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# A Mathematical Formulation to Estimate the Effect of Grain Refiners on the Ultimate Tensile Strength of Al-Zn-Mg-Cu Alloys

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**Abstract:** In this study, the feed-forward (FF) neural networks (NNs) with back-propagation (BP) learning algorithm is used to estimate the ultimate tensile strength of unrefined Al-Zn-Mg-Cu alloys and refined the alloys by Al-5Ti-1B and Al-5Zr master alloys. The obtained mathematical formula is presented in great detail. The designed NN model shows good agreement with test results and can be used to predict the ultimate tensile strength of the alloys. Additionally, the effects of scandium (Sc) and carbon (C) rates are investigated by using the proposed equation. It was observed that the tensile properties of Al-Zn-Mg-Cu alloys improved with the addition of 0.5 Sc and 0.01 C wt.%.

**Keywords:** Al-Zn-Mg-Cu alloys; ultimate tensile strength; modeling; ANN

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## 1. Introduction

Al-Zn-Mg-Cu alloys, using ultra-high strength Al-based alloys, are widely used in aviation, automotive, and aerospace industries due to of their low density, high specific strength, toughness, and fatigue durability [1–7]. The mechanical properties of the alloys can be improved by some important

factors such as: minimizing inclusions, applying thermo-mechanical treatments and changing the composition [8]. One of the most important methods to increase the strength is the refining of the grain size, which is described by the well-known Hall-Petch Equation [9]:

$$\sigma_y = \sigma_0 + kd^{-1/2} \quad (1)$$

where  $\sigma_y$  is the yield stress,  $\sigma_0$  a friction stress,  $d$  the grain size and  $k$  a constant.

The existing knowledge of grain refinement can be divided into three basic methods: mechanical, chemical, and thermal, whereas the addition of master alloys (chemical) is the most economical one. In the chemical method, the addition of master alloys promotes nucleation and hinders growth [10–14]. Grain refiners used in aluminum (Al) and its alloys are Al-Ti, Al-B, Al-Ti-B, Al-Ti-C, Al-Zr, Al-Sc, and Al-Sr [15–20]. A fine equiaxed grain structure leads to better mechanical properties. Moreover, grain refinement can also bring other advantages, such as enhanced machinability, good surface finish, resistance to hot tearing, high toughness and high yield strength [21,22]. The type and addition level of these master alloys also affect the mechanical properties of alloys. The relation among grain size, changing operation parameters and strength are quite complex and non-linear. Therefore, it is very important to select the correct master alloy to obtain the maximum strength. As an experiment, this phenomenon is costly and time consuming. Therefore, an attempt has been made to model the complex grain refinement phenomena in Al-Zn-Mg-Cu alloys.

Successful modelings of artificial neural networks (ANN) have been reported in materials science [23]. Reddy *et al.* [24] used the feed forward neural network (FFNN) with back-propagation (BP) learning algorithm to predict the grain size as a function of titanium (Ti) and boron (B) addition levels, and holding time during grain refinement of Al–7Si alloy. The authors stated that the grain size of Al–7Si alloy, with good learning precision and generalization, can be predicted with the FFNN model. Rashidi *et al.* [25] modeled the effect of the process parameters, namely current density, saccharin concentration, and bath temperature on the grain size of nano-crystalline nickel coating by using a feed-forwarded multilayer perceptron ANN framework. They reported that there is a remarkable agreement between the model prediction and the experimental observation. Tofigh *et al.* [26] used the FFNN in prediction of hardness, tensile and compressive yield stress, ultimate tensile strength, and elongation percentage of Al alloy reinforced alumina nano-particles. The authors notified that the prediction ability of the model is in good agreement with experimental data.

The aim of this work is to obtain an explicit formulation of the ultimate tensile strength as a function of complex master alloys and, theoretically, to investigate the effect of scandium (Sc) and carbon (C) rates on the ultimate tensile strength of the alloys by using the obtained equation.

## 2. Background of Neural Networks

ANNs are computationally in the context of artificial intelligence, which is imitating the neural behavior of human beings. ANNs can be called a computer model of the human brain's neural structure. The basic elements of the ANNs are “neurons”, which are the processing elements of ANNs. The “network” is defined as the structure in which the neurons act simultaneously in a group. The network involves an input layer, hidden layer, and the output layer [27]. In an ANN, neurons are tightly interconnected in various layers, where the adaptive weights are, conceptually, connection strengths

between neurons. The three main characteristics of a NN are weights ( $w_i$ ), bias ( $b_i$ ), and transfer function. Each neuron receives inputs, attached with a weight. Each input data is multiplied by the corresponding weight of the neuron connection. Next, a bias value is added to the summation of inputs and corresponding weights ( $u$ ) according to the following equation:

$$u_i = \sum_{j=1}^H w_{ij}x_j + b_i \quad (2)$$

The summation  $u_i$  is converted as the output with an activation (transfer) function,  $F(u_i)$  yielding a value called the unit's "activation", as the following formula:

$$O = f(u_i) \quad (3)$$

The activation or transfer function that converts a neuron's weighted input to its output activation play the substantial role in the overall performance of an ANN's implementation [28].

