

LA-UR-00-2428

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Title: Does the Sun Still Shine?

Author(s): S. Karataglidis, T-16,
B. Jennings, TRIUMF, 4004 Wesbrook Mall,
Vancouver, British Columbia, Canada, V6T 2A3

Submitted to: 9th International Conference on Nuclear Reaction Mechanisms,
Varennna, Italy, June 5-11, 2000

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Does the sun still shine?

S. Karataglidis^(a), B. Jennings^(b)

(a) *Theory Division, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, U.S.A.*

(b) *TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada, V6T 2A3*

Abstract

The astrophysical S factor for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction at threshold is investigated using simple models to determine the model dependence in the extrapolation of the laboratory data to the region of astrophysical interest near 20 keV. We find that the energy dependence below 400 keV is largely well understood in terms of the Coulomb interaction and constrained by the data for the elastic scattering of low-energy neutrons from ${}^7\text{Li}$.

1 Introduction

The ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction near threshold plays an important role in the production of solar neutrinos. The subsequent decay of the ${}^8\text{B}$ is the source of the high-energy neutrinos to which many solar neutrino detectors are sensitive. However, a problem arises in determining the cross section at ~ 20 keV, the Gamow energy for the reaction in the solar interior, necessary for the calculation of the solar neutrino flux, as the Coulomb barrier provides a lower limit of ~ 100 keV for the measuring of the reaction in the laboratory. Hence, most efforts for extracting the cross section at 20 keV involve extrapolating the available data down to threshold using a model energy dependence. The solar neutrino problem arises from the discrepancy in comparison of the calculated flux with that measured from various detectors. Typically, the calculated flux from standard solar models are of the order of a factor of two to four higher compared with measurement [1].

The cross section for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction decreases exponentially as the energy approaches zero, with the cross section at 100 keV being 3 nb [2]. Thus extrapolation of the cross section is quite difficult. To make extrapolation easier, the cross section is conventionally expressed in terms of the S factor which is defined in terms of the cross sections, σ , by

$$S(E) = \sigma(E)E \exp[2\pi\eta(E)] \quad (1)$$

where $\eta(E) = Z_1 Z_2 \alpha \sqrt{\mu c^2 / 2E}$ is the Sommerfeld parameter, α is the fine structure constant, and μ is the reduced mass. The definition of the S factor eliminates from it most of the energy dependence due to Coulomb repulsion by factoring out the penetration to the origin of a particle in the Coulomb potential of a point charge. This does not make the S factor energy independent, as there is still some dependence due to the structure of the final (bound) state, resonances, and the attenuation of the barrier by the nuclear mean field. To do the extrapolation reliably, we must understand the physics associated with the S factor.

To illustrate the problem of extrapolating the data to astrophysical energies, Fig. 1 displays a fit to the experimental data [2, 3, 4]. Therein, a naive straight line fit (solid line) is used as well as a fit that includes s -wave nuclear distortion through a hard sphere potential of radius 4.1 fm (dashed line). (The latter calculation will be described in more detail below.) The $M1$ resonance from the 774 keV state in ${}^8\text{B}$ is described by a Breit-Wigner form. As can be seen from the figure both fits to the data are equally good: $\chi^2 = 0.9$ in each case. However, there is a marked difference in the S factors at 20 keV: 15.3 and 21.0 eVb for the straight line fit and hard sphere model, respectively, a 37% variation in the extrapolated value. Such a large difference must be understood if reliable extrapolations are to be made and a variety of, sometimes conflicting, models [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] have been developed for this purpose.

