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Perturbation of HESR Lattice due to an e-Cool Insertion [1]

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Abstract. The antiproton beam in the 1.5-15 GeV storage ring HESR, in part designed at Jülich for the FAIR complex at GSI, will contain a section with electron cooling. The electron beam in this section generates a radial electric field that produces a marked defocusing of the antiprotons in both transverse planes. This note presents a model of the cooling beam and describes the corresponding perturbation of the optics.

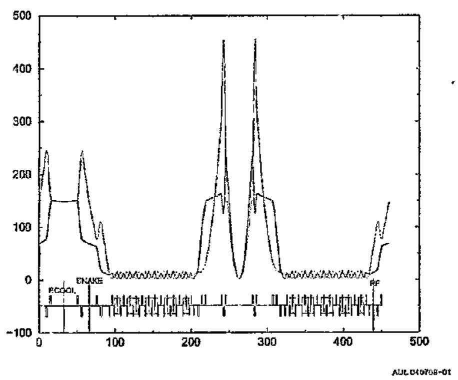

FIGURE 1. HESR. Bare lattice beam envelope (Y.Senitchev)

Main parameters of the game are: electron beam current $I_e = 0.1$ to $1\ A$, anti proton beam energy $\gamma = 0.4$ to 14.5. The unperturbed lattice of HESR [1] is in Fig. 1

Electron cooling is done in a 30 m long straight section. The (round) beam envelope function in this section varies from $\approx 150\ m$ at both ends to $\approx 148\ m$ in its center. The electron beam shape should match the gaussian profile of the beam, with an r.m.s cross section (σ_L = Raleigh length)

$$\sigma_\perp(z) = \sigma_T(0)\left[1+(z/\sigma_L)^2\right]$$

Assume that the e beam has a transverse Gaussian density. The ap-beam is initially larger than the e-beam. Its transverse size decreases during cooling, to ideally become equal to the e-beam size.

Procedures we followed for this problem (1) Integrate the eq. of motion of many ap's through the e-cooling section, in order to calculate the transformation map of the section; (2) insert the map in the *MAD* [2] description of the lattice; (3) run *MAD* to find the perturbation to the lattice; (4) correct the lattice with available quadrupoles.

RELATIVISTIC EQS OF MOTION

The profile of the static electric potential of the e- beam is ($\mu_0 c/2\pi \approx 60\,[V/A]$)

$$\Phi(x,y,z) = \frac{\mu_0 c}{2\pi\beta} I \exp\left(-\frac{x^2+y^2}{2\sigma_T^2}\right)$$

The e-beam electric field is $\vec{E} = -\vec{\Delta}\Phi$.

Using z as indep. coordinate, let's integrate numerically five 1.st order diff. eqs. for $x, \beta_x, y, \beta_y, \beta_z$

$$\begin{cases} dx/dz &= \beta_x/\beta_z \\ d\beta_x/dz &= A\left[(1-\beta_x^2)E_x - \beta_x\beta_y E_y - \beta_x\beta_z E_z\right] \\ dy/dz &= \beta_y/\beta_z \\ d\beta_y/dz &= A\left[-\beta_y\beta_x E_x + (1-\beta_y^2)E_y - \beta_y\beta_z E_z\right] \\ d\beta_z/dz &= A\gamma^2\left[-\beta_z\beta_y E_x - \beta_z\beta_y E_y + (1-\beta_z^2)E_z\right] \end{cases}$$

$$\beta^2 = \beta_x^2+\beta_y^2+\beta_z^2, \quad A = \tfrac{e}{m_p c^2}\tfrac{1}{\beta_z\gamma}\tfrac{1}{\gamma^2}$$

$1/\gamma^2$ for partial balance of electric and magnetic forces.

for a small beam near the axis, transform maps for an insertion can be found as the Jacobian of the transformation. Define a phase space vector $\vec{(v)} = (x,x',y,y')$, with values v at the end and $\hat{v}$ at the start of integration, respectively. To obtain the first order R, track a reference particle (1) and four more, each with only one component of the phase space changed by a small amount δ, in turn. Evaluate first derivatives with two point. The 1.st

