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Robust finite-time control of descriptor Markovian jump systems with impulsive

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Abstract

In this paper, we obtain sufficient conditions ensuring stability of the robust finite-time for descriptor Markovian jump systems with impulsive effects and time-varying norm-bounded disturbance, especially, when the system is in actuator saturation. Using the theory of Lyapunov functions and the concept of convex hull-based representation of saturation function. We design the state feedback controller and obtain estimation of domain of attraction, extending the results to convex optimization problems; the solvability condition of the controller can be equivalent to a feasibility problem of coupled linear matrix inequalities (LMIs). Finally, we present some numerical examples showing the effectiveness of the obtained theoretical results.

Keywords: Descriptor Markovian jump systems; Actuator saturation; Finite-time control; Impulsive systems

1 Introduction

Descriptor systems, also called singular systems, generalized state-space systems, or differential-algebraic systems, have been widely used in many scientific areas because they can better describe the actual system. Descriptor system theory has become an important field in the study of modern control theory. When the structural parameters of a system are randomly mutated, it is naturally modeled as a Markovian jump system or a semi-Markov jump system. Markovian jump systems can be regarded as an extension of single-mode systems to multimodal systems with essentially more complex structure than single-mode systems. During the last decades, Markovian jump systems have attracted great attention in the field of control because they are more suitable for dynamic systems with random changes in the structure of model than single-mode systems. They are widely used in some practical systems, such as manufacturing systems, power systems, economic systems, spare systems, and many other systems [1, 2]. Hence a great number of fundamental notions and substantive results are also emerging. The authors in [3] investigated the stochastic admissibility problems for descriptor Markovian jump systems with partially unknown transition rates, descriptor Markovian jump systems with time-varying delay, and nonlinear descriptor Markovian jump systems with time delay. The problems of the robust exponential stability of uncertain singular Markovian jump time-delay systems were studied in [4]. Shen, Su, and Park [5, 6] extended passive and nonfragile fault detection filtering problem, which is investigated for a class of discrete-time singular Markov jump systems (SMJs) with time-varying delays. Their attention is focused on the design of a general filter that contains the mode-independent and mode-dependent parts to address the filtering issue and on the design of a mode-dependent nonfragile fault detection filter to guarantee the fault detection system to be stochastically admissible with an H_∞ performance index for all admissible uncertainties. In [7] a reliable filtering is designed so that the considered filtering error system in the presence of a time-varying delay and sensor failures is mean-square exponentially admissible with a specified decay rate and simultaneously satisfies an H_∞ performance.

In addition, finite-time stability means that the system is stable in finite or short time, which is first mentioned in [8]. Based on the Lyapunov theory, the transient performance of the finite-time interval internal system is dealt with. Some researchers [9–12] give a definition of finite-time stabilization and finite-time boundedness. The time-domain stability is a special form of time-domain boundedness, and time-domain boundedness is an extended concept of time-domain stability. They are interrelated and different from each other. During the last decades, people paid more attention to the bounded problem of the system state within a limited time. With the advance of time and linear matrix inequality (LMI) technology, scholars had a deeper understanding of the stability of the time domain of a dynamic system and obtained some meaningful results about the stability of the time domain. In particular, [13] focuses on the problem of robust finite-time stabilization for one family of uncertain singular Markovian jump systems. Sufficient conditions for singular stochastic finite-time boundedness are obtained for a class of singular stochastic systems with parametric uncertainties and time-varying norm-bounded disturbance.

Impulsive systems are a kind of discontinuous systems. The impulsive phenomenon exists in different fields of nature and evolutionary processes, which states sudden changes at some points. It is a transient change of state at a certain time in the actual system. The impulsive effect can better describe the evolution process of the system state. From the control point of view, its influence on the stability of the pulse can be divided into two categories, namely, suppression of the stability of an unstable pulse and improvement of the stability of a stable pulse. It is worth mentioning that there have been some important results in time-domain stability for Markov jump systems with impulses. In [14] a new concept of stochastic finite-time stability for a class of nonlinear Markovian switching systems with impulsive effects is introduced. In [15] a stochastic finite-time stability (SFTS) and control synthesis for a class of nonlinear Markovian jump stochastic systems with impulsive effects is proposed. The impulsive of a system can be better described by introducing a time-varying stochastic Lyapunov function with discontinuities at impulse times.

Actuator saturation [16–20] means that if the input of the system actuator reaches a certain limit, then it enters the saturation state. Because further increasing the input cannot affect the output of the actuator, the saturation of the actuator reduces the dynamic performance of the system and even leads to instability of the closed-loop system. Therefore, it is necessary to study the saturation problem. In [21] the robust stochastic problem for discrete-time uncertain singular Markov jump systems with actuator saturation is considered. In [22] the problem of robust exponential stabilization for uncertain impulsive bilinear time-delay systems with saturating actuators is investigated. In [23] the problems of robust linear feedback stabilization and estimation of domain of attraction for a class of uncertain impulsive systems with saturating actuator are investigated.

It can be seen from analysis of the previous literature that, in spite of many studies about the finite-time stability of Markovian jump systems, there are no papers on finite-time stability of systems with actuator saturation and impulse and disturbance effects, which is important and significant in engineering applications. Motivated by these, in this paper, we consider the finite-time stability of systems with simultaneous impulse and saturation effects. Sufficient conditions for the time-domain stability of the system are given by Lyapunov function theory, free weight matrix, LMI, and S-procedure. Based on the previous conditions, a state feedback controller is designed so that the resultant closed-loop system is finite-time stable. Finally, an example is given to solve the problem by MATLAB.