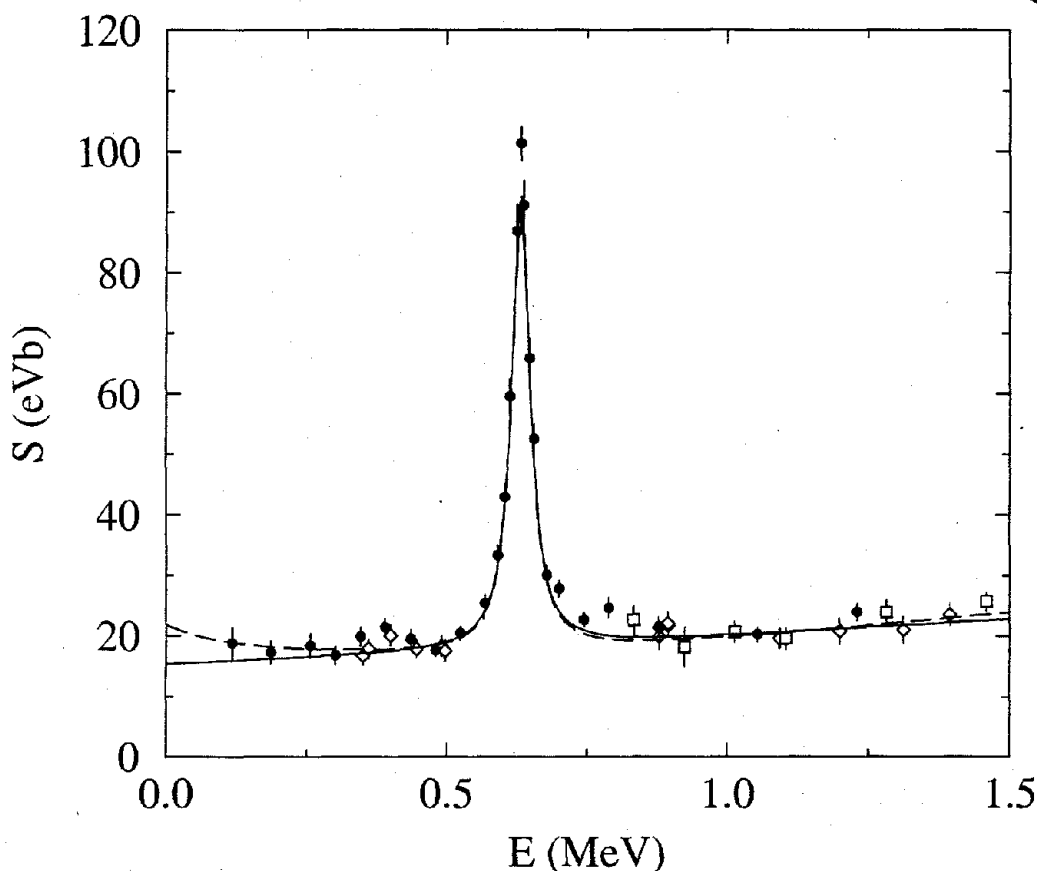


Figure 1: Different fits to the experimental S -factor data. The solid curve is a straight line plus the resonance while the dashed curve is a calculation with a hard sphere cutoff radius of 4.1 fm plus resonance. The data are from Vaughn *et al.* [3] (squares), Filippone *et al.* [2] (circles), and Hammache *et al.* [4] (diamonds).

The dashed curve in Fig. 1 shows an upturn in the S factor at threshold. This is a feature common to all of the model calculations of the astrophysical S factor for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction (known as S_{17}), with the exception of the straight line fit. An understanding of the upturn is central to determining the overall energy dependence.

2 Pole term

To investigate the energy dependence near threshold, we may consider the explicit expansion about zero energy done by Williams and Koonin [16]. They give the first two logarithmic derivatives in a Taylor series expansion of S_{17} as -2.350 MeV^{-1} and 28.3 MeV^{-2} . Using these in a (1,1) Padé approximant gives

$$S_{17} = 0.675 \frac{0.206 + E}{0.139 + E}. \quad (2)$$

This has a pole at $E = -0.139 \text{ MeV}$ and is the source of the upturn at threshold. The pole arises from the subthreshold ground state of ${}^8\text{B}$; the valence proton in ${}^8\text{B}$ is bound by 139 keV.

To understand the pole in terms of the physics we follow the work of Christy and Duck [5].

