

EXPERIMENTAL CHALLENGES FOR QCD - THE PAST AND THE FUTURE

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The past leaves the surprising experimental successes of the simple constituent quark model to be explained by QCD. The future opens the way to new insight into QCD from heavy flavor experiments

1. Introduction

QCD is supposed to explain everything about Hadron Physics - But How?

QED is supposed to explain everything about Superconductivity .

Will explaining Hadron Spectroscopy from QCD be as difficult as explaining Superconductivity from QED?

The Past leaves us with many experimental regularities which await explanation by QCD. The future offers many new experimental opportunities to learn about QCD from heavy flavor physics. .

1.1. *The Past - pre-QCD questions - Challenges for QCD*

1.1.1. *What is a Hadron?*

Present attempts to describe hadrons recall the story of the blind men and the elephant¹ . Each investigation finds one particular property of hadrons and many contradictory conclusions arise that are all correct,

- (1) A pion is a Goldstone Boson and a proton is a Skyrmion,

- (2) A pion is two-thirds of a proton. The simple quark model prediction $\sigma_{tot}(\pi^- p) \approx (2/3) \cdot \sigma_{tot}(pp)$ ^{2,3} still fits experimental data better than 7% up to 310 GeV/c⁴;
- (3) The a_1 is a $q\bar{q}$ pair in a 3P_1 state similar to other 3P states: scalar and tensor (a_2)
- (4) The a_1 is the chiral partner of the ρ coupled similarly to the W .
- (5) The η and η' are orthogonal linear combinations of the same strange and nonstrange ground state wave functions
- (6) The η and η' contain other components like glueballs or radial excitations
- (7) Mesons and Baryons are made of the same quarks. Describing both as simple composites of asymptotically free quasiparticles with a unique effective mass value predicts hadron masses, magnetic moments and hyperfine splittings^{5,6,7}.
- (8) Lattice QCD can give all the answers,
- (9) Lattice calculations disagree on whether the H dibaryon is bound and offer no hope of settling this question until much bigger lattices are available¹.

1.1.2. *What is a good hadronic symmetry? Many contradictions*

- (1) Light (uds) SU(3) symmetry and Heavy Quark symmetry (cbt) are good;
- (2) Light (uds) SU(3) symmetry is bad. All nontrivial hadron states violate SU(3). All light V, A and T mesons have good isospin symmetry with flavor mixing in (u,d) space and no $s\bar{s}$ component; e.g. ρ, ω .
- (3) The s-quark is a heavy quark. Flavor mixing in mass eigenstates predicted by SU(3) is not there. Most nontrivial strange hadron states satisfy (scb) heavy quark symmetry with no flavor mixing; e.g. ϕ, ψ, Υ .
- (4) Light (uds) SU(3) symmetry is the basis of Cabibbo theory of weak interactions and gives excellent description of hyperon decays.
- (5) Violation of the Gottfried sum rule shows the proton sea is not isoscalar.
- (6) Isospin symmetry requires a proton with an isovector sea to have a component with a valence neutron and a charged sea
- (7) SU(3) requires proton with isovector sea to have a component with a valence hyperon and a strange sea to satisfy Cabibbo theory for vector current.

- (8) Experiment shows SU(3) symmetry manifestly broken in proton sea
- (9) No consistent explanation of Gottfried violation, strange sea suppression breaking SU(3) and Cabibbo theory requiring good SU(3).
- (10) Why are the ω and ρ degenerate while the η and π are not? Is there a symmetry beyond SU(3) that forbids octet-singlet splitting for vectors but not for pseudoscalars??

1.1.3. *Why do the constituent quark model and the OZI rule work so well?*

Surprising agreement with experiment from simple Sakharov-Zeldovich model (1966) having quarks with effective masses and hyperfine interaction. Nambu's (1966) Colored quarks with gauge gluons gave mass spectrum with only qqq and \bar{q} bound states.

The topological quark-line OZI rule does not follow from any symmetry and predicts experiments successfully without any solid theoretical justification.

