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Abstract: *Tow ship noise is a major problem plaguing the detection, classification, localization and tracking problems. It is a major contributor to towed array measurement uncertainties that can lead to large estimation errors in any form of signal processing problem aimed at extracting the weak target information. Many sonar-processing approaches ignore this problem relying on narrowband techniques to remove these undesirable interferences at the cost of precious signal-to-noise ratio (SNR). In this paper we address the idea of noise cancellation by formulating it in terms of a joint cancellation/detection problem. It is shown that the joint processor can be designed to perform in a broadband processing environment.*

Keywords: *joint cancellation/detection, signal processing, ship noise cancellation*

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INTRODUCTION

One of the major signal processing problems plaguing towed array processing is the coupling of tow ship noise to each of the array hydrophone sensors. This noise is a major contributor to towed array measurement uncertainty and can lead to large estimation errors in any form of signal processing aimed at extracting weak target information. Ship noise is characterized by propulsion lines from the engines, propellers, gear trains and other transients embedded in broadband noise created by cavitation and ambients [1-2]. Most sonar processing approaches essentially ignore this problem, since they rely on the inherent narrowband processing to remove the ship effects especially when performing such tasks as localization in a shallow ocean environment [3-4]. Other approaches rely on simple filtering techniques to pre-process the hydrophone data hopefully removing these interferences. The effect of this approach is to decrease the overall signal-to-noise ratio (SNR) available along with the noise and therefore potentially degrade the detection performance of the processor when searching for weak acoustic targets.

In this paper, we formulate the problem as a joint cancellation/detection problem by first casting the ship noise canceller into the model-based framework. We then incorporate it in a model-based detection scheme that also includes a target model. Here we use weak planar acoustic targets embedded in broadband noise. Next an optimal solution to the detection problem is developed using the joint processor. It is shown how the joint cancellation/detection evolves into a critical part of a sequential log-likelihood detector that not only enables a more robust processing scheme due to its inherent flexibility, but also improves overall processing performance. We start with the basic problem and then investigate the structure of the processor.

1. OPTIMAL NOISE CANCELLATION

In this section we develop the basic signal and noise models that will be used throughout this paper. We start with the noisy *pressure-field* measurement given by

$$p(r_\ell;t) = s(r_\ell;t) + \eta(r_\ell;t) + v(r_\ell;t)\,, \tag{1}$$

where p is the measured pressure-field at the ℓ^{th}-hydrophone located at spatial location, r_ℓ, and at time t; s is the target or source signal to be detected; η is the ship noise and v is the broadband ambient noise component. We can simplify this notation by expanding over the horizontal array of L-elements, that is,

$$\mathbf{p}(t) = \mathbf{s}(t) + \boldsymbol{\eta}(t) + \mathbf{v}(t)\,, \tag{2}$$

with $\mathbf{p},\mathbf{s},\boldsymbol{\eta},\mathbf{v} \in \mathbf{C}^{L\times 1}$ and $\mathbf{v} \sim N\left(0, R_{vv}(t)\right)$. We decompose this representation further by developing the component signal and noise models. We assume that the *signal* can be characterized by a weak target in the far-field of the array given by

$$s(t) = \alpha_o e^{i(\omega_o t - \mathbf{k}_o \cdot \mathbf{r})} = \alpha_o e^{i\left(\omega_o t - k_o \sin\theta_o (r_o + vt)\right)}, \tag{3}$$

for the source parameters: α_o, ω_o, k_o, θ_o, r_o that are the respective amplitude, temporal frequency, wavenumber, bearing angle and initial sensor location. Since the array is being towed, we include the tow speed, v as well. We can simplify this model by defining the following terms,

$$s(t) = \alpha_o(t) e^{-i\beta_o(t)\sin\theta_o}\,, \tag{4}$$

for $\alpha_o(t) := \alpha_o e^{i\omega_o t}$ and $\beta_o(t) := k_o\left(r_o + vt\right)$. Note that we are not restricting the statistics to be stationary, so we can accommodate the nonstationarities (transients, etc.) that occur naturally in the ocean environment.

