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Calculation of Fragmentation Functions in Two-hadron Semi-inclusive Processes

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Abstract. We investigate the properties of interference fragmentation functions arising from the emission of two leading hadrons inside the same jet for inclusive lepton-nucleon deep-inelastic scattering. Using an extended spectator model for the mechanism of the hadronization, we give a complete calculation and numerical estimates for the examples of a proton-pion pair produced with invariant mass on the Roper resonance, and of two pions produced with invariant mass close to the ρ mass. We discuss azimuthal angular dependence of the leading order cross section to point up favourable conditions for extracting transversity from experimental data.

INTRODUCTION

Because of the still lacking rigorous explanation of confinement, the nonperturbative nature of quarks and gluons inside hadrons can be explored by extracting information from distribution (DF) and fragmentation functions (FF) in hard scattering processes. There are three fundamental DF that completely characterize the quark inside hadrons at leading twist with respect to its longitudinal momentum and spin: the momentum distribution f_1, the helicity distribution g_1 and the transversity distribution h_1. At variance with the first two ones, h_1 is difficult to address because of its chiral-odd nature. A complementary information can come from the analysis of the hadrons produced by the fragmentation process of the final quark, namely from FF. So far, only the leading unpolarized FF, D_1, is partly known, which is the counterpart of f_1. The basic reason for such a poor knowledge is related to the difficulty of measuring more exclusive channels in hard processes. The new generation of machines (HERMES, COMPASS, RHIC) and planned projects (ELFE, EPIC) allow for a much more powerful final-state identification and, therefore, for a wider and deeper analysis of FF, particularly when

Final State Interactions (FSI) are considered. In this context, naive "T-odd" FF naturally arise because the existence of FSI prevents constraints from time-reversal invariance to be applied to the fragmentation process [1]. This new set of FF includes also chiral-odd objects that become the natural partner needed to isolate h_1.

The presence of FSI allows that in the fragmentation process there are at least two competing channels interfering through a nonvanishing phase. However, this is not enough to generate naive "T-odd" FF. Excluding *ab initio* any mechanism breaking factorization, there are basically two ways to describe the residual interactions of the leading hadron inside the jet: assume the hadron moving in an external effective potential, or model microscopically independent interaction vertices that lead to interfering competing channels. In the former case, introduction of an external potential in principle breaks the translational and rotational invariance of the problem. Further assumptions can be made about the symmetries of the potential, but at the price of loosing interesting contributions to the amplitude such as those coming from naive "T-odd" FF [2]. In the latter case, the difficulty consists in modelling a genuine interaction vertex that cannot be effectively reabsorbed in the soft part describing the hadronization. This poses a serious difficulty in modelling the quark fragmentation into one observed hadron because it requires the ability of modelling the FSI between the hadron itself and the rest of the jet [2]. Therefore, here we will consider a hard process, semi-inclusive Deep-Inelastic Scattering (DIS), where the hadronization leads to two observed hadrons inside the same jet. A new set of interference FF arise at leading twist [2] and their symmetry properties are briefly reviewed. For the hadron pair being a proton and a pion with invariant mass equal to the Roper resonance, we have already estimated these FF using an extended version of the diquark spectator model [3]. In this case, FSI come from the interference between the direct production of the two hadrons and the decay of the Roper resonance. Here, we present results for the hadron pair being two pions with invariant mass around the ρ mass.

QUARK-QUARK CORRELATION FUNCTION

In analogy with semi-inclusive hard processes involving one detected hadron in the final state [4], the simplest matrix element for the hadronisation into two hadrons is the quark-quark correlation function describing the decay of a quark with momentum k into two hadrons P_1, P_2, namely

$$\Delta_{ij}(k; P_1, P_2) = \sum_X \int \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)\, a_{P_2}^\dagger\, a_{P_1}^\dagger |X\rangle\, \langle X| a_{P_1}\, a_{P_2}\, \overline{\psi}_j(0)|0\rangle\,, \qquad (1)$$

where the sum runs over all the possible intermediate states involving the two final hadrons P_1, P_2. Since the three external momenta k, P_1, P_2 cannot all be collinear at the same time, we choose for convenience the frame where the total pair momentum

