

THE STRONG COUPLING CONSTANT AT LOW Q^2

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We extract an effective strong coupling constant using low- Q^2 data and sum rules. Its behavior is established over the full Q^2 -range and is compared to calculations based on lattice QCD, Schwinger-Dyson equations and a quark model. Although the connection between all these quantities is not known yet, the results are surprisingly alike. Such a similitude may be related to quark-hadron duality.

1. The strong coupling constant

A peculiar feature of strong interaction is asymptotic freedom: quark-quark interactions grow weaker with decreasing distances. Asymptotic freedom is expressed in the vanishing of the QCD coupling constant, $\alpha_s(Q^2)$, at large Q^2 . Conversely, the fact that $\alpha_s(Q^2)$, as calculated in pQCD, becomes large when $Q^2 \rightarrow \Lambda_{QCD}^2$ is often linked to quark confinement. Since it is not expected that pQCD holds at the confinement scale and since the condition $\alpha_s(Q^2) \rightarrow \infty$ when $Q \rightarrow \Lambda_{QCD}$ is far from necessary to assure confinement¹, it is interesting to study $\alpha_s(Q^2)$ in the large distance domain.

Experimentally, moments of structure functions are convenient objects to extract α_s . Among them, Γ_1^{p-n} is the simplest to use. In pQCD, it is linked to the axial charge of the nucleon, g_A , by the Bjorken sum rule:

$$\Gamma_1^{p-n} \equiv \int_0^1 dx (g_1^p(x) - g_1^n(x)) = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s}{\pi} \right)^3 - 130.0 \left(\frac{\alpha_s}{\pi} \right)^4 - 893.38 \left(\frac{\alpha_s}{\pi} \right)^5 \right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}}{Q^{2i-2}}, \quad (1)$$

where $g_1^p(g_1^n)$ is the first spin structure function for the proton(neutron). The $\mu_i(Q^2)/Q^{i-2}$ are higher twist corrections and become important at lower Q^2 . This series, usually truncated to leading twist and to 3rd order, can be used to fit experimental data and to extract α_s . The higher twists can be computed with non-perturbative models or can be extracted from data, although with limited precision at the moment³. This imprecise knowledge and the break down of pQCD at low Q^2 prevent *a priori* the extraction of α_s at low Q^2 . However, an *effective* strong coupling constants

was defined by Grunberg⁴ in which higher twists and higher order QCD radiative corrections are incorporated. Eq. 1 becomes by definition:

$$\Gamma_1^{p-n} \equiv \frac{1}{6}g_A[1 - \frac{\alpha_{s,g_1}}{\pi}]. \quad (2)$$

This definition yields many advantages: the coupling constant is extractable at any Q^2 , is well-behaved when $Q^2 \rightarrow \Lambda_{QCD}$, is not renormalization scheme (RS) dependent and is analytic when crossing quark thresholds. The price to pay for such benefits is that it becomes process-dependent (hence the subscript g_1 in Eq. 2). However, as pointed out by Brodsky *et al.*⁵, effective couplings can be related to each other, at least in the pQCD domain, by “commensurate scale equations”. These relate, using different Q^2 scales, observables without RS or scale ambiguity. Thus, one effective coupling constant is enough to characterize the strong interaction.

Among the possible observables available to define an effective coupling constant, Γ_1^{p-n} has unique advantages. The generalized Gerasimov-Drell-Hearn (GDH)^{6,7} and Bjorken sum rules predict Γ_1^{p-n} at low and large Q^2 , and Γ_1^{p-n} is experimentally known between these two domains. Hence, α_{s,g_1} can be extracted at any Q^2 . In particular, it has a well defined value at $Q^2=0$. Furthermore, we will see that α_{s,g_1} might best be suited to be compared to the predictions of theories and models.

2. Experimental determination of α_{s,g_1}

A measurement of Γ_1^{p-n} at intermediate Q^2 was reported recently⁸ and was used to extract α_{s,g_1} ⁹. The results are shown by the triangles in Fig. 1, together with α_{s,g_1} extracted from SLAC data¹⁰ at $Q^2=5 \text{ GeV}^2$ (open square). Note that the elastic contribution is not included in Γ_1^{p-n} .