1.2. The Future - Heavy Flavor Decays Give New Insight

- (1) Weak Decays need hadron models and QCD to interpret decays, but have too many diagrams, too many free parameters
- (2) No rigorous QCD results for FSI and strong phases
- (3) Too many decay modes, too much data. Need phenomenologists to choose data for analysis
- (4) Experimental results from B and Charm factories that defy conventional wisdom can provide clues to new physics and inadequacies in hadron models.

2. The Past - The constituent quark model and other pre-QCD Challenges for QCD

2.1. The Sakharov-Zeldovich 1966 Quark model (SZ66)

2.1.1. *The Model*

Andrei Sakharov, a pioneer in quark-hadron physics asked in 1966 “Why are the Λ and Σ masses different? They are made of the same quarks”. Sakharov and Zeldovich⁵. assumed a quark model for hadrons with a flavor dependent linear mass term and hyperfine interaction,

$$M = \sum_i m_i + \sum_{i>j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i \cdot m_j} \cdot v_{ij}^{hyp} \quad (1)$$

where m_i is the effective mass of quark i , $\vec{\sigma}_i$ is a quark spin operator and v_{ij}^{hyp} is a hyperfine interaction with different strengths but the same flavor dependence for qq and $\bar{q}q$ interactions.

This model can be considered analogous to the BCS description of superconductivity. The constituent quarks are quasiparticles of unknown structure with a background of a condensate. They have effective masses not simply related to the bare current quark masses, and somehow including all effects of confinement and other flavor independent potentials. The only contribution to hadron masses not already included is a flavor-dependent two-body hyperfine interaction inversely proportional to the product of these same effective quark masses. Hadron magnetic moments are described simply by adding the contributions of the moments of these constituent quarks with Dirac magnetic moments having a scale determined by the same effective masses. The model describes low-lying excitations of a complex system with remarkable success.

2.1.2. Striking Results and Predictive Power

Sakharov and Zeldovich already in 1966 obtained two relations between meson and baryon masses in remarkable agreement with experiment. Both the mass difference $m_s - m_u$ between strange and nonstrange quarks and their mass ratio m_s/m_u have the same values when calculated from baryon masses and meson masses⁵

$$\langle m_s - m_u \rangle_{Bar} = M_\Lambda - M_N = 177 \text{ MeV} \quad (2)$$

$$\langle m_s - m_u \rangle_{mes} = \frac{3(M_{K^*} - M_\rho) + M_K - M_\pi}{4} = 180 \text{ MeV} \quad (3)$$

$$\left(\frac{m_s}{m_u} \right)_{Bar} = \frac{M_\Delta - M_N}{M_{\Sigma^*} - M_\Sigma} = 1.53 = \left(\frac{m_s}{m_u} \right)_{Mes} = \frac{M_\rho - M_\pi}{M_{K^*} - M_K} = 1.61 \quad (4)$$

Further extension of this approach led to two more relations for $m_s - m_u$ when calculated from baryon masses and meson masses^{6,7}, and to three magnetic moment predictions with no free parameters^{8,9}

$$\langle m_s - m_u \rangle_{mes} = \frac{3M_\rho + M_\pi}{8} \cdot \left(\frac{M_\rho - M_\pi}{M_{K^*} - M_K} - 1 \right) = 178 \text{ MeV}. \quad (5)$$

$$\langle m_s - m_u \rangle_{Bar} = \frac{M_N + M_\Delta}{6} \cdot \left(\frac{M_\Delta - M_N}{M_{\Sigma^*} - M_\Sigma} - 1 \right) = 190 \text{ MeV}. \quad (6)$$

$$\mu_\Lambda = -0.61 \text{ n.m.} = \mu_\Lambda = -\frac{\mu_p}{3} \cdot \frac{m_u}{m_s} = -\frac{\mu_p}{3} \frac{M_{\Sigma^*} - M_\Sigma}{M_\Delta - M_N} = -0.61 \text{ n.m.} \quad (7)$$

$$-1.46 = \frac{\mu_p}{\mu_n} = -\frac{3}{2} \quad (8)$$

$$\mu_p + \mu_n = 0.88 \text{ n.m.} = \frac{M_p}{3m_u} = \frac{2M_p}{M_N + M_\Delta} = 0.865 \text{ n.m.} \quad (9)$$

Also in 1966 Levin and Frankfurt² noted a remarkable systematics in hadron-nucleon total cross sections indicating that mesons and baryons were made of the same basic building blocks. The analysis supporting their ratio of 3/2 between baryon-nucleon and nucleon-nucleon cross sections has been refined³ and consistently confirmed by new experiments⁴. QCD calculations have not yet explained such remarkably successful simple constituent quark model results. A search for new experimental input to guide us is therefore of interest.

