

A three-dimensional validation of crack curvature in muscovite mica

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ABSTRACT: Experimental and computational efforts focused on characterizing crack tip curvature in muscovite mica. Wedge-driven cracks were propagated under monochromatic light. Micrographs verified the subtle curvature of the crack front near the free surface. A cohesive approach was employed to model mixed-mode fracture in a three-dimensional framework. Finite element calculations captured the crack curvature observed in experiment.

1 INTRODUCTION

The mechanical structure of rock can be characterized on multiple length scales. At each length scale, fracture along known interfaces can dominate the global response of the geotechnical structure (Jaeger, 1979). Therefore, the development and validation of numerical methods capable of predicting the fracture of major joints as well as crack growth along grain boundaries are clearly needed. Moreover, these tools must be developed for the inherent, three-dimensional nature of rock.

The fracture of rock can be addressed within the context of a cohesive zone. Cohesive zone approaches provide a general framework for modeling the fracture process (Barenblatt, 1959, Dugdale, 1960, Willis, 1967, Needleman, 1987, Needleman, 1990a,b, Gonzalo, et al., 2000). Of particular interest is the variance in crack tip curvature with confining pressure.

To simplify matters, the authors have chosen to investigate, muscovite mica, a mineral that cleaves along known, crystallographic planes. The goal of the effort was to not only document the crack tip curvature experimentally, but also predict curvature within the context of the finite element method.

2 EXPERIMENTAL EFFORT

2.1 Material selection

Muscovite mica, an alumina silicate, is widely used in brittle fracture and interface studies because of its optical and cleavage properties (Obreimoff, 1930, Bryant, et al., 1963, Bailey, 1967, Wan, et al., 1990).

In 1930, Obreimoff validated the Griffith fracture criterion for brittle materials in a constant displacement double-cantilever beam (DCB) experiment. Most recently, Wan (Wan, et al., 1992) studied environmental effects such as the relative humidity of the atmosphere on the crack propagation and fracture energy of mica.

2.2 Specimen geometry

In this study, crack lengths, and thus energy release rates, were measured for the DCB geometry. Displacement boundary conditions were applied via a narrow wedge. Unlike a constant load or constant moment DCB test, the wedge enforces a constant displacement and yields a constant, stable crack length. The energy release rate, G , is only a function of the wedge height, beam geometry, modulus, and crack length.

2.3 Specimen preparation

Mica is composed of silica tetrahedra that are strongly bonded in layers to aluminum cations. A layer of potassium ions weakly bonds silica-aluminum-silica layers together. Because of its layered crystal structure, mica cleaves along the weak plane of potassium bonds illustrated in Figure 1. Its cleavage properties and optical transparency make mica an ideal brittle material for fracture studies.

To prepare the specimens for the fracture testing, 50 mm by 50 mm by 0.15 mm single crystal muscovite mica sheets were waxed together and cut using a precision saw into 10 mm by 50 mm specimens. The wax was removed in a series of fifteen-minute

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sonic baths of trichloroethylene, acetone, and isopropanol. Because of the layered structure of mica, edge damage was a significant problem and would negatively affect the testing. Therefore, specimens with significant edge damage were discarded.

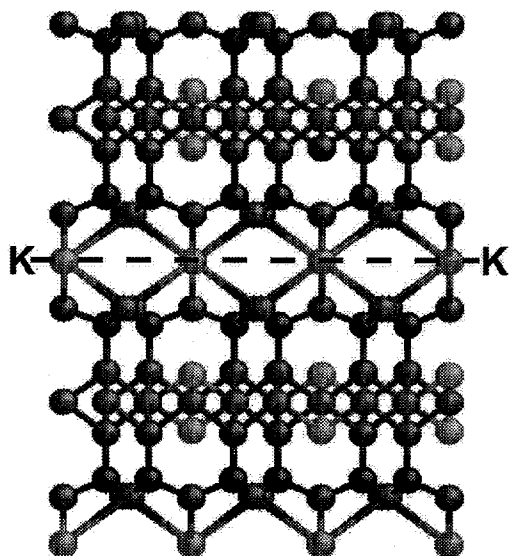


Figure 1. Layered crystal structure of muscovite mica.

