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The CMU Baryon Amplitude Analysis Program

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Abstract. The PWA group at Carnegie Mellon University has developed a comprehensive approach and analysis package for the purpose of extracting the amplitudes for photoproduced baryon resonances. The end goal is to identify any missing resonances that are predicted by the constituent quark model, but not definitively observed in experiments. The data comes from the CEBAF Large Acceptance Spectrometer (CLAS) at Jefferson Lab.

1. Introduction

The non-strange baryon spectrum has been mapped out predominantly through studies of $N\pi$ elastic scattering utilizing phase-shift analysis as the tool of choice. The observed resonances match up very well with the predictions of the constituent quark model (CQM) [1, 2, 3, 4]. While there has been much success with these experimental techniques, the results have still fueled debates in the community, most notably regarding the missing baryons problem. To resolve this discrepancy, researchers have postulated a diquark-system within the baryons [5] or a weak coupling to $N\pi$ and stronger coupling to other states [1, 6, 2]. This is summarized in Fig. 1. The CLAS detector at Jefferson Lab has turned out high-statistics, photoproduction datasets which are optimal for resolving these issues. However, new analytical techniques may be required to deal with this rich physics sector.

The baryon resonances are photoproduced off liquid hydrogen and the CLAS detector allows us to measure a variety of final states. We have access to $n\pi^+, p\pi^0, p\pi^+\pi^-, p\omega, p\eta, p\eta', \Lambda K^+$ and ΣK^+ final states. To handle this diversity, CMU has developed computer code to work within a covariant tensor formalism which allows for amplitudes to be calculated using a very general set of C++ classes. The fitting machinery has also been written with the end goal of a coupled fit in mind. This flexibility lends itself to combining data from multiple current experiments as well as for future meson studies from GlueX.

2. Overview of amplitude analysis.

Because the masses, widths and couplings of the missing states are, by definition, unknown, the goal is to perform the analysis in as model independent way as possible. To this end we perform a *mass-independent analysis*. A toy example is shown in Fig. 1 and discussed in this section.

Suppose we are trying to analyze the photoproduction of some final state. We divide the dataset into narrow bins (10 MeV/c^2) of W ($\sqrt{s}$) so that we may extract energy independent couplings. In each bin, we attempt to describe the distributions of the particles with some "cocktail" of physics. We leave the details of this "cocktail" for later discussion, but in effect we put in terms which represent the s-channel processes of interest as well as some non-resonant

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State	Width (MeV/c^2)	Branching fraction (%)						
		$N\pi$	$\Delta\pi$	$N\rho$	$N\eta$	$N\omega$	ΛK	ΣK
N$\frac{1}{2}^+$(1880)	150	5	49	3	18	14	0	9
N$\frac{1}{2}^+$(1975)	50	8	47	14	0	22	3	1
N$\frac{3}{2}^+$(1870)	190	20	12	2	26	11	0	26
N$\frac{3}{2}^+$(1910)	390	0	75	3	0	17	0	2
N$\frac{3}{2}^+$(1950)	140	12	43	11	0	28	3	1
N$\frac{3}{2}^+$(2030)	90	4	57	15	0	16	1	0
N$\frac{5}{2}^+$(1995)	270	1	89	2	0	3	0	0
N$\frac{5}{2}^+$(2000)	190	0	51	33	4	8	0	0
N$\frac{7}{2}^+$(1835)	50	13	53	3	21	6	0	2
$\Delta\frac{1}{2}^+$(1985)	310	5	63	20	0	0	0	3
$\Delta\frac{3}{2}^+$(1880)	220	5	44	25	0	0	0	5
$\Delta\frac{5}{2}^+$(1990)	350	0	56	10	0	0	0	0

Table 1. Here we show the predicted couplings for the missing N=2 resonances.[3, 7] Note the relatively weak coupling to $N\pi$ and the strength in other decay modes.

terms which may consist of t- or u-channel terms or contact terms. Within each bin, we can run a fit independently, initializing the fit with random starting values. In this way if there is energy dependant physics, we can extract it in a later step, without biasing ourselves. These fits are performed using a maximum likelihood method based on the total intensity for a given event, as opposed to fitting some differential cross sections where the correlations may be lost

