

ELRADGEN: Monte Carlo generator for radiative events in elastic electron–proton scattering

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Abstract

We discuss the theoretical approach and practical algorithms for simulation of radiative events in elastic ep -scattering. A new Monte Carlo generator for real photon emission events in the process of elastic electron–proton scattering is presented. We perform a few consistency checks and present numerical results.

1 Introduction

Radiative effects accompany any processes with charged light particle scattering. These effects are exactly calculable in QED and they should be incorporated as an essential step in the data analysis. For elastic processes, which are the subject of the present report, our starting point for the radiative effect calculation is the technique proposed by Bardin and Shumeiko [1] and later developed for polarized particles as well as for specific elastic and exclusive processes in papers [2, 3, 4, 5]. For other approaches, please see Refs. [6, 7, 8]. In some cases the radiative effects can be calculated analytically. However, often the analytical integration over the photonic phase space is complicated because of specifics of detector geometry or is not desirable because of experimental setup (for example, detector resolution and kinematic cuts). In these situations an approach based on Monte Carlo simulation of the radiative effects is the most adequate. Monte Carlo generator RADGEN [9] for deep inelastic processes was developed on the basis of corresponding analytical code POLRAD [10] and successfully applied by several experimental collaborations analyzing polarized and unpolarized observables in deep inelastic electron–nucleon scattering.

The reason for careful implementation of the radiative correction procedure in elastic ep -scattering is twofold. First, elastic ep -processes are commonly used for detector calibration, and JLab CLAS detector is one of the examples. Second, a new generation of experiments on elastic ep -scattering provide data with very high accuracy (see, for instance,

Refs. [11, 12]), and set new precision requirements to data analysis and, in particular, to the radiative correction procedure. In this report we present a new Monte Carlo generator for elastic unpolarized ep -scattering events taking into account QED radiative effects. The main requirement for construction of the generator is appearance of simulated events (Born and radiative) as well as all event distributions with respect to the kinematic variables in accordance with their contribution to the total cross section.

The contribution to the total cross section from the Born process

$$e(k_1) + p(p_1) \longrightarrow e'(k_2) + p'(p_2) \quad (1)$$

is described by the transfer momentum squared $Q^2 = -(k_1 - k_2)^2$ and the azimuthal angle ϕ . The cross section does not depend on the angle ϕ up to acceptance effects. To describe the phase space of the radiative process

$$e(k_1) + p(p_1) \longrightarrow e'(k_2) + p'(p_2) + \gamma(k), \quad (2)$$

three new kinematic variable have to be considered. We choose them as the proton transfer momentum squared $t = -(k_1 - k_2 - k)^2$, the inelasticity $v = (p_2 + k)^2 - M^2$ and the azimuthal angle ϕ_k between the planes $(\mathbf{q}, \mathbf{k}_1)$ and $(\mathbf{k}_1, \mathbf{k}_2)$. This set of the variables defines the four-vectors of all final particles in any frame. Usually Lab system is used with OZ axis along the beam direction and plane OZX parallel to the ground level.

Thus, the strategy for simulation of one event can be defined as follows.

- For the fixed initial energy and Q^2 the non-radiative and radiative parts of the total cross section are calculated. Note, that together with Born subprocess, the contributions of loop effects and soft photon radiation are also included into the non-radiative part.
- The channel of scattering is simulated for this event in accordance with partial contributions of these two positive parts into the total cross section.
- For any process ϕ is simulated uniformly from 0 to 2π .
- For the radiative event the kinematic variables t , v and ϕ_k are simulated in accordance with their calculated distribution (see discussion below).
- The four-momenta of all final particles in the required system form are calculated.
- If the initial Q^2 has not a fixed value but is instead simulated according to some probability distribution (for example, the Born cross section) then the cross sections have to be stored for reweighting. The Q^2 distribution is simulated over the Born cross section, and realistic observed Q^2 distribution is calculated as sum of weights, they are ratios of the total and Born cross sections.

We note that separation of the bremsstrahlung process into the radiative and nonradiative parts necessarily requires to introduce an additional separating parameter. Usually this quantity is associated with photon energy resolution in the detector. In our program the minimal inelasticity v_{min} is chosen as this parameter.

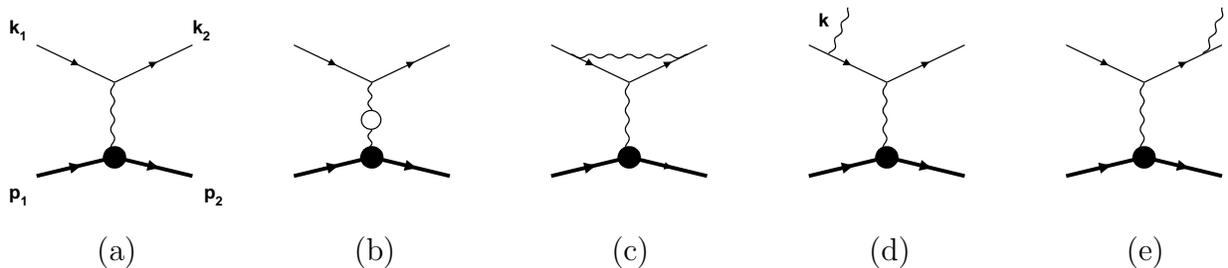


Figure 1: *Feynman graphs contributing to the non-radiated (a, b, c) and the radiated (d, e) parts of the cross sections in lepton-nucleus scattering.*

