

**UNCERTAINTY ANALYSIS FOR CORROSION DEPTH OF
NUCLEAR SPENT FUEL CANISTER**

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ABSTRACT

Uncertainty analysis for corrosion depth of the spent nuclear fuel canister has been studied using differential analysis. It shows that the mean value presents a second-order linear increase while the variance demonstrates a first-order linear increase in 1000-year time history. By respectively incorporating correlation into pitting factor, uniform corrosion factor, oxygen concentration and chlorine concentration, it is found that the covariance of uniform corrosion factor and chlorine concentration has the highest contribution in the variance of total loss.

Keyword: uncertainty analysis, canister, nuclear waste

INTRODUCTION

Uncertainty analysis plays a role in establishing reliability and reasonable assurance of waste package performance in the licensing process. For the unexpected time scale of waste disposal uncertainty evaluation must be conducted to measure performance of waste package. Predictions for the performance of waste repository unit will inevitably include quantifiable parameter, such as uncertainty or sensitivity.

Many studies on waste package performance have been studied by first-order reliability method (Sutcliffe, 1984; Song and Lee, 1989). A first-order approach applied to the stochastic analysis of radioactive waste package performance is useful for cases where statistical information is incomplete and insufficient. The method can be used to solve subsurface environmental problems where only first and second order statistical moments and marginal probability distributions are available (Song and Lee, 1989). A method of uncertainty analysis is illustrated by analyzing canister corrosion in a nuclear waste package (Sutcliffe, 1984). In that research, uncertainty is represented by a probability distribution in the form of a histogram. It results that the method is with less calculation than a Monte Carlo approach. For the preliminary phase of nuclear waste disposal, field's data are incomplete and insufficient in many cases. The mean and variance are substantially well measured than a precise probability distribution. A mean value, or expectation estimator, is a center value of parameter. Variance is a measure of deviation from its mean value. We can detect variance to identify uncertainty and sensitivity of model or parameter (Shih, 2001). This study conduct differential analysis to analytically evaluate uncertainty of NRC corrosion depth of nuclear spent fuel canister (NRC/USA, 1983). Only the mean and variance of system output are required to evaluate uncertainty. Differential analysis has been also discussed and used in other field (NRC/USA, 1989; Shih, 2001).

THEORY

It is assumed that basic statistical model of system parameters are constituted by its expected value and variance. The expected value, or mean estimator can be denoted as

$$E(p_j) = m^{-1} \sum_{i=1}^m p_{j_i}$$

(Eq. 1)

Variance and covariance estimator are

$$V(p_j) = (m-1)^{-1} \sum_{i=1}^m (p_{ji} - E(p_j))^2 \quad (\text{Eq. 2})$$

and

$$\text{Cov}(p_j, p_k) = (m-1)^{-1} \sum_{i=1}^m (p_{ji} - E(p_j))(p_{ki} - E(p_k)). \quad (\text{Eq. 3})$$

where m : sample size.

Differential analysis is based on using Taylor series to approximate the system model under consideration. The base values, ranges and distribution are selected for the input variables $p_j, j=1, 2, \dots, n$. The base value can be represented by the vector

$$\mathbf{p}_0 = [p_{10}, p_{20}, \dots, p_{n0}] \quad (\text{Eq. 4})$$

The Taylor series approximation to y is developed by using restriction of first-order terms, the approximation has the form

$$y(\mathbf{p}) \cong y(\mathbf{p}_0) + \sum_{j=1}^n \frac{\partial f(\mathbf{p}_0)}{\partial p_j} (p_j - p_{j0}) \quad (\text{Eq. 5})$$

In essence, uncertainty analysis is to evaluate variance propagation of input parameters to system response. The expected value and variance of y can be estimated by

$$E(y) \cong y(\mathbf{p}_0) + \sum_{j=1}^n \frac{\partial f(\mathbf{p}_0)}{\partial p_j} E(p_j - p_{j0}) = y(\mathbf{p}_0) \quad (\text{Eq. 6})$$

and

$$V(y) \cong \sum_{j=1}^n \left(\frac{\partial f(\mathbf{p}_0)}{\partial p_j} \right)^2 V(p_j) + 2 \sum_{j=1}^n \sum_{k=j+1}^n \left[\frac{\partial f(\mathbf{p}_0)}{\partial p_j} \right] \left[\frac{\partial f(\mathbf{p}_0)}{\partial p_k} \right] Cov(p_j, p_k) \quad (\text{Eq. 7})$$

where E , V and Cov denote expected value, variance and covariance.

