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Multi-scale Discretization of Shape Contours

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ABSTRACT

We present an efficient multi-scale shape approximation scheme by adaptively and sparsely discretizing its continuous (or densely sampled) contour by means of points. The notion of *shape* is intimately related to the notion of *contour* and, therefore, the efficient representation of the contour of a shape is vital to a computational understanding of the shape. Any discretization of a planar smooth curve by points is equivalent to a piecewise constant approximation of its parameterized X and Y coordinate. Using the Haar wavelet transform for the piecewise approximation yields a hierarchical scheme in which the size of the approximating point set is traded off against the morphological accuracy of the approximation. Our algorithm compresses the representation of the initial shape contour to a sparse sequence of points in the plane defining the vertices of the shape's polygonal approximation. Furthermore, it is possible to control the overall resolution of the approximation by a single, scale-independent parameter.

Keywords: boundary, contour, discrete wavelet transform, Haar wavelet, polygonal approximation, shape representation,.

1. INTRODUCTION

The representation of the boundaries of two-dimensional planar shapes, using approximating polygons, has applications in areas as diverse as computational geometry, image understanding, and video coding. The densely sampled boundary of a planar shape typically contains a large number of points, and renders most shape processing operations, such as triangulation and skeletonization, computationally hard. Merely down-sampling the boundary point set yields an approximation that is not morphologically accurate to the original shape. What is needed is an algorithm that adaptively uses more approximating points where the boundary has significant morphological detail, and fewer points elsewhere. Such a scheme would be useful (for example) for optimally representing and encoding video objects in MPEG-4 scenes.[10] The research described here is part of a larger effort aimed at understanding images, and their contents, and forms the basis of novel skeletonization and shape feature extraction algorithms described elsewhere in these proceedings.

The notion of *shape* is closely related to the notion of contour or boundary. A shape is completely described by the specification of its spatial extent; i.e., the specification of the "inside" and the "outside" of the shape, which is equivalent to specifying the boundary of the shape. The boundary of a shape however, has a continuum of points, and is therefore not, in general, amenable to a finite representation. In order to computationally characterize a shape, it is vital to first obtain a discrete representation of its boundary in a morphologically faithful manner. This representation is necessary for subsequent processing such as triangulation, skeletonization and morphological segmentation.

The continuous boundary of a planar shape can be approximated to any degree of accuracy (i.e., at any *scale* or *resolution*) by a discrete sampling of its boundary points. However, a uniform sampling of the boundary does not distinguish between significant points (i.e., points that are essential to capture the overall shape), and insignificant ones. The significance of a boundary point depends upon the degree of accuracy with which the boundary is to be approximated. Therefore, an ideal approximation algorithm should identify morphologically salient boundary points depending upon the resolution with which the shape is to be represented. We draw on the theory of discrete wavelet transforms[1] to extract significant boundary points that best describe the contour of the planar shape. Specifically, we use the fact that discrete wavelet transforms optimally represent discretely sampled functions at various scales of resolution. To obtain a coarse approximation of the function, we need only eliminate (set to zero) all wavelet transform coefficients in the finer scales.

There have been several attempts at obtaining approximations of planar shapes.[3, 5, 6, 7] None of these attempts has addressed the fact that the significance of a boundary point in conveying the overall shape of the object depends on the resolution at which the shape is characterized. The significance of a boundary point depends on the level of detail desired in the discrete approximation. Conversely, different levels of detail determine different sets of significant boundary points.

We have developed a multi-resolution shape discretization scheme, based on the Haar wavelet transform, for connected planar shapes. The boundary points obtained from this discretization are joined pairwise, in the sequential order in which they occur on the original contour. This scheme yields a hierarchy of polygons, of increasing complexity, which adaptively capture detail in the boundary of the shape at increasing scales of resolution. When the X and Y coordinates of the boundary points are parametrically specified (parameterized by the arc length of the boundary), any discretization of the boundary can be thought of as a piecewise constant approximation of the coordinate functions. The Haar functions, consequently, are ideal for obtaining such approximations.

