HIGHER TWISTS IN SPIN STRUCTURE FUNCTIONS FROM A "CONSTITUENT QUARK" POINT OF VIEW

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We discuss the implications of a "constituent quark" structure of the nucleon for the leading ($1/Q^2$–) power corrections to the spin structure functions. Our basic assumption is the presence of quark–gluon correlations in the nucleon wave function, whose size, $\rho \sim 0.3\,\text{fm}$, is small compared to the nucleon radius, R (two–scale picture). We argue that in this picture the isovector twist–4 matrix element in the proton has a sizable negative value, $M_N^2 |f_2^{u-d}| \sim \rho^{-2}$, while the twist–3 matrix elements are small, $M_N^2 d_2 \sim R^{-2}$. These findings are in agreement with the result of a QCD fit to g_1 world data, including recent neutron data from HERMES and Jefferson Lab Hall A, which gives $M_N^2 f_2^{u-d} = -0.28 \pm 0.08\ \text{GeV}^2$.

The transition from the scaling regime at large Q^2 to the quasi–real regime at small Q^2 in the structure functions of inelastic eN scattering represents a major challenge for theory and experiment. Coming from high Q^2, the onset of the transition manifests itself in power ($1/Q^2$–) corrections to the Q^2 dependence of the structure functions. In QCD, these corrections are related to the interactions of the "active" quark/antiquark with the non-perturbative gluon field in the nucleon, described by nucleon matrix elements of certain quark–gluon operators of twist 3 and 4. What are the scale parameters governing the size of these matrix elements?

There is ample evidence for a constituent quark structure of the nucleon — the presence in the nucleon of small–size extended objects — from hadron spectroscopy and low–energy electromagnetic interactions. The notion of a massive constituent quark of finite size is also intimately related to the spontaneous breaking of chiral symmetry. For instance, the microscopic picture of chiral symmetry breaking based on QCD instantons gives rise

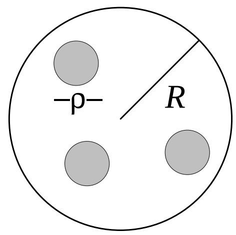

Figure 1.

to constituent quarks of a "size" of $\rho \sim 0.3\,\mathrm{fm}$, which is determined by the average size of the instantons in the vacuum.[1] It is natural to ask if this scale could explain the quark–gluon correlations giving rise to power corrections to polarized deep–inelastic scattering.[a]

In this note[3] we explore the implications of a constituent quark structure of the nucleon for the leading ($1/Q^2$–) power corrections to the nucleon spin structure functions, from a general, model–independent perspective. We think of constituent quarks and antiquarks in the field–theoretical sense, as finite–size correlations in the quark–gluon wave function of the nucleon in QCD, not as elementary objects in the sense of a potential model. We shall try to relate the size of these correlations to the nucleon matrix elements of twist–3 and 4 quark–gluon operators which govern the $1/Q^2$–corrections to the lowest moments of the spin structure functions in QCD.

Our basic assumption is that the size of the "constituent quarks", ρ, be much smaller than the radius of the nucleon, R (see Fig. 1),

$$\rho \;\ll\; R. \tag{1}$$

Various phenomenological considerations point to a constituent quark size of ~ 0.3 fm, which should be compared, say, to the nucleon isoscalar charge radius, $\langle r^2 \rangle^{1/2} = 0.8$ fm. The precise values of these parameters are not the issue here; what is important is that we have a two–scale picture. We stress that the hierarchy (1) is really a logical necessity — if the size of the constituent quark were comparable to that of the nucleon, we would not be "seeing it" as an independent dynamical entity.

In QCD, the leading ($1/Q^2$) power corrections to the lowest moments of the spin structure functions g_1 and g_2 are governed by the matrix elements

[a] Further evidence for constituent quarks of a size $\rho \sim 0.3\,\mathrm{fm}$ comes from the correlations in the transverse spatial distribution of partons in the nucleon, observed in the production of multiple hard dijets in high–energy pp collisions.[2]

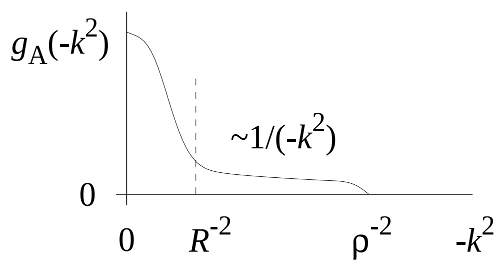

Figure 2.