Γ_1^{p-n} is related to the generalized GDH sums:

$$\Gamma_1^{p-n} = \frac{Q^2}{16\pi^2\alpha}(GDH^p - GDH^n) \quad (3)$$

where α is the QED coupling constant. Hence, at $Q^2=0$, $\Gamma_1^{p-n} = 0$ and

$$\alpha_{s,g_1} = \pi. \quad (4)$$

At $Q^2 = 0$, the GDH sum rule implies:

$$\Gamma_1^{p-n} = \frac{Q^2}{16\pi^2\alpha}(GDH^p - GDH^n) = \frac{-Q^2}{8}(\frac{\kappa_p^2}{M_p^2} - \frac{\kappa_n^2}{M_n^2}) \quad (5)$$

where κ_p (κ_n) is the proton (neutron) anomalous magnetic moment. Combining Eq. 2 and 5, we get the derivative of α_{s,g_1} at $Q^2=0$:

$$\frac{d\alpha_{s,g_1}}{dQ^2} = \frac{3\pi}{4g_A} \times (\frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2}). \quad (6)$$

Relations 4 and 6 constrain α_{s,g_1} at low Q^2 (dashed line in Fig. 1). At large Q^2 , Γ_1^{p-n} can be estimated using Eq. 1 at leading twist and α_s calculated with pQCD. α_{s,g_1} can be subsequently extracted (gray band).

These data and sum rules give $\alpha_{s,g_1}(Q^2)$ at any Q^2 . A similar result is obtained using a model of Γ_1^{p-n} and Eq. 2 (dotted line). The Burkert-Ioffe¹¹ model is used because of its good match with data.

One can compare our result to effective coupling constants extracted using different processes. $\alpha_{s,\tau}$ was extracted from τ -decay data¹² from the OPAL experiment (inverted triangle). It is compatible with α_{s,g_1} . The Gross-Llewellyn Smith sum rule¹³ (GLS) can be used to form α_{s,F_3} . The sum rule relates the number of valence quarks in the hadron, n_v , to the structure function $F_3(Q^2, x)$. At leading twist, it reads:

$$\int_0^1 F_3(Q^2, x) dx = n_v \left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 \right] \quad (7)$$

We expect $\alpha_{s,F_3} = \alpha_{s,g_1}$ at high Q^2 , since the Q^2 -dependence of Eq. 1 and 7 at leading twist are identical. The GLS sum was measured by the CCFR collaboration¹⁴ and the resulting α_{s,F_3} is shown by the star symbols.

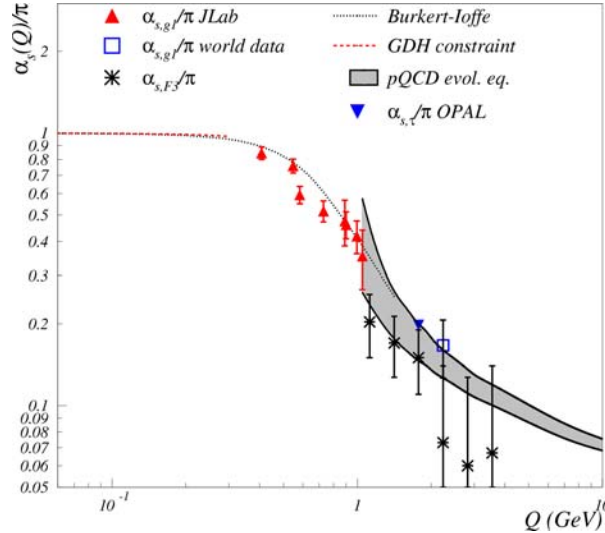


Figure 1. Extracted $\alpha_{s,g_1}(Q)/\pi$ using JLab data (up triangles), the GLS sum rule (stars), the world Γ_1^{p-n} data (open square), the Bjorken sum rule (gray band) and the Burkert-Ioffe Model. $\alpha_{s,\tau}(Q)/\pi$ from OPAL is given by the reversed triangle. The dashed line is the GDH constrain on the derivative of $\alpha_{s,g_1}/\pi$ at $Q^2=0$.

3. Comparison with theory

Just like effective coupling constants extracted experimentally, there are also many possible theory definitions for the coupling constant and, contrarily to the experimental quantities, the relations between the various definitions are not well known. Furthermore, the connection between the experimental and the theoretical quantities is not clear. Hence, the remainder of this paper is to be understood as a candid comparison of quantities *a priori* defined differently, in order to see if they share common features.

Calculations of α_s using Schwinger-Dyson equations (SDE), lattice QCD or quark models are available. Different SDE results are shown in Fig. 2. The pioneering result of Cornwall¹⁵ is shown by the blue band in the top left panel. The more recent SDE results from Fisher *et al.*, Bloch *et al.*, Maris and Tandy, and Bhagwat *et al.* are shown in top left, top right, bottom left and bottom left panels respectively. There is a good match between the data and the result from Fisher *et al.* and a fair match with the curve from Bloch *et al.* The results from Maris-Tandy, Bhagwat *et al.* and Cornwall do not match the data. The Godfrey and Isgur curve in the top right panel of Fig. 2 is the coupling constant used in the framework of hadron spectroscopy²⁰. Q^2 -behavior of coupling constants can also be compared regardless of their absolute magnitudes by normalizing them to π at $Q^2 = 0$ (These curves are not shown here). The Godfrey-Isgur, Cornwall and Fisher *et al.* Q^2 -behavior match well the data. The normalized curves from Maris-Tandy, Bloch *et al.* and Bhagwat *et al.* are slightly below the data (by typically one sigma) for $Q > 0.6$ GeV.

