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Author(s):

Ken Amos, Pradip Deb, Steven Karataglidis, and David G. Madland

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A predictive model of nucleon-nucleus scattering cross sections

Ken AMOS[1,*], Pradip DEB[1], Steven KARATAGLIDIS[2], and David G. MADLAND[2]

[1] *School of Physics, University of Melbourne, Victoria 3010, Australia*

[2] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, U. S. A.*

Nucleon total reaction and neutron total cross sections as well as differential (including spin) observables from 25 to 300 MeV for stable nuclei from ^{6}Li to ^{238}U have been predicted that are in good agreement with measured data. Those predictions have been made using non-local, energy dependent, and complex optical potentials in coordinate space formed by full folding of effective nucleon-nucleon interactions with realistic nuclear ground state densities. By inverse kinematics the same model prescription describes exotic (radioactive) nuclei scattering from hydrogen as a target and the results reveal the extended (neutron) distributions such nuclei can have.

KEYWORDS: nucleon-nucleus cross sections, integrated and differential, microscopic optical model predictions, stable and unstable nuclei.

I. Introduction

Integral and differential observables for both proton and neutron scattering for energies to from 25 to 300 MeV have been predicted by using complex, nonlocal, coordinate-space optical potentials formed by full folding realistic, effective, nucleon-nucleon (NN) interactions with density matrices (hereafter called densities) specified from credible models of the structure of the targets. That predictions agree with observation forebodes well for the use of such calculated values as are needed in a number of fields of study.

An example of such a topical study concerns the transmutation of long-lived radioactive waste into shorter-lived products , which together with energy production, uses accelerator-driven systems (ADS). These systems are being designed in the US, Europe, and Japan, for providing an intense neutron source to a sub-critical reactor. The technology takes advantage of spallation reactions in a thick high-Z target (such as Pb or Bi), where an intermediate-energy proton beam induces nuclear reactions and the secondary nuclear products, particularly lower-energy neutrons and protons, in turn induce further nuclear reactions in a cascade process. The total reaction cross section plays a particularly important role since the secondary particle production cross sections sum to this quantity, which often is an input to intra-nuclear cascade simulations that guide ADS design.

In more basic science, the total reaction cross section is an important ingredient to a number of problems in astrophysics, such as nucleosynthesis in the early universe and for aspects of stellar evolution. The density distribution of neutrons in nuclei are far less well known than that of protons. By considering the integral observables of both proton and neutron scattering from a given nucleus one may seek direct information on the neutron root-mean-square radius.

But most nucleon-nucleus (N-A) total reaction cross sections have not been measured. Thus we must have a reliable method for their prediction. The usual vehicle for specifying N-A total reaction cross sections has been the N-A optical potential; a potential most commonly taken as a local parametrized function, usually of Woods-Saxon type. However, it has long been known that the optical potential must be nonlocal and markedly so, although it has been assumed also that the energy dependence of the customary (phenomenological) model accounts for that. Of more concern is that the phenomenological approach is not truly predictive. The parameter values chosen, while they may be set from a global survey of data analyses, are subject to uncertainties and ambiguities leading, for example, to different values of the total reaction cross section while yielding comparable elastic angular distributions.

We consider a predictive theory of N-A scattering to be one that is *direct* for which all quantities required are defined *a priori*, with no *a posteriori* adjustment of results. With the nucleus viewed as a system of A nucleons, N-A scattering is determined by an optical potential formed by a folding process. That folding we choose to be of realistic interactions of the projectile with each and every nucleon constituting the ground state target density. Such microscopic approaches defining the N-A optical potential have been quite successful in predicting angle-dependent observables of elastic scattering.[1)] In the coordinate space approach, and for analyses that use the DWBA98 programs,[2)] the projectile-target nucleon interaction is not only complex but also energy and density dependent. We have used that program herein to calculate all integral cross sections with realistic effective interactions folded with reasonable descriptions of the densities of nuclei to give credible optical potentials. Using those optical potentials, differential cross sections and spin observables, for proton scattering at many energies in the range 40 to 800 MeV (65 and 200 MeV in particular) and from diverse targets ranging from ^{3}He to ^{238}U have been predicted and found to agree very well with measured values. The (nonlocal) optical potentials are complex and energy dependent; properties that arise from mapping the effective interactions to NN g matrices that are solutions of the Brueckner-Bethe-Goldstone (BBG) equations for nuclear matter. Details of the specifications of the effective interactions, of the folding process that gives the optical potential, and of the successful predictions of differential cross sections and analyzing powers from the scattering of protons at diverse energies and from diverse mass targets, have been

