

## BARYON SPECTROSCOPY AND OPERATOR CONSTRUCTION IN LATTICE QCD

LHP COLLABORATION:

S. BASAK, I. SATO AND S. WALLACE

*Department of Physics, University of Maryland, College Park, MD 20742, USA*

R. EDWARDS AND D.G. RICHARDS

*Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*

R FIEBIG

*Physics Department, Florida International University, Miami, FL 33199, USA*

G.T. FLEMING

*Sloane Physics Laboratory, Yale University, New Haven, CT 06520, USA*

U.M. HELLER

*American Physical Society, Ridge, NY 11961-9000, USA*

C. MORNINGSTAR

*Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA*

This talk describes progress at understanding the properties of the nucleon and its excitations from lattice QCD. I begin with a review of recent lattice results for the lowest-lying states of the excited baryon spectrum. The need to approach physical values of the light quark masses is emphasized, enabling the effects of the pion cloud to be revealed. I then outline the development of techniques that will enable the extraction of the masses of the higher resonances. I will describe how such calculations provide insight into the structure of the hadrons, and enable comparison both with experiment, and with QCD-inspired pictures of hadron structure, such as calculations in the limit of large  $N_c$ .

### 1. Introduction

Spectroscopy is a powerful tool for uncovering the important degrees of freedom of a physical system and the interaction forces between them. The

spectrum of QCD is very rich: conventional baryons (nucleons,  $\Delta$ ,  $\Lambda$ ,  $\Xi$ ,  $\Omega$ , and so on) and mesons ( $\pi$ ,  $K$ ,  $\rho$ , *etc.*) have been known for nearly half a century, but other, higher-lying exotic states, such as glueballs, hybrid mesons and hybrid baryons bound by an excited gluon field, and ‘multi-quark’ states, consisting predominantly of four or five quarks in the case of mesons and baryons respectively, have proved more elusive, partly because our theoretical understanding of such states is insufficient, making their identification difficult.

Interest in excited baryon resonances in particular has been sparked by experiments dedicated to mapping out the  $N^*$  spectrum in Hall B at the Thomas Jefferson National Accelerator Facility (JLab); evidence for the possible existence of a strangeness  $S = 1$   $qqqq\bar{q}$  pentaquark state, discussed in many talks at this workshop, has provided a further incentive for a detailed understanding of baryon resonances.

Much of our current understanding of conventional and excited hadron resonances comes from QCD-inspired phenomenological models. For conventional baryons, the extensive calculations by Isgur, Karl, and Capstick within a non-relativistic quark model<sup>1,2,3</sup> remain influential. However, there are a growing number of resonances which cannot be easily accommodated within quark models. States bound by an excited gluon field, such as hybrid mesons and baryons, are still poorly understood. The natures of the Roper resonance and the anomalously light  $\Lambda(1405)^-$  remain controversial. Experiment shows that the first excited positive-parity spin-1/2 baryon lies below the lowest-lying negative-parity spin-1/2 resonance, a fact which is difficult to reconcile in quark models. The question of the so-called “missing” baryon resonances is still unresolved: the quark model predicts many more states<sup>2,3</sup> than are currently known. Compared to the large number of positive-parity states, there are only a few low-lying negative-parity resonances. A quark-diquark picture of baryons predicts a sparser spectrum<sup>4</sup>. Various bag and soliton models have also attempted to explain the baryon mass spectrum. The search for the  $S = 1$  pentaquark states was spurred by a chiral soliton model. While most models expect the lightest pentaquarks to have positive parity<sup>5,6</sup>, a light, narrow isotensor state of negative parity can be accommodated within the quark model<sup>7</sup>.

Given the current intense experimental efforts in spectroscopy for baryons, the need to predict and understand the baryon spectrum from first principles is clear; lattice QCD calculations provide the means of undertaking such *ab initio* studies. The aim is not merely to obtain a set of masses for the states, but also to gain insight into the quark and gluon

structure of the states and to understand the relevant degrees of freedom; this latter aspect will be an important emphasis of this talk. The freedom to vary quark masses, numbers of quark flavours and even the gauge group enables us not only to relate lattice computations directly to experiment, but also to QCD-inspired pictures of baryon structure. Thus comparison with lattice computations has a central role when considering the rôle of QCD in the large- $N_c$  limit.

