangian and diagonalizing the $N^* F_N$ quadratic terms generated through RGE evolution.

With this insight into how to treat self-energy terms in the RGEs, we can integrate the RGEs for coupling constants through the three thresholds. The renormalization group equations for coupling constants are as given the Appendix of [11],

$$
\begin{aligned}
\frac{dY_\nu}{dt} &= 3Y_\nu K_a + \cdots \\
\frac{dY_\ell}{dt} &= Y_\ell K_a + \cdots \\
\frac{dA_\nu}{dt} &= 4\tilde{K}_a A_\nu + 5A_\nu K_a + \cdots \\
\frac{dA_\ell}{dt} &= 2Y_\ell (Y_\nu)^\dagger A_\nu + A_\ell K_a + \cdots \\
\frac{dm^2_{\tilde{L}}}{dt} &= \{m^2_{\tilde{L}}\ , K_a\} + 2(Y_\nu^\dagger P_a m^2_{\tilde{N}} P_a Y_\nu + m^2_{H_u} K_a + A_\nu^\dagger P_a A_\nu) + \cdots \\
\frac{dm^2_{\tilde{E}}}{dt} &= 2(m^2_{\tilde{E}} Y_\ell^\dagger Y_\ell + Y_\ell^\dagger Y_\ell m_{\tilde{E}}) + 4(Y_\ell^\dagger m^2_{\tilde{L}} Y_\ell + m^2_{H_u} Y_\ell^\dagger Y_\ell + A_\ell^\dagger A_\ell) + \cdots \ ,
\end{aligned}
\tag{31}
$$

where $K_a = Y_\nu^\dagger P_a Y_\nu$ and $\tilde{K}_a = Y_\nu Y_\nu^\dagger P_a$ and the subscript a specifies the energy scale. Note that $m^2_{\tilde{N}}$ is a supersymmetry breaking mass and should not be mistaken for a large supersymmetric neutrino mass M_i. The terms not written explicitly in (31) are terms with flavor structures such as $Y_\nu \cdot \mathrm{tr}[Y_\nu Y_\nu^\dagger]$ that do not contribute to EDMs. Using the prescription that we have explained above, we find different results from those found previously. When we solve for A_ℓ and for Y_ℓ at a scale much smaller than M_1, we find that the imaginary parts of Y_ℓ and A_ℓ/a_0 are identical and are equal to

$$
\mathrm{Im}\left[Y_\ell (K_3 K_2 t_3 t_2 + K_3 K_1 t_3 t_1 + K_2 K_1 t_2 t_1)\right] \ . \tag{32}
$$

Now one more step is needed. As in (10), we need to choose a new basis for the leptons in which Y_ℓ is real diagonal after taking into account the radiative corrections due to the N_i. Since the imaginary parts of Y_ℓ and A_ℓ/a_0 found at the previous stage are identical, this completely removes the imaginary part of A_ℓ, in agreement with our analysis in Section 3.

It is not clear to us how the analyses of [10] and [11] found nonzero diagonal terms of $\mathrm{Im}[A_\ell]$ at this order. The discussion in [10] does not discuss the issue of re-diagonalizing Y_ℓ and M. On the other hand, [11] writes the renormalization group equations in a way that explicitly takes into account the decoupling of heavy states, as we have noted above. In addition, the calculations done in this paper keep Y_ℓ and M diagonal by adding terms to these RGEs following the 'rotating basis' prescription of Brax and Savoy [15]. The full RGEs considered are not written explicitly in [11], but nevertheless they are used to generate the results that are quoted there [16]. One possible difficulty is that this analysis might not remove the terms with decoupling $\langle N^* F_N \rangle$ propagators that we have discussed above (31).

The observation that lepton EDMs are proportional to the commutator of $\mathbf{Y}_0$ and $\mathbf{Y}_1$ is the most important result of the analysis of EHRS [10]. Once this result has been found, it is straightforward to obtain the correct order of magnitude for the contribution to lepton EDMs from the phases of neutrino Yukawa couplings. Thus, the qualitative dependence of EDMs on the underlying supersymmetry parameters is given correctly in this paper, even though the actual terms that produce the lepton EDMs are different.

6 Electric dipole moments

We are now ready to obtain the actual expression for the lepton EDMs by evaluating the class of diagrams shown in Fig. 1.

A general diagram of the form of Fig. 1 evaluates to the form

$$-iev^T(p')c\sigma^{\mu\nu}q_\nu u(p) \cdot \left[-\frac{1}{m_{\ell i}}(F_{2i} + iF_{25i})\right] , \tag{33}$$

where F_2 is the usual magnetic moment form factor and i indexes the lepton flavor. The lepton EDM is then given by

$$\vec{d}_i = -eF_{25i}\frac{\vec{S}}{m_i} = (1.9 \times 10^{-11}\,\mathrm{e\ cm}) \cdot F_{25i} \cdot \frac{m_e}{m_{\ell i}} \cdot \hat{S} . \tag{34}$$

where $\vec{S}$ is the spin of the lepton and $\hat{S} = \vec{S}/(\hbar/2)$.

We would like to find a contribution to the EDM proportional to C_i in (14). For this, we should find a vertex diagram that depends on both A_ℓ and $m^2_{\tilde{L}}$ and insert the flavor-violating corrections found in Section 3. The only such diagram is shown in Fig. 2(a). The value of this diagram, as a contribution to $F_2 + iF_{25}$, is

$$\begin{aligned} F_2 + iF_{25} = \frac{\alpha}{2\pi}\sum_a \left(\frac{V_{01a}}{c_w}\right)\left(\frac{V_{01a}}{c_w} + \frac{V_{02a}}{s_w}\right)(A_\ell - \mu\tan\beta)m^2_{\ell i}m_a \\ \cdot \int_0^1 dz \int_0^1 dx\, \frac{z(1-z)^2}{(zm_a^2 + (1-z)(xm^2_{\tilde{E}} + (1-x)m^2_{\tilde{L}}))^2} . \end{aligned} \tag{35}$$

In this expression, V_0 is the unitary matrix that diagonalizes the neutralino mass matrix:

$$\widetilde{b}^0 = \sum_a V_{01a}\widetilde{\chi}^0_a \qquad \widetilde{w}^0 = \sum_a V_{02a}\widetilde{\chi}^0_a \ , \tag{36}$$

m_a are the neutralino mass eigenvalues (with signs), $c_w = \cos\theta_w$, $s_w = \sin\theta_w$.

