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ANALYZING PERFORMANCE OF SIC WITH MULTIPLE POWER
MULTICHANNEL ALOHA

BY

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THESIS

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ABSTRACT

This thesis analyzes the performance of successive interference cancellation (SIC) when used with multiple power, multichannel Aloha. We use the fluid model to analyze the performance of the system under various circumstances. We also simulate the performance of a SIC system over a single channel. SIC increases the throughput of a multiple power system up to around 100%. The results show that there is a maximum performance increase that can be achieved using predetermined transmission sequences. An infinite increase may be achieved using dynamic or random sequences.

To the most loving parents in the world, for their unconditional love.
To my wife who took care of me while I took care of school.
To my children who have never failed to bring a smile to my face even in
the most hectic of times.

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CHAPTER 1

INTRODUCTION

ALOHAnet was first developed in 1970 at the University of Hawaii. It is one of the simplest medium access control (MAC) protocols. While the protocol itself is currently not widely used, many of its features are still in use in more modern protocols such as 802.11 [1]. While it is rare to find applications that still use the original Aloha protocol, modified versions of the protocol are still in use today. They are used primarily in satellite communications and as a reservation mechanism in CDMA cellular networks.

In satellite networks Aloha is mainly used to transmit short packets or to reserve a channel so that larger packets can be transmitted. The large round trip time (RTT) for satellite links and small packet size pose a problem for the throughput for these types of packets. A user will not receive feedback regarding the success or failure of the transmission until at least one RTT has elapsed.

Many modifications have been proposed for improving the throughput of Aloha, two of which were proposed by Y. Birk [2, 3]. Baron and Birk [2] proposed using multiple copies to improve throughput. In this scheme an increasing number of copies of the packet are sent after each collision. This increases the throughput in high delay networks because it increases the chances of success and eliminates the need to wait a full RTT to find out whether the transmission was a success or a failure.

Birk and Revah [3] proposed using multiple power levels to increase the throughput of the system. In this scheme the system increases the power level after each successive failed transmission. A packet is received successfully if the signal-to-noise ratio (SNR) is above a threshold β . Both proposals achieved significant improvements, but a combination of multiple copies and

multiple power level achieved the best results.

In this thesis we analyze the improvement on the multiple power scheme: using successive interference cancellation (SIC) in a multiple power Aloha. We analyze the protocol using both a fixed point analysis and a simulation. We also compare the results of the fixed point analysis and the simulation and look at the sources of difference.

CHAPTER 2

BACKGROUND

Birk and Revah [3] proposed a multiple power scheme for multichannel Aloha to improve performance. The scheme assumes a packet can be received successfully even if multiple packets are transmitted simultaneously in the same slot. The only constraint is that the signal to interference plus noise ratio (SINR) must be larger than $\beta > 1$, where SINR is

$$SINR = \frac{\text{Signal Power}}{\text{Power of interfering Signals} + \text{Noise Power}} \quad (2.1)$$

The scheme works by increasing the priority of packets that have failed transmissions. This is done by increasing the power level of a packet with each successive failure. As the number of failures increases, so does the power level of the transmission. This improves the chances that the packet will be received correctly because an increase in the signal power results in an increase in the SINR, which increases the chance of $SINR > \beta$.

In this thesis we make two modifications to the assumptions in [3]. First, we allow $\beta < 1$. In practice this can be done by employing techniques such as code division multiple access (CDMA). This technique allows multiple users to share a channel's bandwidth simultaneously. CDMA works by assigning orthogonal codes to users. Each user multiplies its signal with its assigned code so that the result is a scalar multiple of the orthogonal code. When the receiver receives the signal, it can decode the desired signal by using the same code assigned to the user to filter the signal from the interference. This technique increases the required bandwidth but allows simultaneous transmissions to be received [4].

The second modification is the use of successive interference cancellation (SIC). SIC works by decoding the signal with the highest SINR. After the

signal is decoded it can be removed from the original received signal. The process can be repeated until either all signals are decoded or no other signal meets the SINR threshold ($SINR > \beta$). This increases the number of successfully received packets compared to the multiple power scheme at the expense of circuit complexity [5].