Now we are ready to develop the underlying "ship noise" model. Since we know from the acoustic propagation physics that the noise is correlated from sensor-to-sensor, we choose to model the ship noise in the Gauss-Markov framework [5]. Based on noise cancelling principles [6], it is easy to show that the optimal noise cancelling processor can be developed as a solution to a system identification problem [7] with the reference noise, $r(t)$, as input and the ship noise, $\eta(t)$, as output. This relation is given in terms of an optimal coloring or shaping filter with impulse response $H(t)$. That is, for the stationary case, we have

$$\eta(t) = H(t) * r(t) = \sum_{k=1}^{N_H} h_{opt}(k) r(t-k); \quad \text{for } \mathbf{h}_{\text{opt}} = \mathrm{R}_{rr}^{-1} \mathbf{r}_{\eta\mathbf{r}}, \tag{5}$$

and the well-known LMS technique for the nonstationary case [6].

Thus, we define the corresponding Gauss-Markov representation of the coloring or the *Gauss-Markov ship noise model* in terms of the following state-space model as (see Fig. 1)

$$\begin{aligned} \boldsymbol{\xi}(t) &= A_\xi(t-1)\boldsymbol{\xi}(t-1) + B_\xi(t-1)r(t-1) + \mathbf{w}_\xi(t-1) \\ \boldsymbol{\eta}(t) &= C_\xi(t)\boldsymbol{\xi}(t) + \mathbf{v}_\xi(t) \\ \mathbf{p}(t) &= \mathbf{s}(t) + \boldsymbol{\eta}(t) + \mathbf{v}(t) \end{aligned} \quad , \tag{6}$$

with $\boldsymbol{\xi} \in \mathrm{R}^{N_\xi \times 1}$ the inherent colored noise (state vector) and r the scalar reference noise (input) where the additive zero-mean, white gaussian noise sources have respective covariances, $R_{\mathrm{w}_\xi \mathrm{w}_\xi}$ and $R_{\mathrm{v}_\xi \mathrm{v}_\xi}$. Here $A_\xi \in R^{N_\xi \times N_\xi}$, $B_\xi \in R^{N_\xi \times 1}$, $C_\xi \in R^{L \times N_\xi}$ are the system, input and measurement matrices corresponding to the noise cancelling filter parameters. Recall that the impulse response of the state-space model is

$$H_\xi(t,k) = C_\xi(t)\Phi_\xi(t,k)B_\xi(k) \quad \text{for} \quad \Phi_\xi(t,k) = A_\xi(t-k), \;\; t > k, \tag{7}$$

which reduces to

$$H_\xi(t,k) = C_\xi A_\xi^{t-k} B_\xi \quad \text{for} \quad t > k, \tag{8}$$

in the time invariant case. So we see that ship noise can be completely captured by a Gauss-Markov representation in both stationary and nonstationary cases as in Eq. (5).

Using the Gauss-Markov representation of the noise, we can define the underlying *cancellation* problem as

> GIVEN a set of discrete noisy pressure-field and reference measurements, $\{\mathbf{p}(t),\ r(t)\}$, $t = 1, 2, \cdots, N_t$ and the Gauss-Markov model of Eq. (6), FIND the best (minimum variance) estimate of the ship noise, $\hat{\boldsymbol{\eta}}(t \mid t-1)$, such that the canceller output, $\boldsymbol{\varepsilon}_p(t) = \mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t) \approx \mathbf{s}(t)$ is optimal.

The recursive solution to this problem is given by the MBP (Kalman filter) and shown in Table 1 (see [8] for details). This completes the section on modelling and cancellation of ship noise, next we formulate the underlying detection problem.