The layout of the remainder of this talk is as follows. In the next section, I will review lattice results for the lowest-lying baryon states composed of quarks with masses around that of the strange quark. I will then detail the importance of correctly including the effects of the pion cloud, and describe recent progress toward achieving that goal. The final section will outline the recent development of techniques for a comprehensive study of the masses of the higher excitations, and plans to implement these techniques.

## 2. Recent results for the baryon spectrum

The computation of the masses of the lowest-lying states has long been a benchmark calculation of lattice QCD since it provides a direct comparison with well-known experimental quantities; a recent review is provided in <sup>8</sup>. The computation is in principle straightforward:

- (1) Choose an interpolating operator  $O$  that has a good overlap with  $P$ , the state of interest,

$$\langle 0 | O | P \rangle \neq 0,$$

and ideally a small overlap with other states having the same quantum numbers.

- (2) Form the time-sliced correlation function

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x}, t) O^\dagger(\vec{0}, 0) \rangle.$$

- (3) Examine the behavior of the correlator at large Euclidean time

$$C(t) = \sum_P \frac{|\langle 0 | O | P \rangle|^2}{2m_P} e^{-m_P t}, \quad (1)$$

yielding the mass of the lightest state.

However, a precise comparison with experiment requires control and understanding of the systematic uncertainties: the extrapolation of the lattice volume  $V \rightarrow \infty$ , the control over discretisation errors by taking the lattice

spacing  $a \rightarrow 0$ , and finally, and most delicately, the chiral extrapolation in the quark mass from the values at which the computations are performed to the physical quark masses.

Whilst the benchmark calculations reviewed in Ref. 8 are in full QCD, most of the more exploratory studies described below are in the quenched approximation. The use of the quenched approximation introduces a systematic uncertainty of around 10% for most light-hadron quantities. The

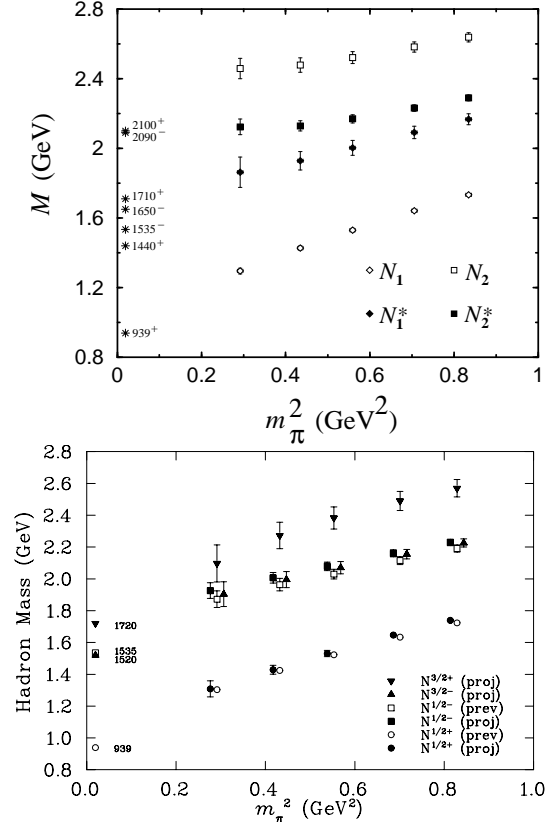


Figure 1. The upper plot shows of the lightest positive- and negative-parity nucleon resonances using two independent interpolating operators<sup>9</sup>, while the lower plot shows also the spin-3/2 and spin-1/2 masses of both parities<sup>10</sup>, obtained using the FLIC fermion action.