Expected value is represented a center value for a group of parameter while variance is a measure of deviation from its center. In the nature, parameter variance can be propagated to system output from Eq. (7). The resultant variance of output response is dependent on the degree of variance and their covariance of the system inputs and the system input parameters.

ANALYSIS

For a typical waste package, the loss of contaminant time is equal to the canister breach time plus the time for leaching and transport of a specified amount of activity past the waste package boundaries. Some conservative assumptions can be cited from other references. As an illustration, we use a formula for corrosion depth, defined by y , used by US/NRC in a sample calculation,

$$y = k_p k_h e^{a/T} [O]^b [Cl]^c t^m \quad (\text{Eq. 8})$$

where

y : corrosion depth (mm)

k_p : pitting factor

k_h : uniform corrosion factor (mm/year^m)

T : absolute temperature (°K)

$[O]$: oxygen concentration (µg/g)

$[Cl]$: chlorine concentration (µg/g)

t : exposure time (year)

m , a , b , and c —dimensionless empirical parameters

In this study, y is system output with independent time variable t by incorporating uncertainty into k_h , k_p , $[O]$, $[Cl]$, and m . T , a , b , and c are constant in calculation.

The mean of corrosion depth is derived using Eq. (6),

$$E(y) = E(k_p)E(k_h)e^{a/T} E([O])^b E([Cl])^c t^m \quad (\text{Eq. 9})$$

Using Eq. (7), the variance of corrosion depth is

$$\begin{aligned} Var(y) = & \alpha^2 Var(k_p) + \beta^2 Var(k_h) + \gamma^2 Var([O]) + \eta^2 Var([Cl]) + \xi^2 Var(m) \\ & + 2\alpha\beta Cov(k_p, k_h) + 2\alpha\gamma Cov(k_p, [O]) + 2\alpha\eta Cov(k_p, [Cl]) + 2\alpha\xi Cov(k_p, m) \\ & + 2\beta\gamma Cov(k_h, [O]) + 2\beta\eta Cov(k_h, [Cl]) + 2\beta\xi Cov(k_h, m) \\ & + 2\gamma\eta Cov([O], [Cl]) + 2\gamma\xi Cov([O], m) + 2\eta\xi Cov([Cl], m) \end{aligned} \quad (\text{Eq. 10})$$

where

$Var()$: variance;

$Cov()$: covariance;

$$\alpha = E(k_h)e^{a/T} E([O])^b E([Cl])^c t^{E(m)} \quad (\text{Eq. 11})$$

$$\beta = E(k_p)e^{a/T} E([O])^b E([Cl])^c t^{E(m)} \quad (\text{Eq. 12})$$

$$\gamma = bE(k_p)E(k_h)e^{a/T} E([O])^{b-1} E([Cl])^c t^{E(m)} \quad (\text{Eq. 13})$$

$$\eta = cE(k_p)E(k_h)e^{a/T} E([O])^b E([Cl])^{c-1} t^{E(m)} \quad (\text{Eq. 14})$$

$$\xi = E(k_p)E(k_h)e^{a/T} E([O])^b E([Cl])^c t^{E(m)} \ln(t)$$

Where $E()$ denote mean or expected value and \ln represents natural logarithm. (Eq. 15)

The variance of corrosion depth is derived with uncorrelated case between k_p , k_h , $[O]$, $[Cl]$, and m as Eq. (16)

$$Var(y) = \alpha^2 Var(k_p) + \beta^2 Var(k_h) + \gamma^2 Var([O]) + \eta^2 Var([Cl]) + \xi^2 Var(m)$$

(16)

In minimum requirements, system uncertainty can be analytically evaluated using only the mean and variance of parameter.

The given value of parameter and constant are listed as Table I. The part of parameters and constants used in Eq. (8) are referred from Soong and Lee (1989). Uncertainty data is assumed for demonstration on Eq. (10), (16), (17), and (18). The NRC's criteria for waste package lifetime of 300-1000 year and 1000-year prewaste emplacement groundwater travel time are proposed at current status (Soong and Lee, 1989). We assume that k_p and k_h are correlated with $[O]$ and $[Cl]$. Exposure time is taken as 1000 year. The mean and variance of corrosion depth are shown in Figure 1. It shows that the mean corrosion depth presents a second-order linear in the early 500 year while a first order linear is shown between 500 and 1000 year. In the case, the variances of corrosion depth demonstrate a first-order linear increase. The variance of corrosion depth reach 2.47 for the correlated case while 0.68 for the uncorrelated case at the time of 1000 years. The variance of correlated case is 3.6 times of uncorrelated case at time of 1000 years. (Figure 1). The uncertainties of correlated case of corrosion depth are more significant than uncorrelated case. It indicates that the uncertainty is propagated and distended with increased variance in time history. It is suggested that correlated case has larger uncertainty than uncorrelated case.