2. SEQUENTIAL MODEL-BASED DETECTION FOR A TOWED ARRAY

In this section, we develop a generic solution to the model-based detection problem for an array of hydrophones towed in a hostile ocean environment contaminated with ship noise. The basic detection problem can be formulated in terms of the Gauss-Markov representations developed in the previous section for both signal and noise. We formulate the problem in terms of the pressure-field measurements as a binary decision problem [9], that is,

$$
\begin{array}{lll}
H_o: & \mathbf{p}(t) = \boldsymbol{\eta}(t) + \mathbf{v}(t) & \text{[Ship and Broadband Noise]} \\
H_1: & \mathbf{p}(t) = \mathbf{s}(t) + \boldsymbol{\eta}(t) + \mathbf{v}(t) & \text{[Signal, Ship, Broadband Noise]}
\end{array}, \tag{9}
$$

where the null hypothesis is noise and the alternate is the signal and noise case. Following the Neyman-Pearson criterion, the optimal solution to the detection problem is satisfied by the *likelihood ratio*, $L(t)$, given by the joint density functions

Table 1. Optimal Noise Cancellation

NOISE ESTIMATOR	
$\hat{\boldsymbol{\xi}}(t\mid t-1) = A_\xi(t-1)\boldsymbol{\xi}(t-1) + B_\xi(t-1)r(t-1)$	[Prediction]
$\hat{\boldsymbol{\eta}}(t\mid t-1) = C_\xi(t)\hat{\boldsymbol{\xi}}(t\mid t-1)$	[Predicted Noise]
$\mathbf{e}_\eta(t) = \boldsymbol{\eta}(t) - \hat{\boldsymbol{\eta}}(t\mid t-1) = C_\xi(t)\tilde{\boldsymbol{\xi}}(t\mid t-1) + \mathbf{v}_\xi(t)$	[Innovation]
$R_{e_\eta e_\eta}(t) = C_\xi(t)\tilde{P}_{\xi\xi}(t\mid t-1)C'_\xi(t) + R_{v_\xi v_\xi}(t)$	[Innovation Covariance]
$\hat{\boldsymbol{\xi}}(t\mid t) = \hat{\boldsymbol{\xi}}(t\mid t-1) + K_\xi(t)\mathbf{e}_\eta(t)$	[Correction]
$K_\xi(t) = \tilde{P}_{\xi\xi}(t\mid t-1)C'_\xi(t)R^{-1}_{e_\eta e_\eta}(t)$	[Gain]
$\tilde{\boldsymbol{\xi}}(t\mid t-1) = \xi(\mathbf{t}) - \hat{\boldsymbol{\xi}}(t\mid t-1)$	
CANCELLER	
$\hat{\boldsymbol{\eta}}(t\mid t) = C_\xi(t)\hat{\boldsymbol{\xi}}(t\mid t)$	[Filtered Noise]
$\boldsymbol{\varepsilon}_p(t) = \mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t\mid t) \approx \mathbf{s}(t)$	[Cancelled Output]
where $\tilde{\boldsymbol{\xi}}(t\mid t-1)$, $\tilde{P}_{\xi\xi}(t\mid t-1)$ are the state error and corresponding covariance.	

$$
L(t) := \frac{\Pr\left(\mathrm{P}_t \mid H_1\right)}{\Pr\left(\mathrm{P}_t \mid H_o\right)} \underset{<^{\text{Accept } H_o}}{\overset{>^{\text{Accept } H_1}}{}} \mathrm{T}, \tag{10}
$$

where $L(t)$ is the sufficient statistic [9], P_t, is the set of vector pressure-field measurements across the array (snapshots) defined by $\mathrm{P}_t := \{\mathbf{p}(1), \cdots, \mathbf{p}(t)\}$ and $\Pr(\cdot)$ are the respective conditional probabilities under each hypothesis. Using Bayes' rule [5] we can expand the joint probability as

$$
\Pr\left(\mathrm{P}_t \mid H_i\right) = \Pr\left(\mathbf{p}(t), \mathrm{P}_{t-1} \mid H_i\right) = \Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_i\right)\Pr\left(\mathrm{P}_{t-1} \mid H_i\right) \quad \text{for} \quad \text{i=0,1} \tag{11}
$$