observation of excited resonances in lattice QCD is invariably more de-

manding than that of the ground states since, for hadrons composed of light quarks, the signal-to-noise ratio for excited-state correlators decreases with increasing excitation energy. None-the-less, there has been a flurry of activity aimed at computing the excited nucleon spectrum, and in particular the masses of the lightest spin-1/2 and spin-3/2 states of both parities<sup>11,12,13,9,10,14,15</sup>. These calculations employ a variety of fermion discretisation, each has quarks with masses around that of the strange quark, uses local interpolating operators, and each finds a spectrum broadly in line with quark-model expectations  $m_N < m_{N^{1/2-}} < m_{N'}$ , where  $N$ ,  $N^{1/2-}$  and  $N'$  are the nucleon, its parity partner, and first radial excitation of the nucleon, the so-called “Roper” resonance, respectively. This is illustrated in Figure 1. In particular, none of these calculations reveal evidence of two of the more puzzling observations in the nucleon spectrum, the anomalously light Roper resonance, and a light  $\Lambda(1405)^-$ .

A crucial realization in recent years has been that QCD at the physical values of the light-quark masses is very different from that for quarks with masses around that of the strange quark mass because of the rôle played by the pion cloud, and the resultant non-analytic behavior with the quark mass. Several groups have embarked on programs aiming at enabling fits to the lattice data so as to correctly incorporate this behaviour, both within full QCD and quenched QCD<sup>16,17</sup>.

In concert with this understanding has been the advent of fermion actions, satisfying the Ginsparg-Wilson relation<sup>18</sup>, possessing an exact analogue of chiral symmetry at a finite lattice spacing, and the development of the computational resources required to exploit these actions. Thus quark masses approaching the physical light-quark masses are now within reach. A particular realization of the action is through overlap fermions, which admit the use of shifted-mass inverters allowing the simultaneous calculation of propagators at a range of quark masses. A Bayesian fit to the standard nucleon interpolating operator obtained in the quenched approximation to QCD using overlap fermions, with pion masses as low as 180 MeV, is reported in Ref. 19. The masses of the nucleon, its parity partner and the first radial excitation of the nucleon are shown in Figure 2, revealing an inversion of the ordering of the states at light pion masses; such a result is consistent with a picture of the experimentally observed  $N(1440)$  Roper as indeed a simple three-quark resonance.

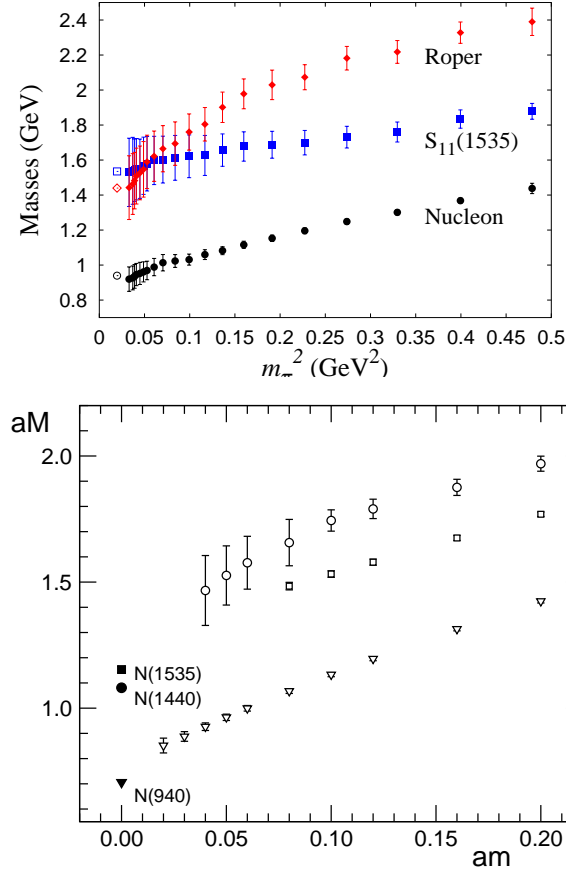


Figure 2. The upper plot shows the masses of the nucleon, its parity partner, and the first radial excitation of the nucleon obtained from a calculation using overlap fermions at a lattice spacing  $a = 0.2$  fm<sup>19</sup>. The lower plot is a calculation of the masses using a variational approach<sup>20</sup>

