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Title: APPLICATION OF FUZZY SET THEORY FOR EXPOSURE CONTROL IN BERYLLIUM PART MANUFACTURING

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Application of Fuzzy Set Theory for Exposure Control in Beryllium Part Manufacturing

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ABSTRACT

We have applied fuzzy set theory to some exposure control problems encountered in the machining and the manufacturing of beryllium parts. A portion of that work is presented in this paper. The major driving force for using fuzzy techniques in this case rather than classical statistical process control is that beryllium exposure is very task dependent and our manufacturing plant is quite atypical. We believe that standard statistical techniques will produce too many false alarms. The beryllium plant produces parts on a daily basis, but every day is different. Some days many parts are produced and some days only a few. Some times the parts are large and sometimes the parts are small. Some machining cuts are rough and some are fine. These factors and others make it hard to define a typical day. The problem of concern, for this study, is the worker beryllium exposure. Even though the plant is new and very modern and the exposure levels are expected to be well below the required levels, the Department of Energy has demanded that the levels for this plant be well below required levels. The control charts used to monitor this process are expected to answer two questions:

1. Is the process out of Control? Do we need to instigate special controls such as requiring workers to use respirators?
2. Are new, previously untested, controls making a difference?

The standard Shewhart type control charts, based on consistent plant operating conditions do not adequately answer this question. The approach described here is based upon a fuzzy modification to the Shewhart X Bar-R chart. This approach is expected to yield better results than work based upon the classical probabilistic control chart.

KEYWORDS: fuzzy logic, control chart, beryllium.

INTRODUCTION

Los Alamos National Laboratory has recently completed a new beryllium part manufacturing facility. The intent of the facility is to supply beryllium parts to the Department of Energy (DOE) complex and also act as a research facility to study better and safer techniques for producing beryllium parts. Exposure to beryllium particulate matter, especially very small particles has long been a concern to the beryllium industry. The industrial exposure limit is set at 2 $\mu g/m^3$ per worker per eight-hour shift. The DOE has set limits of 0.2 $\mu g/m^3$ or ten times lower than the industrial standard for this facility. In addition, they have requested continual quality improvement. In other words, in a short period of time they intend to set even lower limits. Several controls have been implemented to assure that the current low level can be met. But there is a real management concern that the process remains under control and that any further process improvements are truly improvements. Since the facility is a research facility with manufacturing capabilities, the workload and type of work done each day can vary dramatically. This makes the average beryllium exposure vary widely from day to day. This in turn makes it very difficult to determine the degree of control or the degree of improvement with a standard statistical control chart. For this reason we have implemented a fuzzy control chart to improve our perception of the process.

The plant has three workers and seven machines. Each worker wears a device that measures the amount of beryllium inhaled during his or her shift. Each machine is fitted with a similar device. Probably due to the fact that the machines don't inhale, the machine readings are normally a factor of ten lower than the worker readings. The devices are analyzed in the laboratory and the results are reported several days after the exposure has occurred. The machine readings are multiplied by ten and averaged with the worker readings. This provides a sample size of up to ten for each day. A Shewhart-type X Bar-R chart can be constructed with these data and presumably answer the questions of control and quality improvement. Although such a chart can be useful, because of the widely fluctuating daily circumstances, the standard tests for controllability are not very meaningful.

There are four variables that have a large influence upon the daily beryllium exposure. They are the number of parts machined, the size of the part, the number of machine set ups performed, and the type of machine cut (rough, medium, or fine). In our fuzzy model, a semantic description of these four variables and the beryllium exposure are combined to produce a semantic description of the type of day had by each worker and each machine. The day type is then averaged and a distribution is found. These values are then used to produce a fuzzy Shewhart-type X Bar-R chart. This chart is quite consistent and takes into account the daily variability. It provides more realistic control limits and will make it easier to make an honest determination about whether a new control has made a realistic improvement to the system or not.

THE FUZZY SYSTEM

The fuzzy system consists of five input variables or universes of discourse and one output variable. Each input universe has two membership functions and the output universe has five membership functions. The input and the output are connected by thirty-two rules. The five input variables are:

1) Number of Parts – with a standard range of 0 to 10 and membership functions:
   a) Few.
   b) Many.
2) Size of Parts – with a standard range of 0 to 10 and membership functions:
   a) Small.
   b) Large.
3) Number of Set Ups – with a standard range of 0 to 20 and membership functions:
   a) Few.
   b) Many.
4) Type of Cut – with a standard range of 0 to 10 and membership functions:
   a) Fine.
   b) Rough.
5) Beryllium Exposure – with a standard range of 0 to 0.4 and membership functions:
   a) Low.
   b) High.

The output variable is:

1) The Type of Day – with range from 0 to 1 and membership functions:
   a) Good.
   b) Fair.
   c) OK.
   d) Bad.
   e) Terrible.

The ranges on all of the input variables described above and shown in figure 1 are "idealized". They are the ranges used for the membership functions in the computer code. In reality, based on simulations and some data, we found that the membership functions should be different for different groups. The machinist-group uses a different set of input membership functions, but the same rules and the same output membership functions, than the machine-groups use. Machines one and two use the same input membership functions, but they are slightly different than the ones machines three and four use. Machines five and six are similar but their input membership functions are slightly different than those used by the other machines. Machine seven is different than all of the other machines. In order for all of the groups to use the same membership functions in the computer code we multiply their daily reading by factors to make them fit the appropriate membership function in the code. For example, the average beryllium readings for the machines are lower than the average machinist readings by a factor of ten. Therefore, we multiply the machine reading by ten so that we can use the same membership functions for both machinists and machines. Figure 1 represents the idealized or standard input membership functions used in the code. Figure 2 represents the output membership functions.

The rules are based on some simple ideas. For example, if all of the four mitigating input variables indicate that the beryllium exposure should be low, and it is low, then the "type of day" is OK. Likewise, if all four indicate that the exposure should be high, and it is high, then the day is also OK. If all four indicate that the exposure should be low, and it is high, then the day is Terrible. If all four indicate that the exposure should be high, and it is low, then the day is Good. Fair and Bad days fall in between the OK days and the

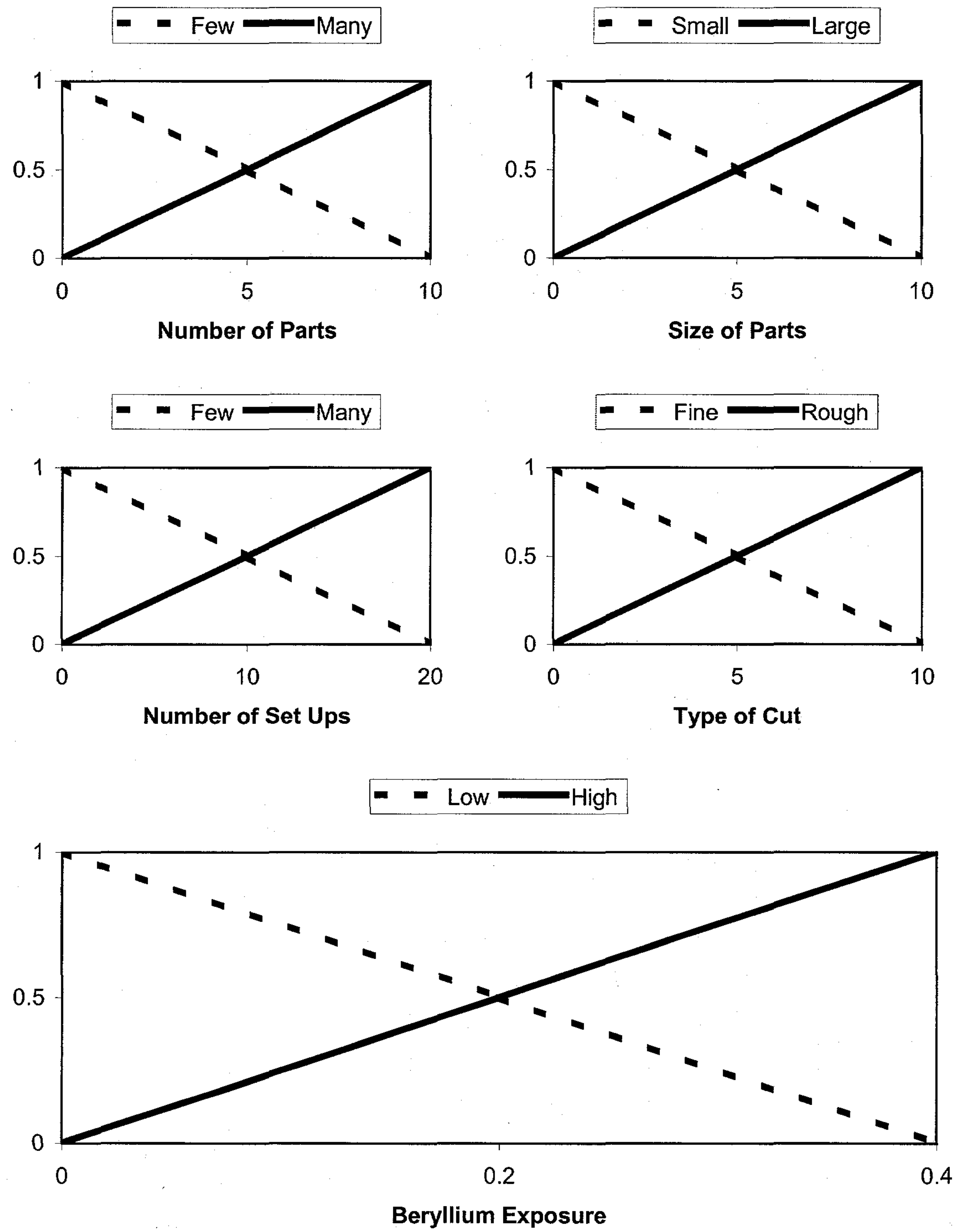

Figure 1. Input membership functions.

