European Particle Accelerator Conference BNL-67373
Vienna, Austria, June 26-30, 2000

CHROMATIC CORRECTION AND OPTICAL COMPENSATION IN THE SNS ACCUMULATOR RING USING SEXTUPOLES*

N. Tsoupas†, C.J. Gardner, Y.Y. Lee, Y. Papaphilippou and J. Wei,
Brookhaven National Laboratory, Upton NY, USA

Abstract

The accumulator ring of the Spallation Neutron Source (SNS) will accumulate a high-intensity beam of 2.1×10^{14} protons in a single bunch with large transverse emittance of 160π mm mrad at 95% beam intensity and energies of 1 to 1.3 GeV. In order to keep low beam losses ($\approx 10^{-4}$) in the ring, it is necessary to control the chromaticity and to minimize the dependence of the optical properties on the momentum spread. In this paper, we describe the procedure to accomplish this by using chromaticity sextupoles. We finally discuss the impact of these sextupoles in non-linear dynamics.

1 INTRODUCTION

The accumulator ring of the Spallation Neutron Source (SNS) [1] is designed to accumulate high-intensity beam (2.1×10^{14} protons) in a single bunch at a maximum energy of 1.3 GeV. The average beam power stored in the beam bunch is about 2 MW at a repetition rate of 60 Hz. Such a high beam power requires optimum design of the accumulator ring with a low uncontrolled beam losses level $\leq 10^{-4}$ at 1GeV proton energy. The large transverse emittance ($\epsilon_x = \epsilon_y = 160$ πmm mrad at 95%) and large momentum spread ($\delta p/p = \pm 0.7\%$) of the circulating beam puts even stricter requirements on the design of the ring. Beam bunches with such large transverse size and momentum spread may bring part of the beam into resonance and/or generate beam instabilities that will induce beam losses. One of the design aspects that will help avoid these undesirable effects and therefore minimize the beam losses in the accumulator ring, is optics control. By optics control we refer to the ability to control the tunes $Q_{x,y}$ and the chromaticities $\xi_{x,y} = -\frac{1}{Q_{x,y}} \frac{\partial Q_{x,y}}{\partial(\delta p/p)|_{\delta p=0}}$. This paper discusses the method for chromaticity control and the necessity for optical compensation by using four families of sextupoles along the ring. Optical compensation refers to the minimization of the derivatives of the various optical functions with respect to the momentum spread of the beam. For the nominal working point $(Qx, Qy) = (6.3, 5.8)$ of the lattice, the required field strength for each sextupole family will be provided, such that both chromatic correction and optimum optical compensation are achieved.

2 CHROMATICITY CONTROL

2.1 Optical compensation

For a synchrotron, the chromaticity function can be computed by the classical approach of Courant and Snyder [2]:

$$\xi_{x,y,N} = -\frac{1}{4\pi} \oint \beta_{x,y}(s) k_0(s) ds \ , \qquad (1)$$

where $\beta_{x,y}$ are the horizontal and vertical beta functions of the ring, $k_0 = G/B\rho$ is the ratio of the quadrupole gradient G along the ring over the beam rigidity $B\rho$ and the integration is along the central orbit of the beam. These are often referred to as the "natural chromaticities". In the case of a regular FODO lattice machine with no long straight sections, they are equal and opposite to the tunes $Q_{x,y}$. A method to control the chromaticity, while keeping the tunes constant, is to introduce two families of multi-poles (higher then quadrupole) in non-zero dispersion areas along the ring. Two families of "chromaticity" sextupoles, for example, placed at locations of the ring where the dispersion function is nonzero, will affect the chromaticity by [2]

$$\xi_{x,y,S} = -\frac{1}{2\pi} \oint \beta_{x,y}(s) b_2(s) \eta_x(s) ds \ , \qquad (2)$$

where $b_2(s)$ is the sextupole strength measured in T/m^2 and η_x is the horizontal dispersion of the ring (the vertical dispersion η_y is assumed here to be zero, as in the SNS lattice). Then, the total horizontal and vertical chromaticity are the sums of the "natural" and the "sextupole generated" chromaticities $\xi_{x,y,T} = \xi_{x,y,N} + \xi_{x,y,S}$. The total chromaticities can be controlled by varying the values of $\xi_{x,y,S}$, i.e. by varying the strength b_2 of the sextupoles. In order to achieve higher values of chromaticity correction with lower sextupole field, two families of sextupoles should be placed at high-beta, high-dispersion regions of the ring.