2.2. *The A.....Z or OZI rule and QCD*

2.2.1. *OZI for light quarks*

No rigorous QCD derivation has yet been found for this flavor-topology rule arising also in duality diagrams of Regge phenomenology where leading t-channel exchanges are dual to s-channel resonances and in more modern planar quark diagrams in large N_c QCD. It has been repeatedly confirmed in a large variety of experimental results and theoretical analyses for strong interaction three-point and four-point functions, beginning with the first controversial prediction relating final states in completely different isospin and flavor-SU(3) multiplets unrelated by any known symmetry.

$$\sigma(K^- p \rightarrow \Lambda \rho^0) = \sigma(K^- p \rightarrow \Lambda \omega) \quad (10)$$

Is this connected with the $\omega - \rho$ degeneracy?

2.2.2. *OZI for heavy flavors - Why is J/ψ narrow?*

One diagram $J/\psi \rightarrow 3G \rightarrow \text{light hadrons}$ fits the narrowness but $J/\psi \rightarrow D\bar{D} \rightarrow \text{light hadrons}$ is larger and neglected along with ad hoc forbidden “hairpin diagrams”. Hand waving explanations with cancellations give predictive power which can be tested with future experimental data and still challenge QCD for explanation.

2.3. *Problems with the η and η'*

The pseudoscalars are conventionally described by adding an additional mass contribution to the SU(3) singlet state, thus breaking U(3) while conserving

SU(3) and leaving SU(3) breaking as entirely due to quark mass differences.. The dynamical origin of this additional singlet contribution is still unclear and controversial, with some models attributing it the annihilation of an $q\bar{q}$ pair into gluons or instantons and no reason to limit the mixing to only ground state $q\bar{q}$ wave functions. Admixtures of radial excitations and glueballs have been considered.

2.4. *How to go beyond SZ66 with QCD*

Many approaches are being investigated to use QCD in the description of hadron spectroscopy¹⁰. The complexity of QCD calculations necessitates the introduction of ad hoc approximations and free parameters to obtain results, thus losing the simplicity of the constituent quark model, with its ability to make many independent predictions with very few parameters, There is also a tendency to lose some of the good results of the constituent quark model; namely

- The universal treatment of mesons and baryons made of the same quarks
- The spin dependence of hadron masses as a hyperfine interaction
- The appearance of the same effective quark masses in hadron masses, spin splittings and magnetic moments
- The systematic regularities relating meson-nucleon and baryon-nucleon cross sections

While none of these results can be considered to have a firm theoretical foundation based on QCD, it is difficult simply to dismiss the striking agreement with experiment and the successful predictive power as purely accidental.

3. The Future - Heavy Flavor Physics Gives New Insight

Weak Decays need hadron models and QCD to interpret decays, but have too many diagrams and too many free parameters. Use of flavor topology can simplify analyses on one hand and challenge QCD to explain them if they work.

3.1. *Experimental systematics challenging conventional wisdom*

3.1.1. *Universality of vector dominance couplings*

The large branching ratios observed¹¹ for the appearance of the $a_1(1260)^\pm$ in all quasi-two-body decays $D \rightarrow a_1(1260)^\pm X$ and $B \rightarrow a_1(1260)^\pm X$ are comparable to those observed for $\pi^\pm X$ and $\rho^\pm X$. No decays to the other p-wave mesons are within an order of magnitude of these values; e.g. the difference between the a_1 and the a_2 . All 24 B decays of the form $B \rightarrow \bar{D}W^+ \rightarrow \bar{D}M^+$, where M can denote $a_1, \rho, \pi, \ell^+\nu_\ell, D_s, D_s^*$, are dominant with branching ratios above 0.3%. Other B -decay modes have upper limits in the 10^{-4} ball park, including the absence with significant upper limits of neutral decays $B^0 \rightarrow \bar{D}^0 M^0$ which are coupled by strong final state interactions to $B^0 \rightarrow D^- M^+$.