The renormalization-group running of the soft supersymmetry breaking masses from the GUT scale to the electroweak scale corrections gives large but flavor-independent corrections to the $\tilde{E}$ and $\tilde{L}$ masses proportional to the GUT-scale gaugino masses. These terms do not contribute to the flavor-violating effects that give the dipole matrix element an imaginary part, but they should be taken into account in the denominator of (35) in evaluating this imaginary part. Thus, we have written (35) as depending on the electroweak-scale values of these masses $m_{\tilde{E}}$ and $m_{\tilde{L}}$. The full expression (35) can be checked against many papers on lepton dipole moments, for example, [17,18,19].

Starting from (35), we replace A_ℓ by $a_0 Y_\ell \delta Z_A$, and we include one mass insertion in the $\widetilde{L}$ line by acting on the integral with

$$\Delta m^2_{\tilde{L}} \frac{\partial}{\partial m^2_{\tilde{L}}} \tag{37}$$

A schematic version of this analysis for general flavor-violating perturbations is described, for example, in [20]. In our model, we take the indicated derivative of (35), assemble the structure $(\Delta Z_A \Delta m^2_{\tilde{L}})$, and replace the imaginary part of this object by (iC_i) as given in (14). We thus obtain an expression for the lepton electric dipole moment of the form of (34), where

$$F_{25i} = \frac{2\alpha}{(4\pi)^5} \sum_a \left(\frac{V_{01a}}{c_w}\right)\left(\frac{V_{01a}}{c_w} + \frac{V_{02a}}{s_w}\right) \frac{m_0^2 m^2_{\ell i} a_0 m_a}{|m_a|^6} \ \frac{([\,\mathbf{Y}_0\,,\,\mathbf{Y}_1\,])_{ii}}{i} g(\frac{m^2_{\tilde{L}}}{m_a^2}, \frac{m^2_{\tilde{E}}}{m_a^2}) \ , \tag{38}$$

where $g(x_L, x_E)$ is given in Appendix C. For comparison with the results of the next section, we might write this result alternatively as

$$\begin{aligned} F_{25i} = \frac{2\alpha}{(4\pi)^5} \sum_a \left(\frac{V_{01a}}{c_w}\right)\left(\frac{V_{01a}}{c_w} + \frac{V_{02a}}{s_w}\right) \frac{m^2_{\ell i} a_0 m_a}{|m_a|^6} g(\frac{m^2_{\tilde{L}}}{m_a^2}, \frac{m^2_{\tilde{E}}}{m_a^2}) \\ \cdot \mathrm{Im}[(Y_\nu^{ki})^* Y_\nu^{kj} (Y_\nu^{mj})^* Y_\nu^{mi}] \cdot (-2m_0^2 \log\frac{M_N^2}{M_k^2}) \ . \end{aligned} \tag{39}$$

There is a curious consequence of this result that follows from the fact that the trace of any commutator is zero. If this effect is the only source of the lepton EDM, we expect that

$$d_e/m_e + d_\mu/m_\mu + d_\tau/m_\tau = 0 \ . \tag{40}$$

It is unclear to us how this simple formula could be tested to the required accuracy.

7 Electric dipole moments for large $\tan\beta$

Masina [11] has argued that, for large $\tan\beta$, a different contribution to the lepton EDM can be the dominant one. Looking back at the diagrams of Fig. 1 and 2 studied in the previous section, we see that it is advantageous at large $\tan\beta$ to drop A_ℓ and keep instead the term $\mu\tan\beta$. We still need a second loop correction to combine with $\Delta m^2_{\tilde{L}}$, but this can come from inserting $\Delta m^2_{\tilde{E}}$ in the right-handed slepton propagator. Since $\Delta m^2_{\tilde{E}}$ arises at order $Y_\ell^2 Y_\nu^2$, the new diagram has a size relative to the previous one of

$$\frac{Y_\ell^2}{(4\pi)^2}\tan\beta \approx \frac{m_\tau^2}{8\pi^2 v^2}\frac{\tan^3\beta}{\sin^2\beta}\ , \tag{41}$$

where $v = 246$ GeV, assuming that the τ lepton dominates the intermediate states in the matrix product. We will see in a moment that, whereas all large logarithms of $M_{\rm GUT}$ cancelled out of the expression for lepton EDM in Section 6, the contribution of the large $\tan\beta$ region is enhanced by two powers of this large logarithm. As a result, the terms we will compute in this section can dominate over those we discussed in Section 6 for $\tan\beta > 10$.

To evaluate this contribution, we need to work out the mass insertion $\Delta m^2_{\tilde{E}}$ in (11). To begin, we must compute δZ_E and $\delta m^2_{\tilde{E}}$ up to the two-loop level. We need only compute those two-loop diagrams that contain the maximum number of Y_ν vertices, since only these diagrams will contain factors of the CP violating phases needed for a contribution to the EDMs.

