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Measurement of the Decelerating Wake in a Plasma Wakefield Accelerator

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Abstract. Recent experiments at SLAC have shown that high gradient acceleration of electrons is achievable in meter scale plasmas. Results from these experiments show that the wakefield is sensitive to parameters in the electron beam which drives it. In the experiment the bunch lengths were varied systematically at constant charge. The effort to extract a measurement of the decelerating wake from the maximum energy loss of the electron beam is discussed.

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INTRODUCTION

Plasma wakefield accelerators (PWFAs) have shown great potential as a mechanism for acceleration of particles to high energy. Experiments have demonstrated both high gradients and long propagation distances[1, 2]. However, for a future collider design a deeper understanding of the experimental relationship between the wake and the electron beam is necessary.

EXPERIMENTAL SETUP

In the SLAC experiment E167, a $42GeV$ beam with $1.8 \cdot 10^{10}$ electrons, $\varepsilon_{N,x} \sim 60\mu m$, and $\varepsilon_{N,y} \sim 6\mu m$ was injected into a confined neutral lithium vapor of density $n \approx 2.7 \cdot 10^{17}$ per cm^3 with a length of 85cm, full width at half max (FWHM). The energy spectrum of the incoming beam was measured by recording the incoherent x-ray synchrotron radiation emitted by the beam in a soft vertical chicane placed in a region of a high horizontal dispersion. This energy spectrum could then be matched in the 2D tracking code LiTrack[3] to recover the temporal profile of the beam[1, 4]. Bunch lengths were found to be $10-50\mu m$[5]. With a transverse spot size of $\sim 10\mu m$ this beam was dense enough to ionize the lithium and drive a non-linear wake[1, 6]. The initial energy spread had a full width of 4%.

The measureable effect of the interaction between the beam and the plasma is the large broadening of the energy spectrum as the bunch length and the plasma wavelength

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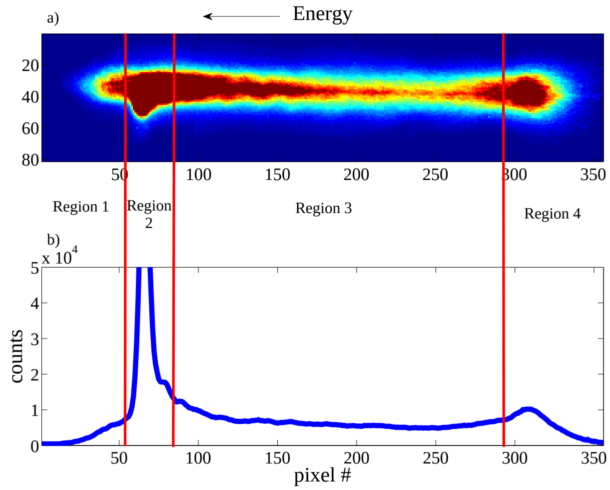

FIGURE 1. An example of a) and image, and b) derived energy profile, from the energy spectrometer diagnostic. Four distinct regions can be seen. Region 1 contains the accelerated charge, region 2 the beam head which has too low a current to ionize, region 3 the beam core driving the wake, and region 4 the lowest energy charge. Region 4 exhibits a peak followed by a lower energy rolloff.

are on the same order, leading to the beam sampling all phases of the wake. The particle energies were measured after exiting the plasma using a magnetic spectrometer and Cerenkov radiation. The electrons were dispersed in energy by a dipole magnet centered 2.18m from the plasma exit with $\int \vec{B} \cdot \mathrm{d}\vec{l} = 12.05 kG \cdot m$. The electrons then passed through an air gap where their Cerenkov radiation was imaged at two locations from the magnet center, 0.85m, for low energies, and 1.85m, for high energies[7]. Figure 1 shows an example of the beam as it appears after the energy spectrometer at the first imaging location. The entire beam can be seen on the image with four distinct regions. Region 1, where the accelerated charge sits, and Region 4, where the maximum decelerated charge is, are of most interest.

ENERGY MEASUREMENTS

Measurement of the accelerating gradient in a PWFA driven by short electron bunches is already well established. The essential problem is identifying the highest energy electrons on the diagnostic images with confidence. There are several strategies, which vary experiment to experiment, for achieving this, from charge contours[1] to two screen methods[2, 7]. The remaining open question, to be addressed here, is how to relate a low energy measurement to the maximum decelerating wake. In the profile shown in Fig. 1 there is a peak in Region 4 where the lowest energy charge sits, followed by a long rolloff of lower energy particles. Understanding this feature will be the key to translating a particle energy measurement into a wake measurement.