2 Notations

Throughout the paper, for real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite; I is the identity matrix of appropriate dimension; the superscript T represents the transpose; diag$\{\cdots\}$ denotes a block-diagonal matrix. For a symmetric block matrix, we use $*$ as an ellipsis for the terms that are introduced by symmetry; E$\{\cdot\}$ denotes the expectation operator with respect to given probability measure P.

3 Modeling

Given a complete probability space (Ω, F, P), the continuous-time descriptor Markovian jump impulsive system is described by

$$\begin{aligned}
&E\dot{x}(t) = \bigl(A\bigl(r(t)\bigr) + \Delta A\bigl(r(t)\bigr)\bigr)x(t) + \bigl(B\bigl(r(t)\bigr) + \Delta B\bigl(r(t)\bigr)\bigr),\\
&\operatorname{sat}\bigl(u(t)\bigr) + G\bigl(r(t)\bigr)\omega(t), \quad t \neq t_k,\\
&x_k^+(t) = A_{d,k}x(t), \quad t = t_k,\\
&x(t_0) = x_0, \qquad r(t_0) = r_0, \quad k = 1, 2, \dots,\\
&y(t) = C\bigl(r(t)\bigr)x(t) + D\bigl(r(t)\bigr)\operatorname{sat}\bigl(u(t)\bigr) + M\bigl(r(t)\bigr)\omega(t),
\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) = R^m$ is the control output, $u(t) \in R^m$ is the control input, $E \in R^{n\times n}$ is a descriptor matrix with $\operatorname{rank}(E) = r \leq n$, $A(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$, $M(r(t))$, and $G(r(t))$ are known matrices of appropriate dimensions depending on $r(t)$, where $\{r(t), t \geq 0\}$ is a continuous-time Markovian stochastic process defined on a probability space and taking values in a finite space; its transition probabilities from mode i at time t to mode j at time $t + 1$ are described as

$$P\bigl(r(t+\delta) = j | r(t) = i\bigr) = \begin{cases} \gamma_{ij} + o(\delta) & \text{if } j \neq i,\\ 1 + \gamma_{ij}\delta + o(\delta) & \text{if } j = i,\end{cases}$$

where $\delta > 0$, $\lim_{t\to 0}(o(\delta)/\delta) = 0$, $\gamma_{ij} \geq 0$ $(i, j \in S, j \neq i)$ is the transition rate from i to j, and $\gamma_{ii} = -\sum_{j\in S, j\neq i} \gamma_{ij}$. The saturating function $\operatorname{sat}: R^p - R^p$ is defined as

$$\begin{aligned}
&\operatorname{sat}\bigl(u(t)\bigr) = \bigl[\operatorname{sat}\bigl(u_1(t)\bigr)\operatorname{sat}\bigl(u_2(t)\bigr)\cdots\operatorname{sat}\bigl(u_p(t)\bigr)\bigr]^T,\\
&\operatorname{sat}\bigl(u_i(t)\bigr) = \operatorname{sign}\bigl(u_i(t)\bigr)\min\bigl\{1, \bigl|u_i(t)\bigr|\bigr\}.
\end{aligned}$$

In the case of unity saturation level, that is, $\text{sat}(u_i(t)) \leq 1$, $i = 1, 2, \ldots, p$; $\Delta A(r(t))$ and $\Delta B(r(t))$ are matrix functions with time-varying uncertainties. Further,

$$\begin{bmatrix} \Delta A(r(t)) & \Delta B(r(t)) \end{bmatrix} = H_e(i)\Delta(t,i)\begin{bmatrix} F_a(i) & F_b(i) \end{bmatrix}, \tag{2}$$

where $H_e(i)$, $F_a(i)$, $F_b(i)$ are the known real constant matrices of appropriate dimensions, and $\Delta(t,i)$ is an unknown analytic function matrix with Lebesgue-measurable elements satisfying

$$\Delta(t,i)^T\Delta(t,i) \leq I. \tag{3}$$

If (2) and (3) are established, then $\Delta A(r(t))$ is called the structural robust uncertainty, and $\Delta B(r(t))$ is said to be permissible.

Moreover, the disturbance $\omega(t)$ satisfies

$$\int_o^\infty \omega^T(t)\omega(t)\,dt \leq d, \quad d \geq 0. \tag{4}$$

For a matrix, we denote the jth row of $F(i)$ as f_{ij} and define $L(F(i))$ as

$$L\big(F(i)\big) = \big\{x(t) \in R^n : \big|f_{ij}x(t)\big| \leq 1, j = 1, 2, \ldots, p\big\}.$$

Let $P \in R^{n\times n}$ be a symmetric matrix such that $E^TPE \geq 0$ and define the set

$$\Omega\big(E^TPE\big) = \big\{x(t) \in R^n : x^T(t)E^TPEx(t) \leq 1\big\}.$$

Let D be the set of $p \times p$ diagonal matrices whose diagonal elements are either 1 or 0. Suppose that each element of D is labeled as D_l, $l = 1, 2, \ldots, 2^p$ and denote $D_l^- = I - D_l$, Clearly, if $D_l \in D$, then $D_l^- \in D$.