In order to be morphologically faithful to the shape, the discretization must be scale-adaptive to local variations of the shape's contour. Now, the discrete Haar wavelet expansion of a function yields piecewise constant approximations of the function at multiple scales. By representing the slowly varying and the rapidly varying parts of the function by Haar scaling functions and wavelets at different scales, the function can be approximated to any degree. Since the Haar wavelet is biorthogonal,[1] we use symmetric signal extensions, and do not pad the number of boundary points to the nearest higher power of 2. This allows us to apply the transform to a point set of any size without introducing (artificial) boundary artifacts. Using this idea, the coordinates of a parametrically specified curve may be jointly and adaptively approximated at varying scales to obtain a discretized representation of the curve.

2. POLYGONAL APPROXIMATION

As mentioned in the preceding section, it is often necessary to represent the continuous boundary of a planar shape using a small number of carefully selected boundary points. This representation yields a polygonal approximation to the continuous contour of the shape. Since the continuous contour has an infinite number of points, it is not amenable, in general, to finite computation. A dense and uniform sampling of the contour, such as that of the outline of a human shape shown in Fig. 1a, yields a finite representation. We will see in this section that it is possible to drastically reduce the number of points required to capture the overall structure of the planar shape by selecting only those along the boundary that yield significant morphological information.

Any discretization of the boundary can be thought of as a piecewise constant approximation of its parametrically specified coordinate functions. This observation forms the basis of our approximation algorithm. A piecewise constant approximation scheme that is scale-adapted to the local variations of the coordinate functions is, thus, the best method for discrete approximation of the continuous boundary of the shape. The resulting discretization is morphologically faithful, and at the same time, economical.

The discrete Haar wavelet expansion of a function yields piecewise constant approximations of a function at multiple scales of resolution. Thus, by representing the slowly varying and the rapidly varying parts of the functions by Haar functions at different scales, the function can be approximated to any degree of accuracy. The Haar basis is the only biorthogonal discrete wavelet basis that is also orthogonal.[1] Implementing the Haar transform as a biorthogonal transform enables the analysis of signals whose length is not a power of 2. This eliminates the need to pad or otherwise extend the size of the boundary point set to the nearest power of 2.

The first step in the approximation algorithm is the parametric representation of the X and Y coordinate functions of the points in the densely sampled boundary. The X and Y coordinate functions are parameterized by point number (the discrete, scale-free equivalent of arc length) starting from an arbitrarily selected point along the boundary.

Next, these periodic functions are expanded in a discrete wavelet basis at a coarse level of resolution as shown in Figs. 1c & 1d. To indicate the "zoom-in" capability of the discrete wavelet transform, Figs. 2c & 2d show the same X and Y functions expanded at the next finer level of resolution. It is easy to see that, in the limit, the wavelet transform can represent the coordinate functions exactly.

In the next step of the algorithm, in each piece wise constant "span" of the approximation, a user-specified threshold is compared against the approximation error. This error is defined as the ratio of the maximum perpendicular distance from a point on the boundary to the length of the approximating planar straight-line segment. This is shown in Fig. 2. Where the approximation error is too large, expanding the boundary points using wavelet functions at the next finer level of resolution refines the corresponding span. Since the refinement is carried out only where the error is too large, this scheme automatically allocates more approximating points where they are needed to reduce the approximation error. A more precise description of the algorithm follows.

Let $\{x_i, y_i\}\ i = 0 \ldots N-1$ represent the sequence of X and Y coordinate pairs corresponding to the points along the boundary of a shape (such as in Fig. 1a, for instance). Thus, $\{x_i\}$ and $\{y_i\}$ represent sequences of X and Y coordinates of the boundary point set parameterized by point number. The discrete wavelet transform (DWT) of the X coordinate sequence

$$W\{x_i\} = \{s_0, d_0, \{d\}_1, \{d\}_2, \ldots, \{d\}_L\}$$

is a multi-scale expansion of $\{x_i\}$ where s_0 is the scaling function coefficient at the coarsest scale of resolution, and $\{d\}_j$ are the wavelet coefficients at scale j. Similarly, $W\{y_i\}$ is a multi-scale expansion of $\{y_i\}$. By setting some or all of the wavelet coefficients, at selected scales of resolution, to zero, we obtain the modified DWT