of twist–3 and 4 operators which measure non-perturbative correlations of the quark and gluon fields in the nucleon (for details, see Refs.[4,5,6]):

$$\begin{array}{llll} d_2 : & \bar{\psi}\gamma_{\{\alpha}\widetilde{F}_{\beta\}\gamma}\psi & & \text{Twist–3} \\ & & & \\ f_2 : & \bar{\psi}\gamma_\alpha\widetilde{F}_{\beta\alpha}\psi & & \text{Twist–4} \end{array} \tag{2}$$

where $\widetilde{F}_{\alpha\beta} = (1/2)\epsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$ is the dual gluon field strength. With the help of the well–known relation

$$F_{\alpha\beta} \;=\; i\,[\nabla_\alpha, \nabla_\beta]\,, \qquad \nabla_\alpha \equiv \partial_\alpha - iA_\alpha \;\; \text{covariant derivative,} \tag{3}$$

and making use of gamma matrix identities and the equations of motion of the quark fields, the twist–4 operator can equivalently be expressed as

$$\bar{\psi}\gamma_\beta\gamma_5(-\nabla^2)\psi. \tag{4}$$

In this form, it can be compared with the axial current operator, which measures the quark contribution to the nucleon spin,

$$g_A : \quad \bar{\psi}\gamma_\beta\gamma_5\psi. \tag{5}$$

We see that the operator (4) measures the correlation of the spin of the quarks with their virtuality (four–momentum squared, k^2) in the nucleon.

The constituent quark picture implies that, generally speaking, the distribution of virtualities of quarks in the nucleon has two components. The bulk of the distribution is governed by the size of the nucleon, $-k^2 \sim R^{-2}$. In addition, there is a "tail" extending up to values of the order of the inverse size of the constituent quark, $-k^2 \sim \rho^{-2}$. This two–component structure, which follows from the Fourier image of the two–scale picture of Fig. 1, is the key to our estimates of higher–twist matrix elements.

Specifically, let us consider the distribution of quark virtualities in the proton's isovector (flavor–nonsinglet) axial charge, g_A, shown schematically in Fig. 2. In the isovector case one can argue that the large–virtuality tail of the distribution is of positive sign and decays as $1/(-k^2)$, until it is "cut off" by the constituent quark size, ρ. This follows from the requirement that in the limit of small size of the constituent quark, $\rho \ll R$, the axial charge should exhibit the logarithmic divergence it has in QCD (with ρ^{-1} acting as the ultraviolet cutoff). We note that exactly this behavior is found also in a field–theoretical chiral model in which the constituent quarks/antiquarks couple to a pion field.[b] Thus, the isovector axial charge, which is the integral of the distribution shown in Fig. 2, behaves parametrically as

$$g_A \;\; = \;\; \int_0^\infty d(-k^2)\; g_A(-k^2) \;\; \sim \log \frac{\rho}{R}. \tag{6}$$

This integral is dominated by virtualities $-k^2 \sim R^{-2} \ll \rho^{-2}$. Consider now the corresponding integral for the isovector (flavor–nonsinglet) twist–4 matrix element, f_2^{u-d}. Since the operator (4) involves an additional contracted derivative, this quantity is determined by the integral with an additional factor k^2, which is parametrically of the order

$$M_N^2 f_2^{u-d} \;\; = \;\; \int_0^\infty d(-k^2)\; g_A(-k^2)\; k^2 \;\; \sim \rho^{-2}. \tag{7}$$

Thus, the isovector twist–4 matrix element in our constituent quark picture is governed by the size of the constituent quark. Furthermore, since $k^2 < 0$ in the integral (7), we can say that

$$f_2^{u-d} \;\; < \;\; 0. \tag{8}$$

To summarize, the constituent quark picture suggests a sizable negative value for $M_N^2 f_2^{u-d}$ in the proton, of the order ρ^{-2}. It is interesting that the estimates of f_2^{u-d} obtained in various QCD–based approaches are in qualitative agreement with this prediction, see Table 1. The QCD sum rule estimates of Refs.[7,8] as well as the instanton vacuum estimate of Ref.[6] both give negative values of the order of $\sim 0.1 - 0.3\ \mathrm{GeV}^2$. These results support the constituent quark interpretation of higher–twist effects.