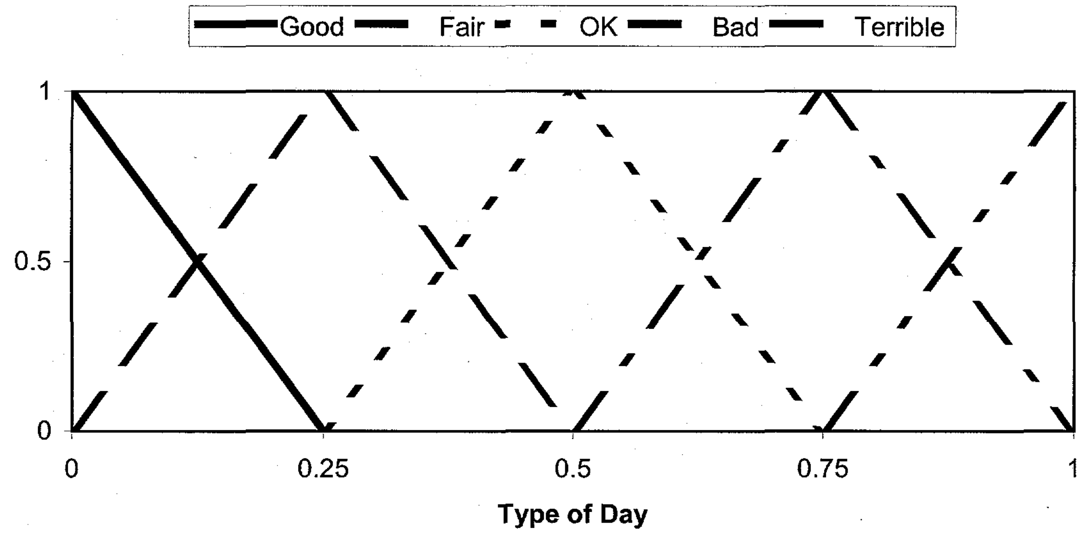

Figure 2. Output membership functions.

Good and Terrible extremes. The thirty-two rules are given in Table I. the form of the rules is:

If (Number of Parts) is …and If (Size of Parts) is … and If (Number of Set Ups) is … and If (Type of Cut) is… and If (Beryllium Exposure) is… Then (The Type of Day) is….

The Size of Parts is determined as the number of parts multiplied by the average diameter of each part, measured in centimeters. The Type of Cut is determined by the number of parts multiplied by the diameter, multiplied by a roughness factor. A fine cut has a roughness factor of 1, a medium cut is 2 and a rough-cut is 3. An example of the use of this technique will follow a discussion of the plant simulation.

PLANT SIMULATION

The Los Alamos Beryllium facility has been completed, but has not yet been put into production. For this reason a computer program was written in order to provide a simulation of the facility operation and provide a demonstration of the fuzzy control chart technique. The results of this study are being supplied to plant workers in order for them to provide input to further improve the technique.

Some actual beryllium exposure data were available for this study. This allowed the investigators to develop some reasonably realistic simulations for each of the intermediate process steps for manufacturing the beryllium parts. The combination of plant data and simulation data were used in this study. The simulation-operator interaction process is iterative and is designed to enhance the beryllium exposure control techniques. A Shewhart-type control chart is used to measure the central tendency and the variability of the data. A process flow diagram of the plant simulation is shown in figure 3.

Table I. Rules

Rule number	Number of Parts	Size of Parts	Number of Set Ups	Type of Cut	Beryllium Exposure	Type of Day
1	Few	Small	Few	Fine	Low	Fair
2	Few	Small	Few	Fine	High	Terrible
3	Few	Small	Few	Rough	Low	OK
4	Few	Small	Few	Rough	High	Terrible
5	Few	Small	Many	Fine	Low	Fair
6	Few	Small	Many	Fine	High	Bad
7	Few	Small	Many	Rough	Low	Fair
8	Few	Small	Many	Rough	High	Terrible
9	Few	Large	Few	Fine	Low	Fair
10	Few	Large	Few	Fine	High	Bad
11	Few	Large	Few	Rough	Low	Fair
12	Few	Large	Few	Rough	High	Terrible
13	Few	Large	Many	Fine	Low	Good
14	Few	Large	Many	Fine	High	Bad
15	Few	Large	Many	Rough	Low	Fair
16	Few	Large	Many	Rough	High	Bad
17	Many	Small	Few	Fine	Low	Fair
18	Many	Small	Few	Fine	High	Bad
19	Many	Small	Few	Rough	Low	Fair
20	Many	Small	Few	Rough	High	Terrible
21	Many	Small	Many	Fine	Low	Good
22	Many	Small	Many	Fine	High	Bad
23	Many	Small	Many	Rough	Low	Fair
24	Many	Small	Many	Rough	High	Bad
25	Many	Large	Few	Fine	Low	Good
26	Many	Large	Few	Fine	High	Bad
27	Many	Large	Few	Rough	Low	Fair
28	Many	Large	Few	Rough	High	Bad
29	Many	Large	Many	Fine	Low	Good
30	Many	Large	Many	Fine	High	OK
31	Many	Large	Many	Rough	Low	Good
32	Many	Large	Many	Rough	High	Bad

The model has the following limitations or boundary conditions:

1. There are three machinists.
2. There are seven machines.
3. Machines 1 and 2 do rough-cuts only.
4. Machines 3 and 4 do both rough-cuts and medium-cuts.
5. Machines 5, 6, and 7 do only fine-cuts.
6. Machine 7 accepts only work from machines 3 and 4.
7. Machines 5 and 6 accept only work from machines 1 and 2.
8. Each machinist does all of the work on one order.
9. All machinists have an equally likely chance of being chosen to do an order.
10. There are ten possible paths through the plant. (At this point all are equally likely. This will probably change as we obtain more data.)

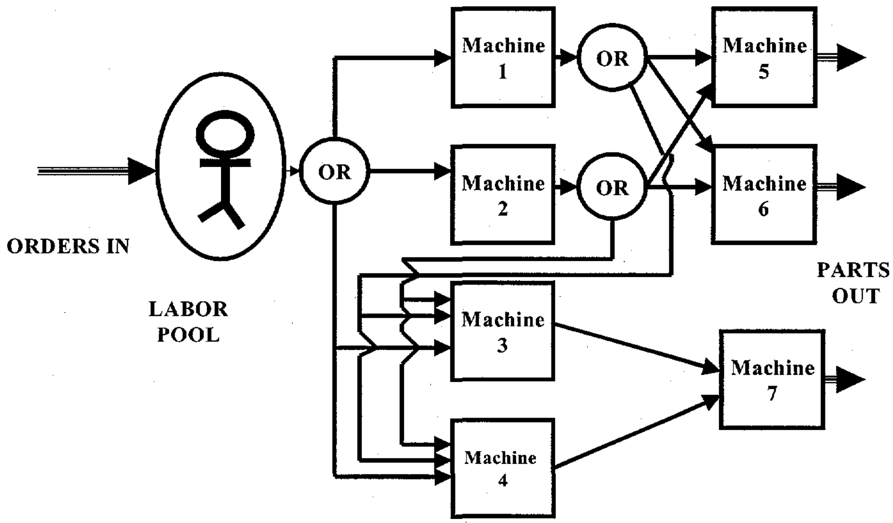

Figure 3. The process flow diagram of the beryllium plant simulation.

The simulation works like this:

1. A random number generator determines how many orders will be processed on a given day. (One to 30.)
2. Another random number generator picks a machinist.
3. A third random number generator picks a part size.
4. A fourth random number generator picks a path through the plant. (For example, Machine 1 to Machine 3 to Machine 7. See figure 3.)
5. The machine and path decide the type of cut, (rough, medium, or fine). Machines 1 and 2 are for rough-cuts only, machines 5, 6, and 7 are for fine cuts only, and machines 3 and 4 do rough-cuts if they are the first machines in the path and medium-cuts if they are the second machines in the path.
6. A random number generator picks the number of set ups for each machine on the path.
7. Another random generator picks the beryllium exposure for the operation. It is distributed appropriately between the machine and the machinist.
8. The above procedure is carried out for each part, each day. The entire procedure is repeated the following day, until the required number of days has passed.

The procedure was run for thirty days to generate some sample control charts. A description of the fuzzy control chart construction follows.

EXAMPLE

In this example, we will work through each step of the process for a specific set of part orders. In our simulation, on day seventeen, fourteen part orders were placed. Machinist one processed four of these orders, machinists two processed three and machinist three processed seven orders. Machine one processed nine parts. Machine two processed three parts. Machine three processed four parts. Machine four processed five parts. Machine five processed two parts. Machine six processed three parts, and machine seven processed nine parts.

We will use machinist one to demonstrate the fuzzy system. The multiplication factors, discussed in the fuzzy system description section, that must be used in order for the machinist-group to be able to use the standard input membership functions shown in figure one are:

- Multiplication factor – machinist-group – Number of Parts = 1,
- Multiplication factor – machinist-group --Size of Parts = 0.41667,
- Multiplication factor – machinist-group – Number of Set-Ups = 0.26316,
- Multiplication factor – machinist-group-- Type of Cut = 0.21986,
- Multiplication factor – machinist-group-- Beryllium Exposure = 1.

The cumulative size of the four parts that machinist one processed on day seventeen was calculated to be 11.66. (We then multiply by the factor 0.41667 to obtain the value ~4.9 that we use with the "size of parts" chart in figure 1.) The number of set ups that he/she performed was 10. After multiplying by 0.26316, we obtain the number ~3.8 to use with the "number of set-ups" chart in figure 1. The numeric value for the type of cuts he/she performed on that day was 23.56. After multiplying by the factor 0.21986, we obtain ~5.2 to use with our input charts in figure 1. Finally, the machinist's beryllium exposure was 0.153 $\mu g/m^3$ for that eight-hour period.

First we note that since all of the input variables are binary, in almost every case, we will fire all thirty-two rules. Upon inserting our pre-multiplied input values into the membership functions shown in figure 1, we get the following values:

For Number of Parts = 4, from figure 1, the membership in Many is 0.4 and the membership in Few is 0.6. For Size of Parts = 4.9, from figure 1, the membership in Small is 0.51 and the membership in Large is 0.49. For Number of Set-Ups = 3.8, from figure 1, the membership in Many is 0.19 and the membership in Few is 0.81. For Type of Cuts = 5.2, from figure 1, the membership in Rough is 0.52 and the membership in Fine is 0.48. The Beryllium Exposure is 0.153. From figure 1, the membership in High is 0.38 and the membership in Low is 0.62. The variable values for set of inputs form figure 1, for all of the rules, are listed in Table II, along with the variable values for the rule outputs.