### 3. Neural Network Studies

An extensive literature survey has been performed for available experimental results, as shown Table 1. The addition of grain refiners promotes the formation of fine equiaxed grains by suppressing the growth of columnar and twin columnar grain. The finer grain size also decreases the size of defects, such as micro-pores and second phase particles, thereby contributing to improved mechanical properties [29]. It is clear from Table 1 that the grain refiners and the applied heat treatment result in an increase in the ultimate tensile strength of the alloys. Ebrahimi and Emamy [30] reported that Al-5Ti-1B and Al-5Zr master alloy decreases the grain size of Al-12Zn-3Mg-2.5Cu aluminum alloy, and Al-5Ti-1B master alloy is more effective than Al-5Zr master alloy in reducing the grain size of Al-12Zn-3Mg-2.5Cu aluminum alloy. They expressed that the mechanical properties of the alloy are improved with the addition of grain refiners and heat treatment, and that this improvement can be attributed the increase to nano-metric precipitates, the homogeneity of the microstructure, and the decrease in grain size.

Table 2 shows the change of the input elements, from minimum to maximum, in training and testing sets of NN. The input variables are silicon (Si), magnesium (Mg), zinc (Zn), copper (Cu), nickel (Ni), iron (Fe), manganese (Mn), chromium (Cr), Ti, zirconium (Zr), Sc, B, C, Al (in wt.%), and heat treatment (HT). The output variable is the ultimate tensile strength (UTS) in MPa. In the program, the alloys with and without heat treatment is coded as "1" and "0", respectively. In ANN system, each input variable is scaled to the range of 0 to 1 by the following the formula:

$$x_N = \frac{x - x_{min.}}{x_{max.} - x_{min.}} \quad (4)$$

where  $x_N$  is the normalized value of variable  $x$ , and  $x_{max}$  and  $x_{min}$  are the maximum and minimum values of the variable, respectively.

**Table 1.** Data set for training and testing.

| Ref.                       | Master Alloys |       | HT  | AGS<br>( $\mu\text{m}$ ) | UTS<br>(MPa) |
|----------------------------|---------------|-------|-----|--------------------------|--------------|
|                            | Type          | wt. % |     |                          |              |
| Ebrahimi and Emamy [30]    | -             | -     | No  | 310                      | 328          |
|                            | -             | -     | Yes | 310                      | 504          |
|                            | Al-5Ti-1B     | 0.05  | No  | 38                       | 352          |
|                            | Al-5Ti-1B     | 0.05  | Yes | 38                       | 621          |
|                            | Al-5Zr        | 0.3   | No  | 150                      | 328          |
|                            | Al-5Zr        | 0.3   | Yes | 150                      | 530          |
| Fakhraei and Emamy [31]    | -             | -     | No  | 305                      | 168          |
|                            | Al-8B         | 0.1   | No  | 155                      | 190          |
|                            | Al-8B         | 0.3   | No  | 130                      | 222          |
|                            | Al-8B         | 0.5   | No  | 113                      | 236          |
|                            | Al-8B         | 1     | No  | 111                      | 197          |
|                            | Al-15Zr       | 0.1   | No  | 215                      | 193          |
|                            | Al-15Zr       | 0.3   | No  | 168                      | 226          |
|                            | Al-15Zr       | 0.5   | No  | 150                      | 243          |
|                            | Al-15Zr       | 1     | No  | 152                      | 202          |
| Fakhraei and Emamy [32]    | -             | -     | No  | 305                      | 168          |
|                            | Al-5Ti-1B     | 0.05  | No  | 178                      | 178          |
|                            | Al-5Ti-1B     | 0.1   | No  | 150                      | 183          |
|                            | Al-5Ti-1B     | 0.3   | No  | 116                      | 200          |
|                            | Al-5Ti-1B     | 0.5   | No  | 100                      | 253          |
|                            | Al-5Ti-1B     | 1     | No  | 88                       | 242          |
| Wang <i>et al.</i> [33]    | -             | -     | Yes | 320                      | 170          |
|                            | Al-5Ti-1B     | 0.2   | Yes | 200                      | 220          |
|                            | Al-5Ti-1B     | 0.5   | Yes | 180                      | 229          |
|                            | Al-5Ti-1B     | 1     | Yes | 100                      | 249          |
|                            | Al-5Ti-1B     | 1.5   | Yes | 80                       | 250          |
|                            | Al-5Ti-1B     | 2     | Yes | 75                       | 241          |
|                            | Al-5Ti-1B     | 3     | Yes | 40                       | 230          |
|                            | Al-5Ti-1B     | 5     | Yes | 70                       | 215          |
|                            | Al-5Ti-0.25C  | 0.2   | Yes | 200                      | 230          |
|                            | Al-5Ti-0.25C  | 0.5   | Yes | 150                      | 260          |
|                            | Al-5Ti-0.25C  | 1     | Yes | 80                       | 267          |
|                            | Al-5Ti-0.25C  | 1.5   | Yes | 65                       | 271          |
|                            | Al-5Ti-0.25C  | 2     | Yes | 40                       | 249          |
|                            | Al-5Ti-0.25C  | 3     | Yes | 50                       | 240          |
|                            | Al-5Ti-0.25C  | 5     | Yes | 80                       | 215          |
| Shabani <i>et al.</i> [34] | -             | -     | Yes | 560                      | 260          |
|                            | Al-5Ti-1B     | 0.05  | Yes | 300                      | 283          |
|                            | Al-5Zr        | 0.3   | Yes | 345                      | 276          |

