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On Graphical Representation of Stability Criteria - Summary

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Abstract — **This paper compares various approaches to stability analysis of linear (and of linearized) circuits. In particular it clarifies the limits of application of a widely used but generally erroneous method. It also compares the criteria formulated in terms of impedances versus those expressed in S-parameters**

Index Terms — **Nyquist Criterion, Passivity, Riemann Sphere, Smith Chart, Stability.**

I. Introduction

The aim of this paper is to discuss the circuit stability criteria, in particular, to explain the limits of the criteria related to the eq. (1), which are widely, and often wrongly, used. A rigorous discussion of stability criteria for 2-ports has been covered in [J,P] and for the 1-ports, related to oscillator analysis, in [J,O]. In [O] simple examples were given that show how some commonly used criteria formulated in terms of S-parameters lead to erroneous results even for very simple circuits, such as shown in Fig.1. A short remark in [O] that the results are equally applicable to impedance/admittance representation seemed to have passed unnoticed, and there are continual claims that the simplified criteria are valid when expressed in terms of immittances. Here we argue that that is not necessarily the case. The perceived differences come from the geometry of the complex plane and can be best explained via the Nyquist loops and via the Riemann sphere. Thus we discuss the effects of passivity and of the dominant poles and provide a unified geometrical interpretation of stability criteria applicable to both immittance and to the S-parameter circuit representation.

$$Y_{res}(j\omega)Z_n = 1 \qquad (1)$$

Contents:

1) Physical and geometrical interpretations of stability criteria and their relation to Nyquist loop
2) Nyquist loop for resonant circuits
3) Nyquist loop for passive circuits
4) Limitations of Nyquist criterion in oscillator analysis
5) S-parameters interpreted on Riemann Sphere

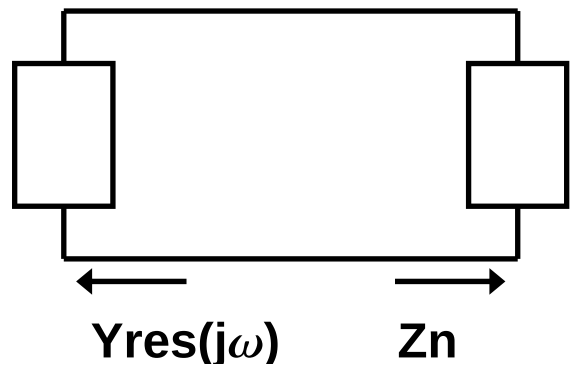

Fig. 1. A one-port model of a linearized oscillator. Immittances determine the (linearized) circuit stability.

II. General Stability Criteria

This section will illustrate the relationship between:

i. Impulse responses that provide intuitive feeling of stability.
ii. Poles (and zeroes) position that correspond to natural resonances (physically) and to system eigenvalues (mathematically).
iii. Nyquist plots that determine the position of the poles and zeroes.

III. Role of Dominant Poles

When the system possesses several dominant poles, then the Nyquist loop(s) have simple shape so that simplified criteria are applicable. In particular the Nyquist loop shown in the Fig. 2 for a single resonance can be determined by a zero crossing, the one with the three resonances requires more caution (however, for the loop shown, the simplified criterion will work).