As an example, let us assume a system receives four packets of power 4 W, 2 W, 1 W and 1 W. Let us also assume the system can successfully decode packets if $SINR \geq \beta$, where $\beta = 1$. If the system is a multiple power level system, it can receive the 4 W packet because $SINR = \frac{4}{2+1+1} = 1 \geq \beta$, but it cannot receive the 2 W or 1 W packets because the SINRs are $\frac{1}{6}$ and $\frac{1}{7}$ respectively. If the system uses SIC then it will first decode the 4 W packet successfully, and then it will cancel the 4 W signal from the received signal. Therefore the system can receive the 2 W signal because $SINR = \frac{2}{1+1} = 1 \geq \beta$. After the system decodes the 2 W system it can cancel it from the received signal and then decode the 1 W signals because $SINR = \frac{1}{1} = 1 \geq \beta$. So the multiple power system can only decode the 4 W signal, whereas the SIC system can decode all four received signals.

Table 2.1 compares the result of receiving multiple signals using both the multiple power and SIC. The table assumes the protocol uses three power levels: high (H), medium (M) and low (L). The signals use power levels $H = 9$, $M = 3$ and $L = 1$. In the example we also use $\beta = 1$. So signals with SINR larger than or equal to one will be received successfully. The table shows all possible combinations that could result in at least one successful transmission.

It can be seen from Table 2.1 that in the case of SIC the higher power packets must be received successfully in order for the lower power packets to be received successfully. So a high power packet must be received successfully in order for a medium or low power packet to be received successfully.

Table 2.1: Possible successful transmissions for systems with SIC and systems without SIC

Transmitted Set	Received Set wo/SIC	Received Set w/SIC
L	L	L
M	M	M
H	H	H
L,L	L,L	L,L
M,L	M	M,L
H,L	H	H,L
M,M	M,M	M,M
M,H	H	M,H
H,H	H,H	H,H
M,L,L	M	M,L,L
H,L,L	H	H,L,L
H,M,L	H	H,M,L
H,M,M	H	H,M,M
M,L,L,L	M	M
H,L,L,L	H	H
H,M,L,L	H	H,M,L,L
H,M,M,L	H	H
M,L,L,L,L	M	M
H,L,L,L,L	H	H
H,L,L,L,L,L	H	H
H,L,L,L,L,L,L	H	H
H,L,L,L,L,L,L,L	H	H
H,L,L,L,L,L,L,L,L	H	H
H,M,L,L,L	H	H
H,M,L,L,L,L	H	M,H
H,M,L,L,L,L,L	H	H,M
H,M,L,L,L,L,L,L	H	H,M

CHAPTER 3

THE MODEL

This thesis uses two models with different assumptions to analyze the system performance. The models differ in the assumption of the number of users in the system.

3.1 Finite user model

The system is composed of a finite number of users, M , that communicate to the central hub (the satellite). Packets are sent to the hub over multiple channels. A user contends with other users to access the channels. The hub acknowledges successful reception via a separate channel so that the acknowledgments (ACKs) do not collide with incoming packets. If a user does not receive an ACK within a specific period of time, its transmission is considered unsuccessful and the packet is either retransmitted or dropped. The model assumes a finite number of users in the system waiting to transmit.

The system divides the time into slots. A packet can only be transmitted at the beginning of a slot. A slot has a length of one round trip time (RTT). The system uses frequency division multiplexing by choosing a random frequency from the available channels for each transmitted packet. This can also be approximated using time division multiplexing because the transmission time for the packets is small compared to the RTT. Therefore many packets can be transmitted in the same slot by dividing a single slot into many mini slots and each user packet is randomly transmitted in one of the mini slots.

As previously explained, a packet is received successfully if the signal to interference plus noise ratio (SINR) is greater than β . The SINR is calculated using Equation (2.1).

In our model we ignore the effect of the noise on the channel, but the noise could be modeled as a nonzero constant. β is the successful reception threshold for the SINR. If the SINR is below the threshold β , then the packet is not received successfully. β can be any positive nonzero value. If β is greater than one, then at most one packet can be received per slot. If β is less than one, then multiple packets can be received in a slot. This can be achieved using techniques such as CDMA.

SIC is used to increase the throughput of the system by removing the successfully received packets from the received signal, thus reducing the interference for the remaining packets. An explanation of how SIC works is given in Chapter 2 of the thesis.