2.4 Test procedure

A crack was initiated in the specimen using a 267 μm blade. The specimens were significantly thinner than the blade; thus, cleaving the mica into two, symmetric beams was difficult. Samples with severely asymmetric beams were not used. Figure 2 represents a schematic of the fracture chamber.

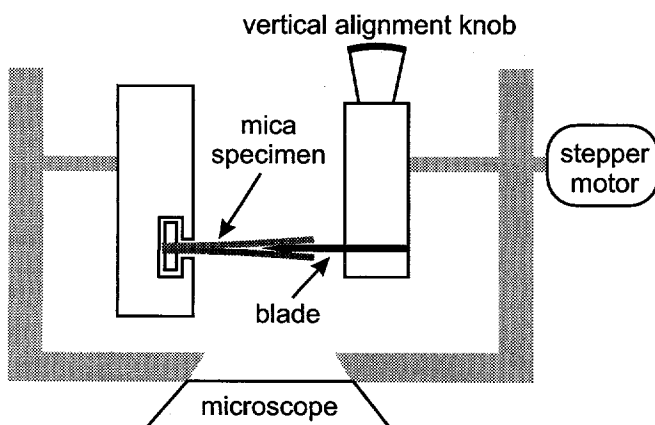
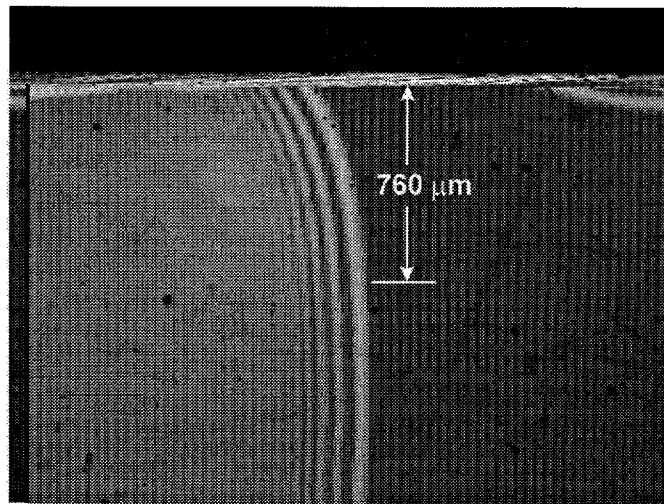


Figure 2. Schematic of the fracture chamber.

The cleaved mica was placed into a specimen holder and the wedge was adjusted vertically to produce a symmetric crack. A stepper motor pushed the mica specimen onto the blade in increments of one micron at user-specified velocities. The crack propagation was viewed with an optical microscope using green light with a wavelength of 550 nm. The crack lengths, measured from the tip of the blade, were measured using a micrometer attached to the microscope stage.

2.5 Experimental results

In Figure 3, the edge effect on the crack tip curvature was readily apparent. In this result, the beam heights were 81 μm and 133 μm . The Young's modulus for muscovite mica was taken to be 169 GPa (McNeil, 1993).



radius of curvature: 1100 μm — 500 μm

Figure 3. Micrograph of crack curvature near the free surface.

3 MODELING EFFORT

For brevity, the computation framework for implementing a cohesive zone formulation will not be presented. Please reference Needleman (1987), and Xu & Needleman (1994) for details. Instead, the authors will concentrate on the analysis of muscovite mica.

3.1 Bulk and interface models

Because experimental evidence indicated a preferential cleavage plane, one could easily separate the bulk and interface models. The bulk model was chosen to be linear elastic. As noted before, the elastic modulus of muscovite mica is 169 GPa (McNeil, 1993). The poisson's ratio was taken to be 0.25.

