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Abstract

A kinetic theory of weakly damped Alfvén Eigenmode (AE) solutions strongly interacting with the continuum is developed for tokamak plasmas with reversed magnetic shear. We show that the ideal MHD model is not sufficient for the eigenmode solutions if the standard causality condition bypass rule is applied. Finite Larmor radius effects are required, which introduce multiple kinetic subeigenmodes and collisionless radiative damping. The theory explains the existence of experimentally observed Alfvénic instabilities with frequencies sweeping down and reaching their minimum (bottom).

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Introduction. In recent years theoretical and experimental studies of a special class of Alfvén plasma oscillations, known as Reversed Shear AEs (RSAEs) attracted a lot of interest. We formulated the forth order differential eigenmode equation for these modes, known as the Orr-Sommerfeld equation, which is used in studies of the hydrodynamic flow stability [1] and localization of the Bernstein waves [2]. Similar equation was analyzed in Refs. [3–6].

Unstable RSAEs have been observed on many tokamak devices [7–9] and are capable of inducing fast ion losses. Understanding RSAEs helps in studying the physics of MHD oscillations such as Alfvén and acoustic branches [10–12]. In addition, observations of RSAEs often serve as a useful diagnostic of such things as safety factor profile of the plasma, $q\left(r\right)$.

Most often RSAEs are observed with sweeping up frequency as minimum $q\left(r\right)$ value, q_{min}, decreases. Its frequency is changing from a minimum stationary value to the TAE frequency [7]. Theoretically and numerically, it was found that sweeping up RSAEs exist in ideal MHD [13, 14]. RSAE instabilities with the frequency sweeping down, are also observed prior to reaching the minimum (see for example [14]). Such relatively rare events present a challenge to theory and may indicate stronger damping or nonexistence of eigenmodes.

Up until now there was no theory that predicts the existence condition of MHD scale RSAEs in the cases of frequency sweeping down or at the bottom of the frequency sweep. In this paper we call these two cases down sweep RSAEs and sweep bottom RSAEs, respectively. Numerically, down sweep RSAEs were modeled with NOVA [15], where ideal MHD RSAEs were found for the down sweep and sweep bottom cases. Theoretical work, Ref. [16], does not predict eigenmode solutions, but finds the propagating solution of the ideal MHD Alfvén eigenmode equation in terms of the "quasi" mode with radiative boundary conditions. Another work, Ref.[6], finds strongly localized KRSAEs with the radial scale on the order of ρ_i, which are potentially stronger damped then solutions found in our work.

In this paper we present an analytical theory for the down sweep and sweep bottom RSAEs. Consistent with previous studies [1, 3] our results indicate that because of strong interaction with the Alfvén continuum the ideal MHD eigenmode equation does not allow for the physical eigenmode solution. The FLR effects are required to remove the singularity at the resonance with the continuum [3–5]. This allows to construct the continuous solution (with continuous derivative) in which the fast varying part of the solution is due to a conversion to the kinetic Alfvén wave (KAW) [17]. Nevertheless, as we demonstrate, properly constructed ideal MHD eigenfrequency and slow varying part of the solution are consistent with their kinetic real counterparts with the accuracy determined by the collisionless damping.

In contrast to previous studies of KTAEs [4] and KRSAEs [6] our RSAE solutions maintain global structure even in the limit of small FLR, which may have a profound effect on the fast ion and thermal plasma transport. Both KTAEs and (K)RSAEs can be weakly damped in this limit.

The existence of singular eigenmodes in ideal MHD approximation is consistent with the results of Ref. [15]. We call these singular solutions eigenmodes (hence, RSAEs) because their dispersion relations can be understood in terms of the quantization condition between two points of a resonance with the continuum.