Table 1. Cont.

| Ref.                      | Master Alloys           |                | HT  | AGS<br>( $\mu\text{m}$ ) | UTS<br>(MPa) |
|---------------------------|-------------------------|----------------|-----|--------------------------|--------------|
|                           | Type                    | wt.%           |     |                          |              |
| Ravi <i>et al.</i> [29]   | -                       | -              | No  | 630                      | 168          |
|                           | -                       | -              | No  | 630                      | 177          |
|                           | Al-5Ti-1B               | 2              | No  | 425                      | 189          |
|                           | Al-1Ti-3B               | 2              | No  | 215                      | 201          |
|                           | Al-1Ti-3B               | 0.5            | No  | 180                      | 227          |
| Kamali <i>et al.</i> [35] | -                       | -              | No  | 190                      | 174          |
|                           | Al-5Ti-1B               | 0.05           | No  | 48                       | 217          |
|                           | Al-5Ti-1B               | 0.1            | No  | 39                       | 225          |
|                           | Al-5Ti-1B               | 0.3            | No  | 41                       | 237          |
|                           | Al-5Ti-1B               | 0.5            | No  | 42                       | 245          |
|                           | Al-5Ti-1B               | 0.05           | Yes | 48                       | 318          |
|                           | Al-5Ti-1B               | 0.1            | Yes | 39                       | 330          |
|                           | Al-5Ti-1B               | 0.3            | Yes | 41                       | 350          |
|                           | Al-5Ti-1B               | 0.5            | Yes | 42                       | 354          |
| Wang <i>et al.</i> [36]   | -                       | -              | No  | 165                      | 167          |
|                           | Al-5Ti-1B               | 0.01           | No  | 87                       | 161          |
|                           | Al-5Ti-1B               | 0.02           | No  | 47                       | 183          |
|                           | Al-5Ti-1B               | 0.05           | No  | 40                       | 181          |
|                           | Al-5Ti-1B               | 0.1            | No  | 52                       | 166          |
| Liu <i>et al.</i> [37]    | Al-0.04Ti               | 0.036          | No  | 110                      | 270          |
|                           | Al-3Sc                  | 0.2            | No  | 172                      | 372          |
|                           | Al-3Zr+Al-3Sc           | 0.15+0.2       | No  | 100                      | 393          |
|                           | Al-0.04Ti+Al-3Zr+Al-3Sc | 0.036+0.15+0.2 | No  | 61                       | 395          |
| He <i>et al.</i> [38]     | -                       | -              | Yes | 370                      | 260          |
|                           | Al-3.8Zr                | 0.1            | Yes | 196                      | 272          |
|                           | Al-3.6Sc                | 0.2            | Yes | 370                      | 296          |
|                           | Al-3.6Sc                | 0.6            | Yes | 72                       | 360          |
|                           | Al-3.8Zr+Al-3.6Sc       | 0.1+0.2        | Yes | 42                       | 398          |

- Any grain refiner was not added in the alloys.

Table 2. Input elements (wt.%).

| Elements | Si   | Mg   | Zn    | Cu   | Ni   | Fe   | Mn  | Cr  | Ti | Zr | Sc  | B | C   | Al   | HT      |
|----------|------|------|-------|------|------|------|-----|-----|----|----|-----|---|-----|------|---------|
| Min.     | 0.03 | 0.02 | 0     | 0.03 | 0    | 0.15 | 0   | 0   | 0  | 0  | 0   | 0 | 0   | Rem. | No = 0  |
| Max.     | 13.2 | 20.4 | 12.24 | 4.47 | 0.95 | 0.45 | 0.2 | 0.1 | 1  | 1  | 0.6 | 1 | 0.2 |      | Yes = 1 |

Output values resulted from ANN are also in the range [0,1], and transformed to their equivalent values, based on a reverse method of the normalization technique [39]. The unnormalized method is as:

$$x = x_N (x_{max} - x_{min}) + x_{min} \quad (5)$$

The two main processing phases of NN include training and testing. The training process is the adjustment of weights and biases in order to obtain an output through applying a proper method. Therefore, the experimental results are divided into training and testing sets. The datasets for training and testing are randomly selected from among the experimental results where 61 sets are training set and

6 sets are testing set. It is well known that increasing the data used in the training process of NN enhances its learning ability. Matlab NN toolbox is employed for the network training. Back-propagation (BP) learning algorithm (Levenberg–Marquardt–Trainlm, Boston, MA, USA), the most popular, and an effective, supervised learning method, and sigmoid function, an act activation function that joins curvilinear, linear, and constant behavior, depending on the values of the input, are used for the training of NN [40,41]. Training of the networks starts with adjusting initial random values for weights and biases. After submitting the input vector, forward propagation of the intermediate results leads to the producing of the output vector. Then, the weights and biases would be modified in order to reduce the error. The network replies to an input, without any change in the structure, within the testing process [42,43]. After the network is trained, the testing dataset is used to verify the effectiveness of the network and to estimate the expected performance in the future.