The interface model employed in this work was proposed by Xu & Needleman (1994). The cohesive tractions are linear-exponential with increasing separation. The local gap is resolved into normal and tangential components, Δ_n and Δ_t . The characteristic length scales associated with normal and tangential de-adhesion are δ_n and δ_t , respectively. For brittle fracture, the critical energy release rate, G_{1c} can be equated to the normal work of separation, ϕ_n . The two-dimensional expressions are detailed in Equation 1 and Equation 2. Mixed-mode separation is

$$T_n = -\frac{\phi_n}{\delta_n} \exp\left(-\frac{\Delta_n}{\delta_n}\right) \left\{ \frac{\Delta_n}{\delta_n} \exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right) + \frac{1-q}{r-1} \left[1 - \exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right) \right] \left[r - \frac{\Delta_n}{\delta_n} \right] \right\} \quad (1)$$

$$T_t = -\frac{\phi_n}{\delta_n} \left(2 \frac{\delta_n}{\delta_t} \right) \frac{\Delta_t}{\delta_t} \left\{ q + \left(\frac{r-q}{r-1} \right) \frac{\Delta_n}{\delta_n} \right\} \exp\left(-\frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right) \quad (2)$$

addressed through the model parameters q and r ,

$$q = \frac{\phi_t}{\phi_n}, \quad r = \frac{\Delta_n^*}{\delta_n} \quad (3)$$

where q relates the normal work of adhesion to the shear work of adhesion and r is a measure of the normal opening associated with complete shear separation. While G_{1c} was an input parameter taken from experiment, 0.82 J/m^2 , the characteristic length scales were chosen to be 100 nm for both normal and tangential separation.

Because the beams were different heights, the neutral axis did not coincide with the fracture plane. Thus, the crack propagated under mixed-mode conditions. Since detailed measurements of the cohesive behavior have not been performed, we assumed $q = 1$ and $r = 0$. Future studies should consider if G_{1c} is truly equal to G_{2c} .

3.2 Spatial discretization

As noted in the experimental section, the mica was pushed onto the wedge at a prescribed rate. Because mica is perfectly brittle, the crack length ahead of the wedge remains constant. This condition produces a "steady-state" crack length of roughly 7 mm . In order to reflect the experiment, a beam length of 10 mm was chosen to span the "steady-state" crack length.

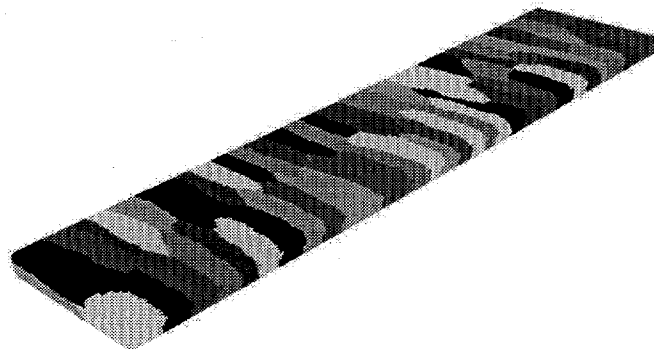


Figure 4. DCB mesh decomposed for 64 processors.

Element spacing along the length was chosen based on the size of the cohesive zone. Defining this zone to span from $0.1\delta_n$ to $4.0\delta_n$, simulations yielded a cohesive zone of 200 microns . An element size of 25 microns guaranteed 8 elements in the zone for integration purposes. Because eight-noded hexagonal elements were employed in bending, ten elements span the thickness. Finally, spacing along the thickness was driven by the curvature shown in Figure 3. Notice the crack curves over approximately 760 microns . In order to place at least ten elements within the area of curvature, elements were chosen to be 75 microns wide. Mesh depth also corresponded to Figure 3, roughly 2.1 mm . These constraints resulted in a mesh with $255,838$ nodes and $235,200$ elements.

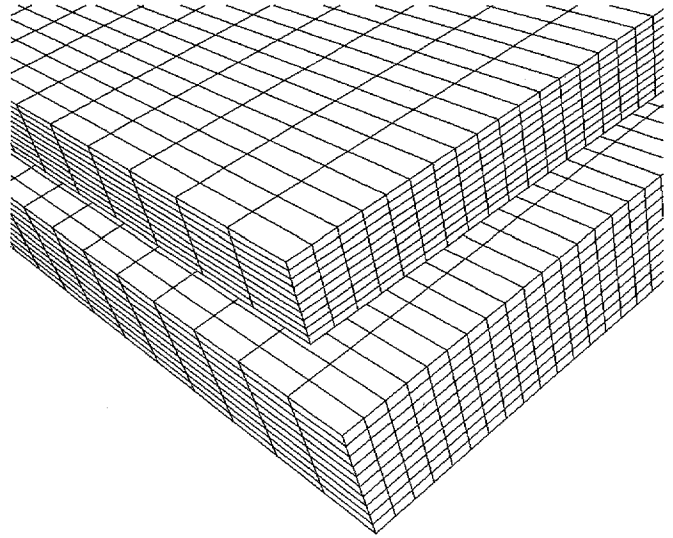


Figure 5. Required mesh spacing for DCB mesh.