$$\{\tilde{x}_i\} = \{s_0, \tilde{d}_0, \{\tilde{d}\}_1, \{\tilde{d}\}_2, \ldots, \{\tilde{d}\}_L\}$$

where $\{\tilde{d}\}_j$ denotes the set of (possibly modified) wavelet coefficients at scale j. Note that we have not altered the number of points in the DWT; only the values of some of the coefficients. By computing the inverse DWT $\{X_i\} = W^{-1}\{\tilde{x}_i\}$ and $\{Y_i\} = W^{-1}\{\tilde{y}_i\}$, we obtain an approximation to the original sequences $\{x_i\}$ and $\{y_i\}$. We now have the polygon $\{(X_i, Y_i)\}$ that approximates the original boundary $\{(x_i, y_i)\}$. By modifying different wavelet coefficients $\{\tilde{d}\}_j$, it is thus possible to obtain more accurate or less accurate representations of the original boundary as desired. Furthermore, it is possible, without any additional computation, to represent different portions of the boundary to different degrees of accuracy.

3. RESULTS

Fig. 1a shows a human shape with 3934 points on the boundary. Fig. 1c shows the parameterized representations of the X and Y coordinates of the boundary points of the human shape. The piecewise constant approximations are obtained by expanding the coordinate functions using the discrete Haar wavelet basis. First, a very coarse scale approximation is obtained for the boundary. This corresponds to approximating the coordinate functions using piecewise constant steps of large width. A user-specified tolerance is then applied to each span of the approximation to identify parts of the boundary that are poorly represented at the current scale of resolution. Boundary points represented in these spans are more accurately approximated at the next scale of resolution (piecewise constant steps of smaller width). Fig. 2b shows a finer approximation to the shape in Fig. 1a obtained by using Haar functions at the next finer resolution than those used to obtain the piece wise constant approximation shown in Figs. 1c & 1d.

The refinement criterion used is shown in Fig 2. In each span of the approximating polygon at a scale j, the ratio of the maximum deviation of the boundary of the original shape from the linear approximation to the length of the linear approximation is computed. If this deviation is greater than $T.2^j$, where T is a user-specified tolerance, the boundary points approximated in this span are more accurately represented by Haar functions at scale j+1.

Fig. 3b shows the result of refining the approximation in Fig. 1b by decreasing the tolerance value. Figs. 3c & 3d now show that the piecewise constant steps are smaller in width than those in Figs. 1c & 1d. Fig. 4 shows a hierarchy of approximating polygons obtained by varying the T. Note from Fig. 4f that at a compression ratio of more than 10:1, the polygonal approximation has captured almost all of the fine details of the boundary of Fig. 1a. Fig. 5 shows the result of applying the approximation algorithm to the boundary of a lizard shape containing 3200 points. Again, at a compression ratio of about 9:1, 5(i) has captured most of the boundary detail that describes the lizard shape.

4. CONCLUSION

We have presented an adaptive, multi-resolution algorithm for approximating the boundary of a planar shape. Our adaptive algorithm yields discrete approximations of densely sampled boundaries of planar shapes. The approximation scheme is hierarchical and may be obtained at any scale of resolution. Morphologically significant boundary points are identified in the Haar transform expansion of the X and Y coordinates of the boundary points obtained by a parametric representation. A user-specified tolerance value controls the accuracy of the approximation. The algorithm uses only addition and subtraction operations and division by 2, and can be implemented very efficiently. The algorithm is adaptive in the sense that only portions of the approximated boundary that are poorly described are selected for further refinement. Results of further geometric processing on the data sets generated by this algorithm are very encouraging, and form the subject of a paper elsewhere in these proceedings.