These experimental systematics suggested a “vector-dominance” model¹² where the initial hadron state i decays to a final state f by emitting a W^\pm which then hadronizes into an a_1^+, ρ^+ or π^+ , along with a universality relation,

$$[if\pi] \equiv \frac{BR[i \rightarrow f\pi^+]}{BR[i \rightarrow f\rho^+]} \approx \left| \frac{W^+ \rightarrow \pi^+}{W^+ \rightarrow \rho^+} \right|^2$$

$$[ifa] \equiv \frac{BR[i \rightarrow fa_1(1260)^+]}{BR[i \rightarrow f\rho^+]} \approx \left| \frac{W^+ \rightarrow a_1^+}{W^+ \rightarrow \rho^+} \right|^2$$

for all states i and f with corrections for phase space.

$$[D^+ \bar{K}^0 \pi] \approx [D^0 K^- \pi] \approx [B^0 D^- \pi] \approx [B^0 D^{*-} \pi] \approx [B^+ \bar{D}^0 \pi] \approx [B^+ \bar{D}^{*0} \pi]$$

$$.44 \pm .17 \approx .35 \pm .09 \approx .38 \pm .08 \approx .41 \pm .20 \approx .40 \pm .06 \approx .30 \pm .07$$

$$[D^+ \bar{K}^0 a] \approx [D^0 K^- a] \approx [B^0 D^- a] \approx [B^0 D^{*-} a] \approx [B^+ \bar{D}^0 a] \approx [B^+ \bar{D}^{*0} a]$$

$$1.2 \pm .5 \approx .68 \pm .12 \approx .8 \pm .4 \approx 1.9 \pm 1.0 \approx .37 \pm .30 \approx 1.2 \pm .4$$

The a_1 data have large errors, but the experimental ratios $[ifa]$ are all consistent with 0.7, and more than order of magnitude higher than other upper limits

$$[D^0 K^- a_2^+] < 0.019 \pm 0.002; \quad [D^+ \bar{K}^0 a_2^+] < 0.045 \pm 0.017$$

That such widely different decays should agree so well is impressive and suggests further investigation. e.g. reducing the experimental errors and looking for more decay modes like $D_s^+ \rightarrow \phi a_1$, $D_s^+ \rightarrow \omega a_1$, $D^+ \rightarrow K^{*0} a_1$ and $D^0 \rightarrow K^{*-} a_1$.

3.1.2. Vector-Dominance Decays of the B_c

The B_c meson is identified against a large combinatorial background by decay modes including a J/ψ . Vector dominance decay modes including the J/ψ are expected to have relatively large branching ratios. These include: $J/\psi\rho^+$, $J/\psi a_1^+$, $J/\psi\pi^+$, $J/\psi D_s^*$, $J/\psi D_{s1A}$, and $J/\psi D_s$. The corresponding modes with a ψ' instead of a J/ψ are expected to have comparable branching ratios.

3.1.3. Puzzles in Singly-Suppressed Charm Decays

Two Cabibbo suppressed D^+ decay modes have anomalously high branching ratios which are not simply explained by any model¹³.

$$BR[D^+ \rightarrow K^*(892)^+ \bar{K}^0] = 3.2 \pm 1.5\% \quad (11)$$

$$BR[D^+ \rightarrow K^*(892)^+ \bar{K}^*(892)^0] = 2.6 \pm 1.1\% \quad (12)$$

These are the same order as corresponding Cabibbo allowed branching ratios

$$BR[D^+ \rightarrow \rho^+ \bar{K}^0] = 6.6 \pm 2.5\% \quad (13)$$

$$BR[D^+ \rightarrow \rho^+ \bar{K}^*(892)^0] = 2.1 \pm 1.3\% \quad (14)$$

The dominant tree diagrams for these corresponding allowed and suppressed decays differ only in the weak vertices $c \rightarrow W^+ + s \rightarrow \rho^+ + s$ and $c \rightarrow W^+ + s \rightarrow K^*(892)^+ + s$ and have the same hadronization of the strange quark s and spectator \bar{d} . These diagrams should show the expected Cabibbo suppression which is not observed.