Definition 1

1. The continuous-time system (1) is said to be uniformly regular if there is a constant s such that the characteristic polynomial $\det(sE - A(r(t)))$ is not identically 0 for any $t \in [0, T]$.
2. The continuous-time system (1) is said to be impulse free in the time interval $[0, T]$ if $\deg(\det(sE - A(r(t)))) = \text{rank}(E)$ for all $t \in [0, T]$.

Definition 2 Given three positive scalars c_1, c_2, T with $c_1 < c_2$, positive definite matrices R_i, $i \in S$, and positive definite matrix-valued functions Γ_i, a descriptor Markovian jump impulsive system is finite-time stable with respect to $(c_1, c_2, T, R_i, \Gamma_i)$ if

$$x^T(0)E^TR_iEx(0) \leq c_1 \quad \Rightarrow \quad x^T(t)E^T\Gamma_iEx(t) < c_2 \quad \forall t \in [0, T]$$

for all admissible uncertainties satisfying (2).

Definition 3 ([13]) Let $V(x(t), r(t), t)$ be a stochastic Lyapunov function of a closed-loop SMJS. We define the operator J by

$$JV\big(x(t), r(t), t\big)$$

$$
\begin{aligned}
&= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \bigl\{ \mathrm{E}\bigl\{ V\bigl(x(t+\Delta t)\bigr), r(t+\Delta t), \\
&\quad t+\Delta t | x(t) = x, r(t) = i\bigr\} - V\bigl(x(t), r(t), t\bigr)\bigr\} \\
&= V_t\bigl(x(t), r(t), t\bigr) + V_x\bigl(x(t), r(t), t\bigr) \\
&\quad + \sum_{j=1}^{k} \pi_{ij} V\bigl(x(t), r(t), t\bigr).
\end{aligned}
$$

Lemma 1 ([24]) *Let $F(i), H(i) \in R^{p\times n}$. Then for any $x(t) \in L(H(i))$,*

$$\operatorname{sat}\bigl(F(i)x(t)\bigr) \in \operatorname{co}\bigl\{D_i F(i)x(t) + D_j^- H(i)x(t), j = 1, 2, \ldots, 2^p\bigr\}$$

or, equivalently,

$$\operatorname{sat}\bigl(F(i)x(t)\bigr) \in \sum_{l=1}^{2^p} a_l(t)\bigl(D_l F(i) + D_l^- H(i)\bigr)x(t),$$

where co *stands for the convex hull, a_l, $l = 1, 2, \ldots, 2^p$, are some scalars satisfying $0 \le a_l \le 1$ and $\sum_{l=1}^{2^p} a_l = 1$.*

Lemma 2 ([25]) *Given a set of suited dimension real matrices T_1, T_2, and $F(t)$ is a time-varying matrix with $F(t)^T F(t) \le I$, Then, we have the following:*

(1) *For any scalar $\varepsilon > 0$,*

$$T_1 F(t) T_2 + {T_2}^T F(t)^T {T_1}^T \le \varepsilon T_1 {T_1}^T + \varepsilon^{-1} {T_2}^T T_2.$$

(2) *For any positive definite matrix G,*

$$T_1 T_2 + {T_2}^T {T_1}^T \le T_1 G {T_1}^T + {T_2}^T G^{-1} T_2.$$

Lemma 3 ([22]) *Let $v(t)$ be a nonnegative function such that*

$$v(t) \le a + b \int_0^t v(s)\, ds, \quad 0 \le t \le T,$$

for some constants $a, b \ge 0$. Then we have the following inequality:

$$v(t) \le a e^{bt}, \quad 0 \le t \le T.$$

In this paper, we consider the state feedback controller

$$u(t) = K\bigl(r(t)\bigr)x(t)$$

such that the closed-loop system is defined by

$$
\begin{aligned}
&E\dot{x}(t) = \sum_{l=1}^{2^p} a_l(t)\big(\bar{A}_i x(t) + \bar{B}_i\big(D_l K_i + D_l^- H_i\big)x(t) + G_i\omega(t)\big), \quad t \neq t_k,\\
&x_k^+(t) = A_{d,k}x(t), \quad t = t_k,\\
&x(t_0) = x_0, \qquad r(t_0) = r_0, \quad k = 1,2,\ldots,\\
&y(t) = \sum_{l=1}^{2^p} a_l(t)\big(C_i x(t) + D_i\big(D_l K_i + D_l^- H_i\big)x(t) + M_i\omega(t)\big).
\end{aligned}
\tag{5}
$$

For convenience, denote the matrix $A(r(t))$ as A_i and

$$
\bar{A}_i = \bar{A}\big(r(t)\big) = A\big(r(t)\big) + \Delta A\big(r(t)\big), \qquad \overline{B}_i = \overline{B}\big(r(t)\big) = B\big(r(t)\big) + \Delta B\big(r(t)\big).
$$