The sextupoles however may strongly affect the first and second order dependence, on the momentum spread of the optics and chromaticity functions (explicit expressions are given in [2]). This dependence of the optics functions on the momentum spread may introduce strong "beta/dispersion waves", which will perturb the dynamics of the ring. In addition these distortions will increase the first and higher order terms of the chromaticity. In order to minimize this effect, additional families of sextupoles are required. The computation of the chromaticities and the various derivatives were performed through the MAD code [3], which employs of the formalism of Courant and Snyder [2]. The minimization process of the various high order derivatives is also done within the MAD computer code, through the HARMON module. For the particular

* Work performed under the auspices of the US Department of Energy
† tsoupas@bnl.gov

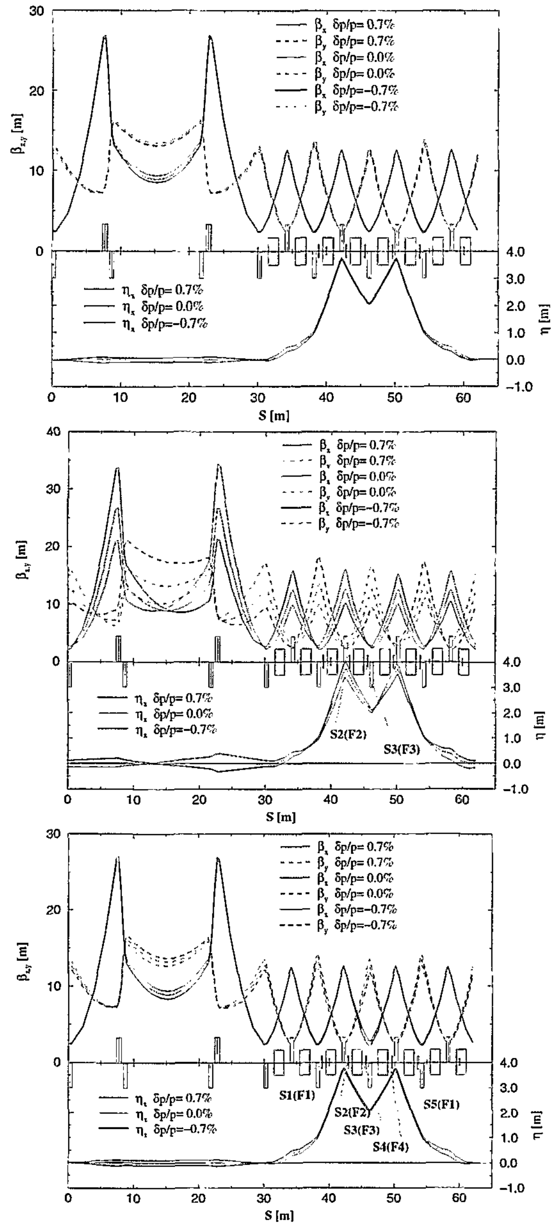

Figure 1: Plot of the optical functions $\beta_{x,y}(s)$ and $\eta_x(s)$ for three different momenta $\delta p/p$ (-0.7%, 0% and +0.7%), along a super-period of the SNS ring, without chromatic correction (top), with two families (middle) and four families (bottom) of chromatic sextupoles, setting the chromaticities to zero. With no sextupoles, there is no strong dependence of the optical functions on the momentum spread. With two families, the optics functions are strongly perturbed for off-momentum cases. With four families, they are almost unchanged.

working point point $(Q_x, Q_y) = (6.3, 5.8)$ of the SNS accumulator ring the natural chromaticities are calculated to be $\xi_{x,N} = -7.74$ and $\xi_{y,N} = -6.40$. Note that their values are not exactly opposite to the tunes, due to the hybrid structure of the lattice (FODO arc and doublet straight sections) [1]. The $\beta_{x,y}(s)$ and $\eta_x(s)$ functions covering one super-period of the accumulator ring are shown in Fig. 1a,