Consider first δZ_E. The one- and two-loop diagrams contributing to the field strength renormalization give

$$\begin{aligned}\delta Z_E^{ji} &= 2Y_{\ell i}^2\delta_{ij}\int\frac{d^4p_E}{(2\pi)^4}\frac{1}{(p_E^2)^2}\\ &\quad -2Y_{\ell i}(Y_\nu^{ki})^* Y_\nu^{kj} Y_{\ell j}\int\frac{d^4p_E}{(2\pi)^4}\int\frac{d^4k_E}{(2\pi)^4}\frac{1}{(p_E^2)[(k_E-p_E)^2+M_k^2](k_E^2)^2}\ .\end{aligned} \tag{42}$$

The two-loop contribution of order $Y_\ell^2 Y_\nu^2$ comes from a diagram of the topology of Fig. 7(a). Notice that the indices of δZ_E are transposed. This is appropriate because, in the figure, the direction of the arrows is reversed on the E lines. In addition to the two-loop diagram, there is a one-loop diagram involving the one-loop δZ_L counterterm. This diagram has topology shown in Fig 7(b) and has the value

$$\delta Z_E^{ji} = 2Y_{\ell i}(Y_\nu^{ki})^* Y_\nu^{kj} Y_{\ell j}\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{(k_E^2)^2}\frac{1}{(4\pi)^2}\frac{1}{\epsilon}\ , \tag{43}$$

where $\epsilon = (4-d)/2$.

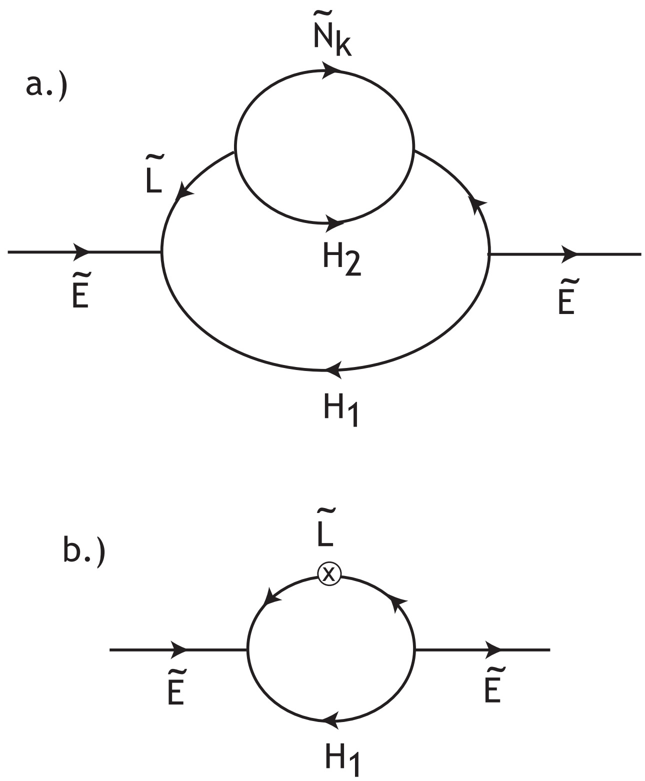

Figure 7: The structure of the two-loop contributions to $\Delta m^2_{\tilde{E}}$ that involve the neutrino Yukawa couplings. All possible particles from each supermultiplet should be put on each line of each diagrams: (a) proper two-loop contributions; (b) one-loop diagrams containing one-loop counterterms for $\Delta m^2_{\tilde{L}}$.

The contributions to $\delta m^2_{\tilde{E}}$ from one-loop diagrams and from two-loop diagrams of the form of Fig. 7(a) are given by

$$\begin{aligned}
\delta m^{2\,ji}_{\tilde{E}} = &-2Y^2_{\ell i}\delta_{ij}\int \frac{d^dp_E}{(2\pi)^d}\frac{2m_0^2+a_0^2}{(p_E^2)^2} \\
&+2\,Y_{\ell i}(Y_\nu^{ki})^*Y_\nu^{kj}Y_{\ell j}\int\frac{d^dp_E}{(2\pi)^d}\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{p_E^2[(k_E-p_E)^2+M_k^2]^2(k_E^2)^2} \\
&\qquad\cdot\left(5m_0^2+4a_0^2-\frac{2m_0^2M_k^2}{(k_E-p_E)^2+M_k^2}\right) \\
&-2Y_{\ell i}(Y_\nu^{ki})^*Y_\nu^{kj}Y_{\ell j}\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{(k_E^2)^2}\frac{1}{(4\pi)^2}\frac{1}{\epsilon}(5m_0^2+4a_0^2)\ ,
\end{aligned}\tag{44}$$

Again, we only consider corrections involving the Y_ν that will contribute to the EDMs. The first line of (44) gives the complete 1-loop contribution. The second line gives the 2-loop contribution proportional to $Y_\ell^2Y_\nu^2$. This contribution comes from diagrams of the topology of Fig. 7(a). To compute $\delta m^2_{\tilde{E}}$, we put $\tilde{E}$ on the external lines and insert m_0^2 into the propagators or a_0^2 into the vertices in all possible ways. The final piece comes from diagrams of the topology of Fig. 7(b) with the counterterms associated with the one-loop corrections to Z_L, $m^2_{\tilde{L}}$, and Z_A.

Note the order of the indices on $\delta m^{2\,ji}_{\tilde{E}}$ in these contributions; this reflects the reversed direction of arrows on the external lines in Fig. 7. Also, note that the integrals over k_E contain superpartners $\tilde{L}$, $\tilde{H}_1$ that have masses of the TeV scale rather than the right-handed neutrino scale. These integrals are potentially infrared divergent, and we will replace $(k_E^2)\to(k_E^2+M^2_{\rm SUSY})$ to regulate this divergence.

