Quark-hadron duality: higher twists and large-x structure functions

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Abstract. I review recent developments in the study of quark-hadron duality in inclusive electron scattering in the resonance-scaling transition region. Results on matrix elements of twist-4 operators extracted from moments of the spin-dependent g_1 structure function suggest that duality violating higher twists are small above $Q^2 \sim 1$ GeV^2. The systematics of the x dependence of local duality are analyzed within a quark model framework, and mechanisms are identified for spin-flavor symmetry breaking which underpin the behavior of structure functions at large x.

INTRODUCTION

The nature of the transition between quark and hadron degrees of freedom in QCD is one of the most fundamental problems in strong interaction physics. Assuming that QCD can ultimately describe the physics of hadrons, the transition from quarks and gluons to hadrons can be considered in principle trivial from the point of view of quark–hadron duality: as long as one has access to a complete set of states, it is immaterial whether physical quantities are calculated in terms of elementary quark or effective hadron degrees of freedom. In practice, truncations are of course unavoidable, and it is the consequences of working with incomplete sets of basis states that allows one to expose the dynamics underlying the quark–hadron transition.

The duality between quarks and hadrons reveals itself in most dramatic fashion in inclusive electron–nucleon scattering, $eN \to eX$. Here the inclusive nucleon structure function measured in the region dominated by low-lying nucleon resonances is observed to follow a global scaling curve describing the high energy data, to which the resonance structure function averages – a phenomenon known as Bloom-Gilman duality [1]. Recent high-precision data on the F_2 structure function from Jefferson Lab [2] have provided spectacular confirmation of this duality for each of the low-lying resonance regions, down to Q^2 values of ~ 1 GeV^2 or below.

More recent studies have explored the spin and flavor dependence of duality, as well as its workings in other reactions. Concurrently there has been considerable progress in the theoretical understanding of duality, with a number of model studies elucidating both the dynamical origins of duality and its phenomenological consequences.

In this talk I first review the basic elements of duality in inclusive electron scattering, and its formulation in terms of the operator product expansion (OPE) in QCD. Using the OPE, I describe how duality violations can be used to extract matrix elements of higher twist operators from recent data on the spin dependent g_1 structure function at intermediate Q^2. The study of local duality (the equivalence of restricted resonance sums with the scaling function over a limited range of x) is illustrated within the quark model, and the conditions for the appearance of duality are explicitly identified.

DUALITY AND QCD

The standard method of analyzing structure functions in QCD is the operator product expansion. For large Q^2 the OPE allows the *moments* of a structure function $F(x,Q^2)$ to be expanded in inverse powers of the hard momentum scale, Q^2,

$$M^{(n)}(Q^2) \equiv \int_0^1 dx\, x^{n-2}\, F(x,Q^2) \quad (1)$$

$$= \sum_{i=2,4\ldots} \frac{A_i^{(n)}}{Q^{i-2}}\,, \quad (2)$$

where the expansion coefficients $A_i^{(n)}$ are matrix elements of operators with a specific twist (dimension – spin). The leading twist (twist-2) term, $A_2^{(n)}$, corresponds to scattering from free partons, and is responsible for the scaling (modulo QCD logarithmic corrections) of the structure functions. The higher twist terms $A_{i>2}^{(n)}$ involve multi-quark and mixed quark-gluon operators, and contain information on long-range, nonperturbative correlations between partons.

As pointed out by De Rújula et al. [3], in the OPE language the approximate independence on Q^2 of the moments is naturally attributed to the dominance of the twist-2 term, and suppression (or cancellation) of the higher twist contributions. This observation reveals two important practical applications of duality: if one finds empirically that higher twists are small down to some scale $Q^2_{\rm min}$, then one can extend leading twist analyses of structure function data to $Q^2_{\rm min}$. For the unpolarized F_2 structure function of the nucleon, $Q^2_{\rm min}$ has been found to be around 1 GeV^2 [2], which is lower than that used in standard parton distribution analyses. If, on the other hand, duality is found to be violated, and if the violations are not overwhelming, then the data can be used to extract matrix elements of higher twist operators. This provides unique information about the strength of nonperturbative, multi-parton interaction effects in the nucleon.