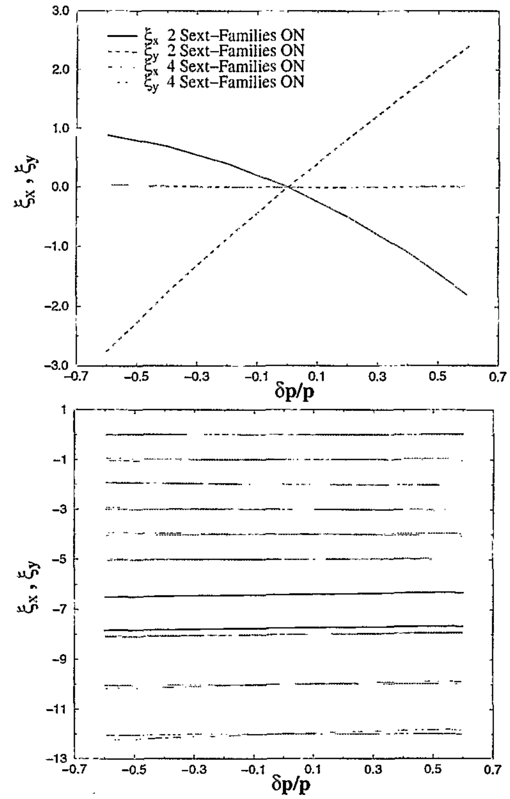

Figure 2: Plot of the chromaticities $\xi_{x,y,T}$ as a function of momentum spread. On the top, the red curves correspond to the chromaticities when two sextupole families are on. The two green curves correspond to the chromaticities when four families are on. On the bottom, the chromaticity range corresponds to the one of Fig. 3, for four sextupole families.

for three different momentum spreads $\delta p/p$ (-0.7%, 0% and +0.7%). It appears that in the case of natural chromaticities there is only a small distortion of the optical functions functions for particles with nonzero momentum spread. Suppose now that we want to keep the same tunes but to change the chromaticities of the ring to $\xi_{x,y,T}|_{\delta p/p=0} = 0$. This change of the chromaticities can be done by using only two families of chromaticity sextupoles. The two sextupoles S2(F2) and S3(F3) shown in Fig. 1b were placed at location of the ring with high values of the β and η functions. The new desired chromaticities can be achieved by adjusting the strength of each sextupole family. The result of the effect of the two family sextupoles on the chromaticity functions is shown in Fig.2a, where the values of the chromaticities are indeed equal to zero at $\delta p/p = 0$, as requested. There is however a strong dependence of the chromaticity functions on the momentum spread. Another undesired effect of using only two families of sextupoles to control the chromaticity functions is shown in Fig. 1b, where we plot the β and η functions along a super-period of the ring for the three different momentum spreads $\delta p/p$. This figure demonstrates that the contribution of the higher order terms is significant. Therefore two families of sextupoles are inadequate to minimize these higher order terms

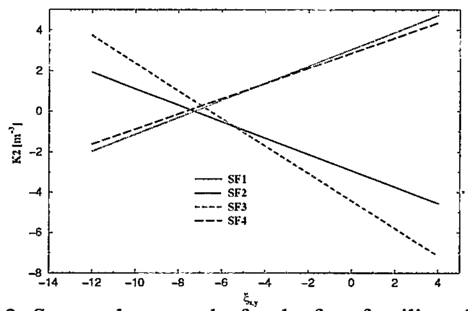

Figure 3: Sextupole strengths for the four families of chromatic sextupoles versus the chromaticities. The strengths are approximately linear with respect to the chromaticity.

while keeping the desired chromaticity values.