The $E1$ contribution to the S factor for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction may be written as

$$S = C(I_0^2 + 2I_2^2)E_\gamma^3 \left(J_{11}\beta_{11}^2 + J_{12}\beta_{12}^2 \right) \frac{1}{1 - e^{-2\pi\eta}}, \quad (3)$$

where

$$I_L = \int_0^\infty r^2 dr r \psi_{iL}(r) \psi_f(r) \quad (4)$$

$$C = \frac{5\pi}{9} \frac{1}{(\hbar c)^3} (2\pi\eta k) e^2 \mu^2 \left(\frac{Z_1}{M_1} - \frac{Z_2}{M_2} \right)^2. \quad (5)$$

In Eq. (3), J_{LS} is the spectroscopic factor for a given angular momentum, L , and channel spin, S , β_{LS} is the asymptotic normalization of the bound state wave function, E_γ is the photon energy, and k is the momentum of the incident proton. The extra factor of r in the integrand comes from the photon wave function, which is $E1$ for real photons. The final bound state wave function $\psi_f(r)$ is normalized in the asymptotic region to $\psi_f(r) = W_{\alpha,l}(\kappa r)/r$ while the initial wave function reduces to the regular Coulomb wave function divided by $kr\sqrt{2\pi\eta}/(e^{2\pi\eta} - 1)$. The unusual choice of normalizations is just to simplify the mathematics and generate integrals that are well-behaved at threshold. The initial state has both Coulomb and nuclear distortions. The Coulomb distortions are large and give the penetration factor included in the definition of the S factor [Eq. 1]. They are included in all calculations. The nuclear distortions are much smaller but they are important and introduce a significant model dependence into the calculations.

The absolute magnitude of the S factor is determined primarily by the spectroscopic factor and the asymptotic normalization (see also Ref. [17]). The spectroscopic factor contains the many-body aspects of the problem and is calculable from standard shell model theory. The asymptotic normalization also depends on the many-body wave function, but is far more difficult to estimate from first principles: it requires detailed knowledge of how the 8-body wave function extends beyond the nuclear potential and its mapping to the Whittaker function in this region. This may be estimated crudely by approximating that behavior by using a suitably chosen Woods-Saxon wave function for the weakly bound proton. Instead we treat the overall factor, $A_n = J_{11}\beta_{11}^2 + J_{12}\beta_{12}^2$, as a free parameter, which is independent of energy, and determined by the S -factor data. For simplicity we will refer to this combination of asymptotic normalization and spectroscopic factor as the asymptotic strength.

To investigate the behavior of the integrals in Eq. (3), we first consider $\psi_f(r) = W_{\alpha,l}(\kappa r)/r$ for all radii and take $\psi_{i0}(r) = F_0(kr)/\{kr\sqrt{2\pi\eta}/(e^{2\pi\eta} - 1)\}$. The s -wave integral then becomes

$$I_0 = \int_0^\infty dr r \frac{W_{\alpha,l}(kr)F_0(kr)}{k\sqrt{2\pi\eta}} (e^{2\pi\eta} - 1). \quad (6)$$

The integral is smooth as k passes through zero and diverges as $k \rightarrow i\kappa$ ($E \rightarrow -E_B$). The nature of the divergence is determined by the asymptotic forms of the Coulomb wave function and Whittaker function for large r . There the Whittaker function is proportional to $r^{-|\eta k|/\kappa} e^{-\kappa r}$ [5] (ηk is independent of k). Above threshold the Coulomb wave function oscillates at large radii, however below threshold it is exponentially growing and is proportional to $r^{|\eta|} e^{|k|r}$. Thus the behavior of the integrand at large radius is

$$r^{1-|\eta k|(1/\kappa - 1/|k|)} \exp[-(\kappa - |k|)r] \quad (7)$$

and the integral diverges as

$$I_0 \sim 1/(\kappa - |k|)^2 \sim 1/(E_B + E)^2 = 1/E_\gamma^2. \quad (8)$$

The first correction term is not simply $1/E_\gamma$ but rather of the form $(1 + c \log E_\gamma)/E_\gamma$, the logarithmic term coming from the $r^{-|\eta k|(1/\kappa - 1/|k|)}$ factor. Both the leading and first correction terms are both determined purely by the asymptotic behavior of the wave functions. The second correction term, of order E_γ^0 , is not determined purely by the asymptotic value of wave function alone but also depends on the wave function at finite r .