2 Non-radiated and radiated contributions

The radiative events are generated in accordance with their contribution to the observed total cross section, that can be separated into non-radiative and radiative parts:

$$\sigma_{obs} = \sigma^{non-rad}(v_{min}) + \sigma^{rad}(v_{min}). \quad (3)$$

Here and later we define $\sigma \equiv d\sigma/dQ^2 d\phi$. The first part includes the Born cross section σ_0 (Fig.1 (a)), loop effects (Fig.1 (b,c)) contributions as well as soft photon radiation, which is restricted by the inelasticity value $v < v_{min}$:

$$\sigma^{non-rad}(v_{min}) = \sigma_0 e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac} + \delta^{add}(v_{min})) + \sigma_R^{add}(v_{min}). \quad (4)$$

The explicit expressions for σ_0 , δ_{inf} , δ_{VR} can be found in [3]. The dependence of the non-radiative part of the cross section on v_{min} appears in the following way:

$$\delta^{add}(v_{min}) = \frac{2\alpha}{\pi} \log\left(\frac{v_{max}}{v_{min}}\right) \left[1 - \log\left(\frac{Q^2}{m^2}\right)\right], \quad (5)$$

$$\sigma_R^{add}(v_{min}) = -\frac{\alpha^3}{2S^2} \int_{t'_1}^{t'_2} dt \sum_i \left[\theta_i \frac{\mathcal{F}_i(t)}{t^2} - 4\theta_i^B \mathcal{F}_{IR} \frac{\mathcal{F}_i(Q^2)}{Q^4} \right]. \quad (6)$$

Here $S = 2k_1 p_1$, $v_{max} = S - Q^2 - M^2 Q^2/S$, $\mathcal{F}_i(t)$ are the squared combinations of the electric and magnetic elastic form factors. The quantities θ_i as well as θ_i^B are the analytical functions [3] of kinematic invariants for the radiative and Born subprocesses, respectively.

The radiative part of the total cross section can be presented as an integral over the real photonic phase space in the following way:

$$\sigma^{rad}(v_{min}) = -\frac{\alpha^3}{2S^2} \int_{t_1}^{t_2} dt \sum_i \frac{\mathcal{F}_i(t)}{t^2} \Theta_i(t), \quad (7)$$

where

$$\Theta_i(t) = \int_{v_1}^{v_{max}} dv \theta_i(t, v) = \int_{v_1}^{v_{max}} dv \int_0^{2\pi} d\phi_k \theta_i(t, v, \phi_k), \quad (8)$$

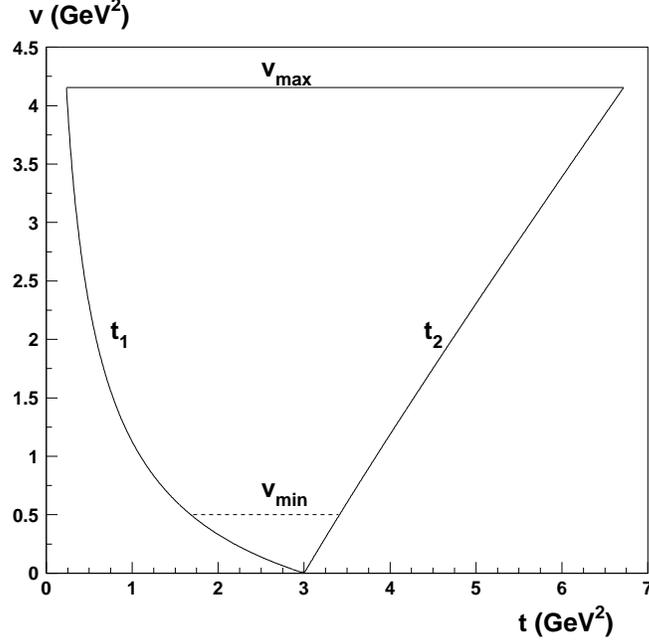


Figure 2: The region of integration over v - and t -variables for JLab kinematics ($Q^2 = 3 \text{ GeV}^2$, $S = 7.5 \text{ GeV}^2$).

and

$$v_1 = \max\left\{v_{\min}, \frac{(t - Q^2)(\sqrt{t} \mp \sqrt{4M^2 + t})}{2\sqrt{t}}\right\}, \quad (9)$$

$$t_{1,2} = \frac{2M^2Q^2 + v_{\max}(Q^2 + v_{\max} \mp \sqrt{(Q^2 + v_{\max})^2 + 4M^2Q^2})}{2(M^2 + v_{\max})}, \quad (10)$$

$$t'_{1,2} = t_{1,2}(v_{\max} \rightarrow v_{\min}). \quad (11)$$

The radiative photonic phase space t - and v -variables are presented in Figure 2.