Formulation of the eigenmode equation. We start from the eigenmode equation for the low frequency Alfvén oscillations [12, 18], which includes finite plasma pressure effects,

$$\hat{L}_4\phi_0 + \hat{L}\phi_0 \; + \; 2m^2\hat{Q}\phi_0 = 0, \tag{1}$$

where

$$\begin{aligned} \hat{Q} \; &= \; \alpha\frac{2\bar{\omega}^2\Delta' - \alpha k_{00}^2}{1-4k_{00}^2q^2} + \frac{\alpha\varepsilon}{2}\frac{q^2-1}{q^4} \\ &+ \; \bar{\omega}^2 \; \frac{\varepsilon\left(\varepsilon+2\Delta'\right) - \delta_{m\partial}\left(-4\Delta'+\varepsilon+\alpha\right)\left(3\varepsilon-\alpha\right)}{1-4k_{00}^2q^2}, \end{aligned} \tag{2}$$

$\alpha \equiv -R_0 q_{min}^2 \beta'$, R_0 is the major radius of the plasma center, β is the ratio of the plasma pressure to the pressure of the magnetic field, prime denotes radial derivative, and $\delta_{m\partial} = 1$ if $\partial_r^2 \gg m^2$ and $\delta_{m\partial} = 0$ if $\partial_r^2 \ll m^2$, $\hat{L} = \partial_r \left[\left(\bar{\omega}+i\eta\right)^2 - k_0^2\right]\partial_r - m^2\left(\left(\bar{\omega}+i\eta\right)^2 - k_0^2\right)$, $\partial_r = rd/dr$. The frequency in Eq.(1) is generalized to include the upshift due to Geodesic Acoustic Mode (GAM) effect $\bar{\omega} \equiv R_0\sqrt{\omega^2-\omega_G^2}/v_A$ [10], ω_G is GAM frequency, v_A is local Alfvén velocity, $k_0 = m/q\left(r\right) - n = m\iota - n$, m and n are the poloidal and toroidal mode numbers, $k_{00} = k_0\left(r_0\right)$, $q\left(r_0\right) = q_{min}$, $\varepsilon = r/R_0$, Δ' is the radial derivative of the Shafranov shift, ϕ_0 is the electric potential of the dominant poloidal harmonic. In this paper we are looking for the solutions at $\omega \geq k_{00} \geq \omega_G$. Here we introduced intrinsic net drive term $\eta < 0$ (neglecting its radial variation), which include excitation and damping (following works [19, 20]). FLR term $\hat{L}_4$ accounts for coupling to small scale kinetic Alfvén wave (KAW) at the resonance with the continuum [17, 20] $\hat{L}_4 = \partial_r\hat{\lambda}^{-2}\partial_r^3$, where $\hat{\lambda}^{-2} = \left[3\bar{\omega}^2/4 + k_0^2\left(1 - i\delta\left(\bar{\omega}/k_0\right)\right)T_e/T_i\right]\rho_i^2/r_0^2 \ll 1$.

In the sweep bottom case, $k_{00} \simeq 0$, second term in the RHS of Eq.(2) is responsible for the existence of RSAEs and is due to the averaged curvature [12, 18].

Eq.(1) is derived for high-n in the vicinity of r_0 (see Refs. [12, 18] for details), so that the radial dependencies of different terms in this equation including $\omega_G(r)$ are neglected, except for the $k_0(r)$, which determines the Alfvénic continuum if density variation is neglected.

We will study unstable modes near the instability threshold, $0 < \gamma \equiv \Im\bar{\omega} \ll |\eta|$, i.e., when modes become observable. Our theory relies on the assumption that net intrinsic mode drive is smaller than the shift of the mode frequency from the continuum $|\eta| \ll |\Re\bar{\omega} + k_{00}|$. This is important because it is the Alfvén continuum that determines the localization and the space scale of the slow varying part of the eigenmode structure. Thus the continuum has to be resolved, which implies that η should be small. The opposite case $|\eta| > |\Re\bar{\omega} + k_{00}|$ corresponds to strongly driven modes such as resonant modes.