* Corresponding author, Fax. +613-9349-2183
E-mail: amos@physics.unimelb.edu.au

recently summarized.[1)]

II. Theoretical facets

To first order in coordinate space, optical potentials are non-local and can be written

$$U(\vec{r}_1, \vec{r}_2; E) = U_D(\vec{r}_1; E) + U_{Ex}(\vec{r}_1, \vec{r}_2; E)\ , \qquad (1)$$

where the direct U_D and non-local exchange U_E terms are

$$U_D = \sum_n \zeta_n \left\{ \delta(\vec{r}_1 - \vec{r}_2) \int \varphi_n^*(\vec{s})\, v_D(\vec{r}_{1s})\, \varphi_n(\vec{s})\ d^3s \right\}$$
$$U_E = \sum_n \zeta_n \left\{ \varphi_n^*(\vec{r}_1)\, v_{Ex}(\vec{r_{12}})\, \varphi_n(\vec{r}_2) \right\} \qquad (2)$$

where v_D , v_{Ex} are combinations of the components of the effective NN interactions, ζ_n are ground state one body density matrices (which often reduce to bound state shell occupancies), and $\varphi_n(\vec{r})$ are single nucleon bound states. All details and the prescription of solution of the associated nonlocal Schrödinger equations are given in the recent review.[1)]

The results to be discussed herein have been obtained by solving the actual nonlocal Schrödinger equations defined with potentials as given by Eq. (1). For the present calculations, the effective NN interactions have been defined by their mapping to the BBG g matrices of the BonnB NN interaction. When those effective interactions are folded with the appropriate ground state target densities we obtain the "g-folding" optical potentials. Calculations have also been made using solely the free NN t matrix in this prescription, the results of which are "t-folding" optical potentials.

The specification of the nuclear ground state density is taken from a given nucleon-based model of structure. For the light nuclei ($A \leq 16$) we use complete (no core) multi-$\hbar\omega$ shell models.[1)] For heavier nuclei simpler structures often must be used, but those considered do give appropriate electron scattering form factors for the nuclear ground states.

The first objective of specifying the optical potentials is to define the S matrix, or equivalently the (complex) phase shifts $\delta_l^\pm(k)$ which relate by ($E \propto k^2$)

$$S_l^\pm \equiv S_l^\pm(k) = e^{2i\delta_l^\pm(k)} = \eta_l^\pm(k) e^{2i\Re\left[\delta_l^\pm(k)\right]} \qquad (3)$$

where

$$\eta_l^\pm \equiv \eta_l^\pm(k) = \left|S_l^\pm(k)\right| = e^{-2\Im\left[\delta_l^\pm(k)\right]}\ , \qquad (4)$$

and the elastic and total reaction cross sections are

$$\sigma_R(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left[1 - \left(\eta_l^+\right)^2 \right] + l \left[1 - \left(\eta_l^-\right)^2 \right] \right\}\ ,$$
$$= \frac{\pi}{k^2} \sum_{l=0}^{\infty} \sigma_l^{(R)}(E)\ , \qquad (5)$$
$$\sigma_{\rm el}(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left|S_l^+ - 1\right|^2 + l \left|S_l^- - 1\right|^2 \right\}\ ,$$

respectively. The total cross section, $\sigma_{\rm TOT}(E)$, is the sum of these two cross sections. Our calculations suggest to us that the partial total reaction cross sections $\sigma_l^{(R)}(E)$ can be described by the simple functional form,

$$\sigma_l^{(R)}(E) = (2l+1) \left[1 + e^{\frac{(l-l_0)}{a}} \right]^{-1} + \epsilon (2l_0 + 1) e^{\frac{(l-l_0)}{a}} \left[1 + e^{\frac{(l-l_0)}{a}} \right]^{-2}\ , \qquad (6)$$

with $l_0(E, A)$, $a(E, A)$, and $\epsilon(E, A)$ varying smoothly with energy and mass. The summation giving the total reaction cross section can be limited to a value l_{max} and the associated form tends appropriately to the known high energy limit. With increasing energy, l_{max} becomes so large that the exponential fall-off of the functional form, Eq. 6, can be approximated as a straight vertical line ($l_0 = l_{max}$). In that case, the total reaction cross section approximates

$$\sigma_R \Rightarrow \frac{\pi}{2k^2} l_{max}(2l_{max} + 1) \approx \frac{\pi}{k^2} l_{max}^2\ . \qquad (7)$$

Then with $l_{max} \sim kR$, at high energies $\sigma_R \Rightarrow \pi R^2$; the geometric cross section as required.