To compute the final mass insertion $\Delta m^2_{\tilde{E}}$, we must now make the redefinitions in (12). If we expand in the Yukawa couplings, we find

$$\Delta m^2_{\tilde{E}} = (\delta m^2_{\tilde{E}} - m_0^2\delta Z_E) + [\ \delta V\ , (\delta m^2_{\tilde{E}} - m_0^2\delta Z_E)\,]\ +\cdots\ , \tag{45}$$

where δV is introduced in (10). To give a nonzero diagonal element in (15), we must expand the quantities in the first term to order $Y_\ell^2Y_\nu^2$. In the second term, we will obtain a nonzero contribution to (15) by taking the one-loop expressions for $\delta m^2_{\tilde{E}}$ and δZ_E together with the one-loop expression for δV that follows from the δZ_L contribution to (10). Note, while the flavor-independent gauge corrections to $\delta m^2_{\tilde{E}}$ and δZ_E commute with δV, the first terms in Eqs. (42) and (44), although flavor-conserving, do not commute with δV. That is why we have dropped the gauge correction in Eqs. (42) and (44) while we have kept the Y_ℓ^2 terms. According to the above equation,

$$\delta V Y_\ell^2 - Y_\ell^2\delta V = Y_\ell(\delta Z_L)^*Y_\ell\ . \tag{46}$$

If we recognize that the one-loop expressions for $\delta m^2_{\tilde{E}}$ and δZ_E are proportional to Y_ℓ^2, we can use this expression to evaluate the second term of (45). Inserting the value of δZ_L given in (17) and transposing the matrix, we find a contribution of the same structure $Y_\ell Y_\nu^\dagger Y_\nu Y_\ell$ that we have in the other contributions to $(\Delta m^2_{\tilde{E}})^T$.

Our final result for $\Delta m^2_{\tilde{E}}$ is then

$$\begin{aligned}
\Delta m^{2\,ji}_{\tilde{E}} &= 2Y_{\ell i}(Y_\nu^{ki})^* Y_\nu^{kj} Y_{\ell j} \\
&\cdot\Big\{\int \frac{d^dk_E}{(2\pi)^d}\frac{d^dp_E}{(2\pi)^d}\frac{1}{(k_E^2+M^2_{\rm SUSY})^2 p_E^2((k_E-p_E)^2+M_k^2)}\Big(6m_0^2+4a_0^2-\frac{2m_0^2M_k^2}{(k_E-p_E)^2+M_k^2}\Big) \\
&\qquad -\int\frac{d^4k_E}{(2\pi)^4}\frac{1}{(k_E^2+M^2_{\rm SUSY})^2}\cdot\frac{1}{(4\pi)^2}(\frac{1}{\epsilon})(6m_0^2+4a_0^2) \\
&\qquad -(\frac{1}{(4\pi)^2}\log\frac{M^2_{\rm GUT}}{M^2_{\rm SUSY}})(\frac{1}{(4\pi)^2}(\log\frac{M^2_{\rm GUT}}{M_k^2}+1))(3m_0^2+a_0^2)\Big\}\ . \qquad (47)
\end{aligned}$$

The two-loop integrals are standard forms that are evaluated, for example, in the Appendices of [21] and [22]. Using these results, we find for the off-diagonal elements of $\Delta m^2_{\tilde{E}}$

$$\begin{aligned}
\Delta m^{2\,ji}_{\tilde{E}} &= \frac{2}{(4\pi)^4}Y_{\ell i}(Y_\nu^{ki})^* Y_\nu^{kj} Y_{\ell j} \\
&\cdot\Big\{(6m_0^2+4a_0^2)\Big[\frac{1}{2}\log^2\frac{M^2_{\rm GUT}}{M_k^2}+\log\frac{M^2_{\rm GUT}}{M_k^2}\log\frac{M_K^2}{M^2_{\rm SUSY}} \\
&\qquad +\log\frac{M^2_{\rm GUT}}{M^2_{\rm SUSY}}+\frac{1}{2}-\frac{\pi^2}{6}\Big] \\
&\quad -2m_0^2\log\frac{M_k^2}{M^2_{\rm SUSY}}-(3m_0^2+a_0^2)\log\frac{M^2_{\rm GUT}}{M^2_{\rm SUSY}}(\log\frac{M^2_{\rm GUT}}{M_k^2}+1)\Big\}\ . \qquad (48)
\end{aligned}$$

This formula is the exact result to order $Y_\ell^2 Y_\nu^2$ with ultraviolet regularization by minimal subtraction at $M_{\rm GUT}$. It does not assume that the right-handed neutrino masses are hierarchial. The dependence on $M_{\rm SUSY}$, with terms of at most one logarithm, is consistent with renormalization group evolution from the heavy neutrino scale to the weak scale. The leading logarithmic terms in this expression are in precise agreement with the result of Masina [11] after correction of a small algebraic error.

The dominant contribution to the EDMs for large $\tan\beta$ is now found by inserting both $(\Delta m^2_{\tilde{E}})^T$ and $\Delta m^2_{\tilde{L}}$ into the vertex diagram as shown in Fig. 2(b). The imaginary part of the diagram is proportional to the structure (15). The contributions to this formula have up to three powers of logarithms. Just as in the evaluation of C_i, we can take advantage of the fact that we are computing the imaginary part of the product of Hermitian matrices, which picks out the antisymmetric product of these matrices. In this case, the result contains the structure

$$\mathrm{Im}[(Y_\nu^{ki})^* Y_\nu^{kj} m^2_{\ell j}(Y_\nu^{mj})^* Y_\nu^{mi}]\ , \qquad (49)$$

contracted by a function of M_k and M_m. Note that the structure in (49) is antisymmetric in the right-handed neutrino flavor indices k and m. When we antisymmetrize the expression contracted with this structure, the leading term with $\log^3(M_{\rm GUT}^2/M_k^2)$ cancels out. However, while for C_i the next subleading logarithm also cancels out, here it does not and so, unlike the previous case, the final result will depend on $M_{\rm GUT}$. More precisely, we find