#### 4. Results and Discussion

The main aim is to obtain the explicit formulation of the ultimate tensile strength as a function of input variables. One of the most difficult duties in NN works is the determination of the number of hidden layers and the number of neurons in the hidden layers. It is well known that NNs are typically in layers, which are made up of a number of interconnected nodes that contain an activation function. The input layer, which communicates to one or more hidden layers, where the actual processing is made via a system of weighted connections. The hidden layers then link to an output layer, where the answer is the output [44]. There is no well-defined procedure to find the optimal parameter settings and the network architecture. The trial and error approach is used to determine the number of neurons in the hidden layer. The various neuron numbers in one hidden layer (5–20) are used in this study. It is observed that the optimal architecture of NN with logistic sigmoid transfer function is 15–17–1. The NN toolbox in Matlab is used to obtain the proposed equation. It should be noted that the developed explicit formulation is valid for the ranges of the training set. Additionally, the correlation coefficient of the proposed equation is also evaluated, and is found to be 0.8509. In other words, the average prediction accuracy of the established ANN model is 85.09%.

$$T = 460 \times \dot{T} + 161 \quad (6)$$

where T is ultimate tensile strength and where:

$$\dot{T} = \left( \frac{1}{1 + e^{-w}} \right)$$

where:

$$\begin{aligned} w = & (0.94889) \times \left( \frac{1}{1 + e^{-u_1}} \right) + (2.13402) \times \left( \frac{1}{1 + e^{-u_2}} \right) + (0.45077) \times \left( \frac{1}{1 + e^{-u_3}} \right) + (-3.32533) \\ & \times \left( \frac{1}{1 + e^{-u_4}} \right) + (2.39273) \times \left( \frac{1}{1 + e^{-u_5}} \right) + (-2.47205) \times \left( \frac{1}{1 + e^{-u_6}} \right) + (0.17732) \\ & \times \left( \frac{1}{1 + e^{-u_7}} \right) + (1.59606) \times \left( \frac{1}{1 + e^{-u_8}} \right) + (1.95837) \times \left( \frac{1}{1 + e^{-u_9}} \right) + (-3.33818) \\ & \times \left( \frac{1}{1 + e^{-u_{10}}} \right) + (-0.36762) \times \left( \frac{1}{1 + e^{-u_{11}}} \right) + (-2.96723) \times \left( \frac{1}{1 + e^{-u_{12}}} \right) + (-2.99594) \\ & \times \left( \frac{1}{1 + e^{-u_{13}}} \right) + (-1.59159) \times \left( \frac{1}{1 + e^{-u_{14}}} \right) + (-5.78338) \times \left( \frac{1}{1 + e^{-u_{15}}} \right) + (-0.19006) \\ & \times \left( \frac{1}{1 + e^{-u_{16}}} \right) + (1.50506) \times \left( \frac{1}{1 + e^{-u_{17}}} \right) + (0.15734) \end{aligned}$$

and

$$\begin{aligned}
 u1 &= (-0.49993 \times X1) + (-0.00354 \times X2) + (-0.61144 \times X3) + (0.49105 \times X4) \\
 &\quad + (0.72856 \times X5) + (-0.06895 \times X6) + (-0.03897 \times X7) + (-0.66021 \times X8) \\
 &\quad + (-0.92799 \times X9) + (0.13060 \times X10) + (0.13452 \times X11) + (0.54220 \times X12) \\
 &\quad + (0.40258 \times X13) + (0.13213 \times X14) + (0.36274 \times X15) + (-0.23276) \\
 u2 &= (-0.35276 \times X1) + (0.18702 \times X2) + (-0.24048 \times X3) + (1.03259 \times X4) + (0.64191 \times X5) \\
 &\quad + (-0.32554 \times X6) + (-2.70935 \times X7) + (0.14497 \times X8) + (-1.54846 \times X9) \\
 &\quad + (1.09702 \times X10) + (-2.60705 \times X11) + (-2.10947 \times X12) \\
 &\quad + (-0.02828 \times X13) + (-0.29674 \times X14) + (0.09758 \times X15) + (-0.26159) \\
 u3 &= (1.69292 \times X1) + (-0.15616 \times X2) + (0.17488 \times X3) + (-2.36728 \times X4) \\
 &\quad + (-0.61096 \times X5) + (-2.04671 \times X6) + (0.02049 \times X7) + (0.59184 \times X8) \\
 &\quad + (0.43818 \times X9) + (-0.31379 \times X10) + (1.14791 \times X11) + (1.12623 \times X12) \\
 &\quad + (-0.77400 \times X13) + (0.25359 \times X14) + (-1.75379 \times X15) + (-0.15834) \\
 u4 &= (-1.19858 \times X1) + (-0.65841 \times X2) + (1.26693 \times X3) + (-0.12854 \times X4) \\
 &\quad + (-0.64606 \times X5) + (0.19024 \times X6) + (1.29375 \times X7) + (-0.54070 \times X8) \\
 &\quad + (-0.85351 \times X9) + (-3.19349 \times X10) + (-1.19332 \times X11) \\
 &\quad + (-0.36499 \times X12) + (-2.22971 \times X13) + (0.79672 \times X14) \\
 &\quad + (-0.28841 \times X15) + (-0.05969) \\
 u5 &= (0.31740 \times X1) + (0.23849 \times X2) + (-0.16464 \times X3) + (0.72392 \times X4) + (0.08065 \times X5) \\
 &\quad + (-0.22394 \times X6) + (-0.45745 \times X7) + (-0.59003 \times X8) + (-0.53258 \times X9) \\
 &\quad + (-0.09952 \times X10) + (-0.59419 \times X11) + (-0.29143 \times X12) \\
 &\quad + (-0.45241 \times X13) + (0.52024 \times X14) + (0.60874 \times X15) + (0.58822) \\
 u6 &= (-1.65634 \times X1) + (-0.28557 \times X2) + (0.74052 \times X3) + (-2.75057 \times X4) \\
 &\quad + (-0.18806 \times X5) + (1.22060 \times X6) + (3.32170 \times X7) + (0.08922 \times X8) \\
 &\quad + (0.75341 \times X9) + (0.77162 \times X10) + (0.61279 \times X11) + (0.99672 \times X12) \\
 &\quad + (1.04996 \times X13) + (-1.24403 \times X14) + (-0.36127 \times X15) + (-1.26639) \\
 u7 &= (0.18672 \times X1) + (-0.21317 \times X2) + (-0.06951 \times X3) + (0.23293 \times X4) + (0.18344 \times X5) \\
 &\quad + (0.86836 \times X6) + (0.15483 \times X7) + (-0.49808 \times X8) + (-0.39467 \times X9) \\
 &\quad + (0.70716 \times X10) + (0.52081 \times X11) + (-0.15581 \times X12) + (0.04996 \times X13) \\
 &\quad + (-0.47618 \times X14) + (0.49369 \times X15) + (0.13833) \\
 u8 &= (0.20311 \times X1) + (-0.18870 \times X2) + (0.71925 \times X3) + (0.59576 \times X4) + (0.04333 \times X5) \\
 &\quad + (-1.38544 \times X6) + (0.22941 \times X7) + (-0.92876 \times X8) + (-0.49392 \times X9) \\
 &\quad + (-0.35084 \times X10) + (-0.85939 \times X11) + (0.69356 \times X12) + (0.92923 \times X13) \\
 &\quad + (0.76245 \times X14) + (0.80903 \times X15) + (0.78233) \\
 u9 &= (0.45508 \times X1) + (0.45380 \times X2) + (-0.53263 \times X3) + (0.62014 \times X4) + (0.48778 \times X5) \\
 &\quad + (-0.42787 \times X6) + (-1.03911 \times X7) + (-0.16981 \times X8) + (0.13517 \times X9) \\
 &\quad + (2.24826 \times X10) + (-0.05111 \times X11) + (-1.26004 \times X12) + (0.30575 \times X13) \\
 &\quad + (1.73001 \times X14) + (-0.22815 \times X15) + (0.74025)
 \end{aligned}$$