In reality the raw image is a convolution of various beam attributes, some due to the nature of the incoming beam and others due to effects in the plasma. The three effects that dominate the interpretation of a low energy measurement are the y-spot size, the peak in the wakefield and radiative losses due to oscillation of beam particles in the plasma. The camera resolution is not large enough to give a contribution. These three effects all contribute to the profile that appears when the image is summed. The strategy for estimating how these affect the measurement will be to reconstruct a theoretical profile, convolving all these effects with some reasonable assumptions, and look at its qualitative and quantitative attributes. The calculated function is $\mathrm{d}N/\mathrm{d}y$ vs. y, where N is the number of particles and y is the dispersion direction. This reconstruction will give a guide as to where to choose the low energy point and how to translate it to the wake.

Spot Size

In the absence of dispersion, the electron beam has a finite spot size at the diagnostic location in the dispersion direction y. This contribution to the measured profile is

$$\frac{\mathrm{d}N}{\mathrm{d}y}|_{spot} \propto \frac{1}{\sqrt{2\pi}\sigma_y} e^{-y^2/2\sigma_y^2}, \tag{1}$$

where σ_y is the beam spot size in y. As the beam is dispersed in y, σ_y cannot be directly measured. As an upper limit for this effect, it is reasonable to use the beam spot in x. Since the y-emittance is an order of magnitude smaller, the y-spot is likely smaller. Measured x spot sizes were $\sim 0.4mm$ RMS width.

Peak in the Wakefield

The shape of the wakefield has an effect on the measurement as at the peak decelerating field G_0, $\mathrm{d}G/\mathrm{d}\tau = 0$, where $\tau = z - ct$ is the beam coordinate. In order to quantify this effect the shape of the decelerating field is taken to be a parabola around this point with no variation as the beam propagates,

$$G = G_0 - A\tau^2, \tag{2}$$

where A defines the sharpness of the field peak. The energy of a beam particle interacting with this field is $E = E_0 - GL$, where E_0 is the original energy and L is the interaction length. Using this expression, and inverting E from above, it is possible to derive the number of particles per energy slice due to the shape of the wake,

$$\frac{\mathrm{d}N}{\mathrm{d}E} = \frac{\mathrm{d}N}{\mathrm{d}\tau} / \frac{\mathrm{d}E}{\mathrm{d}\tau} = \frac{\mathrm{d}N_b}{\mathrm{d}\tau}\frac{1}{2AL\tau} = \frac{\mathrm{d}N_b}{\mathrm{d}\tau}\frac{1}{2\sqrt{AL}\sqrt{G_0L - \Delta E}}, \tag{3}$$

where $\Delta E = E_0 - E$ and $\mathrm{d}N_b/\mathrm{d}\tau$ is the longitudinal density of the electron beam which is assumed to be constant around the wakefield peak. Then, using the expression for the dispersion

$$E = \frac{E_0 \Delta y_{E_0}}{y - y_0}, \tag{4}$$

where Δy_{E_0} is the dispersion and y_0 is the position of a particle of infinite energy. The contribution to $\mathrm{d}N/\mathrm{d}y$ can be calculated, assuming the energy spread imprinted by the plasma is much larger than the initial spread, yielding

$$\frac{\mathrm{d}N}{\mathrm{d}y}|_{wakepeak} = \frac{\mathrm{d}N}{\mathrm{d}E}\frac{\mathrm{d}E}{\mathrm{d}y} \propto \frac{1}{(y-y_0)^2}\frac{1}{\sqrt{G_0L - E_0(1-\frac{\Delta y_{E_0}}{(y-y_0)})}}. \tag{5}$$

This relationship shows that the energy spectrum peaks around the minimum energy.

Radiative Losses Through Propagation in the Plasma

The energy radiated off as a particle oscillates in an ion-column is

$$\frac{\mathrm{d}W}{\mathrm{d}z} = -\frac{1}{12}\frac{r_e mc^2\gamma^2 k_p^4 r_0^2}{e}[eV/m], \tag{6}$$

where W is the particle energy, $r_0 = \sqrt{x_0^2 + y_0^2}$ is the maximum displacement from the propagation axis as the particle oscillates, γ is the relativistic factor of the particle, and k_p is the plasma wavenumber[8], a relationship which has been experimentally verified[9, 10]. This expression is for a single particle and dependent only on the maximum radial displacement from the propagation axis r_0, and not on $x, y, \dot{x}$, or $\dot{y}$ independantly. To extend this to a beam distribution the quantity $\mathrm{d}N/\mathrm{d}r_0$ is needed. Using appropriate transformations of the phase space density for a symmetric beam matched into the ion column[11] and integrating over all angles,

$$\frac{\mathrm{d}N}{\mathrm{d}r_0} = \frac{2N}{\sigma_r^4} r_0^3 e^{-r_0^2/\sigma_r^2}. \tag{7}$$

Using this, it is now possible to determine the effect of radiative losses on the decelerating wake measurement. First, from Eq. 6 an upper limit can be set on the total energy lost to radiation while propagating in the plasma. The relativistic factor γ of the beam and the plasma density n are set as constant. Since the power radiated is proportional to $\gamma^2 n^2$, the actual loss will be less as the particles are also losing energy to the wake as they propagate. The propagation distance is taken to be 90cm as, while the FWHM of the confined plasma was 85cm, deceleration continues past the half density point. This yields the maximum energy lost to radiation, ε, as

$$\varepsilon = \frac{1}{12}\frac{r_e mc^2\gamma^2 k_p^4 r_0^2}{e}[eV]. \tag{8}$$