### 3. Higher excited resonances and variational methods

A comprehensive picture of resonances requires that we go beyond a knowledge of the ground state mass in each channel, and obtain the masses of the lowest few states of a given quantum number. This we can accomplish through the use of variational methods<sup>21,22</sup>. Rather than measuring

a single correlator  $C(t)$ , we determine a matrix of correlators

$$C_{ij}(t) = \sum_{\vec{x}} \langle O_i(\vec{x}, t) O_j^\dagger(\vec{0}, 0) \rangle,$$

where  $\{O_i; i = 1, \dots, N\}$  are a basis of interpolating operators with given quantum numbers. We then solve the generalized eigenvalue equation

$$C(t)u = \lambda(t, t_0)C(t_0)u$$

to obtain a set of real eigenvalues  $\lambda_n(t, t_0)$ . At large Euclidean times, these eigenvalues then delineate between the different masses

$$\lambda_n(t, t_0) \longrightarrow e^{-M_n(t-t_0)} + O(e^{-M_{n+1}(t-t_0)}).$$

The eigenvectors  $u$  are orthonormal with metric  $C(t_0)$ , and a knowledge of the eigenvectors can yield information about the partonic structure of the states.

An early attempt to use these methods to extract the first radial excitation of mesons and baryons employed operators constructed from a local, and from a spatially extended source<sup>23</sup>. A more recent application has been to the calculation of the mass of the Roper resonance, using spatially smeared, gauge-invariant three-quark operators of varying widths<sup>20</sup>. Though the authors are unable to approach the very light pion masses attained in reference<sup>19</sup>, they find the behavior of the masses of the states with pion mass is at least suggestive of the level crossing observed in the Bayesian analysis, as shown in Figure 2. Furthermore, the resulting eigenstates are consistent with a radial node in the wavefunction.

Recently, the LHP Collaboration has developed the techniques to enable the construction of baryon interpolating operators that can easily be extended to include multi-quark operators, and those with excited glue<sup>24,25</sup>. This is delicate, since the cubic symmetry of the lattice admits only three double-valued, irreducible representations (IR's) corresponding to half-integer spins,  $G_1$ ,  $G_2$  and  $H$ , of dimensions 2, 2 and 4 respectively. The irreducible representations  $J$  of the continuum group  $SU(2)$  are reducible under the cubic group  $O$ ; the number of times  $n_\Gamma^J$  that each of these reducible representations occurs in the irreducible representation  $\Gamma$  of  $O$  is shown in Table 1. States with  $J > 5/2$  lie in irreducible representations containing states with lower spins, and furthermore, for a given  $J$ , the different degrees of freedom can lie in different irreducible representations. The masses of the components in these distinct IR's will agree only in the continuum limit. Furthermore, an implicit assumption in previous lattice studies is the increase in ground-states masses with increasing spin; I will

comment further on this below. Parity is easily incorporated, yielding the group  $O_h$ , with the corresponding IR's gaining the labels  $g$  and  $u$  for positive parity and negative parity respectively.

Table 1. The number of times  $n_\Gamma^J$  that the irreducible representation  $\Gamma$  of  $O$  occurs in the reduction of the irreducible representation  $J$  of  $SU(2)$ .

| $J$ | $n_{G_1}^J$ | $n_{G_2}^J$ | $n_H^J$ |
|-----|-------------|-------------|---------|
| 1/2 | 1           | 0           | 0       |
| 3/2 | 0           | 0           | 1       |
| 5/2 | 0           | 1           | 1       |
| 7/2 | 1           | 1           | 1       |
| 9/2 | 1           | 0           | 2       |