HIGHER TWISTS

In the context of global analyses of parton distributions, higher twist effects are often seen as unwelcome complications. However, higher twists contain valuable information on nucleon structure – no less fundamental than that contained in leading twists – and are therefore of tremendous interest in their own right.

Recently several analyses of moments of the spin dependent g_1 structure function of the nucleon have been performed with the aim of extracting so-called *color polarizabilities* of the nucleon, which are related to twist-4 matrix elements involving quark and

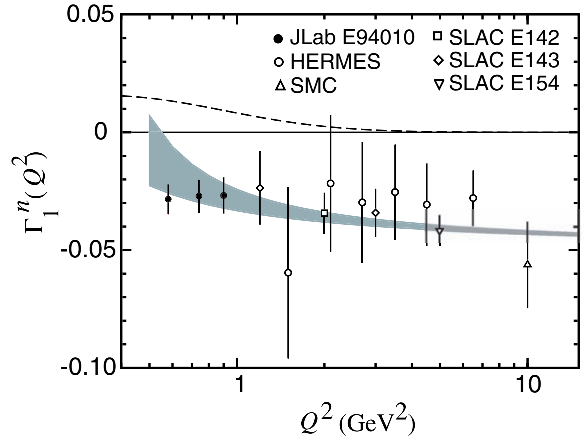

FIGURE 1. Lowest moment of the neutron g_1^n structure function [4]. The shaded band represents the uncertainty on the leading twist contribution due to α_s, and the dashed curve indicates the elastic contribution.

gluon fields. Defining the lowest ($n = 0$) moment of g_1 as

$$\Gamma_1(Q^2) = \int_0^1 dx\, g_1(x,Q^2) \ , \tag{3}$$

the OPE allows one to expand Γ_1 in a series in $1/Q^2$,

$$\Gamma_1(Q^2) \ = \ \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots \tag{4}$$

The leading, twist-2 term μ_2 is related to the spin carried by quarks in the nucleon, and can be expressed in terms of axial vector charges of the nucleon. The coefficient of the $1/Q^2$ correction is given by

$$\mu_4 \ = \ \frac{1}{9} M^2 \left(a_2 + 4 d_2 + 4 f_2 \right) \ , \tag{5}$$

where a_2 is a (twist-2) target mass correction, d_2 is a twist-3 term related to the transverse g_2 structure function, and f_2 is a twist-4 term involving both quark and gluon fields

$$f_2\, M^2 S^\mu \ = \ \frac{1}{2} \sum_q e_q^2\, \langle P,S|\, g\, \overline{\psi}_q\, \widetilde{G}^{\mu\nu} \gamma_\nu\, \psi_q\, |P,S\rangle \ . \tag{6}$$

Here ψ_q is a quark field, $\widetilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$ is the dual gluon field strength tensor, g is the strong coupling constant, and P and S the momentum and spin vectors of the nucleon.

The $1/Q^2$ correction can be determined from Γ_1 data in the intermediate Q^2 region, where Q^2 is neither so large as to completely suppress the higher twists, nor so small as to render the twist expansion unreliable. The result of a recent reanalysis [4] of the world

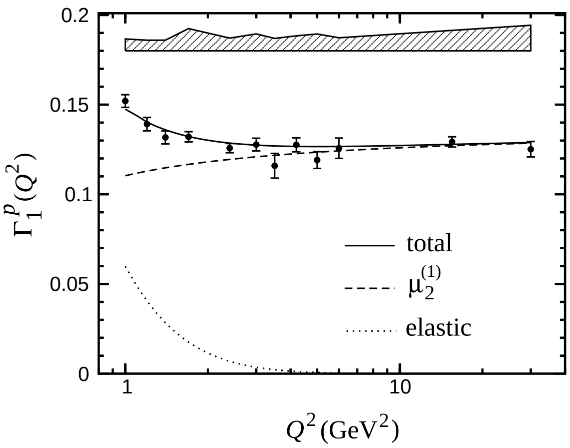

FIGURE 2. Lowest moment of the proton g_1^p structure function [5]. The error bars give statistical uncertainties only, while the systematic and low-x extrapolation errors are given by the shaded band. The shaded band represents the uncertainty on the leading twist contribution due to α_s, and the dashed curve indicates the elastic contribution.

data on the moment of the neutron g_1 structure functions is illustrated in Fig. 1, and for the proton in Fig. 2. Combining the moment data from various experiments is nontrivial since different analyses typically make use of different assumptions about extrapolations into unmeasured regions of kinematics. The structure function moments in Figs. 1 and 2 are therefore extracted using a single set of inputs and assumptions for *all* the data.