In this study we are using the MIN_MAX approach to resolve the "and-or" nature of the rules and we are using the "winner take all" method for defuzification. For example, Rule 1 (Table II) is fired with the following weights:

- Number of Parts --Few = 0.6,
- Size of Parts -- Small = 0.51,
- Number of Set Ups -- Few = 0.81,
- Type of Cut – Fine = 0.48, and

- Beryllium Exposure – Low = 0.62.

Table II. Rules with membership values machinist number 1, day 2 from the simulation study. The rule input values are Number of parts = 4.0, Size of parts = 4.9, Number of set-ups = 3.8, Type of cut = 5.2, and Beryllium exposure = 0.153.

Rule No.	Number of Parts	Size of Parts	Number of Set Ups	Type of Cut	Beryllium Exposure	Type of Day
1	Few = 0.6	Small = 0.51	Few = 0.81	Fine = 0.48	Low = 0.62	Fair = 0.48
2	Few = 0.6	Small = 0.51	Few = 0.81	Fine = 0.48	High = 0.38	Terrible = 0.38
3	Few = 0.6	Small = 0.51	Few = 0.81	Rough = 0.52	Low = 0.62	OK = 0.51
4	Few = 0.6	Small = 0.51	Few = 0.81	Rough = 0.52	High = 0.38	Terrible = 0.38
5	Few = 0.6	Small = 0.51	Many = 0.19	Fine = 0.48	Low = 0.62	Fair = 0.19
6	Few = 0.6	Small = 0.51	Many = 0.19	Fine = 0.48	High = 0.38	Bad = 0.19
7	Few = 0.6	Small = 0.51	Many = 0.19	Rough = 0.52	Low = 0.62	Fair = 0.19
8	Few = 0.6	Small = 0.51	Many = 0.19	Rough = 0.52	High = 0.38	Terrible = 0.19
9	Few = 0.6	Large = 0.49	Few = 0.81	Fine = 0.48	Low = 0.62	Fair = 0.48
10	Few = 0.6	Large = 0.49	Few = 0.81	Fine = 0.48	High = 0.38	Bad = 0.38
11	Few = 0.6	Large = 0.49	Few = 0.81	Rough = 0.52	Low = 0.62	Fair = 0.49
12	Few = 0.6	Large = 0.49	Few = 0.81	Rough = 0.52	High = 0.38	Terrible = 0.38
13	Few = 0.6	Large = 0.49	Many = 0.19	Fine = 0.48	Low = 0.62	Good = 0.19
14	Few = 0.6	Large = 0.49	Many = 0.19	Fine = 0.48	High = 0.38	Bad = 0.19
15	Few = 0.6	Large = 0.49	Many = 0.19	Rough = 0.52	Low = 0.62	Fair = 0.19
16	Few = 0.6	Large = 0.49	Many = 0.19	Rough = 0.52	High = 0.38	Bad = 0.19
17	Many = 0.4	Small = 0.51	Few = 0.81	Fine = 0.48	Low = 0.62	Fair = 0.4
18	Many = 0.4	Small = 0.51	Few = 0.81	Fine = 0.48	High = 0.38	Bad = 0.38
19	Many = 0.4	Small = 0.51	Few = 0.81	Rough = 0.52	Low = 0.62	Fair = 0.4
20	Many = 0.4	Small = 0.51	Few = 0.81	Rough = 0.52	High = 0.38	Terrible = 0.38
21	Many = 0.4	Small = 0.51	Many = 0.19	Fine = 0.48	Low = 0.62	Good = 0.19
22	Many = 0.4	Small = 0.51	Many = 0.19	Fine = 0.48	High = 0.38	Bad = 0.19
23	Many = 0.4	Small = 0.51	Many = 0.19	Rough = 0.52	Low = 0.62	Fair = 0.19
24	Many = 0.4	Small = 0.51	Many = 0.19	Rough = 0.52	High = 0.38	Bad = 0.19
25	Many = 0.4	Large = 0.49	Few = 0.81	Fine = 0.48	Low = 0.62	Good = 0.4
26	Many = 0.4	Large = 0.49	Few = 0.81	Fine = 0.48	High = 0.38	Bad = 0.38
27	Many = 0.4	Large = 0.49	Few = 0.81	Rough = 0.52	Low = 0.62	Fair = 0.4
28	Many = 0.4	Large = 0.49	Few = 0.81	Rough = 0.52	High = 0.38	Bad = 0.38
29	Many = 0.4	Large = 0.49	Many = 0.19	Fine = 0.48	Low = 0.62	Good = 0.19
30	Many = 0.4	Large = 0.49	Many = 0.19	Fine = 0.48	High = 0.38	OK = 0.19
31	Many = 0.4	Large = 0.49	Many = 0.19	Rough = 0.52	Low = 0.62	Good = 0.19
32	Many = 0.4	Large = 0.49	Many = 0.19	Rough = 0.52	High = 0.38	Bad = 0.19

The consequent of the rule, "Fair", takes the minimum value, 0.48. If we observe the last column in table II, we can see that the consequent, Fair, appears ten times with values ranging from 0.19 to 0.49. The MIN_MAX rule assigns the maximum value of 0.49 to the consequent Fair. Similarly, the consequent, "Terrible", appears five times with a maximum value of 0.38. "OK" appears twice with a maximum value of 0.51. "Bad" appears ten times with a maximum value of 0.38, and "Good" appears five times with a maximum value of 0.4. The "winner take all" defuzzification method chooses "OK" from the list of (Fair = 0.48, Terrible = 0.38, OK = 0.51, Bad = 0.38, and Good = 0.4). So on day seventeen, machinist one has an OK "type of day".

The next step is to provide a fuzzy distribution for the entire day based on all of the results from every machinist and every machine. This is easily done using the *extension* *principle* from fuzzy logic and the concept of a triangular fuzzy number (TFN) as described by Kaufmann [1]. A TFN is completely described by a triplet, $T = (t_1, t_2, t_3)$, or in our case the vector $[t_1, t_2, t_3]^T$. The values t_1, t_2, and t_3 are the x-values of the x-y pairs representing the corners of a triangle with the base resting on the x-axis ($y = 0$) and the apex resting on the line $y = 1$. Such a triangle can be described by the three points in the x-y plane $(t_1, 0)$, $(t_2, 1)$, and $(t_3, 0)$. For example, the triangular membership function OK in figure 2, can be described as a TFN with $t_1 = 0.25$, $t_2 = 0.5$, and $t_3 = 0.75$, or $[0.25, 0.5, 0.75]^T$. The other four output membership functions are described as follows:

- Good = $[0.0, 0.0, 0.25]^T$,
- Fair = $[0.0, 0.25, 0.5]^T$,
- Bad = $[0.5, 0.75, 1.0]^T$, and
- Terrible = $[0.75, 1.0, 1.0]^T$.

If we construct the matrix **A** with columns comprised of our five-output membership function TFNs, we obtain:

$$\mathbf{A} = \begin{bmatrix} 0.0 & 0.0 & 0.25 & 0.5 & 0.75 \\ 0.0 & 0.25 & 0.5 & 0.75 & 1.0 \\ 0.25 & 0.5 & 0.75 & 1.0 & 1.0 \end{bmatrix} .$$

Next we construct a five-element vector that we will call **B.** The first element of the **B** vector is the fraction of the daily readings that were Good. The second element is the fraction of the readings that were Fair, and so on. On day seventeen, between the seven machines and three machinists there were no Good days recorded. There were four Fair days, one OK day, four Bad days, and one Terrible day. Then for day seventeen, the **B** vector is calculated to be:

$\mathbf{B} = [0.0, 0.4, 0.1, 0.4, 0.1]^T$.

The product **AB** is a TFN that represents the fuzzy "type of day" distribution for that day. In this case, the TFN is $[0.3, 0.55, 0.775]^T$. This is a triangular distribution that is nearly OK, but a little on the Bad side. Figure 4 shows how day seventeen is distributed on the "type of day" chart. The shaded area is the TFN or fuzzy distribution for day seventeen. A different distribution is obtained every day. In order to construct a control chart, we need to determine values for both a centerline and control limits. There are several metrics that can be used to represent the central tendency of a fuzzy set [2,3]. The one that we have chosen to use is the fuzzy average. The fuzzy average, f_{avg}, is defined by equation 1.

$$f_{avg} = \frac{\int_0^1 x\mu_F(x)dx}{\int_0^1 \mu_F(x)dx} \qquad (1)$$

Where $\mu_F(x)$ is the equation for the membership function or the fuzzy set. In our example for day seventeen $\mu(x)$ is described below.

$$\begin{aligned} \mu_F(x) &= 0, & x &\le 0.3, \\ \mu_F(x) &= 4x - 1.2, & 0.3 &\le x \le 0.55, \\ \mu_F(x) &= -4.444x + 3.444, & 0.55 &\le x \le 0.775, \end{aligned}$$

$$\mu_F(x) = 0, \qquad\qquad x \geq 0.775.$$

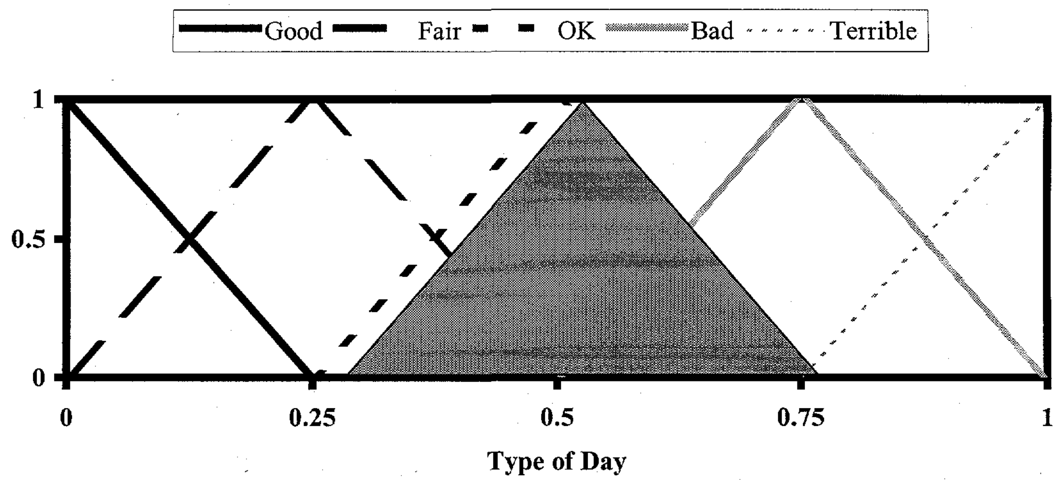

Figure 4. The distribution of day seventeen, shown on the type of day chart. The shaded area is the distribution for day seventeen.