Substituting these expressions into Eq. (10), the likelihood ratio becomes

$$
L(t) := \frac{\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_1\right)}{\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_o\right)}\left[\frac{\Pr\left(\mathrm{P}_{t-1} \mid H_1\right)}{\Pr\left(\mathrm{P}_{t-1} \mid H_o\right)}\right] = L(t-1)\frac{\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_1\right)}{\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_o\right)}, \tag{12}
$$

which is the desired sequential form for this problem. Taking natural logarithms of both sides of the equation, defining $\Lambda(t) := \ln L(t)$, we obtain the *sequential log-likelihood ratio* as

$$\Lambda(t) = \Lambda(t-1) + \ln \Pr(\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_1) - \ln \Pr(\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_o) \tag{13}$$

The general form, therefore, for the binary detection problem is called the sequential probability ratio test [9] implemented in logarithmic form as

$$\begin{array}{lr} \Lambda(t) \geq \ln \mathrm{T}_1 & [\text{Accept } H_1] \\ \ln \mathrm{T}_o < \Lambda(t) < \ln \mathrm{T}_1 & [\text{Continue}] \\ \Lambda(t) \leq \ln \mathrm{T}_o & [\text{Accept } H_o] \end{array} \quad . \tag{14}$$

For the joint cancellation/detection problem, we use the underlying Gauss-Markov canceller representations of the previous section in developing the required density functions to implement the detector of Eq. (13). Therefore, under the null hypothesis, we have that $\Pr(\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_o)$ is a conditionally gaussian distribution, since $\boldsymbol{\eta}$ is characterized by the Gauss-Markov model of Eq. (6) and $\mathbf{v}$ is white, gaussian. Therefore under the null hypothesis it can be shown that the conditional density is given by $\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_o\right) \sim \mathrm{N}\left(\hat{\mathbf{p}}_o(t \mid t-1), R_{e_{\mathrm{p}_o} e_{\mathrm{p}_o}}(t)\right)$ for $\hat{\mathbf{p}}_o(t \mid t-1)$ the *conditional mean estimate* at time t based on the data up to time $t-1$. That is, $\hat{\mathbf{p}}_o(t \mid t-1) := E\{\mathbf{p}(t) \mid \mathrm{P}_{t-1}; H_o\} = \hat{\boldsymbol{\eta}}(t \mid t-1)$, for $\hat{\boldsymbol{\eta}}$ the optimal noise estimate of Table 1. The cancelled output sequence (under H_o) is therefore

$$\begin{aligned} \mathbf{e}_{\mathrm{p}_o}(t) = \mathbf{p}(t) - \hat{\mathbf{p}}_o(t \mid t-1) &= \left(\boldsymbol{\eta}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right) + \mathbf{v}(t) = \tilde{\boldsymbol{\eta}}(t \mid t-1) + \mathbf{v}(t) \\ &= C_\xi(t)\tilde{\boldsymbol{\xi}}(t \mid t-1) + \mathbf{v}_\xi(t) + \mathbf{v}(t) \end{aligned} \tag{15}$$

which yields the corresponding covariance

$$R_{\mathrm{e}_{\mathrm{p}_o}\mathrm{e}_{\mathrm{p}_o}}(t) = R_{e_\eta e_\eta}(t) + R_{\mathrm{vv}}(t) = C_\xi(t)\tilde{P}_{\xi\xi}(t)C'_\xi(t) + R_{\mathrm{v}_\xi \mathrm{v}_\xi}(t) + R_{\mathrm{vv}}(t) \,. \tag{16}$$

Under the alternate hypothesis and gaussian assumptions, we have that $\Pr\left(\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_1\right) \sim \mathrm{N}\left(\hat{\mathbf{p}}_1(t \mid t-1), R_{e_{\mathrm{p}1} e_{\mathrm{p}1}}(t)\right)$. The conditional expectation for this case is therefore, $\hat{\mathbf{p}}_1(t \mid t-1) := E\{\mathbf{p}(t) \mid \mathrm{P}_{t\text{-}1}; H_1\} = E\{\mathbf{s}(t) + \boldsymbol{\eta}(t) + \mathbf{v}(t) \mid \mathrm{P}_{t\text{-}1}\} = \mathbf{s}(t) + \hat{\boldsymbol{\eta}}(t \mid t-1)$. The cancelled output sequence (under H_1) is therefore

$$\mathbf{e}_{\mathrm{p}_1}(t) = \mathbf{p}(t) - \hat{\mathbf{p}}_1(t \mid t-1) = \left(\mathbf{s}(t) - \mathbf{s}(t)\right) + \left(\boldsymbol{\eta}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right) + \mathbf{v}(t) = \tilde{\boldsymbol{\eta}}(t \mid t-1) + \mathbf{v}(t)$$