Since the S factor is proportional to $I_0^2 E_\gamma^3$ [Eq. (3)] the quadratic divergence of I_0 gives rise to a simple pole in S at $E_\gamma = 0$. The presence of a pole suggests the S factor may be parametrized as a Laurent series:

$$S = d_{-1} E_\gamma^{-1} + d_0 + d_1 E_\gamma + \dots \quad (9)$$

The coefficients of the first two terms, d_{-1} and d_0 , are determined purely by the asymptotic forms of the wave functions while the third coefficient, d_1 , is also dependent on the short range properties of the wave functions.

The pole is evident also in the $^{16}\text{O}(p, \gamma)^{17}\text{F}^* \left(\frac{1}{2}^+; \frac{1}{2}, 0.498 \text{ MeV}\right)$ reaction [18, 19]. Fig. 2 displays the S -factor data of Morlock *et al.* [19] for the reaction (top), and for the product $E_\gamma S$ (bottom). In the top panel, the energy dependence of the S factor is well approximated by the form

$$S = n \frac{1 + c_1 E}{E_\gamma} = n \frac{1 + c_1 E}{E + E_B} \quad (10)$$

where the constants $c_1 = 1.18 \text{ MeV}^{-1}$ and $n = 1.59 \times 10^3 \text{ keV}^2 \text{ b}$ are determined from the straight line fit to $E_\gamma S$ shown in the bottom panel. There is remarkable agreement with the data except near the resonance at 2.504 MeV. The form of Eq. (10) is motivated by the form of the pole above.

In the case of the $^{16}\text{O}(p, \gamma)^{17}\text{F}^{\text{ast}}$ reaction, the capture is purely s -wave, stemming from the angular momentum of the final state. For $^7\text{Be}(p, \gamma)^8\text{B}$, as illustrated above, there is also a contribution from d -wave capture. Nuclear distortions may be taken into account by introducing a hard-sphere model [15], where the initial state wave function is zero inside some radius, r_c , and is purely Coulombic outside. The wave function is zero at r_c . This generates a phase shift and is equivalent to having an infinitely repulsive potential with a radius r_c . Fig. 3 displays the s - and d -wave contributions to S_{17} , from which it is clearly seen that the upturn is entirely due to s -wave capture.

The other models used in calculating S_{17} may be assessed in terms of the model presented herein. In order to make such an evaluation, we appeal to the pole and use a rational approximation for the S factor,

$$S(E)/S(0) = \frac{a}{E_b + E} + b + cE, \quad (11)$$

for which $a/E_b + b = 1$. This form is essentially constructed by Taylor expanding $E_\gamma S$. Table 1 shows the coefficients a , b , and c as extracted from various one-body models. Also included are calculations of the first and second logarithmic derivatives. For the most part, there is agreement within the models for the values of the coefficients. The exception to this is for c , where the Williams and Koonin and $r_c = 4.1 \text{ fm}$ models have much lower values compared to the others. In the case of $r_c = 4.1 \text{ fm}$, the value of the radius suggests nuclear distortions outside the nuclear volume ($r_{rms} = 2.48 \text{ fm}$ for ^7Be [21] and so is unrealistic. The derivatives also are largely consistent, with the exception of the the Barker [8] and Adelberger [20] results. Those