III. As a probe of nuclear structure

The results of calculations using our g-folding optical potentials are sensitive to details of the model structure used. That is shown in **Fig. 1** where data are compared with three

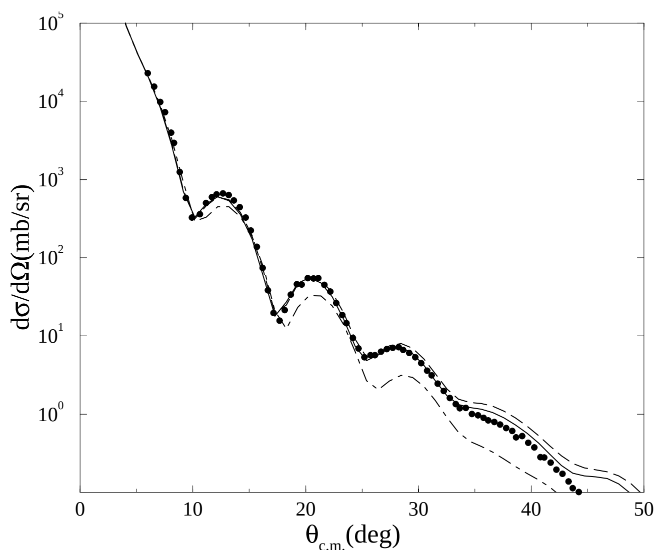

Fig. 1 The differential cross section for 200 MeV p-^{208}Pb scattering.

calculated results of differential cross sections from the elastic scattering of 200 MeV protons from ^{208}Pb. three models for the structure of ^{208}Pb have been used. A standard, packed shell model with harmonic oscillator bound states was used to get the result shown by the dot-dash curve. That description had the same proton and neutron r.m.s. radii (5.45 fm. and 5.6 fm. respectively) as the first of two Skyrme Hartree-Fock (SHF) wave functions[3)] which when used in forming the optical potential gave the result shown by the dashed curve in **f**ig. 1. The solid curve depicts the result we get when the SHF model is adjusted to give a neutron r.m.s. radius 0.26 fm. larger. Clearly the comparison with data selects that second SHF as a better description of ^{208}Pb matter distributions.

We have defined effective NN g matrices for energies 20 to 300 MeV, and by using that at 25 MeV in an analysis of the differential cross-section data form the scattering of 24.5A MeV ^{6}He ions from hydrogen, we obtained the results that are presented in **Fig. 2**. The structure of ^{6}He was defined by a

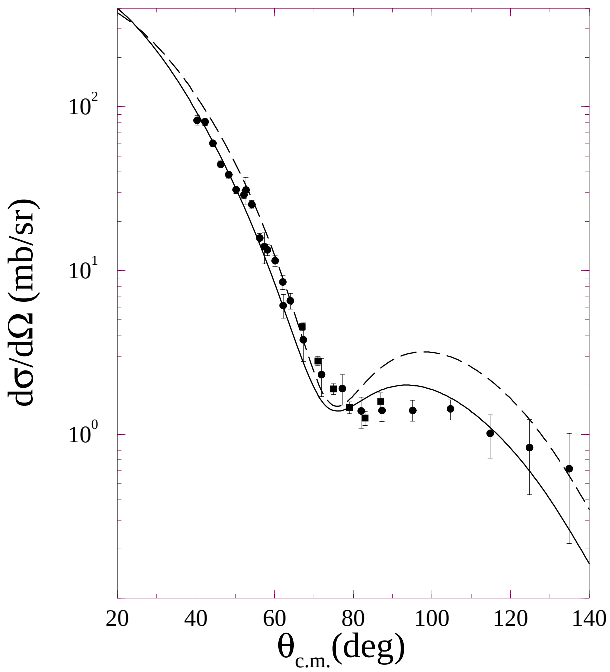

Fig. 2 The differential cross section for 24.5A MeV ^{6}He ions elastically scattering from Hydrogen.

large space (complete $(0+2+4)\hbar\omega$) shell model calculation using the Zheng interaction that gave the individual nucleon shell occupancies and two choices made for the single nucleon bound state wave functions. The first, which equates to what a normal shell model infers, makes ^{6}He have a neutron "skin" around the two protons that it contains. That structure gave the result depicted by the dashed curve in **Fig. 2**. The better result that is depicted by the solid curve, was found when we use wave functions that coincide with a more extended ("halo") neutron distribution in the nucleus.