The starting point for the operator construction is a basis of gauge-invariant terms of the form

$$\Phi_{\alpha i; \beta j; \gamma k}^{ABC} = \varepsilon_{abc} (\tilde{D}_i^{(p)} \tilde{\psi})_{a\alpha}^A (\tilde{D}_j^{(p)} \tilde{\psi})_{b\beta}^B (\tilde{D}_k^{(p)} \tilde{\psi})_{c\gamma}^C, \quad (2)$$

where  $A, B, C$  indicate quark flavor,  $a, b, c$  are color indices,  $\alpha, \beta, \gamma$  are Dirac spin indices,  $\tilde{\psi}$  indicates a smeared quark field, and  $\tilde{D}_j^{(p)}$  denotes the  $p$ -link covariant displacement operator in the  $j$ -th direction; the quark fields are smeared using a three-dimensional gauge-covariant Laplacian. These gauge-invariant operators are now combined into elemental operators having the appropriate flavor structure. The remaining step is to apply group-theoretical projections to obtain operators which transform irreducibly under all lattice rotations and reflection symmetries:

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h}} \sum_{R \in O_h} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^\dagger, \quad (3)$$

where  $\Lambda$  refers to an  $O_h$  IR,  $\lambda$  is the row in the IR,  $g_{O_h}$  is the number of elements in  $O_h$ ,  $d_\Lambda$  is the dimension of the  $\Lambda$  IR,  $D_{mn}^{(\Lambda)}(R)$  is a  $\Lambda$  representation matrix corresponding to group element  $R$ , and  $U_R$  is the quantum operator which implements the symmetry operations.

An initial, exploratory study exploiting this formalism has recently been performed<sup>28</sup>, at only a single value of the quark mass around that of the strange quark, and at a single value of the lattice spacing  $a \simeq 0.1$  fm.



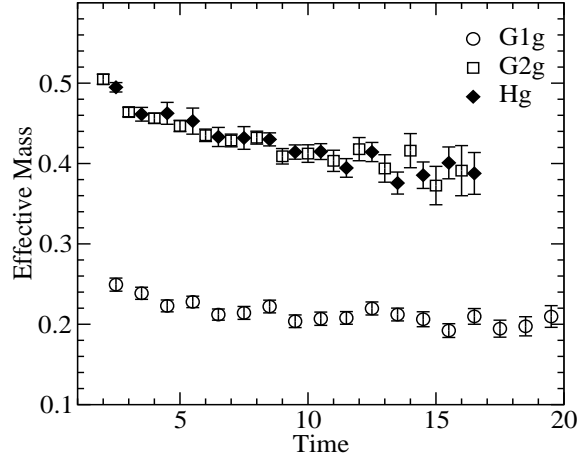


Figure 3. Ground-state effective masses for the each of the IRs  $G_{1g}$ ,  $H_g$  and  $G_{2g}$ , where the subscript  $g$  denotes positive parity<sup>28</sup>.

Figure 3 shows the effective masses, defined by

$$M_{\text{eff}}(t) = \ln C(t)/C(t+1)$$

for the lowest-lying states for each of the three, positive-parity IR's. The apparent coincidence of the effective masses in the  $G_{2g}$  and  $H_g$  channels, containing spins  $J = 5/2, 7/2, \dots$  and  $J = 3/2, 5/2, \dots$  respectively, suggests that the usual identification of a mass extracted from  $H$  with the mass of a state of spin  $3/2$  is somewhat premature. Indeed, experimentally the lowest-lying  $I(J^P) = 1/2(5/2^+)$  state, the  $N(1680) F_{15}$ , is comparable in mass with the lowest-lying  $I(J^P) = 1/2(3/2^+)$  state, the  $N(1720) P_{13}$ , albeit with widths of around 150 MeV. Further lattice studies are essential to definitively identify the quantum numbers of baryon states, requiring the determination of the masses of several states in each channel, and the behavior of these masses in the approach to the continuum limit.

I have emphasized that determinations of the spectrum can provide insight into the structure of states, as well as their masses. Thus future lattice studies of baryons will need to include not only the simple three-quark fields introduced above, but also interpolating fields sensitive to excited glue ('exotics'), to molecular states, and pentaquark or multi-quark states<sup>29</sup>. Perhaps most importantly, these states become unstable with decreasing pion mass, requiring more sophisticated, and computationally demanding, analysis.