In the absence of radiation each longitudinal slice will be at the same energy, as the longitudinal wake has no transverse variation for a symmetric drive beam[12]. Therefore each longitudinal slice has an expression analagous to Eq. 7, which can be used in conjunction with the derivative of Eq. 8 to obtain

$$\frac{\mathrm{d}N_{long}}{\mathrm{d}\varepsilon} = \frac{\mathrm{d}N_{long}}{\mathrm{d}r_0}\frac{\mathrm{d}r_0}{\mathrm{d}\varepsilon} = \frac{12N_{long}}{\sigma_r^4}\left(\frac{e}{r_e mc^2\gamma^2 kp^4}\right) r_0^2 e^{-r_0^2/\sigma_r^2}, \tag{9}$$

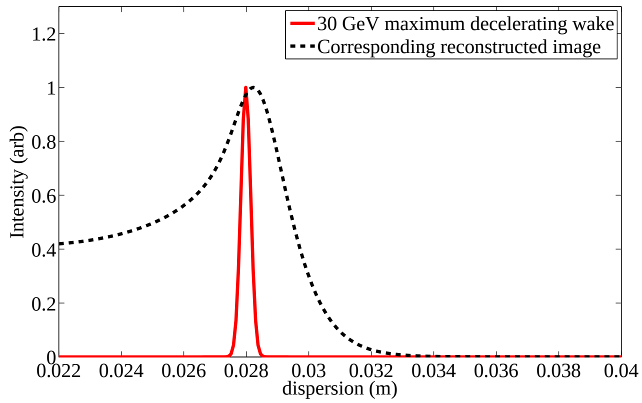

FIGURE 2. A reconstructed profile convolving the dominant effects. The solid red curve shows how a single particle with initial energy of 42 GeV sitting in a 30 GeV decelerating wake would be measured on the diagnostic due to the 2 pixel resolution limit of the optical system. The dotted black curve shows how this maximum decelerating wake actually appears taking into account that it is sampled by a beam with finite transverse and longitudinal size.

where N_{long} is the number of particles in each longitidinal beam slice. Inverting Eq. 8 yields

$$\frac{\mathrm{d}N_{long}}{\mathrm{d}\varepsilon} = N_{long}\alpha^2\varepsilon e^{-\alpha\varepsilon}, \tag{10}$$

where $\alpha = 12e/(\sigma_r^2 r_e mc^2\gamma^2 kp^4)$. This shows that each longitudinal slice develops a tail of lower energy particles as the particles away from the axis of propagation radiate.

To calculate the effect on the energy spectrum, the departure from the nominal energy of each longitudinal slice by an off-axis particle, ε, is taken to be small. Its position can then be calculated through a linearization of the dispersion expression, Eq. 4,

$$P - E_0 = -\varepsilon = -E_0\frac{y}{\Delta y}, \tag{11}$$

where y is the position of the measured particle and $\Delta y = y_{E_0} - y_0$. Then, utilizing Eq. 10,

$$\frac{\mathrm{d}N}{\mathrm{d}y}|_{rad} = \frac{\mathrm{d}N}{\mathrm{d}\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}y} = N_{long}(\frac{E_0}{\Delta y})^2\alpha^2 y e^{-\alpha E_0 y/\Delta y}. \tag{12}$$

RESULTS AND CONCLUSIONS

The separate contributions of the three dominant effects to a low energy measurement have been calculated. Now these effects must be convolved. Figure 2 shows an example of this convolution, where the maximum decelerating gradient was taken to be 30 GeV/m. The peak of the solid red curve shows how a particle influenced by only this field would appear on the diagnostic, while the dotted black curve shows what actually will be measured. The peak of the dotted black curve is therefore the measurement that should be identified with the wake and corresponds to the peak in Region 4 of Fig. 1.

The peak in the dotted black curve is shifted slightly off that in the solid red curve. This shift can be corrected for by making a series of such reconstructions for all possible maximum decelerating wakes. This allows a one-to-one mapping from the peak in the low energy part of the measured profile to the real decelerating wake. The results of this calculation show that the corrections to the effective gradient, calculated using the plasma length and energy at this peak, are small, on the order of 1-2%.

Thus, in order to map the minimum particle energy to the maximum decelerating wake, the location of the peak in the low energy part of the spectrum is the desired measurement. There is a small correction that can be made to account for the shifting of this peak. The particles in the long tail have lower energy due to radiative effects and not the wake by itself, and therefore should be ignored. Using this method, decelerating gradients of $20 - 35GeV$ were measured. These will be combined with accelerating gradient measurements to yield an experimental limit on the transformer ratio.

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