All standard model diagrams that can contribute to these anomalously enhanced decays (11-12) are related by symmetries to a very similar diagrams for one of the following decay modes which show the expected Cabibbo suppression

$$BR[D^+ \rightarrow K^+ \bar{K}^*(892)^0] = 0.42 \pm 0.05\% \quad (15)$$

$$BR[D^0 \rightarrow K^*(892)^+ K^-] = 0.35 \pm 0.08\% \quad (16)$$

$$BR[D^0 \rightarrow K^*(892)^- K^+] = 0.18 \pm 0.01\% \quad (17)$$

$$BR[D^0 \rightarrow K^*(892)^0 \bar{K}^0] < 0.08\% \quad (18)$$

$$BR[D^0 \rightarrow \bar{K}^*(892)^0 K^0] < 0.16\% \quad (19)$$

$$BR[D^0 \rightarrow K^*(892)^0 \bar{K}^*(892)^0] = 0.14 \pm 0.05\% \quad (20)$$

There is no simple diagram that enhances the suppressed diagrams (11-12) without also enhancing others that show no experimental enhancement.

It is therefore of interest to check the branching ratios for the transitions (11-12) and reduce the errors. Using the present data we find:

$$BR[D^+ \rightarrow K^*(892)^+ \bar{K}^0] + BR[D^+ \rightarrow K^*(892)^+ \bar{K}^*(892)^0] = 5.8 \pm 1.9\% \quad (21)$$

This is still large even at two standard deviations. If the large branching ratios are confirmed with smaller errors, there may be good reason to look for a new physics explanation.

3.1.4. Anomalously high η' in charmless strange B decays

The large experimental branching ratio¹¹ $BR(B^+ \rightarrow K^+ \eta') = 6.5 \pm 1.7 \times 10^{-5}$ as compared with $BR(B^+ \rightarrow K^+ \eta) < 1.4 \times 10^{-5}$ and $BR(B^+ \rightarrow K^0 \pi^+) = 2.3 \pm 1.1 \times 10^{-5}$ still has no completely satisfactory explanation and has aroused considerable controversy¹⁴. Also the large inclusive $B^+ \rightarrow K^+ \eta' X$ branching ratio is equally puzzling.

A parity selection rule provides a clear experimental method to distinguish between two proposed explanations with different flavor topologies

1. The OZI-forbidden hairpin diagram¹⁵ predicts a universal parity-independent enhancement for all final states arising from the flavor singlet component of the η' ^{16,17}.

2. Parity-dependent interference between diagrams producing the η' via its strange and nonstrange components¹⁵ predicts a large η'/η ratio for even parity final states like $K\eta$ and $K\eta'$ the reverse for odd parity states like $K^*(892)\eta$ and $K^*\eta'$ ¹⁴. This selection rule agrees with experiment, although so far the $K^*\eta$ has been seen and the $K^*\eta'$ has not.

3.2. Predictions from simple easily-tested assumptions

3.2.1. The Flavor-Topology OZI rule and QCD

Two predictions which challenge QCD if they agree with experiment.

$$BR(B^\pm \rightarrow K^\pm \omega) = BR(B^\pm \rightarrow K^\pm \rho^0) \quad (22)$$

Because the ρ^0 and ω mesons both come only from $\bar{u}u$ this prediction requires only exclusion of hairpin diagram topology and holds even in presence of strong final state rescattering via all other quark-gluon diagrams.

$$\tilde{\Gamma}(B^\pm \rightarrow K^\pm \phi) = \tilde{\Gamma}(B^\pm \rightarrow K^0 \rho^\pm) \quad (23)$$

This prediction also assumes the SU(3) flavor symmetry relation between strange and nonstrange pair production