In Fig. 1 we try to describe what we would see in an idealized case. This is not a real analysis, merely a sketch to illustrate the procedure. The left plot shows the total cross section for the photoproduction of some final state with a single W-bin indicated by the vertical lines. The colored points represent some amplitudes which were used to describe the physics in each bin. In this example there was one non-resonant term, and two s-channel terms which seem to map out Breit-Wigner shapes. But the strength of the analysis is not solely in mapping out the intensities, but in the phase motion. Because the fits are run in each bin independently, there is a freedom in phase from bin to bin. However, if these are truly resonant terms, then the relative phases are well defined terms and would map out a structure as seen in the right plot in Fig. 1.

While much of the fit is model and energy independent as possible, we relax both of these constraints when putting in non-resonant terms. The couplings for a given t-channel process should, of course, be independent of W and so we lock these values down over the whole range. These values are derived from theoretical models or experimental measurements. While form factors and the functional forms of these amplitudes may also be motivated by theoretical or phenomenological models, our procedure and machinery is flexible to allow us to compare a variety of these models with relative ease.

3. Writing out the amplitudes.

Our amplitudes utilize a covariant tensor formalism first set down by Rarita and Schwinger [8] and utilized by Anisovich and others [9]. Unlike the helicity formalism, where amplitudes can be calculated using D-functions, we require computer code which can handle tensor algebra. This code has been written primarily by one of the members of our group, Mike Williams, and has been in extensive use since 2005. The usefulness of this software package cannot be overstated. With a full suite of Dirac spinors and orbital angular momentum tensors, we are able to write out our amplitudes as they are initially written down by theorists, without needing to be simplified to some function of kinematic variables in assorted frames. This package is written in a C++ environment. An example of some of the classes is shown in Table 2 in order to give

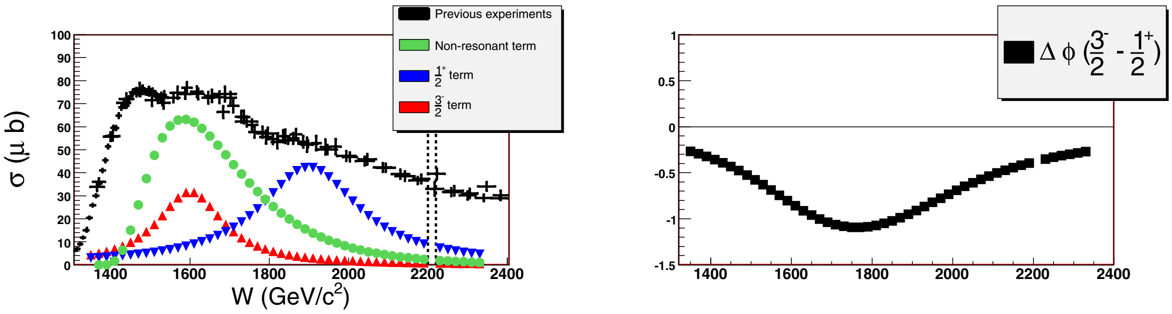

Figure 1. *MOCK DATA AND ANALYSIS.* Here we show an imaginary example of our procedure. In the left figure, the black crosses represent the total cross sections for photoproduction of some final state, for example $p\pi^+\pi^-$. Our procedure will try to describe the physics for *one small energy range*, indicated by the vertical lines. This analysis is then repeated for bins over the entire range. The colored points represent the relative contributions of some non-resonant term and two s-channel terms. The right plot shows the difference in phases of the two s-channel terms. Even if each energy bin is fit independently, if there is resonant behavior in the waves, it will show up in both intensity and phase motion.

the reader the flavor of the software. Being able to directly calculate the amplitude for some process directly from the measured 4-vectors is invaluable when attacking an analysis program as overarching as this one.