IV. Predictions of total reaction cross sections

The total reaction cross sections for proton scattering from ^{12}C up to 300 MeV are presented in **Fig. 3**. The structure for ^{12}C was found using the MK3W interaction in a complete $(0+2)\hbar\omega$ shell model calculation. The solid curves are our g-folding model predictions. Clearly, with the exception of two data points at 61 and 77 MeV, our predictions of the proton total reaction cross sections are quite good. There is a mismatch below 20 MeV but that does not vitiate our method of analysis for higher energies. The spectrum of ^{12}C, and also

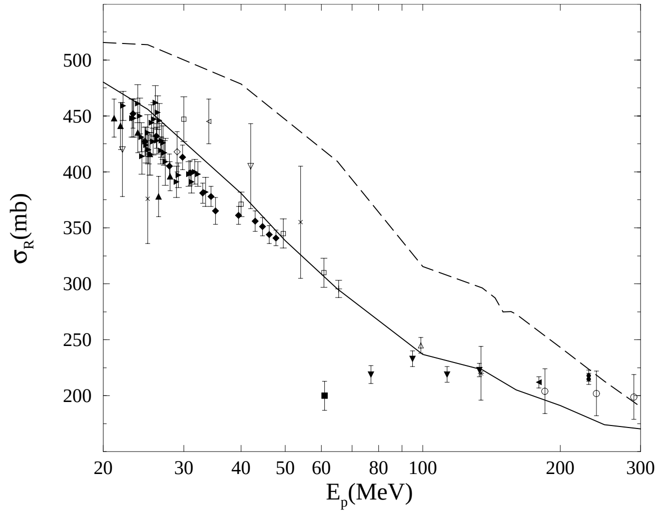

Fig. 3 The reaction cross sections from proton-^{12}C scattering.

of the compound mass-13 system, has many discrete states in that energy regime. So nucleon scattering will reflect effects of (discrete) thresholds, isolated compound nuclear resonances, etc. Our first order folding model of potential scattering has not been designed to accommodate such specific structures and processes. The other result (dashed curve) was found when we ignored the effects of the nuclear medium in specifying the effective interaction. Clearly that is a poor approximation here and it remains so for most other nuclei and at most energies.

The total scattering cross sections of neutrons from ^{12}C and ^{208}Pb are displayed in **Fig. 4**, in the top and bottom parts respectively. The data shown therein are compared with our g

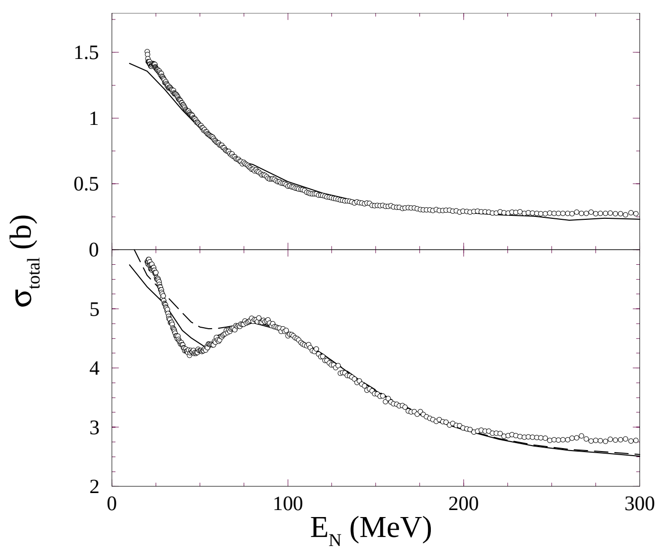

Fig. 4 Total scattering cross sections for neutrons from ^{12}C (top) and from ^{208}Pb (bottom)

folding optical potential predictions. Overall, our predictions are in good agreement with the data. For scattering from ^{208}Pb there are two results shown. The solid curve was obtained by using the SHF prescription[3] for ^{208}Pb and that clearly gives a better reproduction of the Ramsauer diffraction structure in the 30 to 200 MeV region in comparison to the result found with the (simpler) shell model description. More details and other examples are given in a recent publication.[5]

V. A simple model for $\sigma_R(E)$

The g-folding optical potentials for a wide range of target masses and energies have been used to list tables of partial reaction cross sections. Those at 9 energies ranging form 30 MeV to 300 MeV from the scattering of protons from ^{208}Pb are displayed by the dots in **Fig. 5**. The curves shown therein