The fuzzy average for day seventeen in our example is 0.541667. Next we compute the average-average or Grand Average for our thirty-day run. This Grand Average is 0.496261. Figure 5 depicts the daily " type of day" fuzzy distributions relative to the Grand Average line.

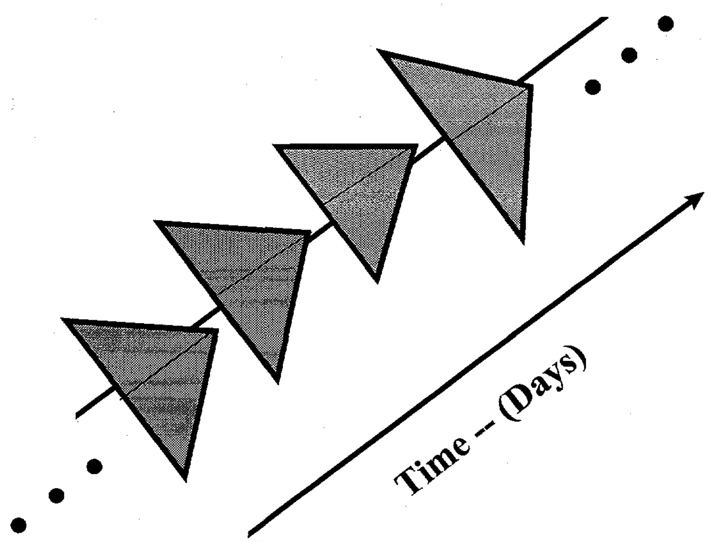

Figure 5. Daily type of day fuzzy distributions spread about the Grand Average line.

The control charts generated from our thirty-day simulation run are shown in figures 6 and 7. Figure 6 is the Shewhart-type X Bar chart and figure 7 is the R or range chart. The upper and lower control limits (UCL and LCL) on these charts were not calculated in the usual X Bar-R manner. These calculations will be discussed in the next section.

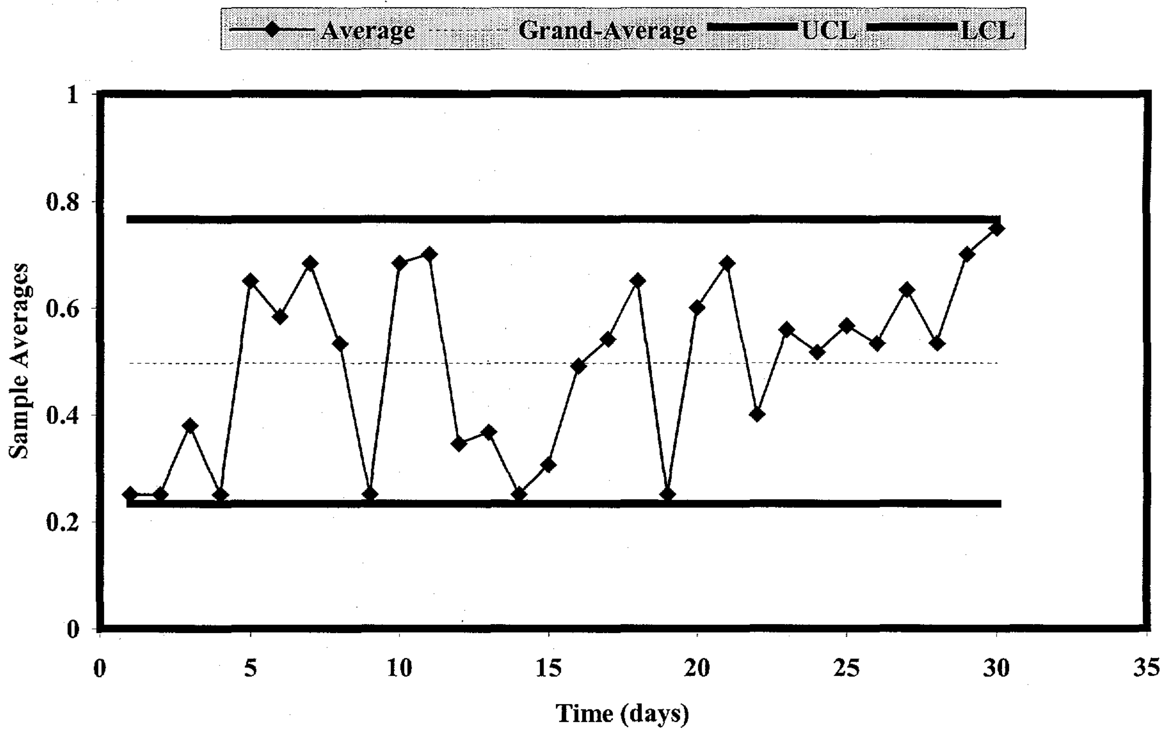

Figure 6. The Shewhart-type X Bar chart, type of day samples, for the example thirty-day simulation.

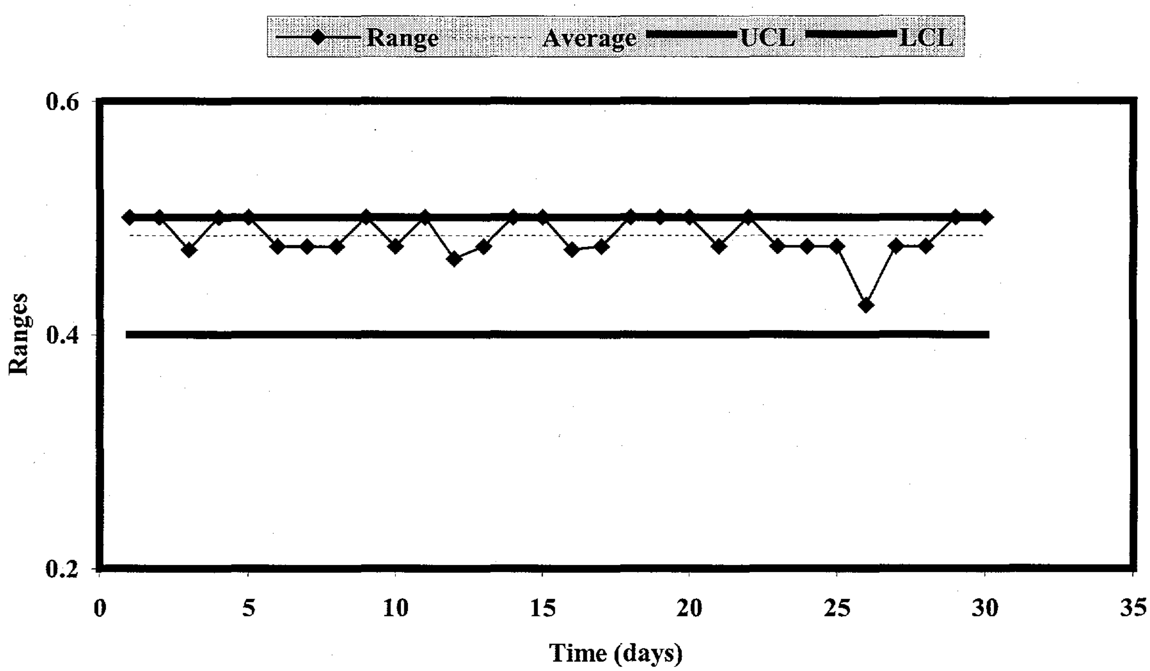

Figure 7. The Shewhart-type R chart, type of day samples, for the thirty-day simulation.

DISCUSSION

The motivation for this project was that beryllium exposure is highly task dependent. In the new Los Alamos beryllium facility, daily tasks will be highly variable. This means that a standard Shewhart X Bar-R chart based on daily beryllium exposure will probably not provide the control information required. In fact, previous beryllium exposure data show that these control charts don't supply the needed information. The simulator was designed, in part, using these data. Figures 8 and 9 show the Shewhart X Bar-R charts for beryllium exposure for the thirty-day simulation run described in the Example section. These charts use upper and lower control limits based on standard X Bar-R techniques, using variable sample sizes. Figure 8 is the X Bar chart and figure 9 is the R chart.