5. REFERENCES

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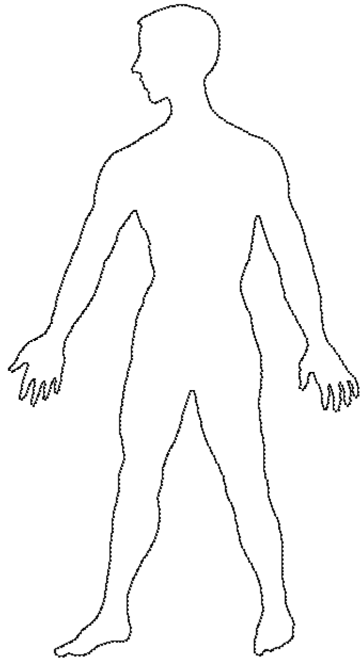
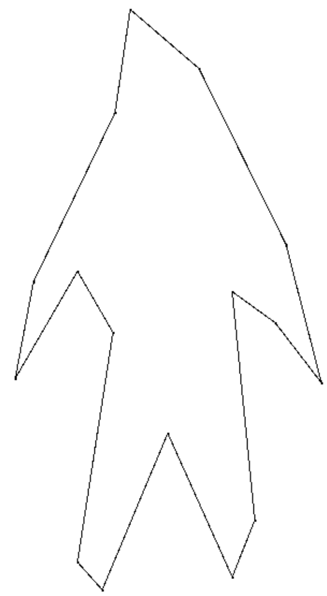
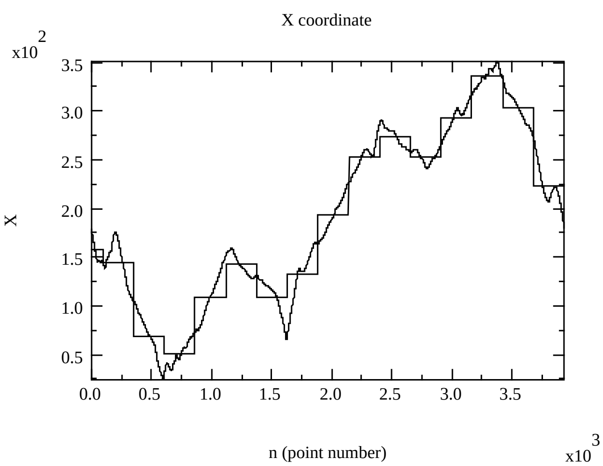


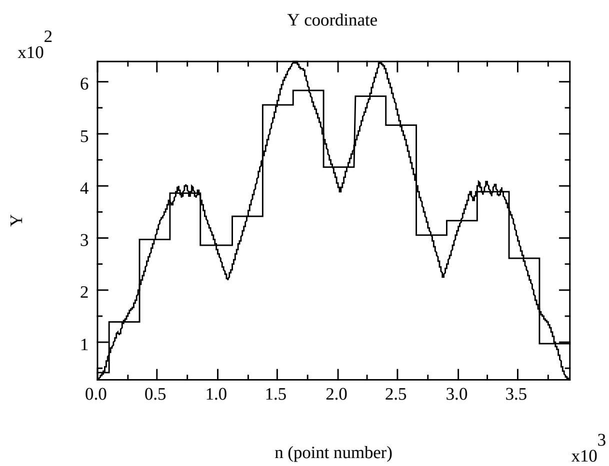
(a) Densely sampled outline of human shape.



(b) Coarse approximation to human shape.

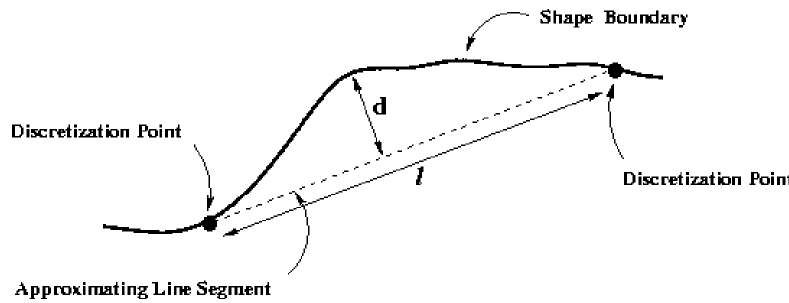


(c) X coordinate function of boundary points.