Without loss of generality we will use a special form of the safety factor profile [18]

$$q(r) = \iota(r)^{-1} = q_{min}/\left(1 - (r - r_0)^2/w^2\right), \tag{3}$$

where one can express the q-profile width parameter as $w^2 = 2q_{min}/\left.q^{(\mathrm{ii})}\right|_{r=r_0}$. For further analysis it is convenient to rewrite Eq.(1) in a reduced form

$$L_4\phi_0 + L\phi_0 \equiv \partial_z\lambda^{-2}\partial_z^3\phi_0 + \{\partial_z D\partial_z - SD + Q\}\phi_0 = 0, \tag{4}$$

where all terms are evaluated at r_0 and we drop min subscript,

$$z^2 = x^2/S \equiv x^2\left(\sqrt{A^2 + 4B} + A\right)/2, \tag{5}$$

$x = (r - r_0)m/r_0$, $\mu = \left(\sqrt{A^2 + 4B} - A\right)S/2 = (\bar{\omega} + k_{00})/(\bar{\omega} - k_{00})$, $D = (1 - z^2)(1 + \mu z^2)$, $Q = 2S\hat{Q}/(\bar{\omega}^2 - k_{00}^2) = 2mqw^2\hat{Q}/r_0^2(\bar{\omega} - k_{00})$,

$$A = \frac{-k_{00}2r_0^2}{mqw^2}\frac{1}{\bar{\omega}^2 - k_{00}^2} = -2k_{00}\sqrt{\frac{B}{\bar{\omega}^2 - k_{00}^2}}, \tag{6}$$

$$B = \frac{r_0^4}{m^2q^2w^4}\frac{1}{\bar{\omega}^2 - k_{00}^2} > 0, \tag{7}$$

$S = \sqrt{\mu/B} = \frac{mqw^2}{r_0^2}(\bar{\omega} + k_{00})$ and $\lambda^{-2} = \Lambda^{-2}\frac{\bar{\omega}^2}{(\bar{\omega}+i\eta+k_{00})^2(\bar{\omega}+i\eta-k_{00})}$, $\Lambda^{-2} = n(\rho_i^2/w^2)[3/4 + (k_0^2/\bar{\omega}^2)(T_e/T_i)(1 - i\delta(\bar{\omega}/k_0))]$.

In a case of down sweep activity we have $\mu \ll 1$ and $q > m/n$, $k_{00} < 0$, and $A > 0$, whereas at the sweep bottom $k_{00} = A = 0$, $q = m/n$, $S = 1/\sqrt{B} = \bar{\omega}mqw^2/r_0^2$, $\mu = 1$, and

$$Q_{bott} \simeq \frac{nw^2}{\bar{\omega}r_0^2}\alpha\varepsilon\frac{q^2 - 1}{q^2}. \tag{8}$$

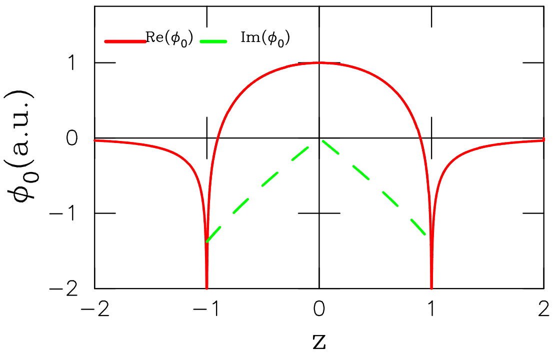

Figure 1: Ideal MHD solution for RSAE as given by Eq.(17) with $l = 2$ and $c_0 = -1\,(a.u.)$.

Nonexistence of ideal MHD down sweep RSAE solution. The ideal MHD limit of Eq.(4), i.e., $L\phi_0 = 0$, can be analyzed analytically for the down sweep case, $\mu \ll 1$, at the near threshold condition, $\bar{\omega} \simeq -k_{00}$. We rewrite the eigenmode equation at the point of expected mode peak, $z = 0$:

$$\frac{\partial}{\partial z}\left(1 - z^2\right)\frac{\partial}{\partial z}\phi_0 + (Q - S)\,\phi_0 = 0. \tag{9}$$