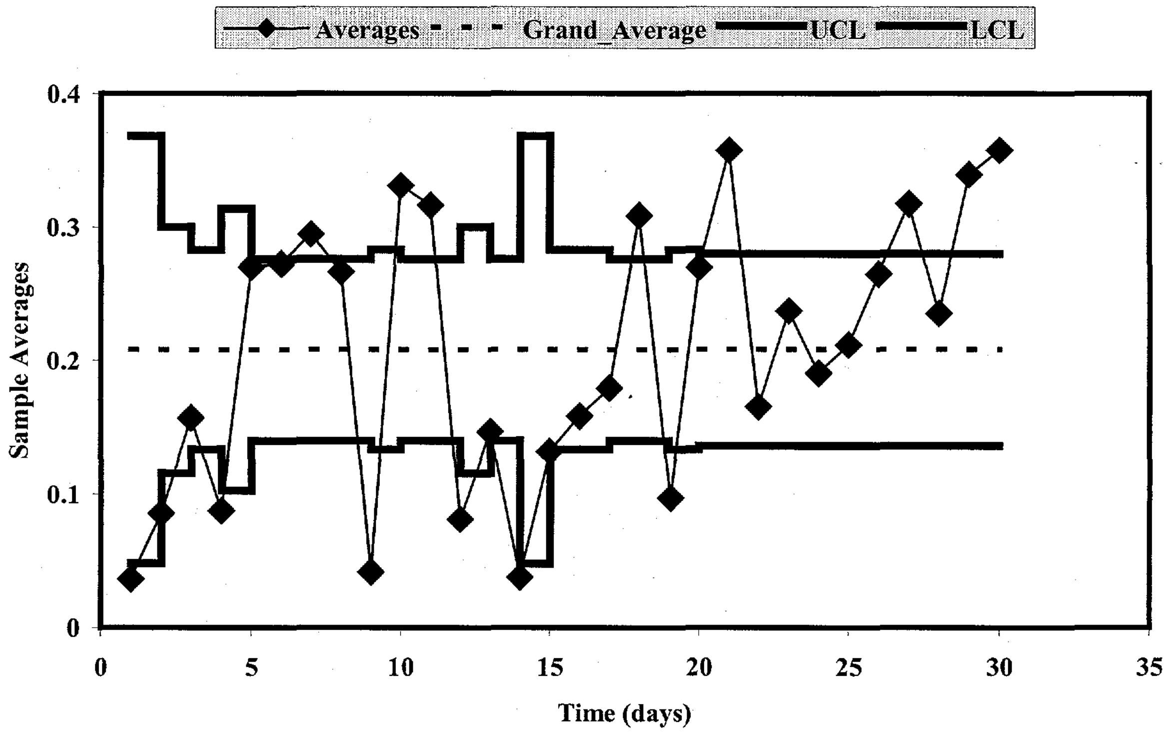

Figure 8. The Shewhart X Bar chart for, beryllium exposure samples, for the thirty-day simulation.

Other than the variable of interest and the manner in which the upper and lower control limits are computed, figures 8 and 9 are quite similar to figures 6 and 7. The formulas used to compute the upper and lower control limits for figures 8 and 9 are found in standard books on statistical process control [4,5], and are give below:

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2 \overline{R} \qquad (2)$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2 \overline{R} \qquad (3)$$

$$UCL_R = D_4\overline{R} \tag{4}$$

$$LCL_R = D_3\overline{R} \tag{5}$$

Where $UCL_{\overline{X}}$, $LCL_{\overline{X}}$, UCL_R, and LCL_R are the upper and lower control limits for the X Bar chart, or plot of sample averages, and for the R chart, or plot of sample ranges, respectively. Since X Bar and R are plotted on separated diagrams the control limits are simply designated as UCL and LCL on their respective plots. $\overline{\overline{X}}$ is the Grand Average or average of the average samples over a given time period, in our case thirty days. $\overline{R}$ is the average of all sample ranges, maximum sample value minus the minimum sample value, over the thirty-day time period. The constants A_2, D_3, and D_4 are taken from Table III that is reproduced from [4].

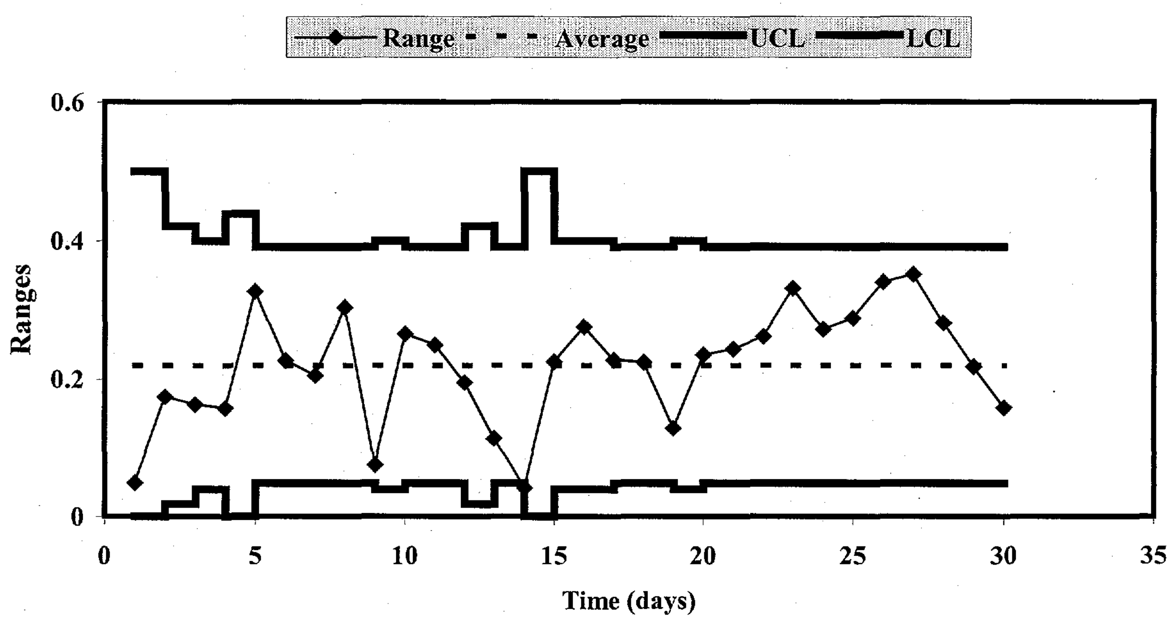

Figure 9. The Shewhart R chart for, beryllium exposure samples, for the thirty-day simulation.

Table III. Table of constants for equations 2, 3, 4, and 5. The values are based on the sample size, n.

n	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A_2	1.88	1.02	0.73	0.58	0.48	0.42	0.37	0.34	0.31
D_3	0	0	0	0	0	0.08	0.14	0.18	0.22
D_4	3.27	2.57	2.28	2.11	2.00	1.92	1.86	1.82	1.78

The upper and lower control limits computed from equations 2, 3, 4, and 5 are supposed to represent the $\pm 3\sigma$, or 3 standard deviations, about the respective means. This is clearly not the case for the beryllium exposure sample averages shown in figure 8. This chart shows the upper control level alarm would be triggered at least eight times in the thirty-day period. In the thirty-day simulation, the plant was supposedly under control. Part of the problem may be because the plant model is not perfect, but most of the problem lies in the nature of the exposure data. This method does not take into account the task dependent exposure problem. The fuzzy " type of day" method does not fair much better with upper and lower control limits defined by equations 2, 3, 4, and 5. But this is, at least in part, for a different reason. The current rules and membership functions used for the fuzzy technique make the large majority of the daily averages either Bad or Fair. This gives us a bimodal distribution that is not Gaussian. A different point of view about what constitutes Bad, Fair and OK days would possibly make the fuzzy model fit the standard Shewhart X Bar-R chart format. Since the rules we used comply with the current thinking, we took another approach to determining the upper and lower control limits for our control charts.

Wheeler [5] points out that there is nothing magical about the $\pm 3\sigma$ range for upper and lower control limits. He states that Shewhart was looking for a range that the measured variable didn't step outside of too often and therefore would not cause excessive trouble shooting. He chose the $\pm 3\sigma$ range because it seemed to fit his goals. Wheeler further mentions the following empirical rules:

- Roughly, 60% to 75% of the data will be located within a distance of one sigma unit on either side of the average.
- Usually 90% to 98% of the data will be located within a distance of two sigma units on either side of the average.
- Approximately 99% to 100% of the data will be located within a distance of three sigma units on either side of the average.

We used these rules and ran a simulation with our model for 1000 days. We chose the 0.995 and the 0.005 percentiles to determine our $\pm 3\sigma$ range. These are the upper and lower control limits that appear in figures 6 and 7. The results from these runs are plotted in figures 10 and 11.

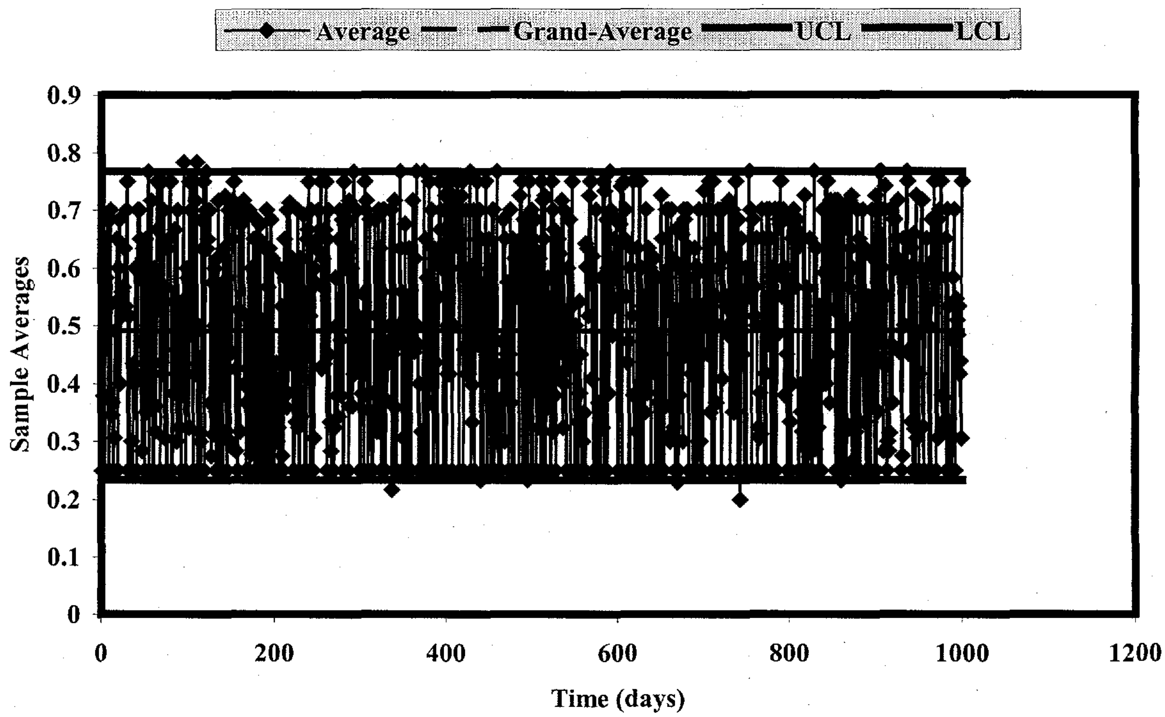

Figure 10. Average data generated from type of day samples for the 1000-day simulation.