$$\begin{aligned}
 u_{10} = & (-3.04675 \times X_1) + (-1.47749 \times X_2) + (0.52853 \times X_3) + (1.76084 \times X_4) \\
 & + (-1.14597 \times X_5) + (4.48201 \times X_6) + (0.88897 \times X_7) + (-2.78018 \times X_8) \\
 & + (-0.76322 \times X_9) + (-1.89132 \times X_{10}) + (-0.37789 \times X_{11}) \\
 & + (-7.27827 \times X_{12}) + (-0.16189 \times X_{13}) + (1.69861 \times X_{14}) \\
 & + (-3.22967 \times X_{15}) + (-1.60096)
 \end{aligned}$$

$$\begin{aligned}
 u_{11} = & (0.38619 \times X_1) + (0.00559 \times X_2) + (-0.13668 \times X_3) + (-0.13627 \times X_4) \\
 & + (-0.00132 \times X_5) + (-0.42122 \times X_6) + (0.12256 \times X_7) + (0.12089 \times X_8) \\
 & + (0.30673 \times X_9) + (-0.19105 \times X_{10}) + (0.09498 \times X_{11}) + (0.33387 \times X_{12}) \\
 & + (0.13216 \times X_{13}) + (-0.34123 \times X_{14}) + (-0.09914 \times X_{15}) + (-0.47796)
 \end{aligned}$$

$$\begin{aligned}
 u_{12} = & (-0.88070 \times X_1) + (-1.26162 \times X_2) + (1.00008 \times X_3) + (0.44580 \times X_4) \\
 & + (-0.69643 \times X_5) + (0.54684 \times X_6) + (1.47863 \times X_7) + (0.26171 \times X_8) \\
 & + (-0.07183 \times X_9) + (-2.17460 \times X_{10}) + (1.83544 \times X_{11}) + (-1.22216 \times X_{12}) \\
 & + (0.19267 \times X_{13}) + (-1.94418 \times X_{14}) + (0.16993 \times X_{15}) + (-0.74403)
 \end{aligned}$$

$$\begin{aligned}
 u_{13} = & (-1.16712 \times X_1) + (-1.05184 \times X_2) + (0.73395 \times X_3) + (0.04286 \times X_4) + (-0.39426 \times X_5) \\
 & + (2.84218 \times X_6) + (1.99864 \times X_7) + (-0.22580 \times X_8) + (1.15608 \times X_9) \\
 & + (-0.03548 \times X_{10}) + (-1.92515 \times X_{11}) + (-4.03396 \times X_{12}) + (0.38855 \times X_{13}) \\
 & + (-0.37459 \times X_{14}) + (1.85893 \times X_{15}) + (-0.99138)
 \end{aligned}$$