In order to compensate the strong dependence of the optics functions to the momentum spread, a set of two additional sextupole families are introduced in the ring. The location of each sextupole family in each super-period is shown in Fig. 1c. The first S1(F1) and the last S5(F1) belong to the same family (F1). As in the case of two family sextupoles, the values of the chromaticity functions $\xi_{x,y,T}$ are indeed zero with the additional benefit that the chromaticity functions depend very little on $\delta p/p$ (almost horizontal curves in Fig. 2a). Another beneficial effect of the four family sextupoles is shown in Fig. 1c which plots the optics functions along a super-period of the ring, for three different momentum spreads (-0.7%, 0% and +0.7%). A comparison of this plot with Fig. 1b demonstrates that the four families of sextupoles reduce significantly the dependence of the β and η functions on $\delta p/p$. Thus, the four family sextupoles does not perturb the linear beam optics and can provide the necessary flexibility during operation with the chromaticities' control functions over a wide range of $\delta p/p$. By varying the strength for each of the four family sextupoles, the ring can attain a wide range of chromaticities, as shown in Fig. 3. The horizontal axis corresponds to the chromaticity settings and the vertical to the values of the sextupole strength of each of the four sextupole families which are required to provide these chromaticities settings. Part of the optical compensation which is accomplished by the four family sextupoles is the minimization of the first and second order terms of the optics function and of the chromaticity. This minimization is shown in Fig. 2b, where the chromaticities $\xi_{x,y,T}$ are plotted as a function of the momentum spread. Note the small slope and curvature of all the plotted curves.

2.2 *Impact to non-linear dynamics*

The introduction of non-linear elements as sextupoles can perturb the motion of particles in the ring. These sextupoles introduce a second order (quadratic in the sextupole strength) tune-shift with amplitude which is linear with the particles' emittance. This tune-shift may be quantified by the anharmonicity coefficients, $a_{hh} = dQ_x/d\varepsilon_x$, $a_{vv} = dQ_y/d\varepsilon_y$ and $a_{hv} = dQ_x/d\varepsilon_y$, the first derivatives of the tune with respect to the emittance. These three quantities have been computed for all range of chromaticity values. In addition to the anharmonicities, we have computed, at first

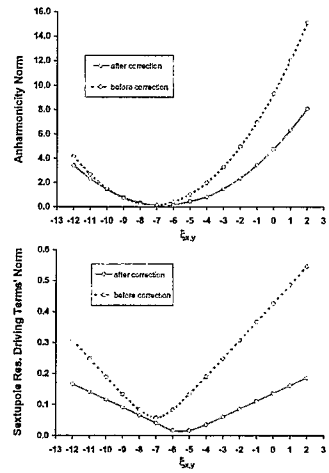

Figure 4: Norm of sextupole resonance driving terms (top) and the anharmonicity (bottom) versus the chromaticities, before correction (dashed line) and after correction (solid line) with dedicated sextupole correctors. The anharmonicity and resonance norms are reduced, after the correction, by as much as a factor of two and four, respectively.

order, the sextupole resonance driving terms $h_{3,0}$, $h_{1,2}$ and $h_{1,-2}$ excited by the chromatic sextupoles. In Fig. 4, we plot the norm of the anharmonicity $\sqrt{a_{hh}^2 + a_{hv}^2 + a_{vv}^2}$ and resonance driving terms $\sqrt{h_{3,0}^2 + h_{1,2}^2 + h_{1,-2}^2}$, as global indicators of the impact of the chromatic sextupoles. The maximum anharmonicity values are found to be a factor of four smaller than the ones introduced by the quadrupole fringe-fields [4], indicating that the introduction of chromatic sextupoles does not have an important non-linear impact on the SNS ring. In order to compensate these resonances, the eight dedicated sextupole correctors of the SNS ring [4] can be used. For all the range of chromaticity settings, we computed the sextupole strengths needed to compensate the sextupole resonances, and at the same time keep the linear optics unperturbed. The resulting anharmonicity and resonance norms are plotted in Fig. 4 (solid curves). A comparison with the uncompensated case suggests that the correction is quite efficient, as it drops the norms by as much as a factor of two and four respectively.

3 CONCLUSION

The chromaticity control is one of the main issues in order to achieve the low beam loss level of 10^{-4} for the SNS ring. Four families of sextupoles are needed in order to adjust the chromaticity and to keep the optical properties of the ring unperturbed. The impact of these magnets with respect to non-linear dynamics is small and can be corrected with the dedicated sextupole correctors.

4 REFERENCES

[1] J. Wei et al., A FODO/Doublet Lattice for the SNS Accumulator Ring, these Proceedings.

[2] E.D. Courant and H.S. Snyder, Ann.Physics 3, 1-48, 1958.

[3] MAD Program User's Reference Manual, CERN/SL/90-13.

[4] Y. Papaphilippou, *et al.*, BNL/SNS Technical Note 72, 2000.