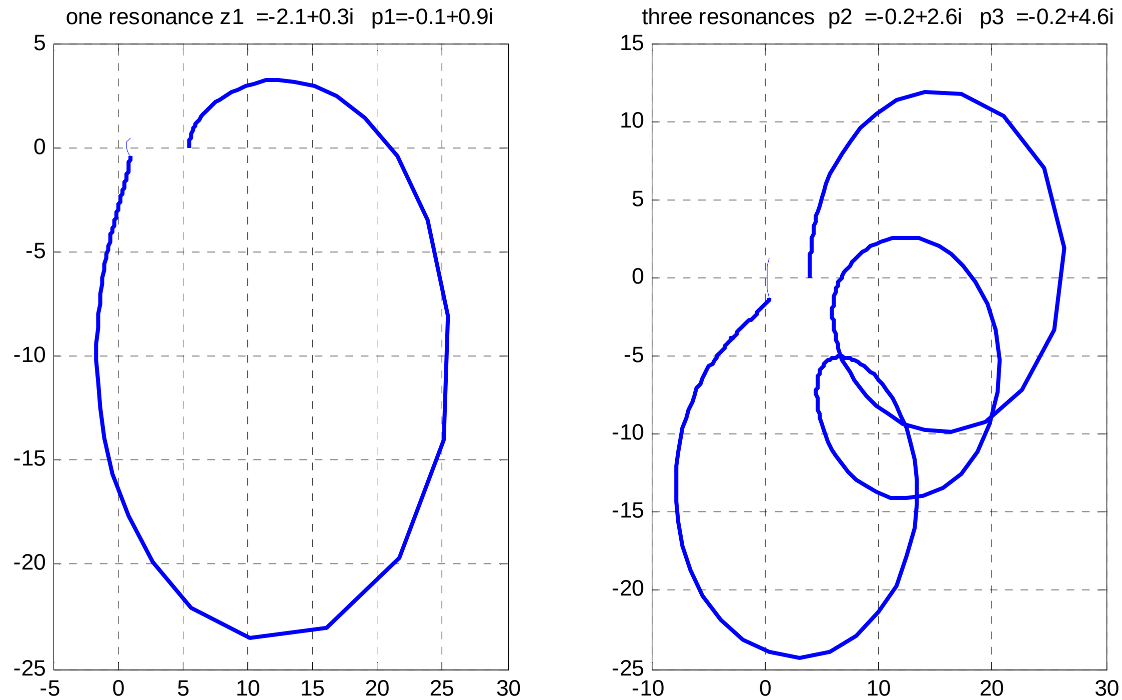

Fig. 2. Nyquist loop for positive frequency: a) one resonance, b) three resonances.

IV. Nyquist Loops for Passive Circuits

There is a well developed theory of immittances of passive 1-ports [ACF, B,G,T]. In this section we interpret their properties in terms of Nyquist loops and relate them to the simplified stability criteria.

A. Properties of passive 1-ports

The immittances of passive 1-ports are expressed by positive real (PR) functions. They have many nice properties, in particular, their Nyquist plots lie in the (closed) right-half plane, see Fig. 3.

B. Nyquist loops for PR functions

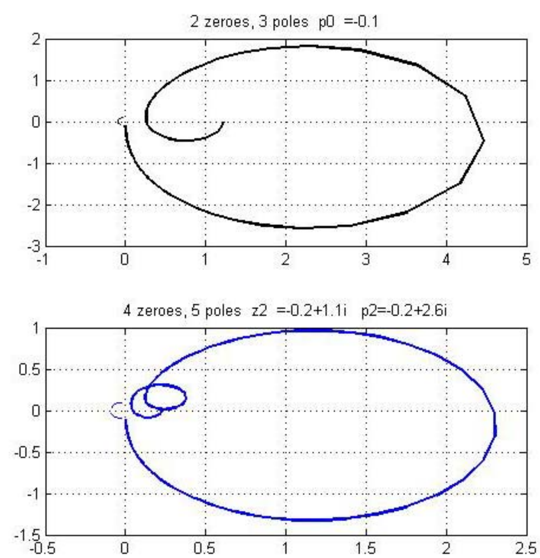

Fig. 3. Nyquist loop for basic immittances. For passive 1-ports they must lie in the closed right half-plane (RHP).

V. LIMITATION OF NYQUIST PLOT

Let us note that the Nyquist plot is only a stability criterion. When applied to oscillator analysis, the loop encirclement does not necessarily relate to the resonant frequency. The frequency of oscillations can be determined from the steady state one-harmonic (describing function) analysis. In this case the resonator characteristics often coincide with the Nyquist plot; however, the physical meaning of the equation (1) is very different.