3.2.2. The “inactive spectator” approach

Many interesting predictions that can be checked experimentally and challenge QCD if they work follow from a simple flavor-topology rule¹⁸. The spectator quark line must flow continuously from initial to final state, emitting and absorbing gluons freely, but not undergoing annihilation or pair creation. One example arises in B decays to final states containing charmonium. The first prediction forbids all decays without the spectator flavor in the final state :

$$A[B_d \rightarrow J/\psi M(\bar{q}s)] = 0 = A[B_s \rightarrow J/\psi M(\bar{q}d)] \quad (24)$$

$$A(B_s \rightarrow J/\psi \rho^0) = A(B_s \rightarrow J/\psi \omega) = A(B_d \rightarrow J/\psi \phi) = 0 \quad (25)$$

where $M(\bar{q}s)$ and $M(\bar{q}d)$ denote any $\bar{q}q$ meson with these constituents. If this selection rule holds all other decays described by two amplitudes

$$B(\bar{b}q) \rightarrow J/\psi \bar{d}q \rightarrow J/\psi M(\bar{d}q) \quad (26)$$

$$B(\bar{b}q) \rightarrow J/\psi \bar{s}q \rightarrow J/\psi M(\bar{s}q) \quad (27)$$

Decay is product of $\bar{b} \rightarrow J/\psi \bar{d}$ or $\bar{b} \rightarrow J/\psi \bar{s}$ decay and hadronization function h

$$\begin{aligned} A[B_d \rightarrow J/\psi M^0(\bar{s}d)] &= A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}d \rightarrow M^0(\bar{s}d)] \\ A[B_s \rightarrow J/\psi M^0(\bar{d}s)] &= A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}s \rightarrow M^0(\bar{d}s)] \\ A[B_d \rightarrow J/\psi M^0(\bar{d}d)] &= A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}d \rightarrow M^0(\bar{d}d)] \\ A[B_s \rightarrow J/\psi M^0(\bar{s}s)] &= A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}s \rightarrow M^0(\bar{s}s)] \\ A[B^+ \rightarrow J/\psi M^+(\bar{s}u)] &= A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}d \rightarrow M^+(\bar{s}u)] \\ A[B^+ \rightarrow J/\psi M^+(\bar{d}u)] &= A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}d \rightarrow M^+(\bar{d}u)] \end{aligned} \quad (28)$$

Decays into charge-conjugate strange final states differ only by weak interaction vertex and kinematic and form factor differences induced by $B_d - B_s$ mass difference. For any partial wave L in a vector-vector final state, we can write

$$A(B_s \rightarrow J/\psi \bar{K}^{*0})_L = F_{CKM}^L \cdot A(B_d \rightarrow J/\psi K^{*0})_L \quad (29)$$

If the weak transition is $\bar{b} \rightarrow \bar{c} + W^+ \rightarrow \bar{c} + c + \bar{q}$ as in the dominant tree diagram.

$$F_{CKM}^L = \frac{A_L(\bar{b} \rightarrow J/\psi \bar{d})}{A_L(\bar{b} \rightarrow J/\psi \bar{s})} \approx V_{cd}/V_{cs} \quad (30)$$

If $F_{CKM}^L \neq V_{cd}/V_{cs}$ other contributions are indicated.

Additional SU(3) assumption gives

$$A_L(B_d \rightarrow J/\psi \rho^0) = A_L(B_d \rightarrow J/\psi \omega) = \frac{A_L(B_s \rightarrow J/\psi \bar{K}^{*0})}{\sqrt{2}} \quad (31)$$

$$A_L(B_s \rightarrow J/\psi \phi) = A_L(B_d \rightarrow J/\psi K^{*0}) \quad (32)$$

$$A_L(B_s \rightarrow J/\psi \rho^0) = A_L(B_s \rightarrow J/\psi \omega) = A_L(B_d \rightarrow J/\psi \phi) = 0 \quad (33)$$

3.2.3. How to test $\eta - \eta'$ mixing

If the $\eta - \eta'$ system satisfies the standard mixing,

$$|\eta\rangle = |\eta_n\rangle \cos \phi - |\eta_s\rangle \sin \phi; \quad |\eta'\rangle = |\eta_n\rangle \sin \phi + |\eta_s\rangle \cos \phi \quad (34)$$