`DiracGamma`	γ^μ : The Dirac Gamma matrices
`DiracSpinor`	$u_{\mu_1\mu_2\ldots\mu_{S-1/2}}(p,m)$: Spin-S spinors, half-integer spin
`PolVector`	$\epsilon_{\mu_1\mu_2\ldots\mu_S}$: Spin-S polarization vectors, integer spin
`LeviCivitaTensor`	$\epsilon_{\mu\nu\rho\sigma}$: Totally anti-symmetric Levi-Civita tensor
`OrbitalTensor`	$L^{(\ell)}_{\mu_1\mu_2\ldots\mu_\ell}$: Orbital angular momentum tensors

Table 2. Examples of the C++ tensor code.

Fig. 2 shows a representation of a standard s-channel process as we would write it out. In this case a photon and proton interact in some electric/magnetic multipole state (E/M) and produce some state of definite isospin, angular momentum and parity (I, J^P). This state then decays to some meson (X) with some relative orbital angular momentum between it and the final state proton (L). The meson then decays to three pions. This part of the amplitude is calculated using the Rarita-Schwinger formalism and *no energy dependence is used to describe the intermediate* IJ^P *state.* The fit procedure will then vary the couplings at the production and decay vertex (α, β) in an attempt to describe the physics. While the decay vertex is fit with a real number, the production vertex is fit with a complex number, the phase of which maps onto a Breit-Wigner phase in the subsequent mass-dependant analysis.

Fig. 3 illustrates one of the strengths of our overall approach. Photoproduction of the ω can have a t-channel, η-exchange contribution. Conversely, photoproduction of the η can have a t-channel, ω-exchange contribution. While the lower vertex is different, the top vertex can be described by *the same coupling* $(g_{\gamma\omega\eta})$. We can fit both final states simultaneously taking advantage of this constraint.

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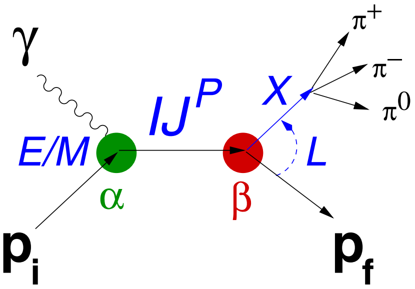

Figure 2. Representation of an s-channel process. The circles at the vertex labeled α and β are fit parameters and the remainder of the amplitude is calculated in terms of the variables shown.

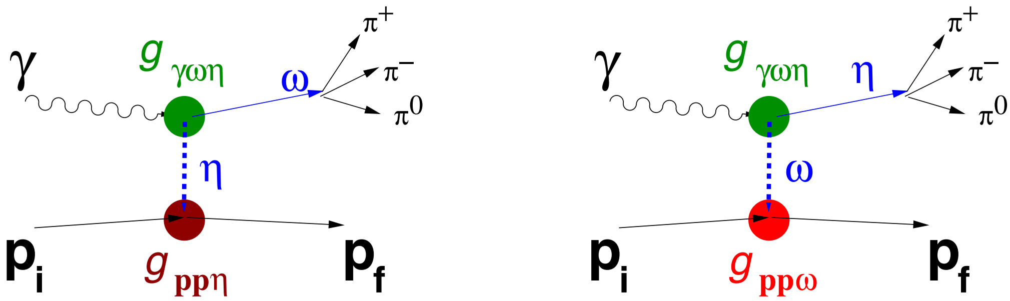

Figure 3. Two t-channel processes for photoproduction of an $\omega(\eta)$ through $\eta(\omega)$ exchange. Note the common coupling at the top vertex.

4. Summary

Our group has developed an extremely flexible software suite that allows us to analyze a variety of final states within the same framework. Instead of fitting cross sections which are then turned over to the theoretical community, we are able to work hand-in-hand with theorists to apply their models and calculations directly to the data analysis. In the course of this analysis, we require a consistency out of the fit parameters across different different final states which should reflect onto the underlying physics. Our fitting code has been constructed that we can even use data from different experiments. This approach to the baryon spectrum should give us the best shot of extracting the underlying amplitudes and answering the missing baryons problem.

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