4 Main results

4.1 Robust finite-time stabilization

Theorem 1 *Consider the closed-loop system* (5) *for $t \in [0,T]$. Let c_1, c_2, T be three positive scalars with $c_1 < c_2$, let R_i, $i \in S$ be positive definite matrices, and let Γ_i be positive definite matrix-valued functions, Suppose that there exist a scalar $\alpha \geq 0$, a set of nonsingular matrices $P_i \in R^{n\times n}$, two sets of symmetric positive definite matrices $Q_2(i) \in R^{d\times d}$, $i \in S$, and $Q_1(i) \in R^{n\times n}$, $i \in S$, such that the following hold*:

$$
E^T P_i = P_i{}^T E \geq 0, \tag{6}
$$

$$
\begin{bmatrix}
\begin{matrix} A_l^T(i)P_i + P_i^T A_l(i) + \varepsilon P_i^T H_e(i)H_e^T(i)P_i + \\ \varepsilon^{-1}F_l^T(i)F_l(i) + \sum_{j=1}^{2^p}\pi_{ij}E^T P_j - \alpha E^T P_i \end{matrix} & P_i^T G_i \\
* & -Q_2(i)
\end{bmatrix} < 0, \tag{7}
$$

$$
A_{d,k}^T E^T P_i A_{d,k} - E^T P_i < 0, \quad t = t_k, \tag{8}
$$

$$
E^T \Gamma_i \leq E^T P_i \leq E^T R_i, \tag{9}
$$

$$
\lambda_{\max}\big(Q_1(i)\big)c_1 e^{\alpha t} + d\lambda_{\max}\big(Q_2(i)\big)e^{\alpha t} < c_2\lambda_{\min}\big(Q_1(i)\big), \tag{10}
$$

where

$$
\begin{aligned}
\bar{A}_i + \bar{B}_i \operatorname{sat}(u_i) &= A_i + \Delta A_i + (B_i + \Delta B_i)\big(D_l K_i + D_l{}^- H(i)\big)\\
&= A_l(i) + H_e(i)\Delta(t,i)F_l(i),\\
A_l(i) &= A_i + B_i\big(D_l K_i + D_l^- H(i)\big),\\
F_l(i) &= F_a(i) + F_b(i)\big(D_l K_i + D_l^- H(i)\big)
\end{aligned}
$$

for all $i,j = 1,2,\ldots,s$, $l = 1,2,\ldots,2^n$, and $\Omega(E^T X_i E) \subset L(H_i)$. Then the closed-loop system (5) *with respect $(c_1, c_2, T, R, d, \Gamma_i)$ is robust finite-time stable within $\bigcap_{i=1}^N \Omega(E^T X_i E)$.*

Proof Firstly, we prove that the closed-loop systems (5) is regular and impulse-free in the time interval $[0,T]$. By the Schur complement and condition (7) we obtain

$$
A_l^T(i)P_i + P_i^T A_l(i) + (\pi_{ij} - \alpha)E^T P(i) < -\sum_{j=1, j\neq i}^{2^p} \pi_{ij}E^T P_j \leq 0. \tag{11}
$$

Choose nonsingular matrices M and N such that

$$\begin{aligned} &MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad MA_l(i)N = \begin{bmatrix} A_{11}(i) & A_{12}(i) \\ A_{21}(i) & A_{22}(i) \end{bmatrix}, \\ &MP(i)N = \begin{bmatrix} P_{11}(i) & P_{12}(i) \\ P_{21}(i) & P_{22}(i) \end{bmatrix}. \end{aligned} \tag{12}$$

Then, according to (6) and (12), it is not difficult to prove that $P_{12}(i) = 0$ and $\det(P_{22}(i)) \neq 0$. Pre- and post-multiplying (11) by N^T and N, we get

$$\begin{aligned} &N^T A_l^T(i)P_iN + N^T P_i^T A_l(i)N + (\pi_{ii} - \alpha)N^T E^T P_i N < 0, \\ &\bigl(MA_l(i)N\bigr)^T (MP_iN) + (MP_iN)^T \bigl(MA_l(i)N\bigr) + (\pi_{ii} - \alpha)(MEN)^T (MP_iN) < 0, \\ &\begin{bmatrix} A_{11}^T(i)P_{11}(i) & A_{12}^T(i)P_{22}(i) \\ A_{21}^T(i)P_{11}(i) & A_{22}^T(i)P_{22}(i) \end{bmatrix} + \begin{bmatrix} P_{11}^T(i)A_{11}(i) & P_{11}^T(i)A_{12}(i) \\ P_{22}^T(i)A_{21}(i) & P_{22}^T(i)A_{22}(i) \end{bmatrix} \\ &\quad + (\pi_{ii} - \alpha) \begin{bmatrix} P_{11}(i) & 0 \\ 0 & 0 \end{bmatrix} < 0. \end{aligned}$$

We can easily obtain that $A_{22}^T P_{22}(i) + P_{22}^T A_{22}(i) < 0$ and $A_{22}(i)$ is nonsingular, which implies that the closed-loop SMJS is regular and impulse-free in the time interval [0,T].

Construct the following Lyapunov function: $V(x(t), i) = x^T(t)E^T P_i x(t)$.