Then

$$r_d = \frac{p_{\eta'}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta)}{p_{\eta}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta')} = \cot^2 \phi; \quad r_s = \frac{p_{\eta'}^3 \Gamma(\bar{B}_s \rightarrow J/\psi \eta)}{p_{\eta}^3 \Gamma(\bar{B}_s \rightarrow J/\psi \eta')} = \tan^2 \phi \quad (35)$$

$$R_\eta = \frac{p_{B\eta}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta)}{p_{Bs\eta}^3 \Gamma(\bar{B}_s \rightarrow J/\psi \eta)} \cdot \frac{p_{BsK}^3 \Gamma(\bar{B}_s \rightarrow J/\psi K^0)}{p_{BK}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \bar{K}^0)} = \cot^2 \phi \quad (36)$$

$$R'_\eta = \frac{p_{B\eta'}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta')}{p_{Bs\eta'}^3 \Gamma(\bar{B}_s \rightarrow J/\psi \eta')} \cdot \frac{p_{BsK}^3 \Gamma(\bar{B}_s \rightarrow J/\psi K^0)}{p_{BK}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \bar{K}^0)} = \tan^2 \phi \quad (37)$$

$$r = \sqrt{r_d r_s} = 1; \quad R_B = \sqrt{R_\eta R'_\eta} = 1 \quad (38)$$

Any large deviation of r or R_B from 1 would indicate evidence of non standard $\eta - \eta'$ mixing¹⁸.

3.2.4. SU(3) Relations between Cabibbo-Favored and Doubly-Cabibbo Suppressed D^0 decays

The SU(3) transformation $d \leftrightarrow s$, also called a Weyl reflection or a U-spin reflection relates Cabibbo-favored \leftrightarrow doubly-cabibbo suppressed charm decays

$$\begin{aligned} d \leftrightarrow s; \quad K^+ \leftrightarrow \pi^+; \quad K^- \leftrightarrow \pi^-; \quad D^+ \leftrightarrow D_s; \\ D^0 \leftrightarrow D^0; \quad K^+ \pi^- \leftrightarrow K^- \pi^+ \end{aligned} \quad (39)$$

If strong interaction final state interactions conserve SU(3) the only SU(3) breaking occurs in the CKM matrix elements.

A simple test of this SU(3) symmetry is

$$\tan^4 \theta_c = \frac{BR(D^0 \rightarrow K^+ \pi^-)}{BR(D^0 \rightarrow K^- \pi^+)} = \frac{BR[D^0 \rightarrow K^*(892)^+ \rho^-]}{BR[D^0 \rightarrow K^*(892)^- \rho^+]} \quad (40)$$

These relations involve only branching ratios and are easily tested. They involve no phases and only branching ratios of decay modes all expected to be comparable to the observed DCSD $D^o \rightarrow K^+ \pi^-$. A similar relation.

$$\tan^4 \theta_c = \frac{BR[D^o \rightarrow K^+ a_1(1260)^-]}{BR[D^o \rightarrow K^- a_1(1260)^+]} \quad (41)$$

may be subject to a different type of SU(3) breaking. A weak vector dominance form factor can enhance

$$D^o(c\bar{u}) \rightarrow (s\bar{u} \rightarrow K^-)_S \cdot (u\bar{d} \rightarrow a_1^+)_W \rightarrow K^- a_1^+ \quad (42)$$

where the subscripts S and W denote strong and weak form factors.

No such enhancement should occur in

$$D^o(c\bar{u}) \rightarrow (d\bar{u} \rightarrow a_1^-)_S \cdot (u\bar{s} \rightarrow K^+)_W \rightarrow a_1^- K^+ \quad (43)$$

If the SU(3) breaking is really due to the difference between products of weak axial and strong kaon form factors and vice versa, the SU(3) relation involving the a_1 can be expected to be strongly broken and replaced by the inequality

$$\frac{BR[D^o \rightarrow K^- a_1(1260)^+]}{BR(D^o \rightarrow K^- \pi^+)} \gg \frac{BR[D^o \rightarrow K^+ a_1(1260)^-]}{BR(D^o \rightarrow K^+ \pi^-)} \quad (44)$$

3.2.5. A problem with strong phases

The $d \leftrightarrow s$ interchange SU(3) transformation also predicts¹⁹ $D^o \rightarrow K^+ \pi^-$ and $D^o \rightarrow K^- \pi^+$ have the same strong phases. This has been shown to be in disagreement with experiment²⁰ showing SU(3) violation.