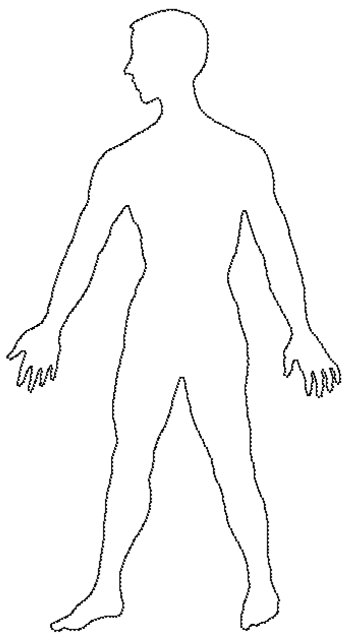
(d) Y coordinate function of boundary points.

Figure 1. Coarse Approximation to densely sampled outline of human shape

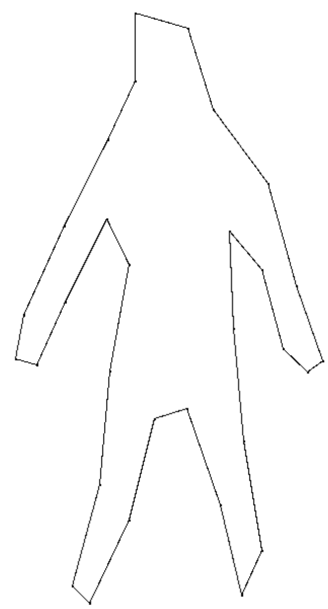


If $d/l > 2^{j}T$, the span is approximated at scale j+1
(T is a user-defined threshold at the coarsest scale)

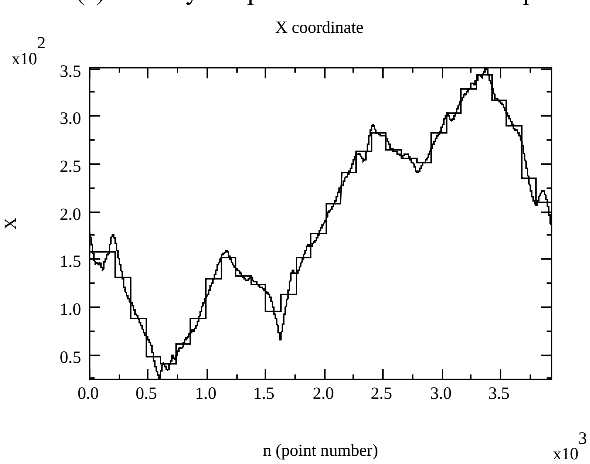
Figure 2. Span refinement criterion



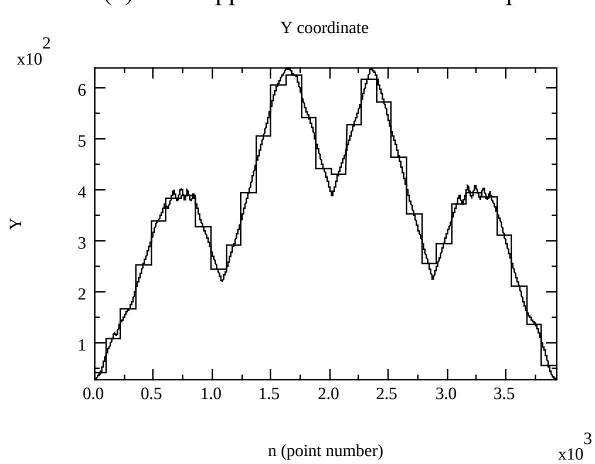
(a) Densely sampled outline of human shape.



(b) Finer approximation to human shape.