When $t \neq t_k$, using Definition 3, we obtain that

$$\begin{aligned} \ell V\bigl(x(t), i\bigr) &= \dot{x}^T(t)E^T P_i x(t) + x^T(t)E^T P_i \dot{x}(t) + x^T(t)\Biggl(\sum_{j=1}^{N} E^T P_j\Biggr)x(t) \\ &= \bigl(E\dot{x}(t)\bigr)^T P_i x(t) + x^T(t)P_i{}^T E\dot{x}(t) + x^T(t)\Biggl(\sum_{j=1}^{N} E^T P_j\Biggr)x(t) \\ &= x^T(t)\Biggl[\bar{A}_i^T P_i + P_i{}^T \bar{A} + \bigl(D_l K_i + D_l^- H_i\bigr)^T \bar{B}_i^T P_i \\ &\quad + P_i{}^T \bar{B}_i\bigl(D_l K_i + D_l^- H_i\bigr) + \sum_{j=1}^{N} \bigl(E^T P_j\bigr)\Biggr]x(t) \\ &\quad + \omega^T(t)G^T(i)P_i x(t) + x^T(t)P^T{}_i G(i)\omega(t) < 0, \end{aligned}$$

where $z(t) = \left[\begin{smallmatrix} x(t) \\ \omega(t) \end{smallmatrix}\right]$, so that

$$\ell V\bigl(x(t), i\bigr) = z^T(t)\begin{bmatrix} \begin{matrix} (\bar{A}_i^T + D_l K_i + D_l^- H_i{}^T \bar{B}_i^T)P_i + \\ P_i^T(\bar{A}_i + \bar{B}_i D_l K_i + D_l^- H_i) + \sum_{j=1}^{2^p} \pi_{ij} E^T P_j \end{matrix} & P_i^T G_i \\ * & 0 \end{bmatrix} z(t).$$

By Lemma 1, Lemma 2, and (2), this formula is equivalent to

$$\ell V\big(x(t),i\big) = z^T(t)\begin{bmatrix} A_l^T(i)P_i + P_i^T A_l(i) + \varepsilon P_i^T H_e(i)H_e^T(i)P_i + \\ \varepsilon^{-1}F_l^T(i)F_l(i) + \sum_{j=1}^{2^p}\pi_{ij}E^T P_j & P_i^T G_i \\ * & 0 \end{bmatrix} z(t).$$

According the last formula and (7), we can obtain that $\ell V(x(t), r(t) = i, t) < 0$ and

$$\ell V\big(x(t),i\big) < \alpha V\big(x(t),i\big) + \omega^T(t)Q_2(i)\omega(t). \tag{13}$$

When the system depends on the state to jump, applying (8), we have

$$V\big(t,x_k^+\big) - V(t,x) = x^T(t)\big[A_{d,k}^T E^T P(t)A_{d,k} - E^T P(t)\big]x(t) < 0.$$

So we derive that $V(t,x)$ is strictly decreasing on T.

Integrating (13) from 0 to t and using Lemma 3, we have

$$\begin{aligned} \mathrm{E}\big\{V\big(x(t),i\big)\big\} &< V\big(x(0),i\big) + \alpha\int_0^t V\big(x(s),i\big)\,ds + \int_0^t \omega^T(t)Q_2(i)\omega(t)\,d\tau \\ &< V\big(x(0),i\big) + \alpha\int_0^t V\big(x(s),i\big)\,ds + d\lambda_{\max}\big(Q_2(i)\big) \\ &< \big[V\big(x(0),i\big) + d\lambda_{\max}\big(Q_2(i)\big)\big]e^{\alpha t} \\ &< V\big(x(0),i\big)e^{\alpha t} + d\lambda_{\max}\big(Q_2(i)\big)e^{\alpha t}, \end{aligned} \tag{14}$$

where $Q_1(i) = E^{-T}R_i^{-\frac{1}{2}}E^T P_2 R_i^{\frac{1}{2}}E^{-1}$, and $\lambda_{\max}(Q_1(i))$ and $\lambda_{\min}(Q_1(i))$ are the maximum and minimum eigenvalues of $Q_1(i)$. Thus

$$\begin{aligned} E\big\{V\big(x(t),i\big)\big\} &= E\big\{x^T(t)E^T R_i^{\frac{1}{2}}Q_1(i)R_i^{\frac{1}{2}}Ex(t)\big\} \\ &\geq \lambda_{\min}Q_1(i)E\big\{x^T(t)E^T\Gamma_i Ex(t)\big\}. \end{aligned}$$

On the other hand,

$$\begin{aligned} V\big(x(0)\big)e^{\alpha t} &= x^T(0)E^T R_i^{\frac{1}{2}}Q_1(i)R_i^{\frac{1}{2}}Ex(0)e^{\alpha t} \\ &\leq \lambda_{\max}Q_1(i)E^T x^T(0)R_i Ex(0)e^{\alpha t} \leq \lambda_{\max}Q_1(i)c_1 e^{\alpha t}. \end{aligned}$$

Therefore

$$E\big\{x^T(t)E^T\Gamma_i Ex(t)\big\} \leq \frac{\lambda_{\max}(Q_1(i))c_1 e^{\alpha t} + d\lambda_{\max}(Q_2(i))e^{\alpha t}}{\lambda_{\min}(Q_1(i))} < c_2.$$

This proves that system (5) is robust finite-time stable. □

Theorem 2 *Let* (5) *be a closed-loop system with* $\omega(t) = 0$ *for* $t \in [0,T]$, *let* c_1, c_2, T *be three positive scalars with* $c_1 < c_2$, *let* R_i, $i \in S$, *be positive definite matrices, and let* Γ_i *be positive definite matrix-valued functions. Suppose that there exist a scalar* $\alpha \geq 0$, *a set of* *nonsingular matrices $P_i \in R^{n\times n}$, and two sets of symmetric positive definite matrices $Q_2(i) \in R^{d\times d}$, $i \in S$, and $Q_1(i) \in R^{n\times n}$, $i \in S$, such that the following hold*:

$$\begin{aligned}
&E^T P_i = {P_i}^T E \geq 0,\\
&A_{d,k}^T E^T P_i A_{d,k} - E^T P_i < 0, \quad t = t_k,\\
&E^T \Gamma_i \leq E^T P_i \leq E^T R_i,\\
&A_l^T(i)P_i + P_i^T A_l(i) + \varepsilon P_i^T H_e(i)H_e^T(i)P_i + \varepsilon^{-1}F_l^T(i)F_l(i) + \sum_{j=1}^{2^p}\pi_{ij}E^T P_j - \alpha E^T P_i < 0,\\
&\lambda_{\max}\bigl(Q_1(i)\bigr)c_1 e^{\alpha t} < c_2\lambda_{\min}\bigl(Q_1(i)\bigr),
\end{aligned}$$

for all $i,j = 1,2,\dots,s$, $l = 1,2,\dots,2^n$, and $\Omega(E^T X_i E) \subset L(H_i)$. Then the closed-loop system (5) *with respect $(c_1, c_2, T, R, d, \Gamma_i)$ is robust finite-time stable within $\bigcap_{i=1}^N \Omega(E^T X_i E)$.*

This result can be proved in much the same way as Theorem 1.

4.2 Robust state feedback controller

Theorem 1 gives a set of conditions to judge if some initial state is in the domain of attraction in mean square sense. To further facilitate the synthesis procedure, we will state these conditions in terms of LMIs.

Theorem 3 *Suppose that, for each mode $i \in S$ and given scalars $\varepsilon_i > 0$, there exist a positive definite symmetric matrix $X_i > 0$, matrices Y_i and H_i for $i = 1,2,\dots,2^n$, and $\Omega(E^T P_i E) \subset L(H_i)$. Then for all uncertainties satisfying* (2) *and* (3), *the closed-loop system is robust finite-time stable within $\bigcap_{i=1}^N \Omega(E^T X_i E)$, and the state feedback controller gain matrix is given by $K_i = L_i X_i^{-1}$, $i = 1,2,\dots,N$.*

Let $X_R \subset R^n$ be a prescribed bounded convex set containing origin that can be represented as the polyhedron $X_R = \mathrm{co}\{x_0^1, x_0^2, \dots, x_0^q\}$, where $x_0^1, x_0^2, \dots, x_0^q$ are a priori given initial states in R^n. To see if the initial states $x_0 \subset R^n$ are in the domain of attraction in the mean square sense, we can formulate the following optimization problem:

$$\begin{aligned}
&\max_{P_i>0, F_i, H_i, Q_i, \varepsilon_i} \alpha\\
&\text{s.t.}\quad \text{(i)}\quad \alpha x_0^j \in \Omega\bigl(E^T P_i E\bigr), \quad j = 1,2,\dots,s,\\
&\qquad\quad \text{(ii)}\quad \text{Inequalities (7), (10)},\\
&\qquad\quad \text{(iii)}\quad \Omega\bigl(E^T P_i E\bigr) \subset L\bigl(H(i)\bigr),
\end{aligned}$$

where h_{iq} denotes the qth row of H_i. If $\max|\alpha| > 1$, then $x_0 \in \Omega(E^T P_i E)$. Noticing that the optimization problem is nonconvex, we need to formulate this problem into a convex optimization problem.

Let $P_i = X_i^{-1}$, $L_i = K_i X_i$, $E^T P_i = {P_i}^T E = P_i^{-1} H(i) P_i^{-T}$. Condition (i) is equivalent to

$$\alpha^2 \bigl(x_0^j\bigr)^T E^T P_i E x_0^j \leq 1, \quad j = 1,2,\dots,s.$$

By the Schur complement it can be converted to

$$\begin{bmatrix} \alpha^{-2} & (x_0^j)^T E^T X_i \\ X_i E x_0^j & X_i \end{bmatrix} \geq 0, \quad j = 1, 2, \ldots. \tag{15}$$

Letting $\beta = \alpha^{-2}$, (15) can be rewritten as

$$\begin{bmatrix} \beta & (x_0^j)^T E^T X_i \\ X_i E x_0^j & X_i \end{bmatrix} \geq 0, \quad j = 1, 2, \ldots. \tag{16}$$

Inequality (7) can be transformed into

$$\begin{bmatrix} \sum & P_i^T He(i) & F(i)^T \\ * & -\varepsilon_i^{-1} I & 0 \\ * & * & \varepsilon_i I \end{bmatrix} < 0, \tag{17}$$

where

$$\sum = A_l^T(i) P_i + P_i^T A_l(i) + \sum_{j=1}^{N} \pi_{ij} E^T P_j - P_i^T G(t) Q_2 G(t)^T P_i - \alpha E^T P_i.$$

Pre- and postmultiplying (17) by te diagonal matrix $\operatorname{diag}\{X_i, I, I\}$, we obtain

$$\begin{bmatrix} X_i \sum X_i & X_i P_i^T He(i) & X_i F(i)^T \\ * & -\varepsilon_i^{-1} I & 0 \\ * & * & \varepsilon_i I \end{bmatrix} < 0.$$