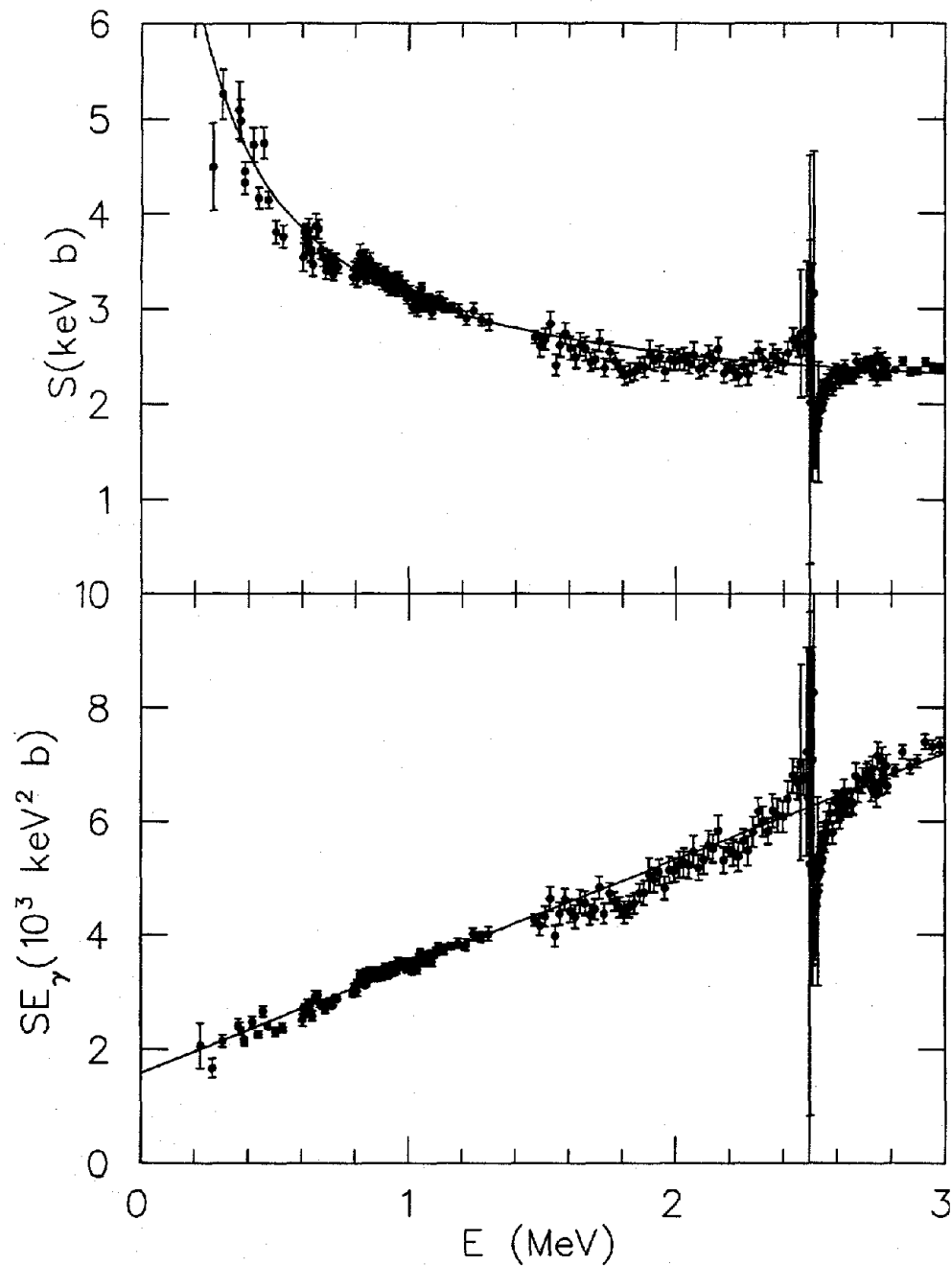


Figure 2: S factor (top) and $E_\gamma S$ (bottom) for the $^{16}\text{O}(p, \gamma)^{17}\text{F}^{\text{ast}}$ reaction. The data of Morlock *et al.* [19] are compared to the fit as described in the text (solid line).