$P_h = P_1 + P_2$ has no transverse component. By generalizing the Collins-Soper light-cone formalism [5] for fragmentation into multiple hadrons, the cross section for two-hadron semi-inclusive emission is a linear combination of projections $\Delta^{[\Gamma]}$ of Δ by specific Dirac structures Γ, after integrating over the (hard-scale suppressed) light-cone component k^+ and, consequently, taking ζ as light-like [2]. At leading order, we get

$$\begin{aligned}
\Delta^{[\gamma^-]}(z_h, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) &\equiv D_1 \\
\Delta^{[\gamma^-\gamma_5]}(z_h, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) &\equiv \frac{\epsilon_T^{ij}\, R_{Ti}\, k_{Tj}}{M_1\, M_2}\, G_1^{\perp} \\
\Delta^{[i\sigma^{i-}\gamma_5]}(z_h, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) &\equiv \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2}\, H_1^{\sphericalangle} + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2}\, H_1^{\perp}\ ,
\end{aligned} \tag{2}$$

where $\epsilon_T^{\mu\nu} = \epsilon^{-+\mu\nu}$. The functions $D_1, G_1^{\perp}, H_1^{\sphericalangle}, H_1^{\perp}$ are the interference FF that depend on how much of the fragmenting quark momentum k is carried by the hadron pair ($z_h = z_1 + z_2$), on the way this momentum is shared inside the pair ($\xi = z_1/z_h$), and on the "geometry" of the pair, namely on the transverse relative momentum of the two hadrons ($\mathbf{R}_T^2$) and on the relative orientation between the pair plane and the quark jet axis ($\mathbf{k}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T$, see also Fig. 1). The different

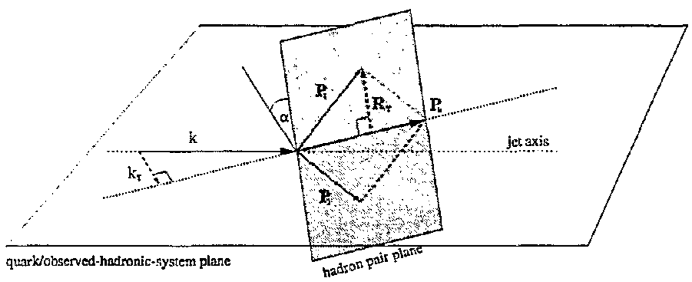

FIGURE 1. Kinematics for a fragmenting quark jet containing a pair of leading hadrons.

Dirac structures Γ are related to different spin states of the fragmenting quark and lead to the nice probabilistic interpretation at leading order [2]: D_1 is the probability for an unpolarized quark to produce a pair of unpolarized hadrons; $G_1^{\perp}$ is the difference of probabilities for a longitudinally polarized quark with opposite chiralities to produce a pair of unpolarized hadrons; $H_1^{\sphericalangle}$ and $H_1^{\perp}$ both are differences of probabilities for a transversely polarized quark with opposite spins to produce a pair of unpolarized hadrons. $G_1^{\perp}$, $H_1^{\sphericalangle}$ and $H_1^{\perp}$ are (naive) "T-odd" and do not vanish only if there are residual interactions in the final state. In this case, the constraints from time-reversal invariance cannot be applied. $G_1^{\perp}$ is chiral even; $H_1^{\sphericalangle}$ and $H_1^{\perp}$ are chiral odd and can, therefore, be identified as the chiral partners needed to access the transversity h_1. Given their probabilistic interpretation, they can be considered as a sort of "double" Collins effect [6].

NUMERICAL RESULTS

In order to make quantitative predictions, we adopt the formalism of the spectator model, specializing it to the emission of a hadron pair. The basic idea is to replace the sum over the complete set of intermediate states in Eq. (1) with an effective spectator state with a definite mass M_D, momentum P_D. Consequently, the correlator simplifies to

$$\Delta_{ij}(k;P_1,P_2) \sim \frac{\theta(P_D^+)}{(2\pi)^3}\,\delta\left((k-P_h)^2-M_D^2\right)\ \langle 0|\psi_i(0)|P_1,P_2,D\rangle\langle D,P_2,P_1|\overline{\psi}_j(0)|0\rangle\ , \tag{3}$$

where the additional δ function allows for a completely analytical calculation of the Dirac projections (2). For the hadron pair being a proton and a pion with invariant mass the mass of the Roper resonance, results have been published in Ref. [3]. In this case, the spectator state has the quantum numbers of a scalar or axial diquark. FSI arise from the interference between the direct production and the decay of the Roper resonance. Here, we show results for the hadron pair being two pions with invariant mass in the range $[m_\rho-\Gamma_\rho, m_\rho+\Gamma_\rho]$, $m_\rho = 768$ MeV and $\Gamma_\rho \sim 250$ MeV. The spectator states becomes an on-shell quark with mass $m_q = 340$ MeV. The quark decay is specialized to the set of diagrams shown in Fig. 2, and their hermitean conjugates, where the naive "T-odd" FF now arise from the interference between the direct production of the two π and the decay of the ρ. Diagram 2a

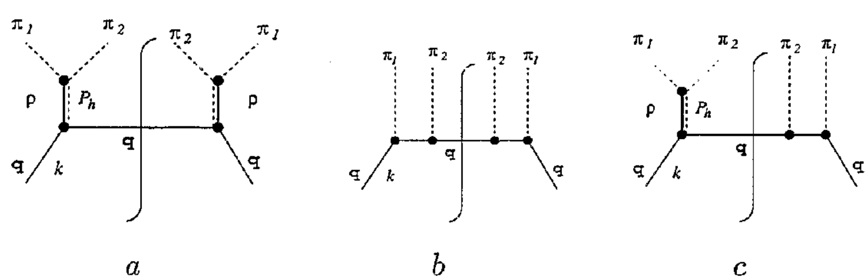

FIGURE 2. Diagrams for quark q decay into two pions through a direct channel or a ρ resonance.