$$\begin{aligned} D_i = \mathrm{Im}\Big\{ & \frac{4}{(4\pi)^6}\frac{m_{\ell i}}{v^2\cos^2\beta}(Y_\nu^{ki})^* Y_\nu^{kj} m_{\ell j}^2 (Y_\nu^{mj})^* Y_\nu^{mi} \\ & \cdot\Big[m_0^4\Big(9\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_{\rm GUT}^2}{M_k^2}\log\frac{M_N^2}{M_k^2} + 9\log^2\frac{M_N^2}{M_k^2}\log\frac{M_N^2}{M_m^2} \\ & \qquad +6\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_N^2}{M_k^2} + 3\log^2\frac{M_N^2}{M_k^2} + (7-\pi^2)\log\frac{M_N^2}{M_k^2}\Big) \\ & +a_0^2 m_0^2\Big(9\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_{\rm GUT}^2}{M_k^2}\log\frac{M_N^2}{M_k^2} + 9\log^2\frac{M_N^2}{M_k^2}\log\frac{M_N^2}{M_m^2} \\ & +14\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_N^2}{M_k^2} + 5\log^2\frac{M_N^2}{M_k^2} + 4\log\frac{M_N^2}{M_k^2}\log\frac{M_N^2}{M_{\rm SUSY}^2} + (7-3\pi^2)\log\frac{M_N^2}{M_k^2}\Big) \\ & +a_0^4\Big(2\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_{\rm GUT}^2}{M_k^2}\log\frac{M_N^2}{M_k^2} + 2\log^2\frac{M_N^2}{M_k^2}\log\frac{M_N^2}{M_m^2} \\ & \qquad +4\log\frac{M_{\rm GUT}^2}{M_N^2}\log\frac{M_N^2}{M_k^2} + 2\log^2\frac{M_N^2}{M_k^2} + (2-\frac{2}{3}\pi^2)\log\frac{M_N^2}{M_k^2}\Big)\Big]\Big\} \ . \end{aligned} \tag{50}$$

The parameter M_N is a mean right-handed neutrino mass. The precise definition of this mass is unimportant, because, as in (23), the various factors of M_N cancel out of (50) when we use the antisymmetry of the structure $\mathrm{Im}[Y_\nu^4]$. It is convenient to choose M_N as the geometric mean of the M_k to minimize the individual logarithms in (50).

If the right-handed neutrino masses are strongly hierarchial, as was assumed by [11], (50) is enhanced by three large logarithmic factors. The formula we have given here is valid for any right-handed neutrino spectrum; for a spectrum without large hierarchies, the leading term still has two large logarithms. We also confirm the result of [11] that the logarithmic dependence on $M_{\rm SUSY}$ cancels in the leading order of logarithms, though a small dependence does remain in a subleading term. The coefficient of our leading term is almost identical to that found by Masina, with only small algebraic corrections in the terms involving a_0.

From this expression we obtain lepton EDMs of the form of (34) with

$$F_{25i} = -\frac{8\alpha}{(4\pi)^7}\left(\frac{V_{01a}}{c_w}\right)\left(\frac{V_{01a}}{c_w}+\frac{V_{02a}}{s_w}\right)\frac{\mu m_{\ell i}^2 m_a}{|m_a|^8 v^2}\ \frac{\tan\beta}{\cos^2\beta}$$

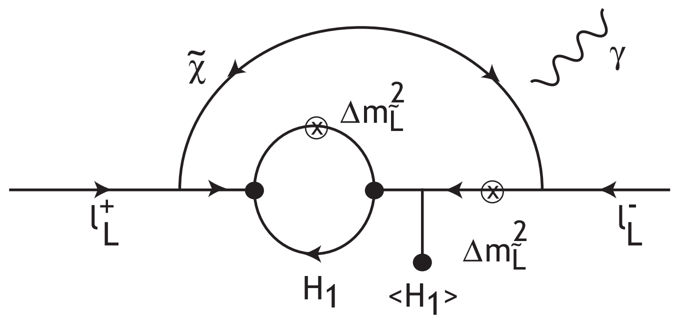

Figure 8: Sub-dominant diagram contributing to EDMs. The vertices marked by heavy dots are A_ℓ vertices, and the marked insertion is a one-loop correction to $m^2_{\tilde{L}}$. All of the momenta flowing through the indicated loops are of order $M_{\rm SUSY}$.

$$\cdot {\rm Im}[(Y_\nu^{ki})^* Y_\nu^{kj} m^2_{\ell j} (Y_\nu^{mj})^* Y_\nu^{mi}]\ h(\frac{m^2_{\tilde{L}}}{m^2_a}, \frac{m^2_{\tilde{E}}}{m^2_a})$$
$$\cdot \Big[(9m_0^4 + 9a_0^2 m_0^2 + 2a_0^4)\Big(\log\frac{M^2_{\rm GUT}}{M^2_N}\log\frac{M^2_{\rm GUT}}{M^2_k}\log\frac{M^2_N}{M^2_k} + \log^2\frac{M^2_N}{M^2_k}\log\frac{M^2_N}{M^2_m}\Big)\Big]\Bigg\}\ . \tag{51}$$

where $h(x_L, x_E)$ is given in Appendix C. In the above formula, we have kept only the leading logarithmic terms, that is, terms with two large logarithms in the case of a general right-handed neutrino mass spectrum and with three large logarithms in the case of a hierarchial mass spectrum. If we wish to give an expression valid, in the general case, at the level of one large logarithm, we should include the corrections to D_i from the TeV threshold, replacing the $M_{\rm SUSY}$ by the actual masses of $\tilde{L}$ and H_1. At the same time, we must include an additional contribution, shown in Fig. 8, involving a two-loop integral with momenta at the TeV scale. An analysis to this accuracy is beyond the scope of this paper.