Eq.(9) is symmetric in z and we are looking for ideal MHD localized RSAE solution with zero boundary conditions at the infinity ($n \gg 1$). Real symmetric solutions within $0 < |z| < \infty$ satisfying zero boundary conditions at the infinity are Legendre functions [21] $\phi_0 = c_0 Q_{l-1}\,(z)$, where $l = \left[\sqrt{1 + 4\,(Q - S)} + 1\right]/2$, which implies the dispersion relation $Q - S = l\,(l - 1)$ with l positive integer. For complex frequency, applying the standard procedure with the causality condition, $\eta = 0$ and $\Re\bar{\omega} \gg \Im\bar{\omega} > 0$ we integrate $L\phi_0 = 0$ through the singular points from $\pm(1 + \varepsilon)$ to $\pm(1 - \varepsilon)$ $(0 < \varepsilon \ll 1)$ with the rule $2Q_l\,(z) \simeq P_l\,(z)\ln(\pm z - 1) \to P_l\,(z)\ln(1 \mp z) - i\pi P_l\,(z)$. Because of opposite parities of Q_l and P_l, complex ideal solution approaches the origin, $z = 0$, with discontinuities:

$$\phi_0 = c_0\phi_{0M} \equiv c_0\left[\Re Q_{l-1}\,(z) + \frac{i\pi H\,(1 - |z|)\,sign\,(z)}{2}\Re P_{l-1}\,(z)\right]. \tag{10}$$

This solution has a discontinuity in ϕ_0 (odd l) or in ϕ_0' (even l) near origin, which means that the eigenmode does not exist. If l is not integer the discontinuity is in both ϕ_0 and ϕ_0'. This can be seen from the figure 1. A similar line of arguments was used earlier to show the nonexistence of stable ideal MHD eigenmodes in a plasma with the nonuniform density [1].

As we will show the FLR term can make ϕ_0' continuous at $z = 0$ (even l), but not ϕ_0

(odd l). Hence, the ideal MHD solution and dispersion should be used to describe the slow varying part of the eigenmode solution. To improve the ideal MHD dispersion (to account for S term) we employ the quadratic form minimization. It is obtained by multiplying equation $L\phi_0 = 0$ (see Eq.(4)) by ϕ_0^* and integrating it over z. The resulting form is minimized with regard to the mode amplitude for even modes (even $l \equiv 2l'$, $l' = 1, 2, ...$) to yield

$$Q - i_{l'} S = 2l'(2l' - 1), \tag{11}$$

where for the lowest radial mode numbers $i_{l'} = 0.401,\ 0.489,\ 0.496, ...$ In the case of sweep bottom, $\mu = 1$, because of weak z-variation of the coefficient $D = 1 - z^4$ following simple WKB dispersion provides good approximation for the RSAE frequency

$$Q - S = \pi^2 (2l' - 1)^2 / 4, \tag{12}$$

where by analogy with the down sweep case we left only even radial solutions. Direct application of the numerical shooting technique to equation $L\phi_0 = 0$ shows good agreement with the dispersions Eq.(11,12) over the range of plasma parameters [22].

Conversion to KAW. We will make use of the method developed for the analysis of the Bernstein waves [2] to solve the following equation

$$\lambda^2 \partial_z \lambda^{-2} \partial_z^3 \phi_0 + \lambda^2 \left\{ \partial_z D \partial_z - SD + Q \right\} \phi_0 = 0. \tag{13}$$

We define three regions for z: outer region I $|\Re z| > 1 + \varepsilon$, nonideal region II $|\Re z - 1| < \varepsilon$, and inner region III $|\Re z| < 1 - \varepsilon$, $\varepsilon = O\left(\lambda^{-2/3}\right)$. We are looking for the solution of the weakly unstable modes with $0 < \Im \bar{\omega} \ll |\eta| \ll \bar{\omega} + k_{00}$. In this case $\Re \lambda > 0$, $\Re \lambda \gg -\Im \lambda > 0$. In the outer region I the slow varying part of the solution, ϕ_{0M}, is well described by MHD, such as down sweep solution, Eq.(17). It is the nonideal region that determines the coupling of the slow and fast varying components of the eigenmode or, in other words, MHD conversion to KAW. To analyze region II we define the new variable $y = 1 - z$. In the vicinity of $z = 1$ we can rewrite the eigenmode equation

$$\partial_y^4 \phi_0 + p\lambda^2 \left\{ y \partial_y^2 + \partial_y + Q/p \right\} \phi_0 = 0, \tag{14}$$