Notice that in (6) as well as in (7) the structure functions \mathcal{F}_i depend on only one integration variable, namely, on t . Therefore to apply our program to different fits or models of nucleon form factors in (6) and (7), the numerical integration over the variable t is used. On the other hand, to speed up the process of event generation, integration of the expressions θ_i in (8) over the other photonic variables v and ϕ_k has been performed analytically because the quantity $\theta_i(t, v, \phi_k)$ appears as a result of the calculation of the matrix element squares with the real photon emission and do not depend on any model for elastic electron–nucleon scattering.

3 Simulation of radiative events

To simulate v and ϕ_k in accordance with the analytically calculated distributions without any numerical integration over t , the first step is simulation of the variable t . Then taking into account the simulated value t , the distribution function for v can be constructed. Finally,

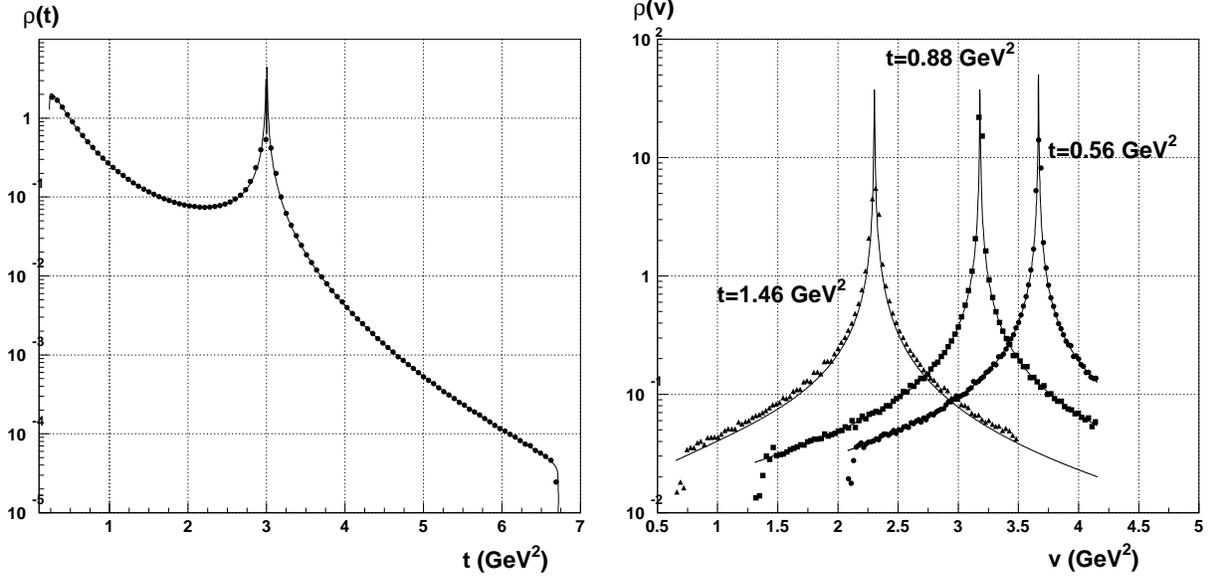


Figure 3: t - and v -histograms and the corresponding probability densities for the kinematics $Q^2 = 3 \text{ GeV}^2$ and $S = 7.5 \text{ GeV}^2$.

ϕ_k is simulated according to the simulated values of t and v . If the event generation is performed successfully, the event distributions over these three variables has to follow the corresponding probability densities:

$$\begin{aligned}
 \rho(t) &= \frac{1}{N_t} \sum_i \mathcal{F}_i(t) \Theta_i(t) / t^2, & N_t &= \int_{t_1}^{t_2} dt \sum_i \mathcal{F}_i(t) \Theta_i(t) / t^2 \\
 \rho(v) &= \frac{1}{N_v} \sum_i \mathcal{F}_i(t_g) \theta_i(t_g, v), & N_v &= \sum_i \mathcal{F}_i(t_g) \Theta_i(t_g) \\
 \rho(\phi_k) &= \frac{1}{N_{\phi_k}} \sum_i \mathcal{F}_i(t_g) \theta_i(t_g, v_g, \phi_k), & N_{\phi_k} &= \sum_i \mathcal{F}_i(t_g) \theta_i(t_g, v_g)
 \end{aligned} \tag{12}$$

As an example, the simulated distributions of the kinematic variables t and v together with their probability densities for representative kinematics at CEBAF energies are compared in Figure 3.

In summary, we presented the first results of the FORTRAN code ELRADGEN, which is a Monte Carlo generator of elastic unpolarized ep -scattering that includes electromagnetic radiative events.

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