8 Discussion

In this paper, we have re-evaluated the contributions from neutrino Yukawa couplings to the lepton EDMs. In contrast to previous studies, we have shown that, in the mass basis of charged leptons, up to two-loop level, neutrino Yukawa couplings do not induce an imaginary part to the diagonal elements of the A_ℓ term. However, complex neutrino Yukawa couplings can create EDMs for charged leptons through differences in the renormalization of the A_ℓ terms and the slepton masses terms, through the diagrams shown in Figs. 1 and 2. Our expressions for the lepton EDMs have the same structure in terms of the neutrino Yukawa couplings as those previously given in [10] and [11]. However, the form of the integrals contributing to F_{25i} is different because there is an extra mass insertion. Further, the overall size of the effect is decreased from the previous estimates, especially in the region of low $\tan\beta$.

There is an important test for the origin of lepton EDMs in the neutrino sector. While complex a_0 and μ induce EDMs both for the charged leptons and for the neutron, effects from the neutrino sector give zero EDM for the neutron while making nonzero contributions for the leptons. However, an EDM present only for leptons could in principle arise from an imaginary part to the A_ℓ coefficient or the neutrino B term as well as from loop effects involving Y_ν. It is interesting to compare the magnitudes of the effects from loop level or tree level CP-violating contributions.

In the low $\tan\beta$ region, the effect that we have computed in (38) gives a lepton EDM of the order of magnitude

$$d_i \sim 10^{-29}\, Y_\nu^4 \, \log \frac{M_3^2}{M_1^2} \left(\frac{200 \;\; \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \left(\frac{m_{\ell i}}{m_e}\right) \;\text{e cm}. \tag{52}$$

The effect from the large $\tan\beta$ region has a double logarithmic enhancement with respect to this value. If we estimate $\log^2(M_{\mathrm{GUT}}^2/M_N^2) \sim 200$, the effect that we have computed in (51) gives a lepton EDM of the order of magnitude

$$d_i \sim 10^{-29} \left(\frac{\tan\beta}{10}\right)^3 Y_\nu^4 \, \log \frac{M_3^2}{M_1^2} \left(\frac{200 \;\; \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \left(\frac{m_{\ell i}}{m_e}\right) \;\text{e cm}. \tag{53}$$

These estimates can be compared to the current best limit on the electron EDM, $d_e < 1.6 \times 10^{-27}$ e cm [23]. To achieve an EDM close to the current bound, we would need to have $Y_\nu^4 \log(M_1/M_3) \sim 100$. However, the experimental limit on the rate of $\mu \to e\gamma$ places a limit on the Y_ν matrix elements [1],

$$Y_\nu^{*ke} Y_\nu^{k\mu} \log \frac{M_{\mathrm{GUT}}^2}{M_k^2} < 0.1 \tan\beta \;, \tag{54}$$

so it seems unlikely to have such large values of the Y_ν. On the other hand, the effect of the neutrino B term leads to a potentially much larger estimate for electron EDM,

$$d_i \sim 10^{-27} \frac{\mathrm{Im}(B_\nu)}{M_{\mathrm{SUSY}}} Y_\nu^2 \left(\frac{200 \;\; \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \frac{m_{\ell i}}{m_e} \;\text{e} \;\; \text{cm} \;. \tag{55}$$

This could easily saturate the present bound. Also, we expect $d_\mu \sim m_\mu/m_e d_e$, so if d_e is close to its present bound, d_μ should also be observable in future muon storage ring experiments [24]. If complex Yukawa couplings are the only source of CP-violation and $Y_\nu \sim 1$, the electron EDM should still be observed in the next generation of experiments, which aim for sensitivity to $d_e \sim 10^{-29}$ e cm [25].

Over the longer term, CP-violating effects of complex neutrino Yukawa couplings can also be probed by lepton flavor oscillation in slepton production at colliders [26], and perhaps also in sneutrino-antisneutrino oscillation [27]. Better understanding of the systematics of leptogenesis can also play a role on constraining the neutrino Yukawa couplings. All of this information will complement the knowledge that we are gaining from neutrino oscillation experiments to help us build a complete picture of the neutrino flavor interactions.

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A Corrections to the lepton A term

In any given loop order, the contributions of the heavy singlet lepton N will be a polynomial in the Yukawa coupling coefficients in (1). Since in the basis we are using Y_ℓ is real and diagonal, nontrivial flavor effects will come only from polynomials in Y_ν^{ij}. In particular, for the problem at hand, we are interested in polynomials that contribute to the A_ℓ term and have a nonzero imaginary part.

A given diagram with only one N line could, in principle, contain structures $Y_\nu \cdot Y_\nu^*$, $Y_\nu \cdot Y_\nu$, or $Y_\nu^* \cdot Y_\nu^*$. The vertex A_ℓ conserves the number of H_2 (in fact H_2 has no A_ℓ coupling). However, the vertices A_ν and Y_ν change the number of H_2 by one unit. Since we ignore the masses of L and H_2 in diagrams involving N, the L and H numbers are conserved by internal propagators. Therefore, any radiative correction to A_ℓ has equal numbers of Y_ν and Y_ν^*.

Consider a diagram contributing to A_ℓ with only one N line and $(n+m)$ Y_ℓ vertices. From the above result, we see that the most general polynomial that can appear in such diagrams is

$$(Y_\ell^j)^n \sum_k (Y_\nu^{kj})^* Y_\nu^{ki} f(M_k) (Y_\ell^i)^m \tag{56}$$

whose diagonal elements are purely real. Notice that in the case of one-loop diagram shown in Fig. 4, $n=1$, $m=0$ and the matrix δZ_A [defined in (7)] is Hermitian.

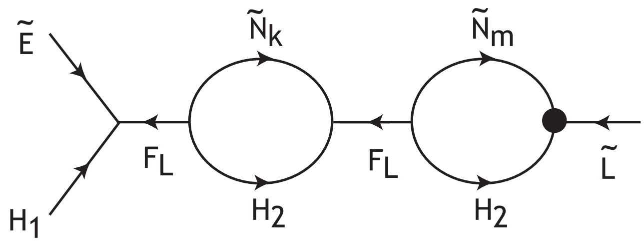

Figure 9: A contribution to the renormalization of A_ℓ in two-loop order from two one-loop diagrams.