CONCLUSION

This study use differential analysis to analytically evaluate uncertainty of NRC proposed performance assessment for corrosion depth of nuclear spent fuel canister in time history. The results show that the mean of corrosion depth represents a first order and second order linear increase in the late and early time history, respectively. The uncertainties of corrosion depth demonstrate a linear increase in 1000 year time history. By incorporating correlation into pitting factor, uniform corrosion factor, oxygen concentration and chlorine concentration, it is found that the covariance of uniform

corrosion factor and chlorine concentration has the highest contribution in the variance of total loss. The differential analysis is suggested to evaluate for uncertainty propagation of analytical system with incomplete and insufficient data.

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Table I . The given value of parameters and constants.

Parameter	value or range	
k_p^*	$E = 4.7$	$Var = 1.66^2$
k_h^*	$E = 0.1706$	$Var = 0.01^2$
$[O]^*$	$E = 7.0$	$Var = 0.1^2$
$[Cl]^*$	$E = 6.6$	$Var = 0.8^2$
M^*	$E = 0.4677$	$Var = 0.0313^2$
a^*		-1402.0
b^*		0.2
c^*		0.543
T^*		373
t^*		0 - 1000
$Cov(k_p, k_h)^{\#}$		0.0
$Cov(k_p, [O])^{\#}$		0.3
$Cov(k_p, [Cl])^{\#}$		0.5
$Cov(k_p, m)^{\#}$		0.0
$Cov(k_h, [O])^{\#}$		0.2
$Cov(k_h, [Cl])^{\#}$		0.4
$Cov(k_h, m)^{\#}$		0.0
$Cov([O], [Cl])^{\#}$		0.0
$Cov([O], m)^{\#}$		0.0
$Cov([Cl], m)^{\#}$		0.0

(*Song and Lee, 1989; #Assumed in this study)

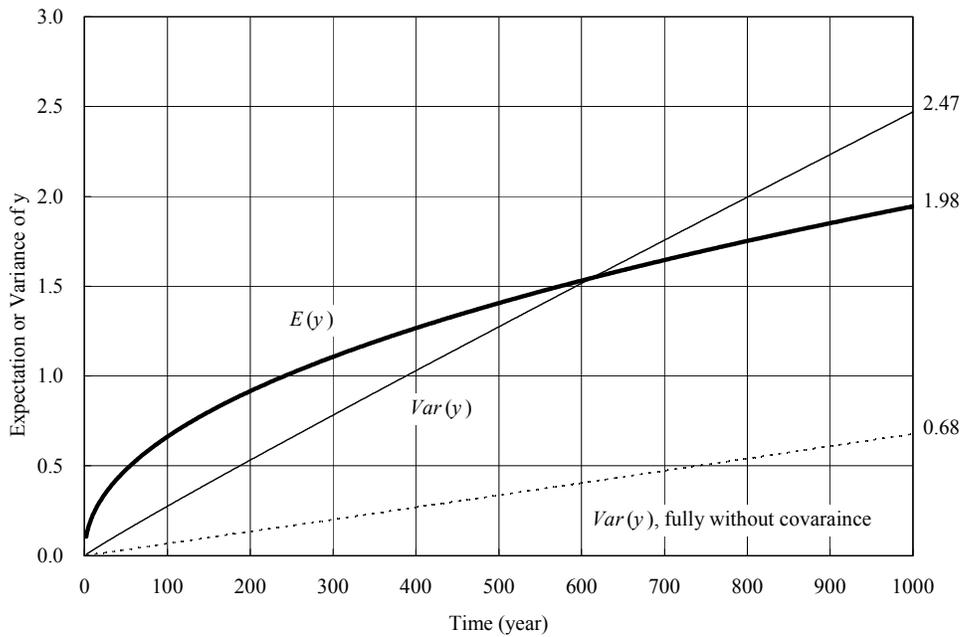


Fig. 1 Expectation and variance of corrosion depth with exposure time.