where p is 2 for down sweep mode or 4 for sweep bottom cases. Since $\Im y < 0$ ($\eta < 0$) it can be shown that the solution just outside of region II can be described by a combination of four independent functions [23]. Only one combination of these functions has required logarithmic behavior in the outer region: $B_3 + A_1 + Cb_0$ (notations are from Ref.[23]). In contrast to Ref.[2], due to our choice of y, function B_3 alone lies in sectors S_2 and S_1 as defined in Ref.[23] and has the same asymptotics as ideal MHD solution. Hence it has the same problem of matching at the origin as in the MHD case described above. On the other hand the combination $B_3+A_1+Cb_0$ implies following the transition rule through the turning point (details will be published [22])

$$P_l \ln(z-1) \rightarrow P_l \ln(1-z) + i\pi P_l + \frac{\sqrt{\pi}}{p^{1/4}\sqrt{\lambda} y^{3/4}} e^{-3i\pi/4 - 2i\lambda\sqrt{p} y^{3/2}/3}. \tag{15}$$

The first two terms match to ϕ_{0M} in regions I and III. The fast varying part, ϕ_{0f}, propagates into region III and allows for matching the imaginary part of the solution at $z = 0$. To find ϕ_{0f} we utilize the WKB method described in Ref. [2] and apply the anzats $\phi_{0f} = e^{i\int k(z)dz}$, where $k = O(\lambda)$. Substituting it into Eq.(13) and keeping terms up to $O(\lambda^3)$ we find the following solution $k(z) = \pm\int \lambda\sqrt{D}dz + 3i\partial_z\left[\ln(\lambda^2 D)\right]/4$. Then matching it to RHS asymptotic in Eq.(15) we find

$$\phi_{0f} = \frac{\sqrt{\pi p}\lambda_1}{2\lambda^{3/2} D^{3/4}} e^{-3i\pi/4 - i\int_z^1 \lambda\sqrt{D}dz}, \tag{16}$$

where subscript 1 means that the value is taken at the resonance point. Note that the fast solution has direction of the k propagation from $z = 0$ toward the turning points and can be directly measured via diagnostic of the phase of density perturbation to inferr the conversion to KAW. Then it follows that the solution is

$$\phi_0 = c_0\left[\phi_{0M}^* - \phi_{0f}\right]. \tag{17}$$

With ϕ_{0f} it is possible to match left and right solutions at the origin, but only for even modes (even l). In the down sweep case the first derivative matching condition is $\left[i\pi c_M/2 - \phi'_{0f}(0-\varepsilon)\right]|_{\varepsilon\rightarrow 0} = \left[-i\pi c_M/2 - \phi'_{0f}(0+\varepsilon)\right]|_{\varepsilon\rightarrow 0}$, where $c_M = P'_{2l'-1} = (-1)^{l'+1}(l'-1/2)!/(l'-1)!(1/2)!$. Since Eq.(15) is also valid for the sweep bottom case it is easy to show that ϕ_{0M} has the same singularity as in the down sweep case. The matching condition is essentially the same for the sweep bottom case as we verified numerically for low l' values. The real part of the matching condition then implies

$$\Re\int_0^1 \lambda\sqrt{D}dz = 2\pi\left(j + \frac{l'}{2} - \frac{3}{8}\right), \tag{18}$$

where $j \gg 1$ is an integer. Matching the imaginary part introduces the dissipation

$$\Im \int_0^1 \lambda \sqrt{D} dz = \ln \frac{\sqrt{\lambda_0 \pi}}{\lambda_1 \sqrt{p}}. \tag{19}$$

The latter equation gives a radiative collisionless damping rate, which is $\gamma_{rad} = \gamma + \eta < 0$:

$$\gamma_{rad} = \frac{2}{\pi \Re \Lambda_0} \frac{\sqrt{p\bar{\omega}} \ln \left(\sqrt{\lambda_0 \pi} / \lambda_1 \sqrt{p} \right)}{\sqrt{\frac{p}{2}} + \left(1 - \frac{1}{p}\right) \left[4 - p + \frac{B\left(\frac{1}{4}, \frac{3}{2}\right)}{\pi} (p-2) \right]}, \tag{20}$$

where $B\left(\frac{1}{4}, \frac{3}{2}\right)$ is the beta function. The collisionless damping is negligible in the MHD limit, in which KAW conversion is necessary for proper matching at the origin. As follows total RSAE growth rate is $\gamma = \gamma_{rad} - \eta$.