Gluon bremsstrahlung and vertex corrections contribute to the running of α_s . Modern SDE calculations include those²¹ but it is *a priori* not the case for the α_s used in the one gluon exchange term of the Godfrey and Isgur quark model, or for older SDE works. If so, pQCD corrections should be added to these calculations. The effect of those corrections (*on* α_{s,g_1}) is given by the ratio of α_{s,g_1} extracted using Eq. 2 to α_{s,g_1} extracted using Eq. 1 at leading twist. For both Eq. 1 and 2, Γ_1^{p-n} is given by a model¹¹. Since model and data agree well, no strong model dependence is introduced. The difference between results using Eq. 1 up to 4th and 5th order is taken as the uncertainty due to the truncation of the pQCD series. The resulting α_s are shown in the bottom right panel of Fig. 2.

Finally, we can compare lattice QCD data to our results. Many lattice results are available and are in general consistent. We chose to compare with the results of Furui and Nakajima²², see bottom left panel in Fig. 2.

They match well the data. The lowest Q^2 point is afflicted by finite size effect and should be ignored.

The match between our data and the various calculations might be surprising since these quantities are defined differently. We can try to understand this fact. Choosing Γ_1^{p-n} minimizes the rôle of resonances, in particular it fully cancels the Δ_{1232} contribution which usually dominates the moments at low Q^2 . By furthermore excluding the elastic contribution, we obtain a quantity for which coherent reactions (elastic and resonances) are suppressed and we are back to a DIS-like case in which the interpretation is straightforward. One can also possibly invoke the phenomenon of quark-hadron duality to explain why the extraction of $\alpha_{s,g1}$, using a formalism developed for DIS¹², seems to also work at lower Q^2 .

4. Conclusion

We have extracted, using JLab data at low Q^2 together with sum rules, an effective strong coupling constant at any Q^2 . A striking feature is its loss of Q^2 -dependence at low Q^2 . We compared our result to SDE and lattice QCD calculations and to a coupling constant used in a quark model. Despite the unclear relation between these various coupling constants, data and calculations match in most cases, especially for relative Q^2 -dependences. This could be linked to quark-hadron duality.

Acknowledgments

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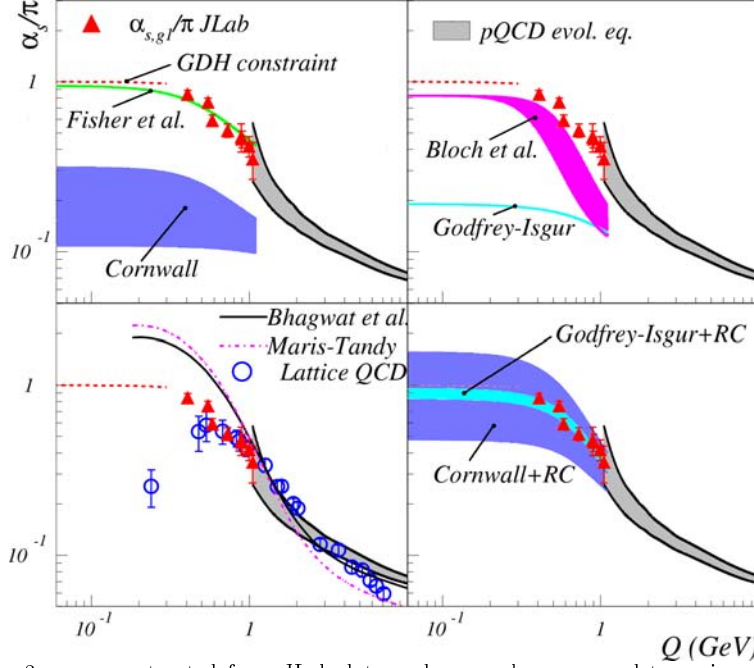


Figure 2. $\alpha_{s,g1}$ extracted from JLab data and sum rules compared to various calculations: top left panel: SDE calculations from Fisher *et al.* and Cornwall; top right panel: Bloch *et al.* (SDE) and Godfrey-Isgur (quark model); bottom left: Furui and Nakajima (lattice QCD), Maris-Tandy (SDE) and Bhagwat *et al.* (SDE); bottom right: the Godfrey-Isgur and Cornwall results with pQCD radiative corrections added.

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