$$\begin{aligned}
 u_{14} = & (-0.04539 \times X_1) + (-0.07940 \times X_2) + (0.32185 \times X_3) + (-0.79070 \times X_4) \\
 & + (-0.20130 \times X_5) + (1.03768 \times X_6) + (1.17849 \times X_7) + (0.17644 \times X_8) \\
 & + (1.38321 \times X_9) + (-1.21757 \times X_{10}) + (-1.73489 \times X_{11}) + (-2.77648 \times X_{12}) \\
 & + (-0.46636 \times X_{13}) + (0.18136 \times X_{14}) + (0.82488 \times X_{15}) + (-0.16546)
 \end{aligned}$$

$$\begin{aligned}
 u_{15} = & (-0.97802 \times X_1) + (-1.35732 \times X_2) + (0.46732 \times X_3) + (-1.94090 \times X_4) \\
 & + (0.45079 \times X_5) + (0.86831 \times X_6) + (0.96056 \times X_7) + (0.86766 \times X_8) \\
 & + (-0.27002 \times X_9) + (2.17786 \times X_{10}) + (-3.34955 \times X_{11}) + (1.57693 \times X_{12}) \\
 & + (-0.51299 \times X_{13}) + (1.69850 \times X_{14}) + (-0.16939 \times X_{15}) + (-1.56104)
 \end{aligned}$$

$$\begin{aligned}
 u_{16} = & (-0.37956 \times X_1) + (-0.13332 \times X_2) + (0.05849 \times X_3) + (-0.34892 \times X_4) \\
 & + (0.22692 \times X_5) + (-0.26460 \times X_6) + (-0.97223 \times X_7) + (0.30502 \times X_8) \\
 & + (0.22059 \times X_9) + (-0.10065 \times X_{10}) + (-0.65479 \times X_{11}) + (0.50434 \times X_{12}) \\
 & + (-0.54470 \times X_{13}) + (-0.16661 \times X_{14}) + (-0.27592 \times X_{15}) + (-0.04856)
 \end{aligned}$$

$$\begin{aligned}
 u_{17} = & (0.18078 \times X_1) + (0.16781 \times X_2) + (-0.13242 \times X_3) + (1.59311 \times X_4) + (0.28523 \times X_5) \\
 & + (-0.66845 \times X_6) + (-0.07740 \times X_7) + (0.03953 \times X_8) + (-0.20656 \times X_9) \\
 & + (-0.12734 \times X_{10}) + (-0.86300 \times X_{11}) + (0.42924 \times X_{12}) + (0.24858 \times X_{13}) \\
 & + (-0.62146 \times X_{14}) + (0.95215 \times X_{15}) + (0.52058)
 \end{aligned}$$

where  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$ ,  $X_5$ ,  $X_6$ ,  $X_7$ ,  $X_8$ ,  $X_9$ ,  $X_{10}$ ,  $X_{11}$ ,  $X_{12}$ ,  $X_{13}$ ,  $X_{14}$ , and  $X_{15}$  are normalized input data of Al, Si, Mg, Zn, Cu, Ni, Fe, Mn, Cr, Ti, Zr, Sc, B, C wt.%, and HT.

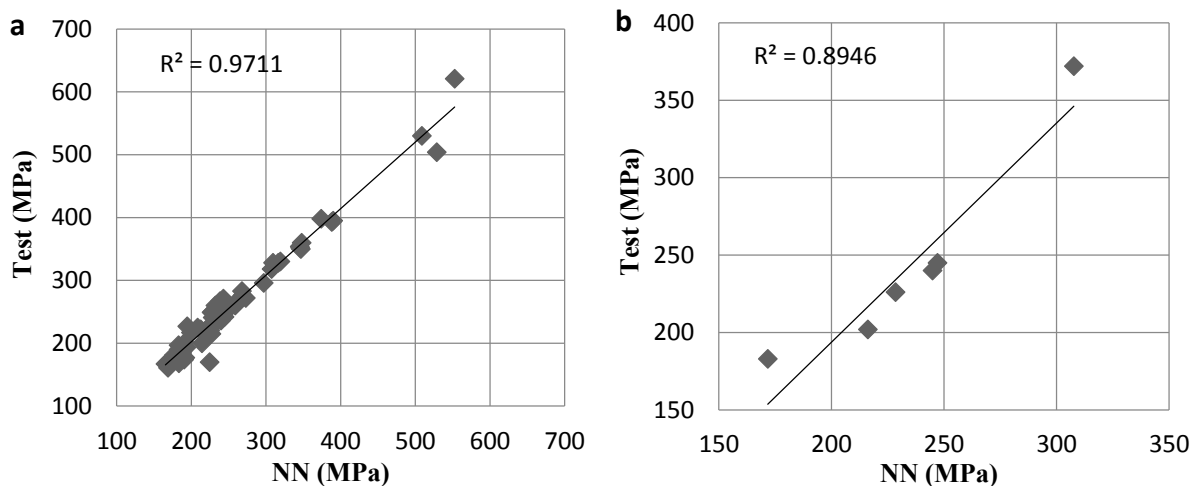
Table 3 shows statistical parameters of training and test data sets. The performance of NN is evaluated by the correlation coefficient (R). Mean absolute error (MAE) and mean square error (MSE) are also used as error evaluation criteria in order to facilitate the comparison between predicted values and desired values.