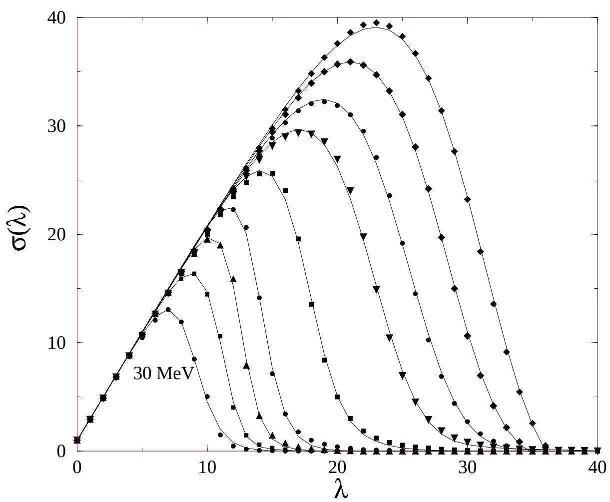

Fig. 5 The partial reaction cross sections for proton scattering from ^{208}Pb. Results are shown for a range of energies.

are the best fir values we could find using the simple functional form for $\sigma_l^R(E)$ as given in Eq. 6. A table of the parameter values for this case and ^{12}C has been published recently.[6] We have now made such analyses for over 50 nuclei and for energies between 20 and 300 MeV wherever data could be found and the best fit values of the parameters $l_0, a, \text{and } \epsilon$ are shown graphically in **Fig. 6** and **Fig. 7**. Clearly there are quite simple variations of those parameter values with both energy and mass with the variation of l_0 being essentially linear with $kA^{1/3}$. The variations of a and ϵ are simple as well but do not have any grossly simple function form. However all these values for 20 to 300 MeV and for $9 \leq A \leq 238$ targets form a compact data set from which simple interpolation could give values to predict cross sections as yet unmeasured.

VI. Conclusions

An optical potential formulation for nucleon scattering from a nucleus based upon defined effective projectile-bound nucleon interactions has been formed that gives successful predictions of integrated as well as differential observables in scattering with energies between 25 and 300 MeV.

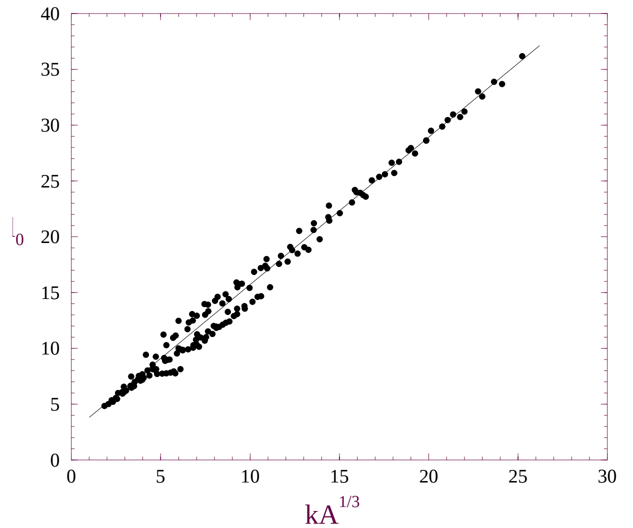

Fig. 6 l_0 variation for masses 9 to 238 and energies 30 to 300 MeV.

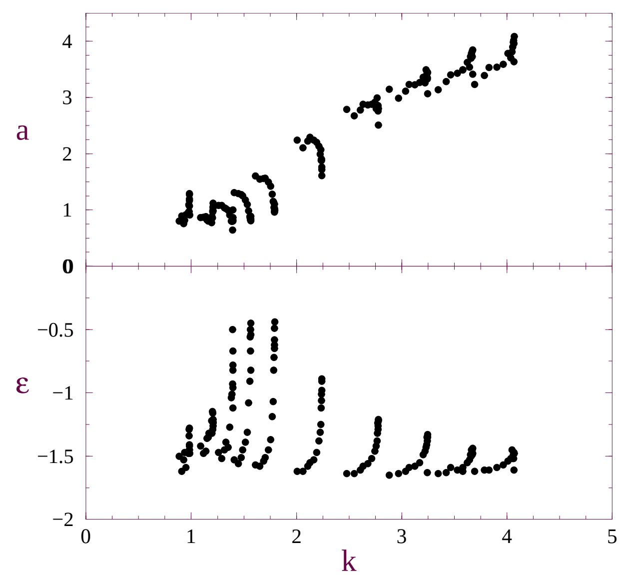

Fig. 7 Plots of a and ϵ; masses and energies as in **Figure 6**

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