Now let us focus on two-loop diagrams with more than one N line. In such diagrams four A_ν-vertices are involved. A contribution from a product of two one-loop diagrams, as shown in Fig. 9, has the polynomial structure

$$\sum_{mn} Y_\nu^{m\alpha} Y_\nu^{mk*} Y_\nu^{nk} Y_\nu^{n\beta*} f_1(M_m) f_2(M_n) \ . \tag{57}$$

It is non-trivial but easy to show that the functions f_1 and f_2 are of the same form. As a result, the matrix in (57) is Hermitian.

This brings us to irreducible two-loop diagrams contributing to A terms. The structures of these diagrams fall into three categories: 1) they can be of the form

$$\sum_{mn} Y_\nu^{mi} Y_\nu^{mk*} Y_\nu^{nk} Y_\nu^{nj*} g_1(M_n, M_m) \ , \tag{58}$$

2) they can be of the form

$$\sum_{mn} Y_\nu^{mi} (Y_\nu^{mj})^* Y_\nu^{nk} (Y_\nu^{nk})^* g_2(M_n, M_m) \ , \tag{59}$$

3) or they can be of the form

$$\sum_{m,n,k} Y_\nu^{mi} Y_\nu^{mk} Y_\nu^{nk^*} (Y_\nu^{nj})^* g_3(M_m, M_n) \ , \tag{60}$$

where g_1, g_2, g_3 are real functions of M_n and M_m. The structure shown in (59) is manifestly Hermitian. If the functions $g_1(M_m, M_n)$ and $g_3(M_m, M_n)$ are symmetric under $M_m \leftrightarrow M_n$, the structures appearing in (58) and (60) will be Hermitian also. It is not very obvious that these functions have the required symmetry. But it is not difficult to show this by explicit examination of the diagrams. All the relevant diagrams are shown in Fig. 10. Since the momenta propagating in the loops are of order of M_N, we can neglect the external momenta, which for our purposes are of order of $M_{\rm SUSY}$. With this simplification, it can be seen that all these diagrams are symmetric under $M_m \leftrightarrow M_n$. This completes the proof that, up to two-loop level, the diagonal elements of A_ℓ remain real.

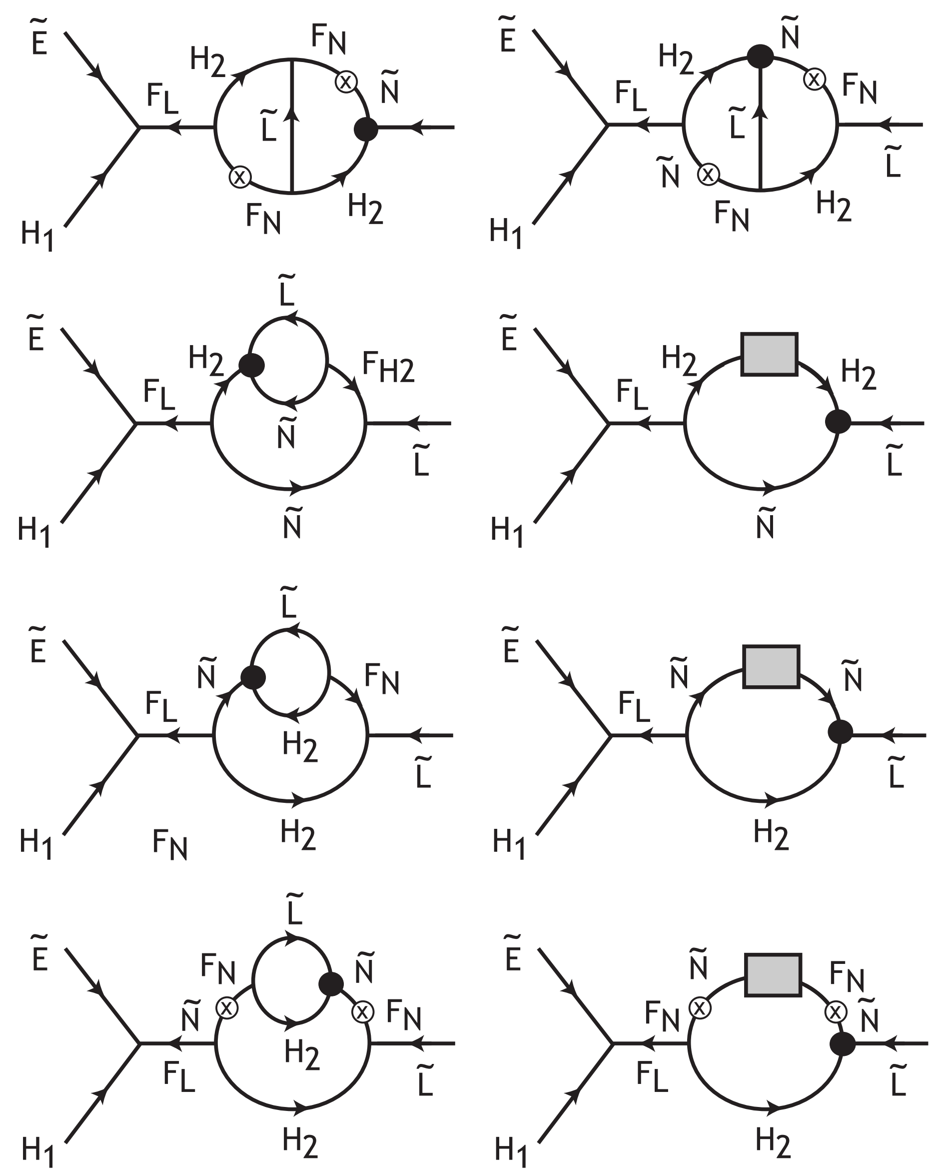

Figure 10: Irreducible two-loop diagrams contributing to A_ℓ. The shaded boxes represent the full one-loop propagator corrections from the N, H_2 supermultiplets.