**Table 3.** Statistical parameters.

| Datasets     | R     | MSE   | MAE   |
|--------------|-------|-------|-------|
| Training set | 0.985 | 0.141 | 2.606 |
| Testing set  | 0.946 | 0.528 | 6.313 |

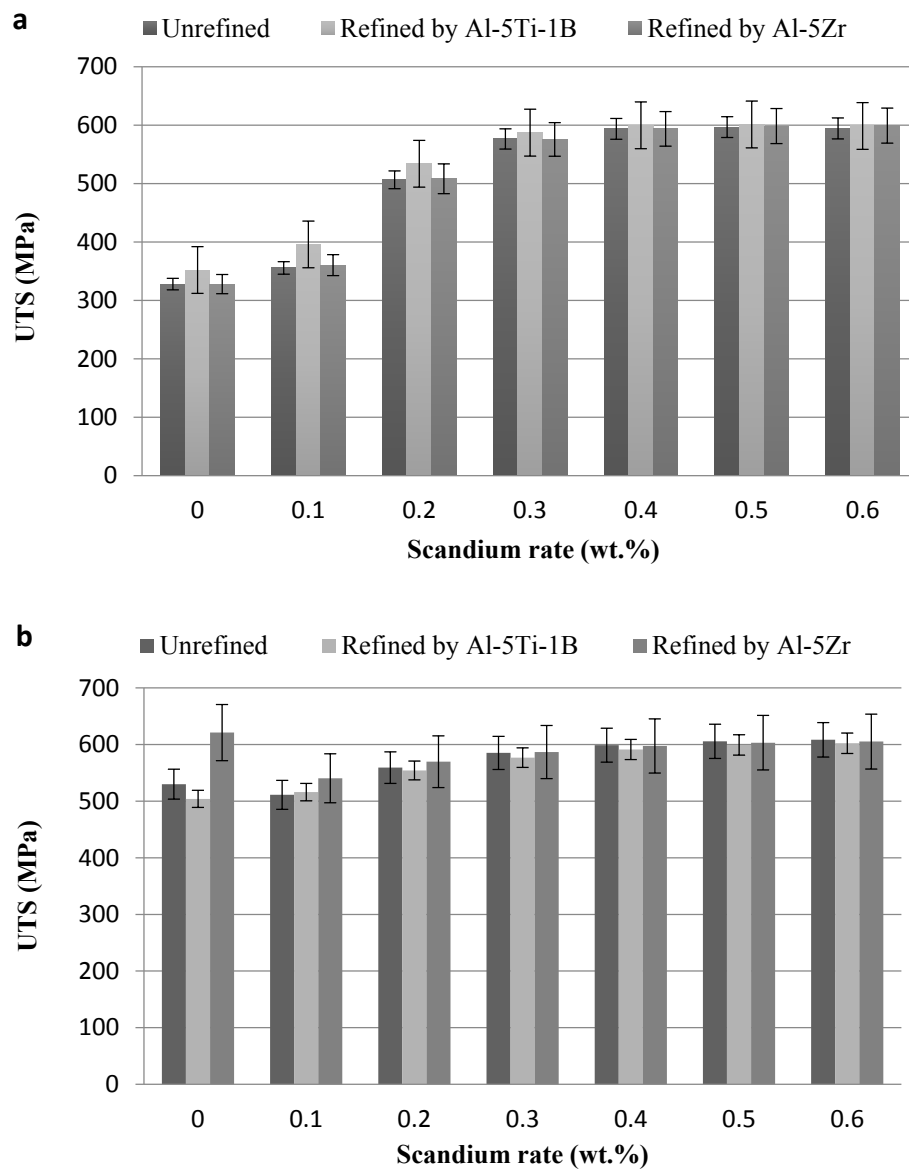
The correlation coefficients of training and testing sets are 0.985 and 0.946, respectively, which means that the performance of the NN model is quite high and acceptable. MAE and MSE values for the training set are 2.606 and 0.141, and are 6.313 and 0.528 for testing set. If the MSE reaches zero, the performance of the model is regarded as excellent [45]. It can be said that the proposed NN model is in good agreement with the experimental data, and that all the errors are within acceptable ranges.

Figure 1 illustrates the correlation of NN and test data for the training and testing sets. The “test” means the determined experimental value of UTS and “NN” means the predicted value of UTS by NN, as shown in Figure 1.  $R^2$  values of training and testing are 0.9711 and 0.8946, respectively.  $R^2$  value compares the accuracy of the model to the accuracy of a trivial assessment model. A high  $R^2$  (1) value tells that all points lie exactly on the curve with no scatter, and the results are perfect. It is clear that all the values are higher than 0.89. It can be said that the proposed NN model can predict the ultimate tensile strength of Al-Zn-Mg-Cu alloys with high accuracy and reliability. A few minor deviations between the experimental and theoretical results are observed in the training and testing sets of NN, which can be attributed to the variation in experimental conditions.