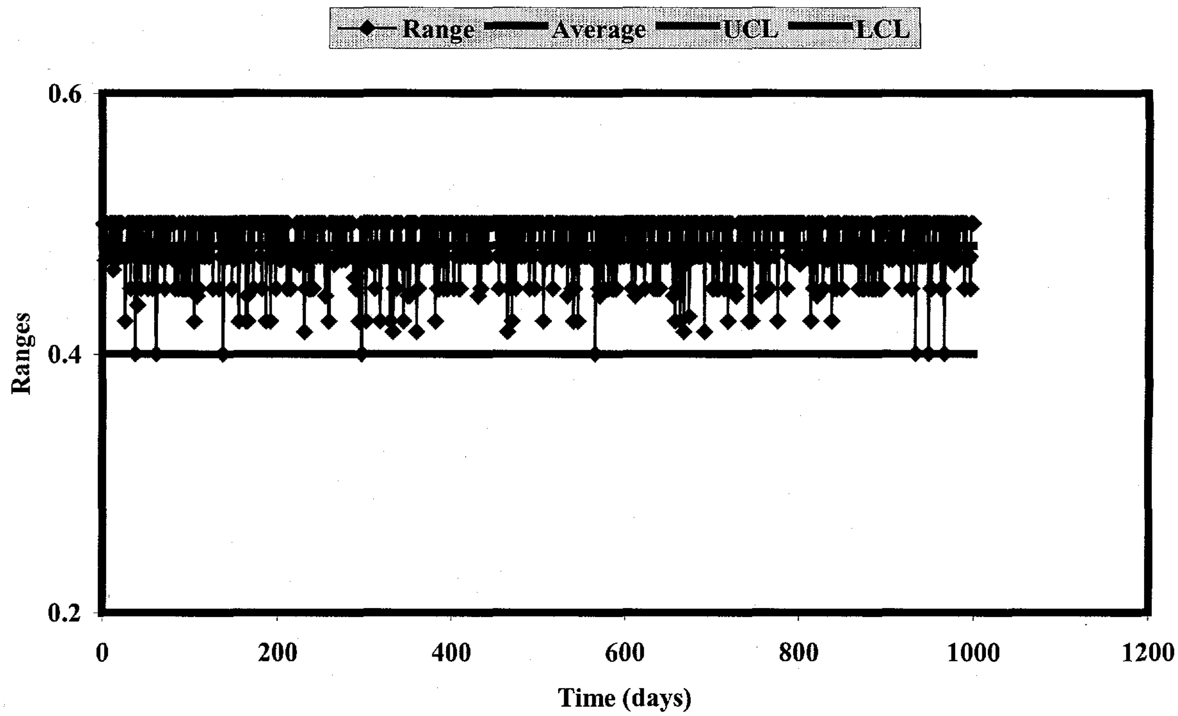

Figure 11. Range data generated from type of day samples from the 1000-day simulation.

Even though the beryllium exposure data does not fit the standard Shewhart X Bar-R chart format for a different reason than the fuzzy "type of day" data, we followed the same procedure for the beryllium exposure data in order to obtain fair upper and lower limits. Table IV contains the pertinent data obtained from 1000-day simulations for both the beryllium exposure and fuzzy "type of day" problems. We used the 0.975 and the 0.025 percentiles to determine our $\pm 2\sigma$ range, and the 0.835 and 0.165 percentiles for the $\pm 1\sigma$ range. We also obtained the 1000-point sample standard deviation, **s,** in order to make comparisons with percentile estimates and the mean values ± 3**s.**

Table IV. Data obtained from 1000-day simulations.

	Type of day Average	Type of day Range	Be exposure Average	Be exposure Range
Mean value, θ	0.490451	0.483191	0.203927	0.21793
.995 percentile	0.766667	0.5	0.367726	0.392627
.005 percentile	0.233333	0.4	0.023639	0.013508
.975 percentile	0.75	0.5	0.353436	0.365705
.025 percentile	0.25	0.425	0.035289	0.041151
.835 percentile	0.70	0.5	0.317372	0.306201
.165 percentile	0.25	0.472222	0.085465	0.125549
s	0.178408	0.021929	0.098909	0.086302
Θ +3s	1.025674	0.548978	0.500653	0.476836
Θ – 3s	-0.04477	0.417403	-0.0928	-0.04098

It is worthwhile to notice that the Θ ±3s data ranges are much wider than the 0.995 and 0.005 percentile ranges. This is because these percentile ranges are truly smaller than the true $\pm3\sigma$ range, but the Θ ±3s ranges are essentially meaningless with the 1000-day simulation data.

The thirty-day simulation was run again, but this time we imposed conditions on days thirteen, fourteen, and fifteen that produced high beryllium exposure, but with justification. The number of parts processed was very high, and the average size of the parts was generally large. On day fourteen, we assumed that two machinists were out sick and the remaining machinist had a heavy workload. On day fifteen, we assumed that one of the machines was out of service, causing other machines to carry heavier than expected loads. These conditions provided days that were defined to be partially OK and partially Bad, not Terrible, in spite of the high beryllium exposure. Figures 12, 13, 14, and 15 show the Shewhart-type X Bar-R charts for this simulation. Figures 12 and 13 were developed from fuzzy " type of day" samples and figures 14 and 15 were developed from beryllium exposure samples.

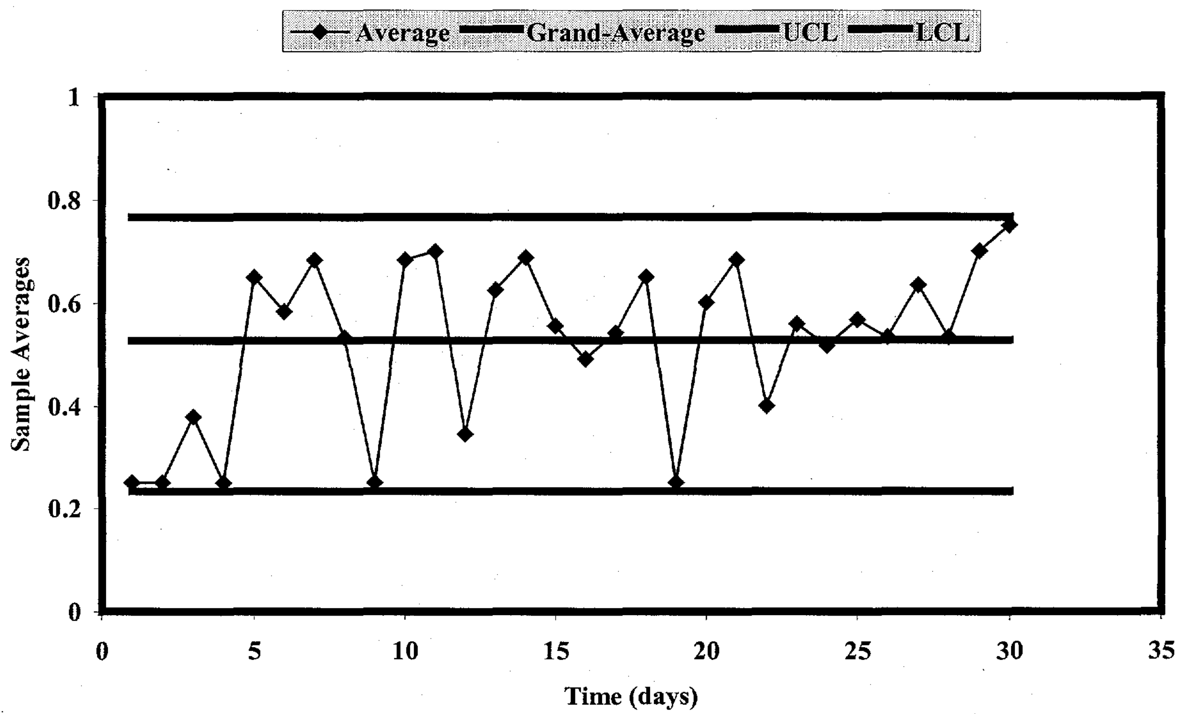

Figure 12. The Shewhart-type X Bar chart for type of day samples, giving high beryllium exposure under heavy-duty operations for days 13, 14, and 15, using a thirty-day simulation.

We notice that the "type of day" control charts shown in figures 12 and 13 take into account the task dependency of the beryllium exposure and do not trigger the upper control limit alarms. This is the desired result. The R chart plays a slightly different role in these observations than it does in the standard Shewhart X Bar-R chart. In the standard case, the R chart is used to calculate the upper and lower control limits, as well as another metric to observe. In these calculations the R chart is used as just another interesting control metric.