Eq.(18) predicts small kinetic splitting of the eigenfrequency due to a change in j and large eikonal (LHS of the equation), i.e., due to the introduction of the small scale length. Given the internal drive $|\eta| \gg \gamma$ one can find the frequency range over which slow varying MHD solution is valid. Such frequency range determines the number of subeigenmodes. We demonstrate this for the case of a down sweep solution at $l' = 1$. Its imaginary part becomes $-i\pi z$ at $0 < z < 1 - \eta / (\bar{\omega} + k_{00})$. Thus the range of ν in which linear in z solution exists determines the required frequency range. Based on results of Ref.[21] one can show that the linear in z solution can be constructed as $P_\nu - (\nu - 1) Q_\nu$. It is linear in z within $0 < z < 1 - \eta / (\bar{\omega} + k_{00})$ if the logarithmic contribution from Q_ν is smaller than $P_\nu(1) \simeq 1$. To be more definite we require the contribution from Q_ν to be as small as 1/3: $|(\nu - 1) \ln [\eta / (\bar{\omega} + k_{00})]| = 1/3 \ll 1$, which implies that the range of ν is $\Delta\nu = -1/3 \ln [\eta / (\bar{\omega} + k_{00})]$. Thus the ideal MHD solution is satisfied within

$$\frac{\Delta \bar{\omega}_M}{\bar{\omega} + k_{00}} \simeq -1 / \left(0.4S + Q \frac{\bar{\omega} + k_{00}}{\bar{\omega} - k_{00}} \right) \ln \left(\frac{\eta}{\bar{\omega} + k_{00}} \right). \tag{21}$$

We expect that $\Delta \bar{\omega}_M$ to be on the same order for the sweep bottom RSAEs. With Eq.(18) we find that the MHD frequency validity range is wide enough to allow for multiple eigenmodes. The number of modes is

$$N_{kRSAE} = \frac{(\bar{\omega} + k_{00}) \left(4 - p + \frac{B\left(\frac{1}{4}, \frac{3}{2}\right)}{\pi} (p-2) \right) \Re \Lambda_0}{\left(0.4S + Q \frac{\bar{\omega} + k_{00}}{\bar{\omega} - k_{00}} \right) \left| \ln \left(\frac{\eta}{\bar{\omega} + k_{00}} \right) \right| 4p \sqrt{p\bar{\omega}}}. \tag{22}$$

In realistic plasma conditions with finite FLR it has finite value.

Summary and discussion. One important implication of our theory is the existence criterion for the down sweep RSAEs, which follows from the requirement that the eigenfrequency is above the continuum, $\bar{\omega} > -k_{00}$. For the modes at the existence threshold $\bar{\omega} = -k_{00}$, $S = 0$ and $\Re Q = 2$ is exact criterion. It can be rewritten as $\bar{\omega}_{thr} = -k_{00} = \frac{nw^2}{r_0^2}\frac{\epsilon\alpha}{4}\frac{q^2-1}{q^2}$ if the frequency is small. One can see from Eqs.(5,8) that for the sweep bottom RSAE, $k_{00} = 0$, there is no such threshold. This is consistent with the experimental data [8]. It can be seen that this value is directly connected to the characteristic flatness of the safety factor profile, w. The observations of $\bar{\omega}_{thr}$ can serve as a useful diagnostic tool.

We observe that the smallest characteristic scale length coming from ϕ_{0f} is near $z = 0$, so that

$$k_r^2\rho_i^2 = \frac{\bar{\omega}^2 - k_{00}^2}{\bar{\omega}^2}\frac{1}{3/4 + (T_e/T_i)\,k_{00}^2/\bar{\omega}^2}. \tag{23}$$

It contains a small factor, which is the proximity of the eigenfrequency to the continuum minimum point, which justifies the truncation of Eq.(1) beyond the forth derivative terms. In cases when $k_r^2\rho_i^2$ is of an order unity, higher order corrections may be required.