This matrix can be transformed into

$$\begin{bmatrix} \Pi & X_i P_i^T He(i) & X_i F_i^T & \Pi_1 \\ * & -\varepsilon_i^{-1} I & 0 & 0 \\ * & * & \varepsilon_i I & 0 \\ * & * & * & -\Pi_2 \end{bmatrix} < 0, \tag{18}$$

where

$$\begin{aligned}
\Pi &= X_i A_m^T(i) P_i X_i + X_i P_i^T A_m(i) X_i - X_i P_i^T G(t) Q_2 G^T(t) P_i, \\
\bar{A}_i + \bar{B}_i \operatorname{sat}(u_i) &= A_i + \Delta A_i + (B_i + \Delta B_i)\bigl(D_l L_i X_i^{-1} + D_l^- H(i)\bigr) \\
&= A_m(i) + H_e(i) \Delta(t, i) F_m(i), \\
A_m(i) &= A_i + B_i\bigl(D_l L_i X_i^{-1} + D_l^- H(i)\bigr), \\
F_m(i) &= F_a(i) + F_b(i)\bigl(D_l L_i X_i^{-1} + D_l^- H(i)\bigr), \\
\Pi_1 &= [\sqrt{\pi_{i,1}} X_i, \ldots, \sqrt{\pi_{i,i-1}} X_i, \sqrt{\pi_{i,i+1}} X_i, \ldots, \sqrt{\pi_{i,k}} X_i], \\
\Pi_2 &= \operatorname{diag}\left\{ \begin{matrix} P^T(1) H^{-1}(1) P(1), \ldots, P^T(i-1) \\ H^{-1}(i-1) P(i-1) P^T(i+1) H^{-1}(i+1) \\ P(i+1), \ldots, P^T(k) H^{-1}(k) P(k) \end{matrix} \right\}.
\end{aligned}$$

Let $Q_i = Q_1^{-1}(i) = R_i^{-1/2} X_i R_i^{-1/2}$ and

$$\lambda_{\max}(Q) = \frac{1}{\lambda_{\min}(Q_1(i))},$$
$$\lambda < \lambda_{\min}\bigl(Q_1(i)\bigr), \qquad \lambda_{\max}\bigl(Q_1(i)\bigr) > 1, \qquad \lambda_{\max}\bigl(Q_2(i)\bigr) > 1.$$

Thus (10) can be rewritten as

$$\frac{c_1 + d}{\lambda_1} - c_2 e^{-\partial t} < 0$$

or, equivalently,

$$\begin{bmatrix} -c_2 e^{-\partial t} & \sqrt{c_1 + d} \\ \sqrt{c_1 + d} & -\lambda \end{bmatrix} < 0. \tag{19}$$

Since $\Omega(E^T P_i E) \subset L(H(i))$, we have

$$x^T(t) h_{iq}^T h_{iq} x(t) \le x^T(t) E^T P_i E x(t),$$

which is equivalent to

$$h_{iq}^T h_{iq} - P_i \le 0, \quad i = 1, 2, \ldots, s, q = 1, 2, \ldots, m.$$

Using the Schur complements, we have

$$\begin{bmatrix} -P_i & h_{iq}^T \\ * & -I \end{bmatrix} \le 0, \quad q = 1, \ldots, m, \tag{20}$$

and the optimization problem can be transformed into the following linear matrix inequality problem:

$$\begin{cases} \min_{X_i > 0, Y_i, H_i} \beta \\ \quad \text{s.t.} \quad \text{LMIs (16), (18), (19), and (20),} \end{cases}$$

where $\varepsilon_i > 0$ is a given scalar. If $\min \beta < 1$, then $x_0 \in \Omega(E^T P_i E)$. The state feedback controller gain $K_i = L_i X_i^{-1}$ can be obtained by solving the linear matrix inequality problem directly.

5 Simulation example

Let us consider the robust finite-time stability for system (1) with the following coefficient matrices:

$$E = \begin{bmatrix} 2.5 & 5 & 2.5 \\ 0 & 1.25 & 1.25 \\ 0 & 0 & 0 \end{bmatrix},$$

mode 1:

$$A(1)=\begin{bmatrix}2 & 2.4 & 1.2\\ 1.8 & 1.1 & 1.1\\ 0.5 & 0.2 & 0.5\end{bmatrix},\qquad B(1)=\begin{bmatrix}0.6\\ 0.4\\ 0.1\end{bmatrix},$$

$$F_a(1)=\begin{bmatrix}0.05 & 0.06 & 0.02\end{bmatrix},\qquad F_b(1)=0.03,$$

$$He(1)=\begin{bmatrix}0.01\\ 0.02\\ 0.03\end{bmatrix},\qquad W_1=\begin{bmatrix}0.6 & 0.3 & 1.3\\ 0.5 & -0.2 & 0.25\\ 1 & 1.5 & 0.5\end{bmatrix},$$

$$\begin{bmatrix}\Delta A(1) & \Delta B(1)\end{bmatrix}=\begin{bmatrix}0.0005 & 0.0006 & 0.0002 & 0.0003\\ 0.0010 & 0.0012 & 0.0004 & 0.0006\\ 0.0015 & 0.0018 & 0.0006 & 0.0009\end{bmatrix};$$

mode 2:

$$A(2)=\begin{bmatrix}1 & -1 & 0\\ 2.25 & 0.75 & 2\\ 1.25 & 0.5 & 0.75\end{bmatrix},\qquad B(2)=\begin{bmatrix}1.5\\ -1\\ -0.75\end{bmatrix},$$

$$F_a(2)=\begin{bmatrix}0.02 & 0.04 & 0.05\end{bmatrix},\qquad F_b(1)=0.04,$$

$$He(2)=\begin{bmatrix}0.03\\ 0.05\\ 0.06\end{bmatrix},\qquad W_2=\begin{bmatrix}0.2 & 0.3 & 1.3\\ 0.5 & -0.2 & 0.25\\ 1 & 1.5 & 0.5\end{bmatrix},$$

$$\begin{bmatrix}\Delta A(2) & \Delta B(2)\end{bmatrix}=\begin{bmatrix}0.0006 & 0.0012 & 0.0015 & 0.0012\\ 0.0010 & 0.0020 & 0.0025 & 0.0020\\ 0.0012 & 0.0024 & 0.0030 & 0.0024\end{bmatrix}.$$