The sequential log-likelihood function follows from the conditional densities, that

$$\begin{aligned} \Lambda(t) = \Lambda(t-1) + \Delta K &+ \frac{1}{2}\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right)' R^{-1}_{e_{\mathrm{p}_o} e_{\mathrm{p}_o}}(t)\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right) \\ &- \frac{1}{2}\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1) - \mathbf{s}(t)\right)' R^{-1}_{e_{\mathrm{p}1} e_{\mathrm{p}1}}(t)\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1) - \mathbf{s}(t)\right) \end{aligned} \quad , \tag{17}$$

where $\Delta K := K_1 - K_o = \frac{1}{2}\left(\ln\left|R_{e_o e_o(t)}\right| - \ln\left|R_{e_1 e_1(t)}\right|\right)$, but $R_{e_\mathrm{p} e_\mathrm{p}}(t) = R_{e_{\mathrm{p}_o} e_{\mathrm{p}o}}(t) = R_{e_{\mathrm{p}1} e_{\mathrm{p}1}}(t)$.

Therefore Eq. (17) becomes

$$\begin{aligned} \Lambda(t) = \Lambda(t-1) + \Delta K &- \frac{1}{2}\mathbf{s}'(t) R^{-1}_{e_\mathrm{p} e_\mathrm{p}}(t)\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right) \\ &- \frac{1}{2}\left(\mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t-1)\right)' R^{-1}_{e_\mathrm{p} e_\mathrm{p}}(t)\mathbf{s}(t) + \mathbf{s}'(t) R^{-1}_{e_\mathrm{p} e_\mathrm{p}}(t)\mathbf{s}(t) \end{aligned} \tag{18}$$

where $R_{e_pe_p}(t)$ is defined in Eq. 16. It is also clear that Eq. (18) can be simplified further by placing all known terms in the threshold expression of Eq. (14). So we see that the log-likelihood detector uses the noise canceller algorithm of Table 1 for its implementation. Thus, we have shown (theoretically) how the joint cancellation/detection problem can be solved using a model-based approach. This completes the theoretical results.

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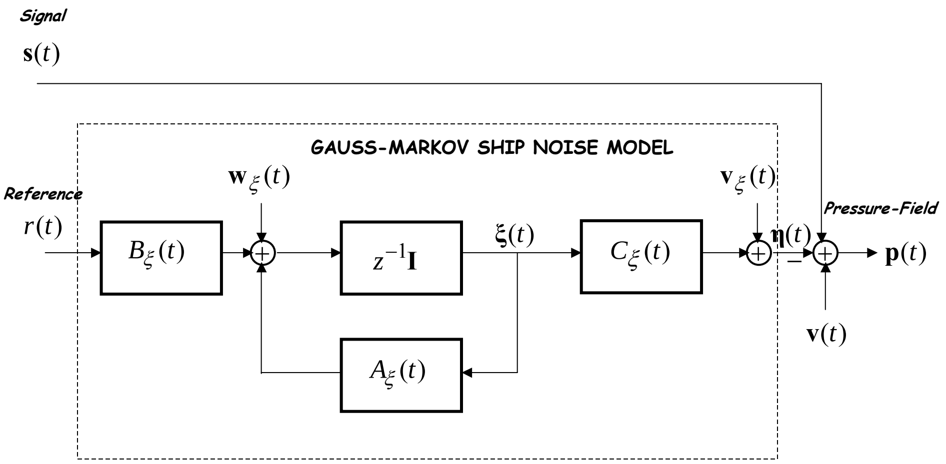

Fig. 1. Gauss-Markov Representation of Ship Noise Cancelling Problem.

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