Fitting the neutron and proton data in Figs. 1 and 2, the extracted values for the f_2 matrix elements are

$$f_2^n \;=\; 0.034 \pm 0.043 \,, \tag{7}$$
$$f_2^p \;=\; 0.039 \pm {}^{0.038}_{0.043} \,, \tag{8}$$

where the error includes statistical and (the more dominant) systematic uncertainties, as well as from the $x \to 0$ extrapolation and uncertainty in α_s at low Q^2. For the neutron, combining the $1/Q^2$ correction with the extracted $1/Q^4$ term, one finds that in fact the total higher twist contribution to Γ_1^n is almost exactly zero at $Q^2 = 1$ GeV^2 [4].

The twist-3 (d_2) and 4 (f_2) operators describe the response of the collective color electric and magnetic fields to the spin of the nucleon. Expressing these matrix elements in terms of the components of $\widetilde{G}^{\mu\nu}$ in the nucleon rest frame, one can relate d_2 and f_2 to color electric and magnetic polarizabilities. These are defined as [6, 7]

$$\chi_E \, 2M^2\vec{S} \;=\; \langle N|\; \vec{j}_a \times \vec{E}_a \;|N\rangle \,, \tag{9}$$
$$\chi_B \, 2M^2\vec{S} \;=\; \langle N|\; j_a^0 \, \vec{B}_a \;|N\rangle \,, \tag{10}$$

respectively, where $j_a^\mu = -g\bar{\psi}\gamma^\mu t_a \psi$ is the quark current, t_a are color SU(3) matrices, and $\vec{E}_a$ and $\vec{B}_a$ are the color electric and magnetic fields, respectively. In terms of d_2 and

f_2 the color polarizabilities can be expressed as

$$\chi_E = \frac{2}{3}\left(2d_2 + f_2\right) \ , \quad \chi_B = \frac{1}{3}\left(4d_2 - f_2\right) \ . \tag{11}$$

With the above values for f_2, and the results for d_2^p from the global analysis in Ref. [5] and d_2^n from the SLAC E155 measurement [8], one finds

$$\chi_E^n = 0.033 \pm 0.029 \quad , \quad \chi_B^n = -0.001 \pm 0.016 \ , \tag{12}$$
$$\chi_E^p = 0.026 \pm 0.028 \quad , \quad \chi_B^p = -0.013 \mp 0.014 \ , \tag{13}$$

These results indicate that both the color electric and magnetic polarizabilities in the proton and neutron are relatively small, with the central values of the color electric polarizabilities being positive, and the color magnetic zero or slightly negative.

The small values of the higher twist corrections in polarized as well as unpolarized structure functions suggest that the long-range, nonperturbative interactions between quarks and gluons in the nucleon are not as dominant at $Q^2 > 1$ GeV^2 as one may have expected. This means that there are strong cancellations between nucleon resonances resulting in the dominance of the leading twist contribution to the moments. In order to see how such cancellations can take place, in the following we examine a model in which the resonance transitions can be evaluated exactly and the degree to which duality holds quantified.

LOCAL DUALITY

While duality for structure function moments can be analyzed in terms of the OPE, no such simple interpretation exists for the x dependence of the functions themselves. For this one must resort to theoretical & phenomenological models, which can be used to study how a scaling function can arise from a sum over resonances.

To understand the generation of a scaling function entirely out of resonances, each of which is described by form factors that fall rapidly with increasing Q^2, one must address the question of how coherent contributions ("square of sums of quark charges") can yield results consistent with incoherent scattering ("sum of squares of quark charges"). Close and Isgur [9] elucidated this problem by deriving the necessary conditions for duality to occur within the spin-flavor symmetric quark model (although the argument can be generalized to more complex systems). They found that for duality to hold at least one complete set of even and odd parity resonances must be summed over.