[1] work performed in part under the auspices of the US Department of Energy

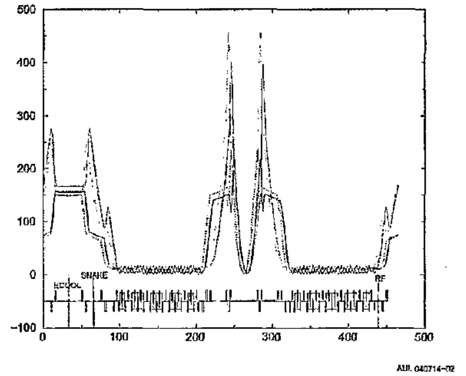

FIGURE 2. HESR modified by the e-cool. Bare: dotted line

order Jacobian is

$$R_{ij} = \frac{\partial v_i}{\partial \hat{v}_j} \approx \frac{v_i^{(2)} - v_i^{(1)}}{\delta}.$$

For the second order T, one needs the function calculated in three points. We will use 1+8 particles and the expression of one of the second derivatives is

$$T_{ijk} = \frac{\partial^2 v_i}{\partial \hat{v}_j \partial \hat{v}_k} \approx \frac{v_i^{(m)} - 2v_i^{(1)} + v_i^{(n)}}{2\delta^2}, \quad m,n = \{2...8\}$$

In a first study, transport maps were calculated for a thin beam as a Jacobian. For an electron current $I_e = 0.27$ and a beam energy $\gamma = 16$, obtain a matrix

$$R = \begin{pmatrix} 1.01016 & 30.49740 & 0.01667 & 0.15831 \\ 0.00068 & 1.01024 & 0.00111 & 0.01682 \\ 0.01667 & 0.15832 & 1.01627 & 30.55544 \\ 0.00111 & 0.01682 & 0.00108 & 1.01640 \end{pmatrix},$$

and a second order map.

With the map R shown above, as element type 'MATRIX', *MAD* gives the following results, first line w/o e-cool, second line with e-cool

ν_x	ν_y	ν'_x	ν'_y	$\beta_{x,max}$	$\beta_{y,max}$	$D_{x,rms}$	$D_{y,rms}$
8.1899	8.1367	0.3578	-0.0357	229.65	448.02	1.9844	0.0000
8.1820	8.1234	0.6479	0.6753	217.12	399.49	1.9845	0.0006

We could exactly match the e-cool section by fine tuning (with additional trim coils) the existing quadrupole doublets existing across the e-cool section.

PHASE SPACE FIGURE MATCHING

For this problem, the Jacobian has a very limited validity. The ap-beam is initially much larger than the e-beam, and will shrink to reach the same transverse size. In the process, an increasing larger fraction of the ap-beam is subjected to defocusing in both planes, producing distortions of the phase space figure. Also, the process is not conservative and the emittance of the beam increases.

A complete treatment would involve high order non symplectic maps. Here, we limited ourselves to first order, and did the following: (1) propagate a number of particles; (2) match the starting and final phase space with an r.m.s. ellipse; (3) find the linear transformation of the starting ellipse (rotation and dilation) that brings it to coincide with the final ellipse

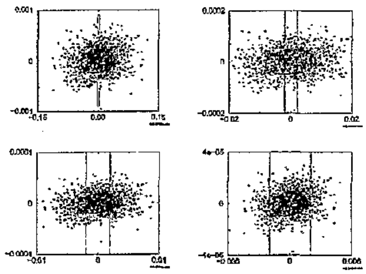

FIGURE 3. $I_e = 1A$, $\gamma = 16$, for $\sigma_a/\sigma_e = 10, 4, 2, 1$. The rms of the electron beam is also shown

The r.m.s. ellipse representing the phase space distribution in each transverse plane is (in x, x')

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon,$$

with Courant-Snyder parameters $\gamma, \alpha, \beta, \varepsilon$ that can be calculated from the covariance dispersion matrix of the transverse distribution

$$\begin{pmatrix} \beta_x\varepsilon_x & -\alpha_x\varepsilon_x & <xy> & <xy'> \\ -\alpha_x\varepsilon_x & \gamma_x\varepsilon_x & <x'y> & <x'y'> \\ <yx> & <yx'> & \beta_y\varepsilon_y & -\alpha_y\varepsilon_y \\ <y'x> & <y'x'> & -\alpha_y\varepsilon_y & \gamma_y\varepsilon_y \end{pmatrix}.$$

High Energy

At the highest end of the ap-beam energy in the HESR, for hig electron current, variable defocusing during e-cool is illustrated by a series of images created by tracking 1000 Gaussian random particles through the e-beam.

The defocusing lens (in both planes) equivalent to the e-beam is increasingly stronger as the beam shrinks (Fig. 3). The effect is small at high particle energy, due to the factor $1/\gamma^3$ in the equation of motion.