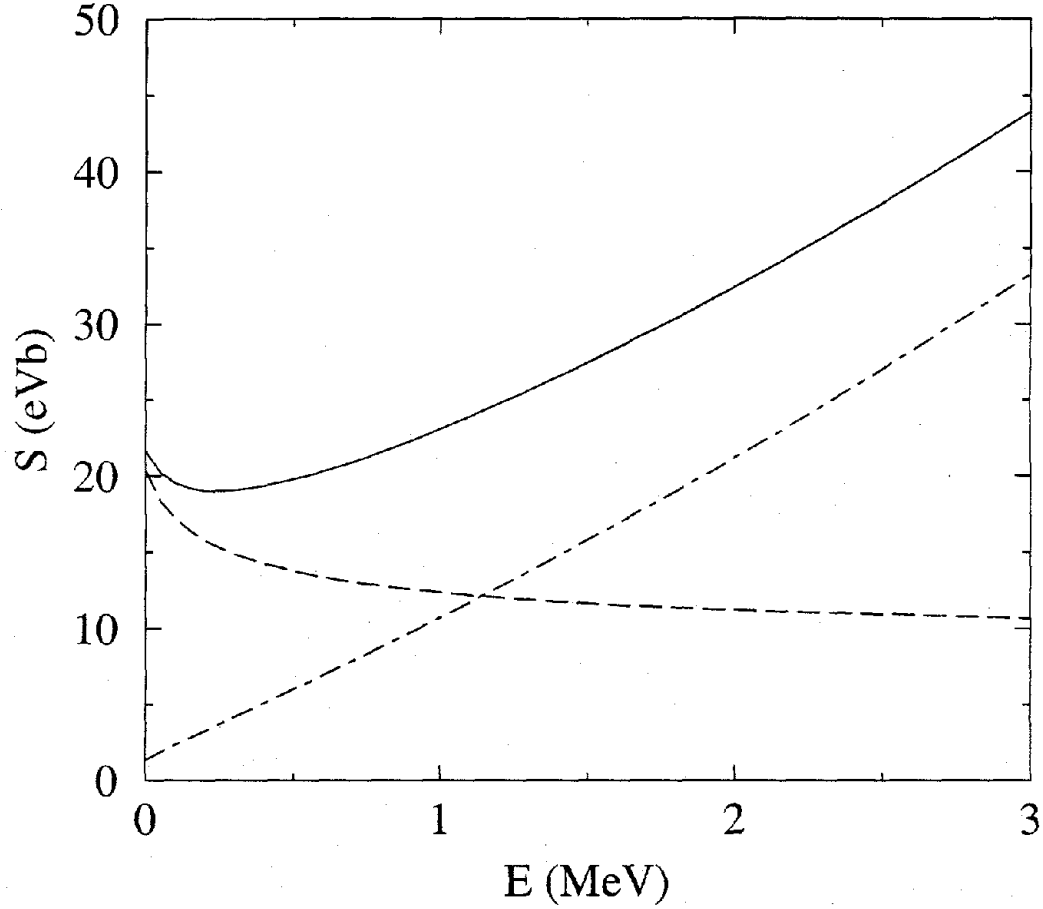


Figure 3: The S factor, as calculated with $r_c = 2.4$ fm. The total result is displayed by the solid line, while the s - and d -wave components are shown by the dashed and dot-dashed lines, respectively.

Table 1: Coefficients of the expansion, Eq. (11). Also shown are the first and second logarithmic derivatives at threshold. The first four rows are for the hard sphere model with different cutoff radii. The fifth row contains the results from Williams and Koonin [16], while the sixth, seventh and eighth rows are with Woods-Saxon potentials. The last two rows contain values of the derivatives from previous work.

Model	a	b	c	$d \log S / dE$ (MeV $^{-1}$)	$d^2 \log S / dE^2$ (MeV $^{-2}$)
$r_c = 0.0$ fm	0.0409	0.702	0.390	-1.77	28.3
$r_c = 1.0$ fm	0.0409	0.703	0.343	-1.82	28.1
$r_c = 2.4$ fm	0.0408	0.703	0.239	-1.92	27.7
$r_c = 4.1$ fm	0.0407	0.704	0.067	-2.09	27.0
W & K [16]	0.0425	0.687	0.050	-2.35	28.3
B1 [7]	0.0420	0.695	0.310	-1.91	28.6
B2 [7]	0.0417	0.697	0.367	-1.84	28.7
T [10]	0.0409	0.703	0.200	-1.96	27.6
Barker [8]	—	—	—	-1.41	10.2
Adelberger et al[20]	—	—	—	-0.70	3.3

stem from quadratic fits to either S or $\ln S$ within an energy region which does not include zero energy. Extrapolating to outside the fit region produces the errors as shown.