Table 1 gives the relative strengths of the contributions to the proton and neutron spin averaged and spin dependent structure functions from the $N \to N^*$ transition matrix elements, for the lowest even parity $\mathbf{56}^+$ and odd parity $\mathbf{70}^-$ representations of SU(6). The coefficients λ and ρ denote the relative strengths of the symmetric and antisymmetric contributions of the SU(6) ground state wave function, and the SU(6) limit corresponds to $\lambda = \rho$.

Summing over all of the states in the $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets gives rise to a neutron to proton ratio $R^{np} \equiv F_1^n/F_1^p = 2/3$, and polarization asymmetries $A_1^p \equiv g_1^p/F_1^p = 5/9$

TABLE 1. Relative strengths of electromagnetic $N \to N^*$ transitions in the SU(6) quark model [9, 10]. The coefficients λ and ρ denote the relative strengths of the symmetric and antisymmetric contributions of the SU(6) ground state wave function. The SU(6) limit corresponds to $\lambda = \rho$.

SU(6) rep$^{\mathrm{n}}$.	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

and $g_1^n/F_1^n = 0$, just as in the quark-parton model in which the structure functions are calculated in terms of partonic (rather than N^*) degrees of freedom.

While the SU(6) predictions for the structure functions hold approximately at $x \sim 1/3$, significant deviations are observed at larger x. It is important therefore to examine the conditions under which combinations of resonances can reproduce, via quark-hadron duality, the behavior of structure functions in the large-x region where SU(6) breaking effects are most prominent.

The most immediate breaking of the SU(6) duality could be achieved by varying the overall strengths of the coefficients for the $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets as a whole. However, since the cancellations of the $N \to N^*$ transitions for the case of g_1^n occur within each multiplet, a non-zero value of A_1^n can only be achieved if SU(6) is broken *within* each multiplet rather than *between* the multiplets. Some intuition is needed therefore on sensible breaking patterns within the supermultiplets.

If the mass difference between the nucleon and Δ is attributed to spin dependent forces, the energy associated with the symmetric part of the wave function will be larger than that of the antisymmetric component. A suppression of the symmetric (λ) configuration at large x will then give rise to a suppressed d quark distribution relative to u, which in turn leads to the famous neutron to proton ratio $R^{np} \to 1/4$ [11, 12].

On the other hand, duality implies that structure functions at large x are determined by transition form factors at high Q^2. At large enough Q^2 one expects these to be constrained by perturbative QCD, which predicts that photons predominantly couple to quarks with the same helicity as the nucleon. Since for massless quarks helicity is conserved, the helicity-3/2 cross section is expected to be suppressed relative to the helicity-1/2 cross section. The helicity-3/2 suppression scenario predicts that $A_1 \to 1$ for both protons and neutrons, and that the neutron to proton ratio $R^{np} \to 3/7$. This latter result is identical to that obtained in the classic quark level calculation of Farrar & Jackson [13] on the basis of perturbative QCD counting rules! Whether the $x \to 1$ behavior of structure functions follows the λ-suppression scenario or is governed by helicity conservation is being addressed experimentally at Jefferson Lab, where the 12 GeV energy upgrade will allow definitive tests of the properties of structure functions at large-x.

OUTLOOK

The discussion of duality in the simple quark model provides a clear illustration of how parton model results for structure functions can be replicated by explicit sums over nucleon resonances. More sophisticated models can of course be considered, and an important challenge will be to consistently include the effects of both resonances and the nonresonant background in the same framework.

Future experimental exploration of duality will focus on determining its flavor, spin and target dependence, as well as its workings when probed by the weak interaction, in neutrino–nucleon scattering. Another important avenue will be to explore duality in semi-inclusive reactions. Confirmation of duality here would open the way to an enormously rich semi-inclusive program in the preasymptotic regime, allowing unprecedented access to the spin and flavor distributions of the nucleon, especially at large x.

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