But the $K^+ \pi^-$ and $K^- \pi^+$ final states are charge conjugates of one another and strong interactions conserve charge conjugation. SU(3) can be broken in strong interactions without breaking charge conjugation only in the quark - hadron form factors arising in hadronization transitions like

$$\begin{aligned} D^o(c\bar{u}) &\rightarrow (s\bar{u} \rightarrow K^-)_S \cdot (u\bar{d} \rightarrow a_1^+)_W \\ &\rightarrow K^- a_1^+ \rightarrow K^- \pi^+ \end{aligned} \quad (45)$$

$$\begin{aligned} D^o(c\bar{u}) &\rightarrow (d\bar{u} \rightarrow a_1^-)_S \cdot (u\bar{s} \rightarrow K^{*+})_W \\ &\rightarrow a_1^- K^+ \rightarrow \pi^- K^+ \end{aligned} \quad (46)$$

with the SU(3) breaking given by the inequality (44).

The a_1 and π wave functions are very different and not related by SU(3). The $K^\mp a_1^\pm \rightarrow K^\mp \pi^\pm$ transition can proceed via ρ exchange

3.2.6. $SU(3)$ relations between D^+ and D_s decays

Both of the following ratios of branching ratios

$$\frac{BR(D_s \rightarrow K^+ K^+ \pi^-)}{BR(D_s \rightarrow K^+ K^- \pi^+)} \approx \frac{BR(D^+ \rightarrow K^+ \pi^+ \pi^-)}{BR(D^+ \rightarrow K^- \pi^+ \pi^+)} \approx O(\tan^4 \theta_c) \quad (47)$$

are ratios of a doubly Cabibbo forbidden decay to an allowed decay and should be of order $\tan^4 \theta_c$. The $SU(3)$ transformation $d \leftrightarrow s$ takes the two ratios (47) into the reciprocals of one another. $SU(3)$ requires the product of these two ratios to be EXACTLY $\tan^8 \theta_c$ ²¹.

$$\frac{BR(D_s \rightarrow K^+ K^+ \pi^-)}{BR(D_s \rightarrow K^+ K^- \pi^+)} \cdot \frac{BR(D^+ \rightarrow K^+ \pi^+ \pi^-)}{BR(D^+ \rightarrow K^- \pi^+ \pi^+)} = \tan^8 \theta_c \quad (48)$$

Most obvious $SU(3)$ -symmetry-breaking factors cancel out in this product; e.g. phase space¹¹ show

$$\frac{BR(D^+ \rightarrow K^+ \pi^- \pi^+)}{BR(D^+ \rightarrow K^- \pi^+ \pi^+)} \approx 0.65\% \approx 3 \times \tan^4 \theta_c \quad (49)$$

Then $SU(3)$ predicts

$$\frac{BR(D_s \rightarrow K^+ K^+ \pi^-)}{BR(D_s \rightarrow K^+ K^- \pi^+)} \approx \frac{\tan^4 \theta_c}{3} \approx 0.07\%. \quad (50)$$

If this $SU(3)$ prediction is confirmed experimentally some new dynamical explanation will be needed for the order of magnitude difference between effects of the final-state interactions in D^+ and D_s decays.

If the final state interactions behave similarly in D_s and D^+ decays, the large violation of $SU(3)$ will need some explanation.

New physics enhancing the doubly suppressed decays might produce a CP violation observable as a charge asymmetry in the products of above the two ratios; i.e between the values for D^+ and D_s decays and for D^- and \bar{D}_s decays.

An obvious caveat is the almost trivial $SU(3)$ breaking arising from resonances in the final states. But sufficient data and Dalitz plots should enable including these effects. In any case the $SU(3)$ relation and its possible violations raise interesting questions which deserve further theoretical and experimental investigation. Any really large $SU(3)$ -breaking final state interactions that we don't understand must cast serious doubts on many $SU(3)$ predictions.

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