We have thus used the hard sphere models to extract values for $S(0)$ and $S(20)$. The fits obtained were to each data set separately and then averaged. The best determinations we obtained are

$$\begin{aligned} S(20) &= 18.4 \pm 1.0 \pm 0.2 \text{ eVb;} \\ S(0) &= 19.0 \pm 1.0 \pm 0.2 \text{ eVb} \end{aligned}$$

when fitting in the region $E < 0.4$ MeV. Fitting to $E < 1.5$ MeV we obtain

$$\begin{aligned} S(20) &= 17.6 \pm 0.7 \pm 0.4 \text{ eVb;} \\ S(0) &= 18.1 \pm 0.7 \pm 0.4 \text{ eVb.} \end{aligned}$$

The first error in each case is statistical, the second comes from the error in the model dependence. The fits with $E < 0.4$ MeV are preferred as the effect of d -wave contribution is minimal in this region, and so the first set of values are to be adopted. The value for $S(0)$ in this fit region is consistent with the determination by Adelberger *et al.* [20], but the error is much decreased.

3 Summary

We have determined that the energy dependence of the S_{17} factor at threshold is due to the presence of a pole arising from the subthreshold ^8B ground state. The pole is s -wave in nature and so the s -wave phase shift partly determines the nature of the pole.

The value of $S_{17}(0)$ is consistent with those previously determined and used in calculations of the flux of ^8B solar neutrinos. As that is the case, the solar neutrino problem still exists and its solution most probably lies elsewhere, in neutrino physics or in better understanding of solar processes.

This work was partly supported by D.O.E. contract no. W-7405-ENG-36.

References

- [1] J. N. Bahcall and M. Pinsonneault, *Rev. Mod. Phys.* **67**, 781 (1992).
- [2] B. W. Filippone *et al.*, *Phys. Rev. C* **28**, 2222 (1983).
- [3] F. J. Vaughn *et al.*, *Phys. Rev. C* **2**, 167 (1970).
- [4] F. Hammache *et al.*, *Phys. Rev. Lett.* **80**, 928 (1998).
- [5] R. F. Christy and I. Duck, *Nucl. Phys.* **24**, 89 (1961).
- [6] C. W. Johnson, E. Kolbe, S. E. Koonin, and K. Langanke, *Astrophysical Journal* **392**, 320 (1992).
- [7] F. C. Barker, *Aust. J. Phys.* **33**, 177 (1980).
- [8] F. C. Barker, *Phys. Rev. C* **28**, 1407 (1983).
- [9] F. C. Barker, *Nucl. Phys.* **A588**, 693 (1995).

- [10] T. A. Tombrello, Nucl. Phys. **A71**, 459 (1965).
- [11] R. G. H. Robertson, Phys. Rev. C **7**, 543 (1973).
- [12] K. H. Kim, M. H. Park, and B. T. Kim, Phys. Rev. C **35**, 363 (1987).
- [13] P. Descouvemont and D. Baye, Nucl. Phys. **A567**, 341 (1994).
- [14] F.M. Nunes, R. Crespo and I.J. Thompson, Nucl. Phys. **A615**, 69 (1997); **A627**, 747 (1997); **A634**, 527 (1998).
- [15] B.K. Jennings, S. Karataglidis, and T.D. Shoppa, Phys. Rev. C **58**, 579 (1998); **58**, 3711 (1998).
- [16] R. D. Williams and S. E. Koonin, Phys. Rev. C **23**, 2773 (1981).
- [17] H. M. Xu, C. A. Gagliardi, R. E. Tribble, A. M. Mukhamedzhanov and N. K. Timofeyuk, Phys. Rev. Lett. **73**, 2027 (1994).
- [18] C. Rolfs, Nucl. Phys. **A217**, 29 (1973).
- [19] R. Morlock *et al.*, Phys. Rev. Lett. **79**, 3837 (1997).
- [20] E. Adelberger *et al.*, Rev. Mod. Phys. **70**, 1265 (1998).
- [21] I. Tanihata *et al.*, Phys. Rev. Lett. **55**, 2676 (1985).