The only exception in Table 1 is the bag model, which gives a positive result for f_2^{u-d}. This model, however, does not respect the QCD equations of motion [*i.e.*, the two forms of the QCD operator, (2) and (4), would

[b] In the isoscalar case the behavior is different, due to the presence of the $U(1)$ anomaly. The following arguments do not apply in this case.

	$M_N^2 f_2^{u-d}$ [GeV^2]
QCD sum rules (Balitskii et al.) [7]	-0.18
QCD sum rules (Stein et al.) [8]	-0.06
Instantons [6]	-0.22
Bag model [5]	+0.1

give inequivalent results], and therefore cannot claim to give a realistic description of quark–gluon correlations in the nucleon.

When applying the same reasoning as above to the twist–3 matrix element, d_2, we find that after the substitution (3) the quark–gluon operator does not produce a contracted covariant derivative. In this operator, all derivatives are "kinematical", *i.e.*, they are needed to support the spin of the matrix element. This operator does not probe the virtuality of the quarks. Its matrix element is not determined by the size of the constituent quark, but by the size of the nucleon,

$$M_N^2 d_2 \quad \sim \quad R^{-2}. \tag{9}$$

Thus, we find that our two–scale picture implies (see also Ref. [9])

$$\underset{\text{Twist–3}}{|d_2|} \qquad \ll \qquad \underset{\text{Twist–4}}{|f_2|}. \tag{10}$$

It is interesting to see to which extent the qualitative predictions of the constituent quark picture are supported by the experimental data. The twist–3 matrix element, d_2, can be extracted in a model–independent way from the spin structure function g_2 (with the Wandzura–Wilczek contribution subtracted). The SLAC E155X experiment[10] and the recent Jefferson Lab Hall A analysis[11] report values of $d_2^{p,n}$ of few times 10^{-3}, which are more than an order of magnitude smaller than the theoretical estimates for f_2^{u-d} given in Table 1 and our estimate for f_2^{u-d} from g_1 data (see below), in agreement with the parametric ordering implied by the constituent quark picture (10).

The twist–4 matrix element, f_2, can only be extracted from the power corrections to the Q^2–dependence of the structure function g_1. The QCD expression for the $1/Q^2$ corrections to the first moment contains the sum of the matrix elements d_2 and f_2; however, d_2 is known from independent measurements. In fact, the parametric ordering suggested by the constituent quark picture implies that one should ascribe the $1/Q^2$ corrections of the first moment of g_1 entirely to the twist–4 matrix element, f_2.

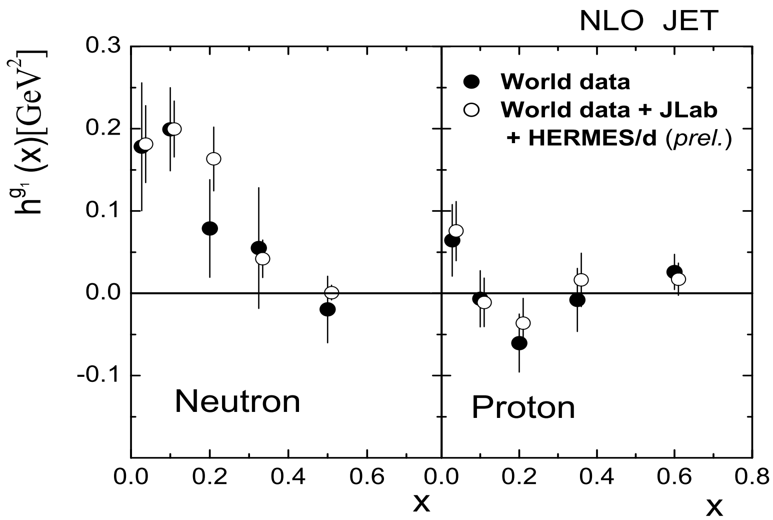

Figure 3.