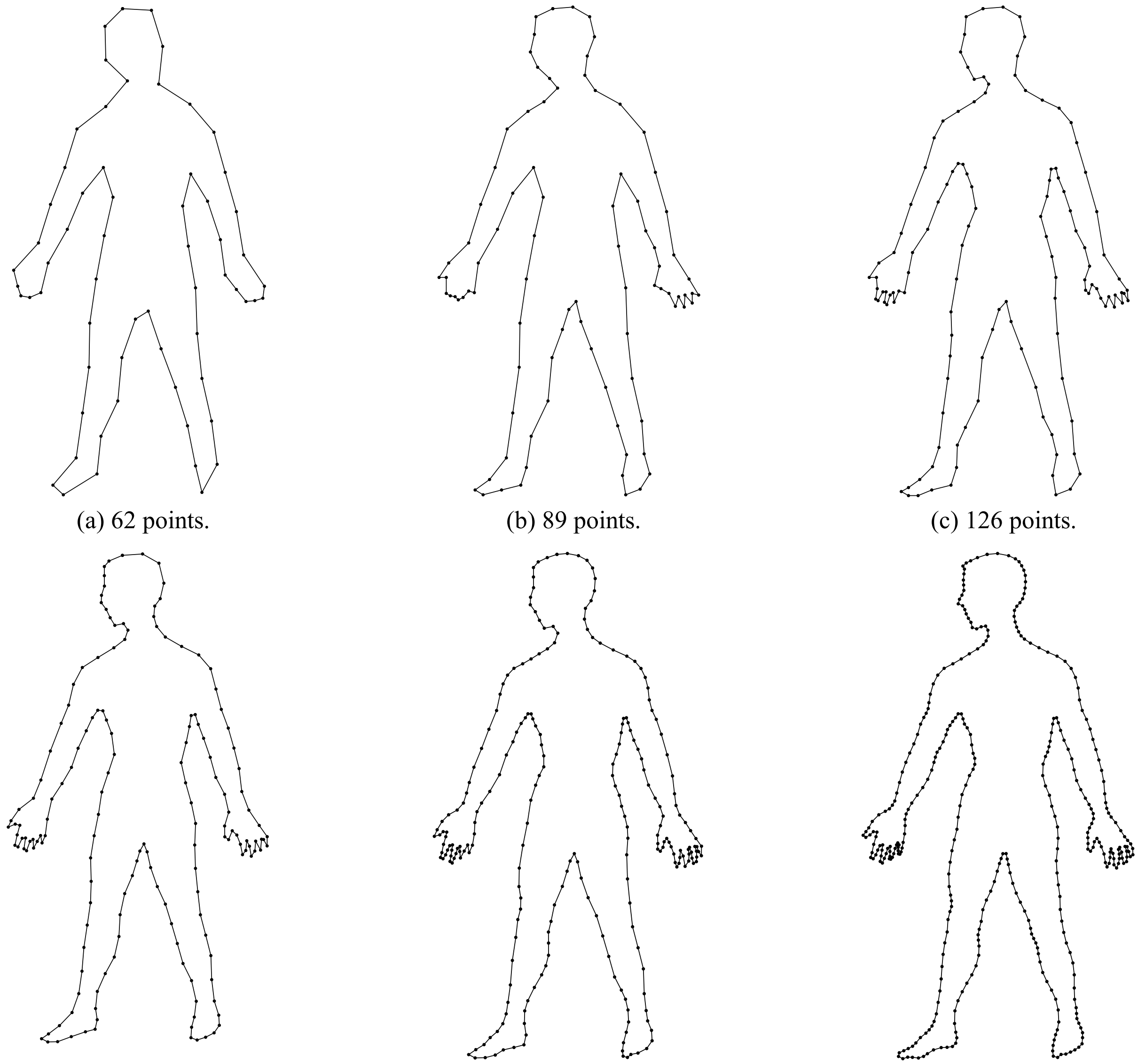


(c) X coordinate function of boundary points.



(d) Y coordinate function of boundary points.