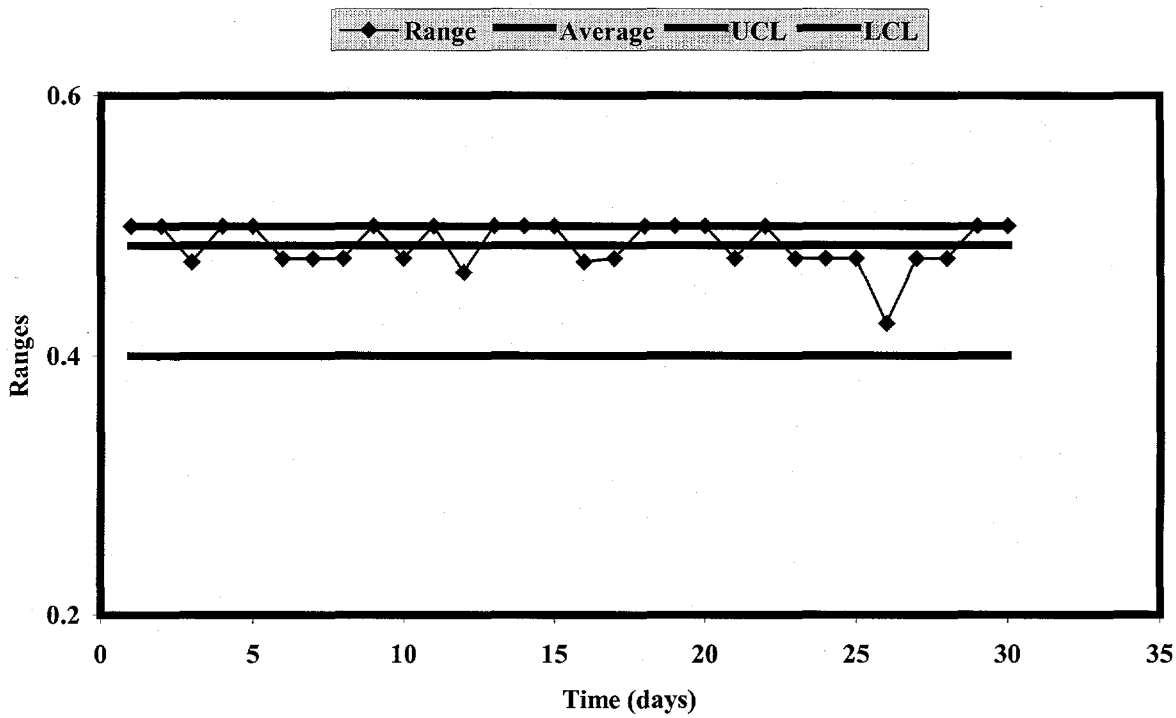

Figure 13. The Shewhart-type R chart for type of day samples, giving high beryllium exposure under heavy-duty operations for days 13, 14, and 15, using a thirty-day simulation.

The beryllium exposure control charts shown in figures 14 and 15 do not take into account the task dependency of the beryllium exposure. The X Bar chart, figure 14, triggers the upper level control alarm for each of the three days. This is not the desired result since there is good reason for the high exposure readings. The R chart does not trigger an alarm but does show the narrow range of the daily beryllium exposures, which is probably valuable information.

The thirty-day simulation was run a third time, this time we imposed conditions on days five and six that produced high beryllium exposure, but with no task dependent justification. We were assuming low load conditions with a possible malfunction in the plant pressure differential system that went unnoticed. The control charts for this simulation are shown in figures 16, 17, 18, and 19. Figures 16 and 17 were developed from fuzzy "type of day" samples and figures 18 and 19 were developed from beryllium exposure samples.

Again we notice that the "type of day" control charts shown in figures 16 and 17 take into account the task dependency of the beryllium exposure and do trigger the control limit alarms. This is the desired result because there was no task performed that should cause high beryllium exposure readings. Figure 16, the "type of day" X Bar chart triggers the upper control limit alarm for high beryllium exposure and figure 17, the "type of day" R chart, triggers the lower control limit alarm, indicating a "type of day" with a very narrow range.

The beryllium exposure control charts shown in figures 18 and 19 also recognize the high exposure on days five and six. The X Bar chart, figure 18, triggers the upper level control alarm for both days. This is the desired result. The R chart does not trigger an

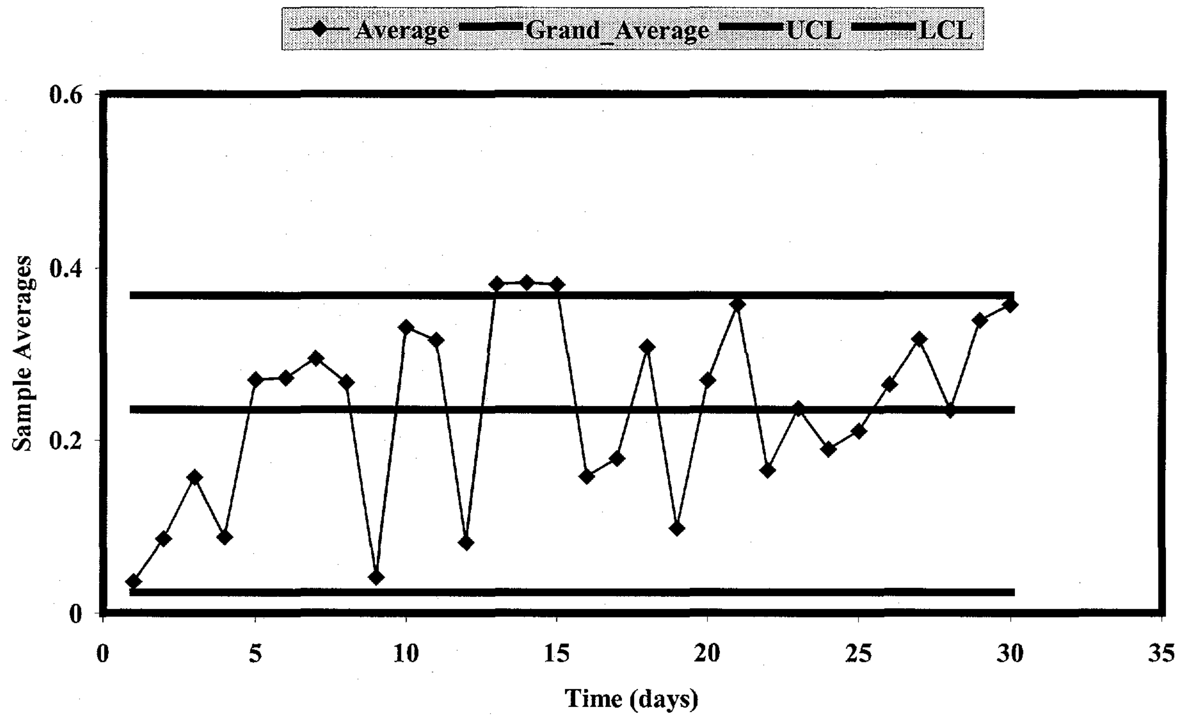

Figure 14. The Shewhart-type X Bar chart for beryllium exposure samples, giving high beryllium exposure under heavy-duty operations for days 13, 14, and 15, using a thirty-day simulation.

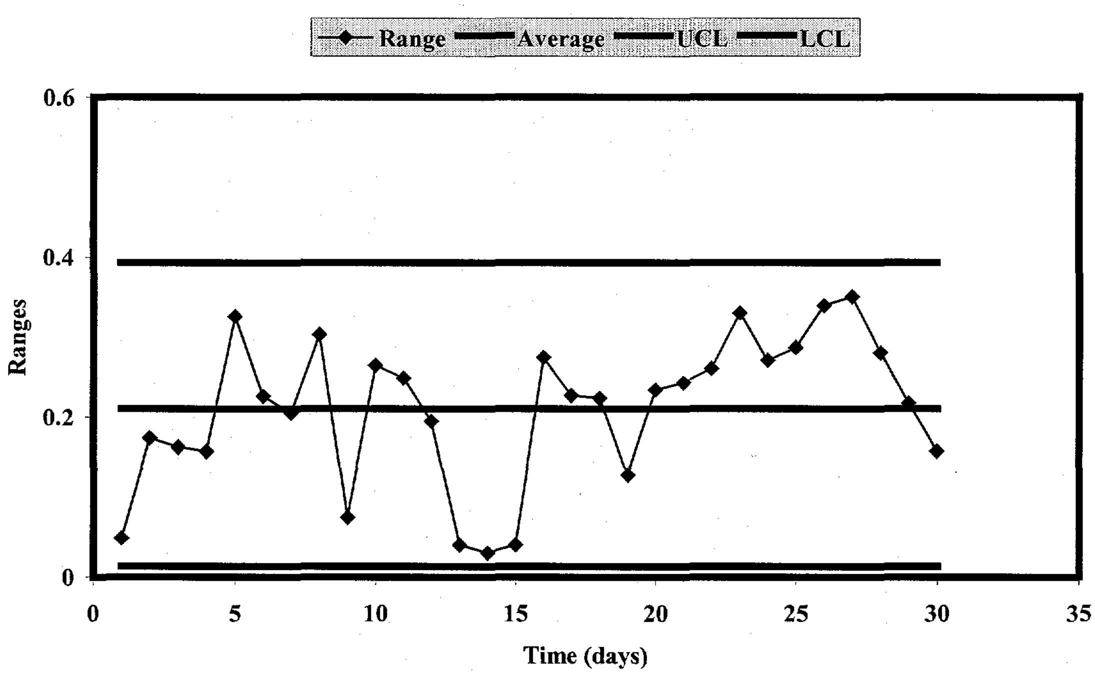

Figure 15. The Shewhart-type R chart for beryllium exposure samples, giving high beryllium exposure under heavy-duty operations for days 13, 14, and 15, using a thirty-day simulation.

alarm but again shows the narrow range of the daily beryllium exposures. This is valuable information.

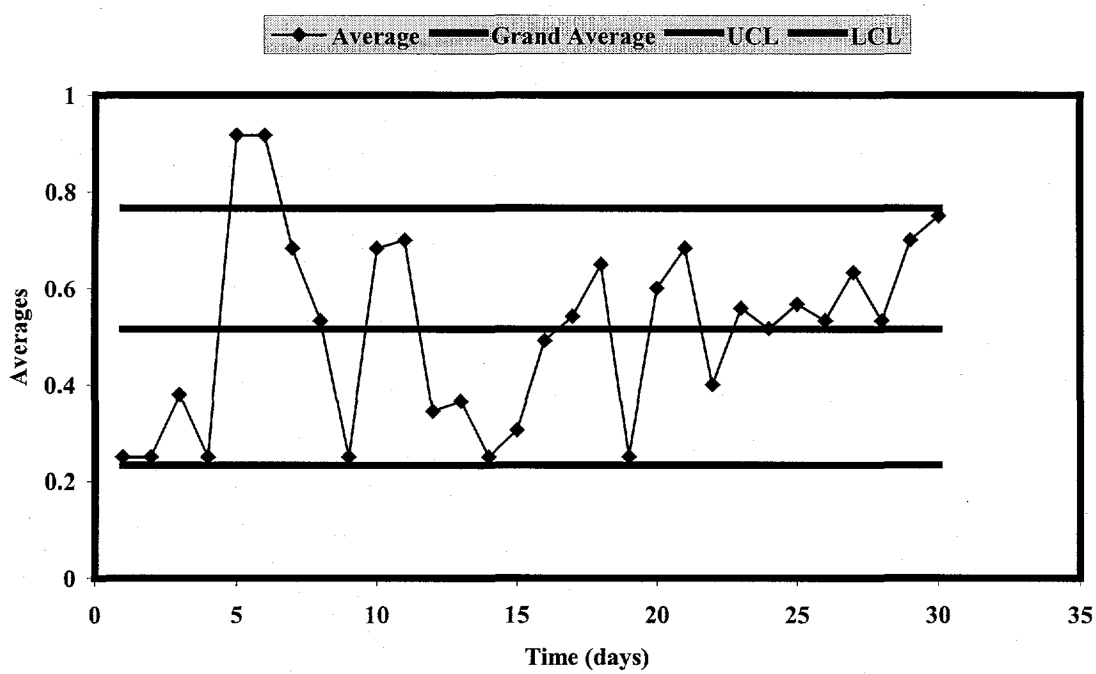

Figure 16. The Shewhart-type X Bar chart for type of day samples, giving high beryllium exposure under light-duty operations for days 5 and 6, using a thirty-day simulation.