We have shown that the kinetic effects are necessary to allow the RSAE modes to exist with the down sweep and sweep bottom frequencies. Eigenfrequencies can be found approximately within the ideal MHD dispersion relation, which are also derived. We demonstrated that the slow varying part of the mode structure is described by the ideal MHD, such as shown in Fig. 1. The theory provides collisionless radiative damping, which becomes small in the ideal MHD limit. In the case of weakly unstable persistent instabilities $\gamma \ll |\eta| = |\gamma_{rad}|$ the frequency range of unstable modes may provide the information about the internal net drive and the collisionless radiative damping. Because of weak radiative damping of multiple RSAEs and their close frequencies the transport of fast ions and background plasma can be significantly affected, which has to be further investigated.

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[1] A. V. Timofeev, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1986), vol. 9, pp. 265–298.

[2] A. L. Peratt and H. H. Kuehl, Phys. Fluids **15**, 1117 (1972).

[3] S. M. Mahajan, Phys. Fluids **27**, 2238 (1984).

[4] R. R. Mett and S. M. Mahajan, Phys. Fluids B **4**, 2885 (1992).

[5] S. Ohsaki and S. M. Mahajan, Phys. Plasmas **11**, 898 (2004).

[6] S. V. Konovalov, A. B. Mikhailovskii, M. S. Shirokov, E. A. Kovalishen, and T. Ozeki, Phys. Plasmas **11**, 4531 (2004).

[7] S. E. Sharapov, B. Alper, H. L. Berk, D. N. Borba, and et. al, Phys. Plasmas **9**, 2027 (2002).

[8] R. Nazikian, B. Alper, H. L. Berk, D. Borba, and et.al., in *Proceedings of 20th IAEA Fusion Energy Conference, Vilamoura, Portugal* (2004), IAEA-CN-116/EX/5-1, pp. 1–9.

[9] E. D. Fredrickson, N. A. Crocker, N. N. Gorelenkov, W. W. Heidbrink, S. Kubota, F. M. Levinton, H. Yuh, J. E. Menard, and R. E. Bell, Phys. Plasmas **14**, submitted (2007).

[10] M. S. Chu, J. M. Greene, L. L. Lao, A. D. Turnbull, and M. S. Chance, Phys. Fluids B **11**, 3713 (1992).

[11] B. N. Breizman, S. E. Sharapov, and M. S. Pekker, Phys. Plasmas **12**, 112506 (2005).

[12] G. Y. Fu and H. L. Berk, Phys. Plasmas **13**, 052502 (2006).

[13] B. N. Breizman, H. L. Berk, M. S. Pekker, S. D. Pinches, and S. E. Sharapov, Phys. Plasmas **10**, 3649 (2003).

[14] G. J. Kramer, R. Nazikian, B. Alper, M. de Baar, and et. al., Phys. Plasmas **13**, 056104 (2006).

[15] G. J. Kramer, N. N. Gorelenkov, R. Nazikian, and C. Z. Cheng, Plasma Phys. Contr. Fusion **46**, L23 (2004).

[16] B. N. Breizman, AIP conf. proceedings **871**, 15 (2006).

[17] A. Hasegawa and L. Chen, Phys. Rev. Letter **35**, 370 (1975).

[18] N. Gorelenkov, G. Kramer, and R. Nazikian, Plasma Phys. Control. Fusion **48**, 1255 (2006).

[19] A. B. Mikhailovskii, [Sov. Phys. JETP **41**, 890 (1975)] Zh. Eksp. Teor. Fiz. **68**, 1772 (1975).

[20] M. N. Rosenbluth and P. H. Rutherford, Phys. Rev. Lett. **34**, 1428 (1975).

[21] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions* (National Bureau of Standards, Washington, USA, 1972), tenth ed., ISBN 0-486-61272-4.

[22] N. N. Gorelenkov, E. D. Fredrickson, G. J. Kramer, R. Nazikian, and L. E. Zakharov, in *21st US Tokamak Transport Force Workshop, Boulder, Colorado: http://fusion.gat.com/conferences/ttf08* (Office of Science, DOE, 2008), to be published.

[23] A. L. Rabenstein, Arch. Ratl. Mech. Anal. **1**, 418 (1958).

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