The dynamical higher–twist contribution has been extracted from NLO QCD fits to the world data on the structure functions $g_1^p(x, Q^2)$ and $g_1^n(x, Q^2)$ (with $Q^2 \geq 1$ GeV^2).[12,13] They are based on the ansatz

$$g_1(x, Q^2) \;=\; g_1(x, Q^2)_{\mathrm{LT\,+\,TMC}} + \frac{h^{g_1}(x)}{Q^2}, \tag{11}$$

where the leading–twist contribution (including target mass corrections) is calculated using the Leader–Stamenov–Sidorov parametrization of polarized parton densities, and $h^{g_1}(x)$ parametrizes the dynamical higher–twist corrections. The results of Ref. [12] for $h^{g_1}(x)$ for proton and neutron are shown in Fig. 3 (filled circles). The second fit[13] (open circles) includes also the new g_1^n data from Jefferson Lab Hall A[14], as well as the preliminary deuteron data from HERMES. One sees that the results for $h^{g_1}(x)$ obtained with the two data sets are nicely consistent. The new data allow to significantly reduce the statistical uncertainty in the higher–twist contribution. Integrating the higher–twist contribution over x we get

$$\int_0^1 dx\, h^{g_1}(x) \;=\; \begin{cases} 0.007 \pm 0.010 \text{ GeV}^2 & \text{(proton)} \\ 0.049 \pm 0.007 \text{ GeV}^2 & \text{(neutron)} \end{cases} \tag{12}$$

Comparing with the QCD expression, neglecting the twist–3 contribution, we obtain

$$M_N^2 f_2^{u-d} \;=\; -0.28 \pm 0.08\,\mathrm{GeV}^2. \qquad (13)$$

This estimate agrees both in sign and in order–of–magnitude with the predictions of the constituent quark picture, (7) and (8).

Our result agrees well with that obtained by Deur *et al.*[15] in a recent analysis of power corrections to the Bjorken sum rule (their $f_2^{p-n} \equiv \frac{1}{3} f_2^{u-d}$ in our conventions). It disagrees in sign with the result of Kao *et al.*[16], who use a resonance–based parametrization of the structure function at low Q^2. The origin of this discrepancy remains to be understood.

The constituent quark picture allows to draw some interesting conclusions about the "global" properties of the transition from high to low Q^2 in the nucleon spin structure functions (*i.e.*, going beyond the leading $1/Q^2$ corrections). Since the characteristic mass scale for the power corrections is set by the size of the constituent quark, one should expect the twist expansion to break down at momenta of the order $Q^2 \sim \rho^{-2}$. For the extraction of the leading ($1/Q^2$–) corrections from QCD fits to the data this implies that one should restrict oneself to the range $Q^2 \gg \rho^{-2}$, where the leading term in the series dominates. Physically speaking, in this region the scattering process takes place "inside" the constituent quark.

When Q^2 is decreased to values of the order R^{-2}, the scattering process probes the motion of the constituent quarks in the nucleon. This is the region dominated by nucleon resonances. In the constituent quark picture, these are changes of the state of motion of the constituent quarks at the scale R, which do not affect the internal structure of the constituent quark at the scale ρ. Thus, our two–scale picture implies a clear distinction between resonance and higher–twist contributions. It is close in spirit to the parametrization of the Q^2 dependence of the first moment of g_1 by Ioffe and Burkert[17], in which the contribution from the Delta resonance is separated from the continuum, and the leading power corrections are associated with the continuum contribution.[c]

To summarize, we have shown that the assumption of a two–scale "constituent quark structure" of the nucleon implies certain qualitative statements about the twist–3 and 4 matrix elements, which are in agreement with present polarized DIS data. For more quantitative estimates, one

[c]It is interesting to note that the mass scale governing the power corrections in the Ioffe–Burkert parametrization[17], M_ρ^2, is numerically close to value associated with the constituent quark size, $\rho^{-2} \sim (0.3\,\mathrm{fm})^{-2} = (600\,\mathrm{MeV})^2$.

eventually has to turn to dynamical models. A consistent realization of the scenario developed here is provided by the instanton model of the QCD vacuum, in which the hierarchy (1) follows from the diluteness of the instanton medium.[6] In particular, this model incorporates the chiral dynamics at the scale R, *i.e.*, the binding of the constituent quarks and antiquarks in the nucleon[1], and thus can serve as the basis of an "interpolating" description connecting the scaling region at large Q^2 with the photoproduction point.

Finally, the constituent quark picture suggested here can be applied also to the power corrections to the unpolarized structure functions; the relation of our approach to that of Petronzio *et al.*[18] will be discussed elsewhere[3].

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