B Expansion of $\Delta\mathcal{A}$, eq. (12), to order Y_ν^4

In Section 3, we claimed that the diagonal matrix element

$$\Big[(1+\delta U)(1+\delta Z_L)^{1/2}(1+\delta Z_A)(1+\delta Z_L)^{-1/2}(1+\delta U)^{-1}\Big]^{ii} \tag{61}$$

is real through order Y_ν^4. To order Y_ν^2, this is easy to see: The matrix element is the matrix element of the sum

$$(\delta U + \frac{1}{2}\delta Z_L + \delta Z_A - \frac{1}{2}\delta Z_L - \delta U) = \delta Z_A \tag{62}$$

and δZ_A is Hermitian.

Working to order Y_ν^4, we first consider the separate contributions of order Y_ν^4 from each factor of δZ_A, δZ_L, and δU. The factors of δU cancel. The contributions from δZ_A and δZ_L are diagonal elements of Hermitian matrices and thus manifestly real.

In addition, we must look at contributions in which two of these objects at a time are expanded to order Y_ν^2. To analyze these terms, we need the expressions for δZ_L and δZ_A given in (20). We also need an expression for δU. The definition of $(1+\delta U)$ is that it diagonalizes the matrix

$$(1+\delta Z_L)^{-1/2} Y^2 (1+\delta Z_L)^{-1/2} \ . \tag{63}$$

Using first-order quantum-mechanical perturbation theory, we find that $(\delta U)_{ii} = 0$ and, for $i \neq j$,

$$(\delta U)_{ij} = \frac{Y_i^2 + Y_j^2}{Y_i^2 - Y_j^2}\frac{1}{2(4\pi)^2}(Y_\nu^{ki})^* Y_\nu^{kj}\left(\log\frac{M_{\rm GUT}^2}{M_k^2} + 1\right) \ . \tag{64}$$

Then any diagonal element of a product of any two of δZ_L, δZ_A, δU is of the form of the quantity

$$\left((Y_\nu^{ki})^* Y_\nu^{kj}[\log\frac{\Lambda^2}{M_k^2}+1]\right)\left((Y_\nu^{pj})^* Y_\nu^{pi}[\log\frac{\Lambda^2}{M_p^2}+1]\right) \tag{65}$$

multiplied by a real-valued expression. No such term has an imaginary part.

C Mass dependence of dipole matrix elements

As we have explained in Sections 6 and 7, the dipole matrix elements that contribute to lepton EDMs contain derivatives of the expression

$$\frac{1}{m_a^4} f(x_L, x_E) = \int_0^1 dz \int_0^1 dx \, \frac{z(1-z)^2}{(z m_a^2 + (1-z)(x m_{\tilde E}^2 + (1-x) m_{\tilde L}^2))^2}$$

$$= \int_0^1 dz \, \frac{z(1-z)}{m_{\tilde{E}}^2 - m_{\tilde{L}}^2} \left(\frac{1}{zm_a^2 + (1-z)m_{\tilde{L}}^2} - \frac{1}{zm_a^2 + (1-z)m_{\tilde{E}}^2} \right) . \quad (66)$$

where $x_L = m_{\tilde{L}}^2/m_a^2$, $x_E = m_{\tilde{E}}^2/m_a^2$. This expression evaluates to

$$f(x_L, x_E) = \frac{1}{2} \frac{1}{x_E - x_L} \left(\frac{1 - x_L^2 + 2x_L \log x_L}{(1-x_L)^3} - \frac{1 - x_E^2 + 2x_E \log x_E}{(1-x_E)^3} \right) . \quad (67)$$

To make one insertion of $\Delta m_{\tilde{L}}^2$, we need

$$\frac{1}{m_a^6} g(x_L, x_E) = \frac{\partial}{\partial m_{\tilde{L}}^2} \frac{1}{m_a^4} f(x_L, x_E) . \quad (68)$$

This has the value

$$g(x_L, x_E) = \frac{1}{2(x_E - x_L)^2} \left(\frac{1 - x_L^2 + 2x_L \log x_L}{(1-x_L)^3} - \frac{1 - x_E^2 + 2x_E \log x_E}{(1-x_E)^3} \right) + \frac{1}{2(x_E - x_L)} \left(\frac{5 - 4x_L - x_L^2 + 2(1 + 2x_L) \log x_L}{(1-x_L)^4} \right) . \quad (69)$$

To make one further insertion of $\Delta m_{\tilde{E}}^2$, we need

$$\frac{1}{m_a^8} h(x_L, x_E) = \frac{\partial}{\partial m_{\tilde{E}}^2} \frac{1}{m_a^6} g(x_L, x_E) . \quad (70)$$

This has the value

$$h(x_L, x_E) = -\frac{1}{(x_E - x_L)^3} \left(\frac{1 - x_L^2 + 2x_L \log x_L}{(1-x_L)^3} - \frac{1 - x_E^2 + 2x_E \log x_E}{(1-x_E)^3} \right) - \frac{1}{2(x_E - x_L)^2} \left(\frac{5 - 4x_L - x_L^2 + 2(1 + 2x_L) \log x_L}{(1-x_L)^4} + \frac{5 - 4x_E - x_E^2 + 2(1 + 2x_E) \log x_E}{(1-x_E)^4} \right) . \quad (71)$$

For $m_{\tilde{L}}^2$, $m_{\tilde{E}}^2 \gg m_a^2$, we find

$$f(x_L, x_E) \approx \frac{1}{2x_L x_E} \qquad g(x_L, x_E) \approx -\frac{1}{2x_L^2 x_E} \qquad h(x_L, x_E) \approx \frac{1}{2x_L^2 x_E^2} , \quad (72)$$

as we might have expected.

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