Let $c_1 = 1$, $c_2 = 11$, $T = 3$, $\alpha = 0.1$,

$$x_0=\begin{bmatrix}-0.56\\ 0.36\\ 0\end{bmatrix},\qquad R=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix},$$

$$\Gamma_i=\begin{bmatrix}1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.5\end{bmatrix}.$$

We use the LMI toolbox of MATLAB software to solve the optimization problem $\beta = 0.218 < 1$. The gain of the state feedback controller is:

$$K_1=\begin{bmatrix}-1.5983 & -1.5495 & 0.0503\end{bmatrix},$$

$$K_2=\begin{bmatrix}-0.4159 & -0.0910 & -0.0306\end{bmatrix}.$$

Then the MATLAB simulation is shown in the following image.

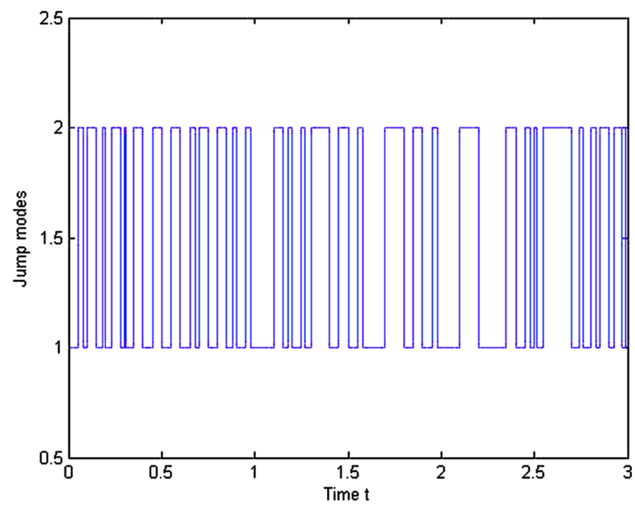

Figure 1 The jumping modes

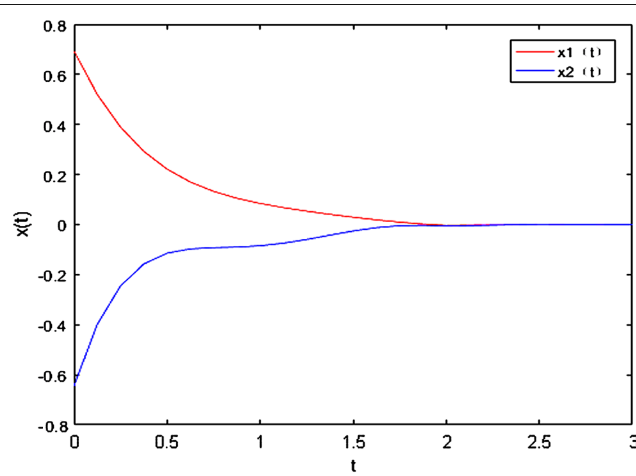

Figure 2 The trajectory of states

One possible realization of Markovian jumping mode is given in Fig. 1. The simulation results under this mode-dependent controller are shown in Fig. 2. We see that the state responses are satisfactory when the saturation appears. The corresponding state trajectory is shown in Fig. 3.

6 Conclusion

The robust finite-time control for descriptor Markov jump systems with impulsive effects, actuator saturation, and time-varying norm-bounded disturbance have been investigated. A sufficient condition for the finite-time stability of systems is given according to Lyapunov function theory and LIMs. Based on the conditions above, a state feedback controller is designed such that the resultant closed-loop system is finite-time stable. The simulation results are given by MATLAB. The results in this paper can be applied in communication engineering and other fields. It has important theoretical significance for further study of some problems of descriptor Markovian jump systems (also, see [1–26]).

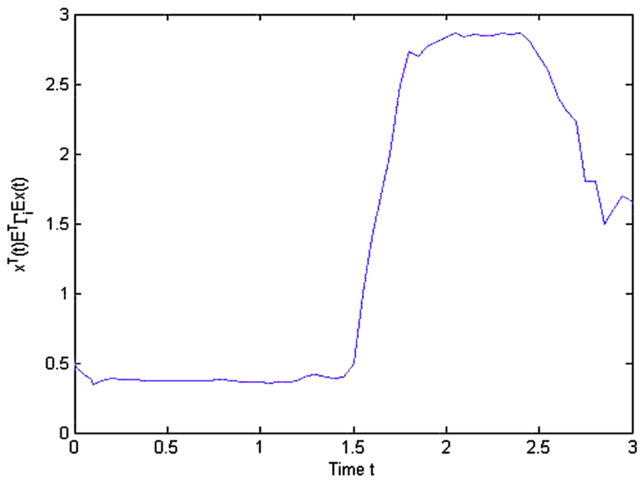

Figure 3 The trajectory of $x^T(t)E^T\Gamma_i Ex(t)$

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Authors' contributions
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