VI. S-PARAMETERS

It is well known [O] that even very simple circuits, when expressed in terms of scattering parameters, may have a counterintuitive Nyquist loop. The effect is most visible for unusual choices of the characteristic impedance Zo. In the past, when we measured devices in 50 ohm systems, the results agreed with intuition. However, the ability to simulate circuits with arbitrary Zo proved confusing. The effect of Zo is best explained by identifying the complex plane with a sphere as shown in the Fig. 4. The concept is called Riemann Sphere, and the mapping is the stereographical projection. It has been fruitfully used in the theory of functions of single complex variable for more than 100 years. It is interesting to note that it has been recently rediscovered in relation to Smith Chart analysis [MM].

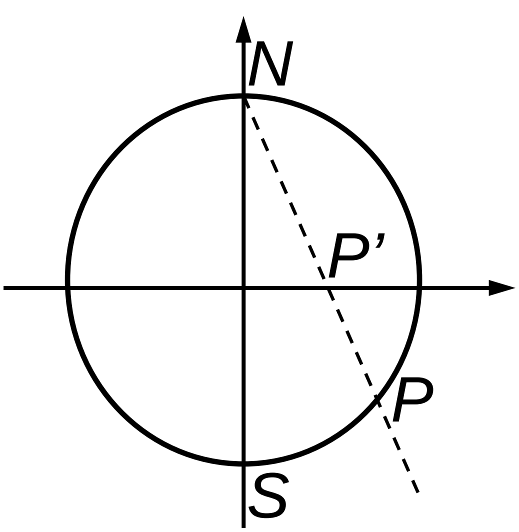

Fig. 4. The stereographic projection

Among many useful features, this transformation elucidates the duality between the zero and infinity (which turn into South and North poles) and also between the straight lines and circles (which all turn into circles on the sphere.

It is given by a simple formula:

$$x = 2X / (1 + X^2 + Y^2),$$
$$y = 2Y / (1 + X^2 + Y^2), \qquad (2)$$
$$z = (-1 + X^2 + Y^2) / (1 + X^2 + Y^2),$$

where $Z = X + jY$, are the complex plane coordinates, and x, y, z are the space coordinates.

Clearly for so defined x, y, z, we have $x^2 + y^2 + z^2 = 1$, so that the projected points lie on a sphere.

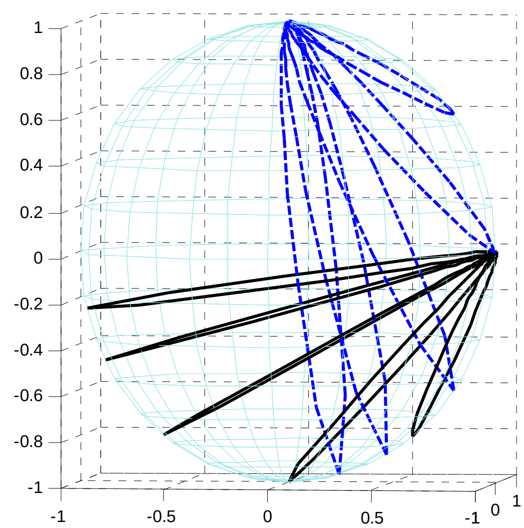

Fig. 5. The stereographic projection onto a Riemann sphere: Images of lines of constant resistance from the complex plane (dotted blue) and from the Smith chart (continuous black).

VII. CONCLUSION

We discussed the relationship between the rigorous and the simplified stability criteria which are widely, but often incorrectly, used. It turns out that the immittances of the passive resonant circuits (i.e., the circuits with the dominant poles) have simple shapes of the Nyquist loop, which makes them amenable to the simplified criteria. The relation of immittance versus S-parameter representation of the stability criteria was explained by interpreting the complex numbers as points on the Riemann sphere.

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