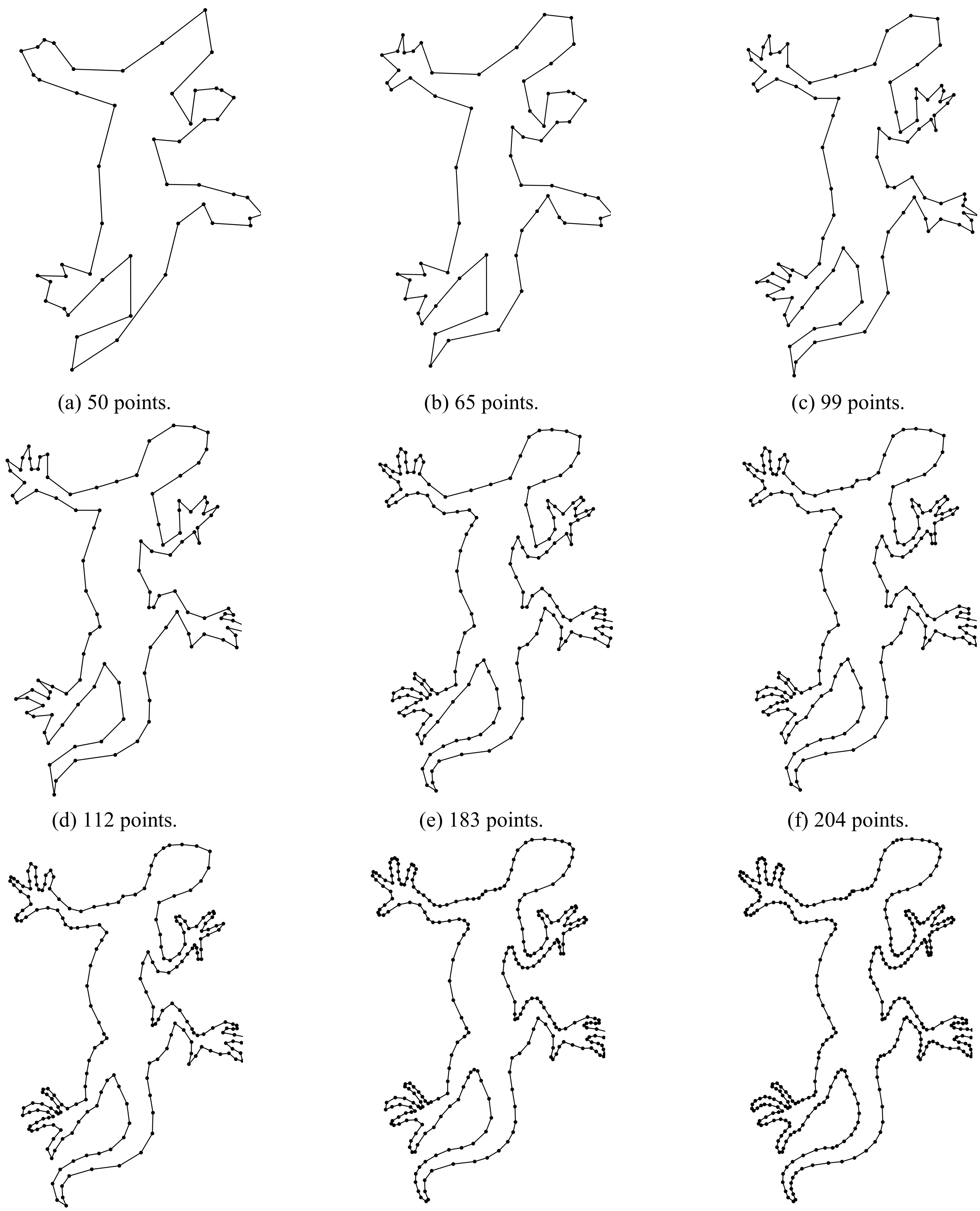
Figure 3. Finer approximation of densely sampled human shape



(a) 62 points. (b) 89 points. (c) 126 points.

(d) 161 points. (e) 239 points. (f) 365 points.

Figure 4. Successive approximations of the human shape



(a) 50 points. (b) 65 points. (c) 99 points.

(d) 112 points. (e) 183 points. (f) 204 points.

(g) 234 points. (h) 289 points. (i) 365 points.

Figure 5. Successive approximations of a "lizard" shape