Figure 4 illustrates the decomposed mesh for parallel processing. A deformed view of the mesh is displayed in Figure 5.

3.3 Boundary conditions

While the boundary conditions reflect the experiment, no attempt was made to enforce the wedge geometry. Rather, an end displacement was applied to the central surface of each beam. The far end of the beam was pinned at the lower corner. Symmetry was applied along the thickness. Although the detailed shape of the blade edge would be important during initiation, our effort focused on crack curvature during "steady state" propagation.

4 RESULTS

Because future work will involve dynamic processes, all simulations presented were performed using explicit dynamics time integration with loading rates selected to minimize inertial effects.

4.1 Two-dimensional simulations

Two-dimensional simulations verified that the imposed velocity approximated the quasi-static solution. Two-dimensional simulations also verified that the crack length was only governed by the energy release rate. The size of the zone, however, was determined by both the critical energy release rate, G_{1c} , and the length scales associated with de-adhesion, δ_n and δ_t . Keeping G_{1c} constant and varying δ_n and δ_t , one could obtain numerous zone sizes with nearly identical crack lengths.

4.2 Three-dimensional simulations

The model detailed in Section 3.2 was run in parallel for 200,000 time steps. The computed crack opening displacements were used to generate a simulated fringe pattern for green light. Figure 6 illustrates the fringes associated with the crack front. Scale markers were placed for comparison to the experimental micrograph, Figure 3. Notice that in both cases, the

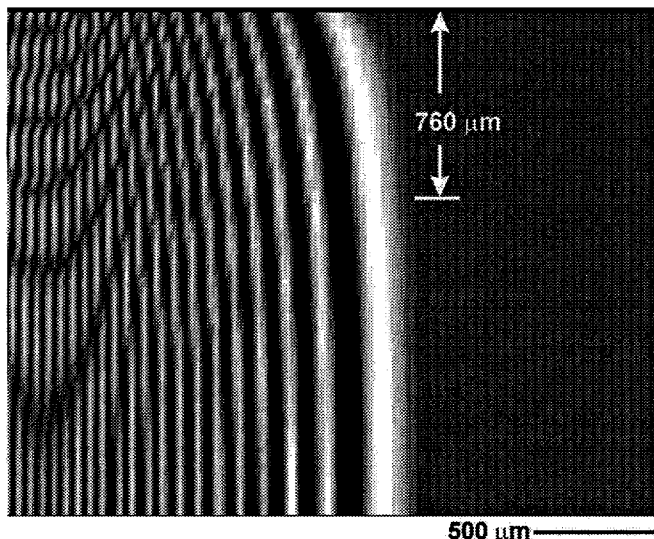


Figure 6. Simulated crack front curvature in muscovite mica. crack curved over approximately 760 μm . This curvature can be attributed to a corresponding variation in the local energy release rate.

The first few fringes in both the simulation and experiment were indicative of the local crack opening displacement profile. Because the experimentally observed crack front was "smeared" over a simulated cohesive zone size of 200 μm , fringes observed in experiment were narrower than those generated by the prediction. Smaller zone sizes could be obtained through a reduction in δ_n and δ_t . A smaller zone size would produce narrower fringes and clarify crack front curvature. However, as noted in Section 3.2, the mesh size would be changed accordingly. More realistic zone sizes would require models on the order of several million degrees of freedom.

5 CONCLUSION

A cohesive approach to fracture was validated through comparison to crack curvature in the muscovite mica mineral system. Experimental results clearly illustrated the nature and length scale of crack front curvature. Because mica cleaved along pre-determined planes, mixed-mode fracture was successfully modeled through a layer of cohesive zone elements. Three-dimensional finite element simulations yielded a curved crack front that compared favorably with experimental findings.

More work is needed. Future simulations should explore a reduction in the cohesive zone size and its effect on the local crack opening displacement profile and crack curvature.

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