accounts for almost all the strength of $\pi-\pi$ production in the relative P-channel. We have explicitly checked that diagram 2b reproduces the experimental transition probability for $\pi-\pi$ production in the relative S-channel. Hence, we believe this choice represents most of the $\pi-\pi$ strength for invariant mass in the considered interval. We define Feynman rules for the $\rho\pi\pi$, $q\pi q$ and $q\rho q$ vertices introducing cut-offs to exclude large virtualities of the quark while keeping the asymptotic behaviour of FF at large z_h consistent with the quark counting rule. We infer the vertex form factors from previous works on the spectator model [7]. However, there numbers should be taken as indicative, since the ultimate goal is to verify that nonvanishing "T-odd" FF occur, particularly when integrating on some of the kinematical variables and possibly washing all interference effects out. Results of analytical calculation of Eq.(3) show that $H_1^{\perp} = 0$ and $H_1^{\sphericalangle} = -2m_q G_1^{\perp}/m_\pi$. After integrating over $\mathbf{k}_T, \mathbf{R}_T^2$ while keeping $\mathbf{R}_T$ in the horizontal plane of the lab (usually identified with the scattering plane), we still get nonvanishing FF. As an example, $H_1^{\sphericalangle}(z_h, M_h)$ is shown in Fig. 3 for the fragmentation $u \to \pi^+\pi^-$, where M_h is the invariant mass of the two pions. The cross section for the deep-inelastic

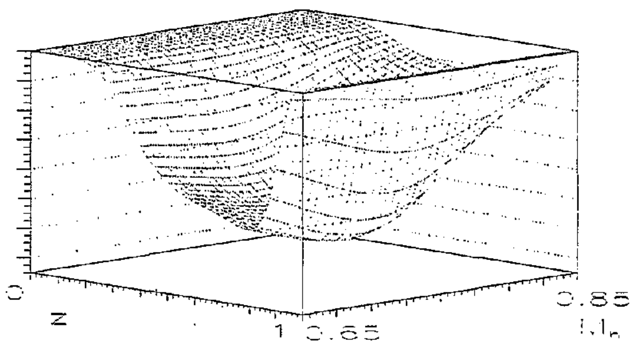

FIGURE 3. $H_1^{\sphericalangle}(z_h, M_h)$ for the fragmentation of a quark u into π^+ and π^-.

scattering of an unpolarized electron on a polarized proton target where two pions are detected in the final state, contains, after integrating all the transverse dynamics $(\mathbf{P}_{hT}, \mathbf{k}_T, \mathbf{R}_T^2)$, an unpolarized contribution proportional to $D_1(z_h, M_h)$ and a term proportional to $H_1^{\sphericalangle}(z_h, M_h)$ which depends on the transverse target polarization S_T. Therefore, by flipping the polarization of the target, it is possible to build the following azimuthal asymmetry

$$\mathcal{A} \propto \frac{S_T}{2m_\pi} \sin(\phi_{R_T} + \phi_{S_T}) \frac{h_1(x)}{f_1(x)} \frac{H_1^{\sphericalangle}(z_h, M_h)}{D_1(z_h, M_h)} , \tag{4}$$

where ϕ_{R_T}, ϕ_{S_T} are the azimuthal angles of $\mathbf{R}_T, \mathbf{S}_T$ with respect to the scattering plane, respectively. The asymmetry shows indeed the familiar sinusoidal azymuthal dependence. Noteworthy is the factorization of the chiral-odd, naive "T-odd" $H_1^{\sphericalangle}$ from the chiral-odd transversity h_1. Therefore, such asymmetry measurement allows for the extraction of h_1 using a model input for the FF.

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REFERENCES

1. De Rújula A., Kaplan J.M., and de Rafael E., *Nucl. Phys.* **B35**, 365 (1971).
2. Bianconi A., Boffi S., Jakob R., and Radici M., *Phys. Rev.* D **62**, 034008 (2000).
3. Bianconi A., Boffi S., Jakob R., and Radici M., *Phys. Rev.* D **62**, 034009 (2000).
4. Mulders P.J., and Tangerman R.D., *Nucl. Phys.* B **461**, 197 (1996).
5. Collins J.C., and Soper D.E., *Nucl. Phys.* B **194**, 445 (1982).
6. Collins J.C., *Nucl. Phys.* B **396**, 161 (1993).
7. Jakob R., Mulders P.J., and Rodrigues J., *Nucl. Phys.* A **626**, 937 (1997).