**Figure 1.** Correlation of NN and Test (a) Training set (b) Testing set.

One of the most effective elements promoting grain refinement in aluminum alloys is scandium. Aluminum and  $\text{Al}_3\text{Sc}$  have the same lattice structure and the misfit between lattice parameters is about 1.3%. The grain refinement mechanism of primary aluminum with scandium is due to heterogeneous nucleation, and the addition of scandium has been shown to have a beneficial effect on the strength of a cast Al-Mg alloy [46,47]. Figure 2 demonstrates the effect of Sc rates on the ultimate tensile strength of unrefined Al-Zn-Mg-Cu alloys and refined the alloys by Al-5Ti-1B and Al-5Zr using the proposed formulation above (Equation 6). In the case where there is no applied heat treatment, the ultimate tensile strength of Al-Zn-Mg-Cu alloys clearly increases with the addition of Sc. The main reason for this

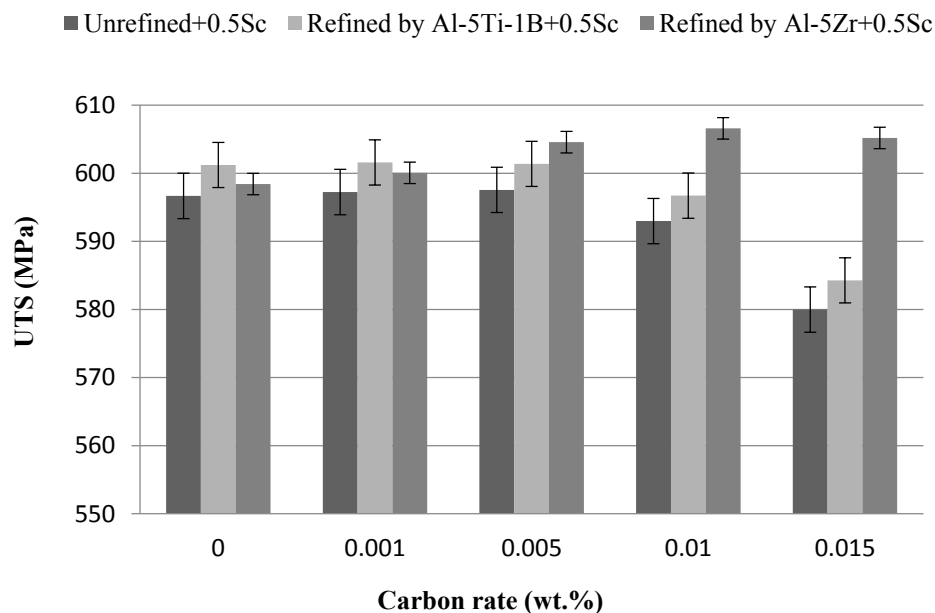
improvement can be attributed to the reduced grain size and the precipitation of more primary complex particles from the melt [48].



**Figure 2.** Effect of Sc rate on UTS of Al-Zn-Mg-Cu alloys: (a) non heat treatment (b) heat treatment.

It is clearly seen that, with the tensile properties of the alloys, first, the applied heat treatment reduces, then increases, the addition of Sc. The comparison of the UTS values revealed that the addition of 0.5% Sc is the most effective method to enhance tensile properties. It is concluded that the ultimate tensile strength of Al-Zn-Mg-Cu alloys, refined by Al-5Ti-1B or Al-5Zr, can be improved with the addition of 0.5% Sc without the need for heat treatment. Figure 3 shows the effect of the rate of C on the ultimate tensile strength of Al-Zn-Mg-Cu alloys containing 0.5 wt.% Sc. It is clear that the ultimate tensile strength of Al-Zn-Mg-Cu alloys, refined by Al-5Zr, enhances with the addition of C to a degree, but that the ultimate tensile strength of unrefined the alloys and refined the alloys by Al-5Ti-1B reduces with the addition of C. The optimum amount of is observed to be 0.01 wt.%C. It is well known that Al-Ti-C

master alloys, which contain  $\text{TiAl}_3$  and  $\text{TiC}$  particles, are good refiners for aluminum alloys. These particles in Al-Ti-C master alloys remain stable in the melt and act the active nucleant substrates. These particles very fine and act quickly, but rapidly dissolve in molten aluminum. The morphology, dimension, and distribution of these particles have major impacts on the grain refining efficiency. The homogeneous distribution of these particles increases the mechanical properties of aluminum alloys [49,50].



**Figure 3.** Effect of C rate to UTS of Al-Zn-Mg-Cu alloys.

## 5. Conclusions

This work proposes an approach for the prediction of ultimate tensile strength of Al-Zn-Mg-Cu alloys refined by Al-5Ti-1B and Al-5Zr. Sixty-seven data are used in this model. Feed-forward NN with back propagation is used for the training process, and the proposed NN model shows good agreement with experimental results. Therefore, the mathematical function is derived in an explicit form by using ANN outputs. The effects of Sc and C on the ultimate tensile strength of Al-Zn-Mg-Cu alloys are predicted with a high success rate. It is suggested that the strength of the alloys, refined by Al-5Ti-1B or Al-5Zr, can be increased with the addition of 0.5% Sc without heat treatment. All R and  $R^2$  values for training, testing, and formula are larger than 0.94 and 89, respectively. This notes that the composed prediction model has a high reliability rate. The mean absolute error for the predicted values does not exceed 6.4%. Hence, it can be concluded that considerable savings, in terms of cost and time, could be obtained by using the advanced neural network model.

## Author Contributions

All authors contributed equally to this work. All authors analyzed and interpreted the data, discussed the conclusions, and prepared the manuscript.

## Conflicts of Interest

The authors declare no conflicts of interest.

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