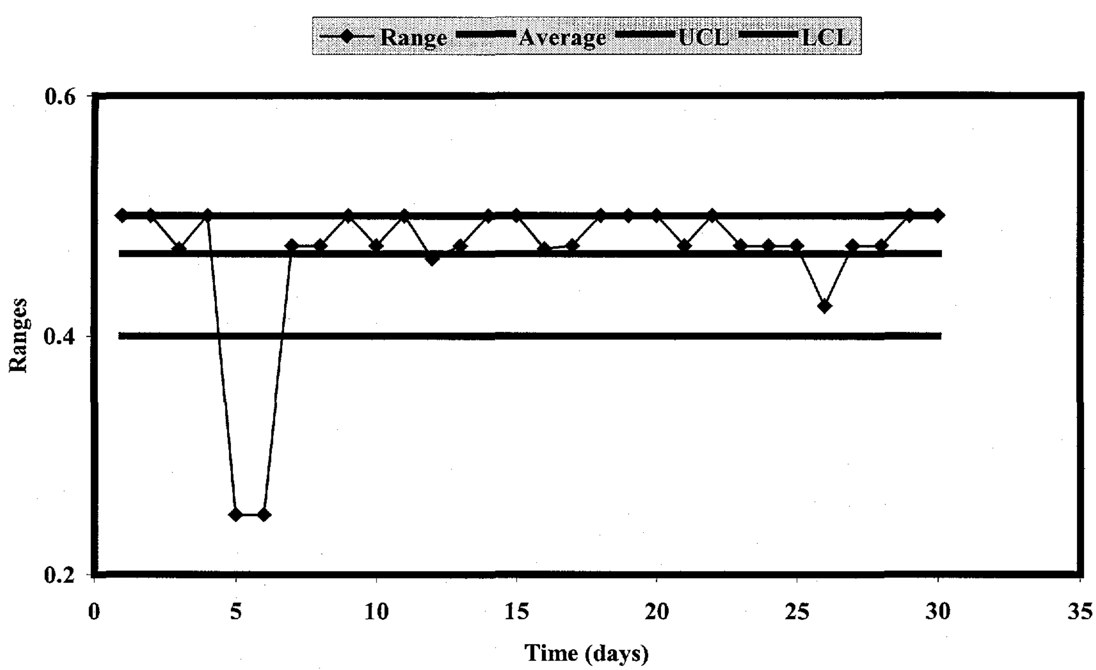

Figure 17. The Shewhart-type R chart for type of day samples, giving high beryllium exposure under light-duty operations for days 5 and 6, using a thirty-day simulation.

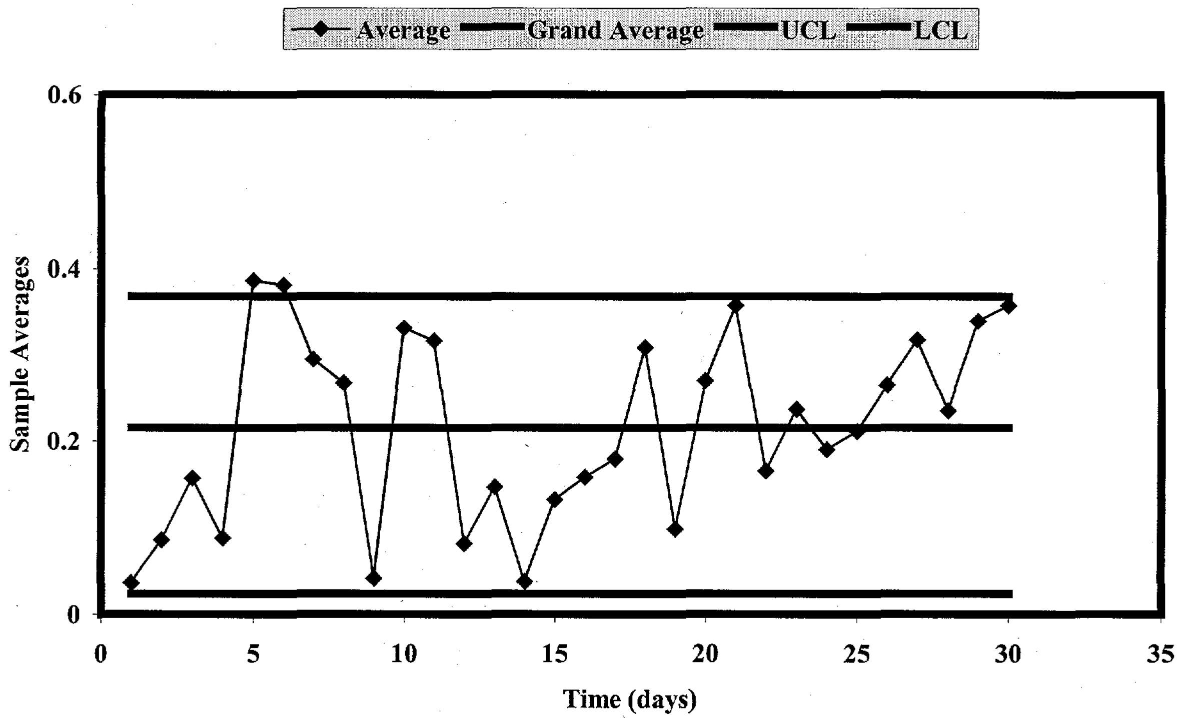

Figure 18. The Shewhart-type X Bar chart for beryllium exposure samples, giving high beryllium exposure under light-duty operations for days 5 and 6, using a thirty-day simulation.

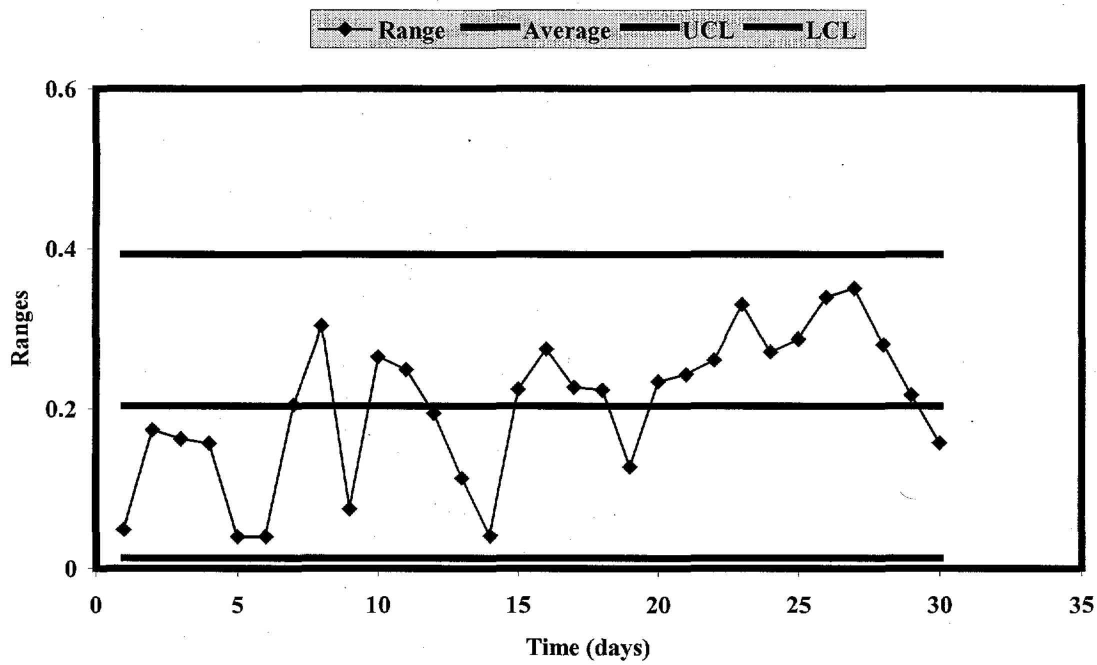

Figure 19. The Shewhart-type R chart for beryllium exposure samples, giving high beryllium exposure under light-duty operations for days 5 and 6, using a thirty-day simulation.

CONCLUSIONS

We have shown that the fuzzy "type of day" Shewhart-type X Bar-R control chart has the potential to take into account the task dependency beryllium exposure in our beryllium plant operations. Based upon the studies completed to this point, we believe these control charts will provide more realistic information than the standard or modified X Bar-R chart using only beryllium exposure information. Because of the ability to take into account task dependency, "the type of day" chart can be used to determine the significance of plant improvements (our second largest concern) as well as trigger "out of control" alarms.

More work needs to be done on the technique to make it even more useful and we feel that we will be able to make these improvements once our beryllium plant is actually running. Until this happens, one method of improving the technique would be to improve the fidelity of our plant model, although ultimately we intend to rely on actual plant data rather than simulation results.

Some suggestions for further work, include adjusting the rules and membership functions so that "type of day" distributions more closely resemble Gaussian distributions, so that we can use the distribution ranges for estimating the $\pm 3\sigma$ range. We also intend to investigate defuzzification techniques other than "winner take all", which might improve the distribution of our sample averages.

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