The lens can be represented by the coefficients of a simple rotation that brings the starting uprigth ellipse to coincide with the final rotated ellipse.

$$\begin{cases} \hat{x} = (\cos\theta)x + (\sin\theta)x' \\ \hat{x'} = (-\sin\theta)x + (\cos\theta)x' \end{cases}$$

An example of initial to final x-ellipse matching is shown in Fig. 4. Matrices for three cases ($\sigma_a/\sigma_e = 1.6, 1.2, 0.8$, respectively) are

$$\begin{pmatrix} 1 & -0.0011 \\ 0.0011 & 1 \end{pmatrix} \begin{pmatrix} 1 & -0.0012 \\ 0.0012 & 1 \end{pmatrix} \begin{pmatrix} 1 & -0.0014 \\ 0.0014 & 1 \end{pmatrix}$$

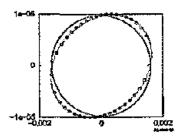

FIGURE 4. Starting and final phase space x-ellipse. Dots represent the dashed start ellipe after rotation

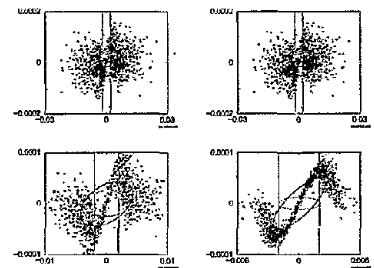

FIGURE 5. $I_e = 1A$, $\gamma = 4$. $\sigma_a/\sigma_e = 10, 4, 2, 1$

MAD results with these maps in the HESR lattice are compared with a case with no e-cooling

$\sigma_a/\sigma_e =$	no/ecool	1.6	1.2	0.8
$\nu_x =$	8.1878	8.159	8.157	8.155
$\nu_y =$	8.1346	8.106	8.104	8.101

Low Energy, High e-Current

Tracking at low energy and high electron current is shown in Fig. 5. At this beam energy the HESR lattice had to be deeply retuned, because the defocusing in both plane brought the tune close to an integer (8), where the lattice was unstable.

Resulting tunes from *MAD* for four cases of low energy, high I_e were

σ_a/σ_e	inf	10	4	2	1
ν_x	7.869	7.805	7.797	7.780	7.707
ν_y	8.178	8.215	8.207	8.185	8.070

Tracking points on ellipses yield the phase space of Fig. 6

Low Energy, Low e-Current

For low energy, decreasing the e-beam current, we obtain the results of Figs. 7 and 8

Resulting tunes from *MAD* for the four cases were

σ_a/σ_e	inf	10	4	2	1
ν_x	7.869	7.805	7.797	7.780	7.707
ν_y	8.178	8.215	8.207	8.185	8.070

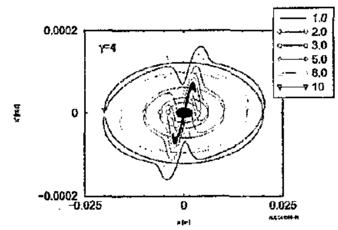

FIGURE 6. Labels mean ratio a-beam/e-beam width

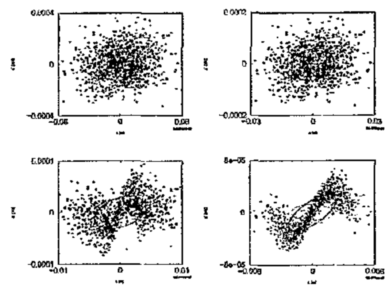

FIGURE 7. $I_e = 500mA$, $\gamma = 4$. $\sigma_a/\sigma_e = 10, 5, 2, 1$

CONCLUSIONS

Preliminary results show that the effects of the e-cooling on the HESR lattice appear large at electron currents exceeding 1 A or at anti proton energies below, say, $\gamma = 10$. To achieve a fast cooling at low energies may prove difficult

ACKNOWLEDGMENTS

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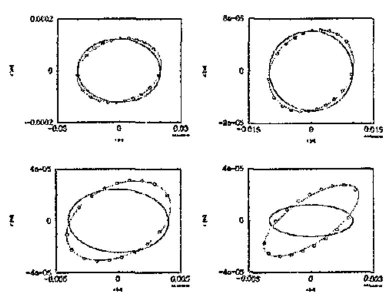

FIGURE 8. Ellipse match. same case of Fig. 7.

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