#### 4. Conclusions

In this talk, I have tried both to review recent lattice QCD calculations of baryon spectroscopy and structure, and to indicate future directions. The ground-breaking theoretical developments enabling computations at light quark masses are being matched by commensurate computational resources, germinating in the US from the Department of Energy's SciDAC Initiative. Studies of spectroscopy are now approaching the regime where the physics of the pion cloud should be manifest.

A more complete picture of the spectrum of QCD in both the meson and baryon sectors will follow the development of improved hadronic operators, together with the use of variational methods. Lattice studies will be in a position both to guide and interpret the exciting experimental program in hadronic physics. Furthermore, comparison of lattice computations with QCD-inspired theories will enable the range of validity of such theories to be determined.

#### Acknowledgments

This work was supported in part by DOE contract DE-AC05-84ER40150 under which the Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility.

#### References

1. N. Isgur and G. Karl, *Phys. Lett.* **B72**, 109 (1977).
2. N. Isgur and G. Karl, *Phys. Rev.* **D18**, 4187 (1978).
3. S. Capstick and N. Isgur, *Phys. Rev.* **D34**, 2809 (1986).
4. M. Oettel, G. Hellstern, R. Alkofer and H. Reinhardt, *Phys. Rev.* **C58**, 2459 (1998), [nucl-th/9805054].
5. D. Diakonov, hep-ph/9802298.
6. R. L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91**, 232003 (2003), [hep-ph/0307341].
7. S. Capstick, P. R. Page and W. Roberts, *Phys. Lett.* **B570**, 185 (2003), [hep-ph/0307019].
8. K. I. Ishikawa, hep-lat/0410050.
9. W. Melnitchouk *et al.*, *Phys. Rev.* **D67**, 114506 (2003), [hep-lat/0202022].
10. J. M. Zanotti *et al.*, *Phys. Rev.* **D68**, 054506 (2003), [hep-lat/0304001].
11. F. X. Lee and D. B. Leinweber, *Nucl. Phys. Proc. Suppl.* **73**, 258 (1999), [hep-lat/9809095].
12. S. Sasaki, T. Blum and S. Ohta, *Phys. Rev.* **D65**, 074503 (2002), [hep-lat/0102010].
13. M. Gockeler *et al.*, *Phys. Lett.* **B532**, 63 (2002), [hep-lat/0106022].

14. Y. Nemoto, N. Nakajima, H. Matsufuru and H. Suganuma, *Phys. Rev.* **D68**, 094505 (2003), [hep-lat/0302013].
15. D. Brommel *et al.*, *Phys. Rev.* **D69**, 094513 (2004), [hep-ph/0307073].
16. D. B. Leinweber, A. W. Thomas and R. D. Young, *Phys. Rev. Lett.* **92**, 242002 (2004), [hep-lat/0302020].
17. M. Frink and U.-G. Meissner, *JHEP* **07**, 028 (2004), [hep-lat/0404018].
18. P. H. Ginsparg and K. G. Wilson, *Phys. Rev.* **D25**, 2649 (1982).
19. N. Mathur *et al.*, *Phys. Lett.* **B605**, 137 (2005), [hep-ph/0306199].
20. T. Burch *et al.*, *Phys. Rev.* **D70**, 054502 (2004), [hep-lat/0405006].
21. C. Michael, *Nucl. Phys.* **B259**, 58 (1985).
22. M. Luscher and U. Wolff, *Nucl. Phys.* **B339**, 222 (1990).
23. C. R. Allton *et al.*, *Phys. Rev.* **D47**, 5128 (1993), [hep-lat/9303009].
24. S. Basak *et al.*, hep-lat/0409093.
25. S. Basak *et al.*, hep-lat/0409080.
26. R. C. Johnson, *Phys. Lett.* **B114**, 147 (1982).
27. J. E. Mandula and E. Shpiz, *Nucl. Phys.* **B232**, 180 (1984).
28. S. Basak *et al.*, hep-lat/0409082.
29. G. Fleming, to appear in the First Meeting of the APS Topical Group on Hadronic Physics (GHP2004), 24-26 October 2004, Fermilab.