The maximum number of transmission attempts per packet is n . Each attempt is at a power level equal to or higher than the previous attempt. If all n transmissions are unsuccessful, then the packet is dropped and leaves the system. We denote the probability of success of a transmission at each phase as $P_{SUC,X}$, where X is the phase of the transmission sequence. So the probability of a packet being dropped is

$$P_{Drop} = (1 - P_{SUC,1})(1 - P_{SUC,2})...(1 - P_{SUC,n}) \quad (3.1)$$

Note: The power levels are the levels received at the satellite.

Packet arrival for each user is modeled as a Bernulli random variable with probability of arrival Q_a . Arrivals occur whether or not the system already has a packet. So the total system arrival rate is MQ_a packets per slot.

A user can only process one packet at a time. Therefore, new arrivals are buffered until they are ready to be transmitted. The buffers are assumed to have infinite length.

We assume the network operates in a stable region. This assumption holds for small enough values of Q_a because packets are discarded after a certain number of trials. Furthermore, the offered load can be controlled by the hub

by reducing the number of transmitting users by communicating through the contention-free channel and requesting that users stop transmitting [6].

3.2 Infinite user model

In the infinite user model we assume an infinite number of users and a very large number of channels. This leads to the assumption of Poisson arrivals per channel per slot. It also implies independence of arrivals between channels and slots.

Because the number of users in the infinite model is infinite, each user in the system only receives one packet throughout its lifetime. This eliminates the need for a buffer at the users' end. Therefore any packet in the system will add to the congestion of the system.

CHAPTER 4

FIXED POINT ANALYSIS

In this chapter we look at the fixed point model for three different scenarios of SIC systems. The first will be the simple case of a three-power-level system, then we look at the general case of an n power-level system, and finally we look at the special case of n -power levels of very large power ratios.

4.1 The fluid model

In the fixed point analysis we divide time into fixed time slots. A user can only transmit at the beginning of a time slot. We assume there is an infinite number of users in the system, and that the system has an arrival rate λ . At the point of equilibrium, the rate of arrival of packets to each power level should be equal to the rate of exit of packets from each level. A packet can exit a level in one of two ways: via a successful transmission in which case it leaves the system, or via an unsuccessful transmission in which case it moves on to the next level. Figure 4.1 shows the state and transition map for the three-power-level fluid model.

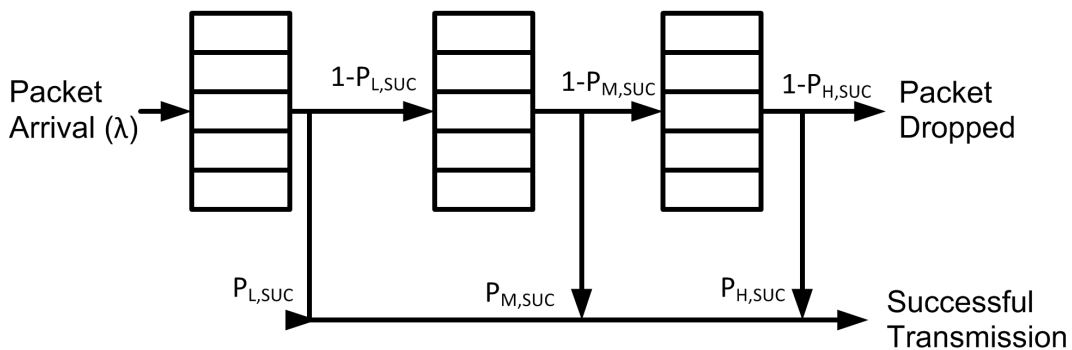


Figure 4.1: The states and transition map for the fluid model

We define g_k as the average rate of flow of packets into power level k . For example, g_3 is the rate of flow of packets between levels two and three. Also note that at equilibrium the stream of arrival for any power level g_i is assumed to be independent of the stream of arrival of all other power levels. We also define the $P_{SUC,k}$ as the average probability of success of a packet transmitted at power level k . The throughput of the system is the sum of the successful transmissions at all power levels.

When the system reaches equilibrium, the rate of packets flowing into any of the levels will be equal to the rate of packets flowing out of the level. Therefore we can find the equilibrium equations and solve for the rates and probabilities of success for each of the levels.

4.2 Simple scenario of three power levels

In this scenario the system has only three power levels: H, M and L. A packet is first transmitted at power level L. If it is unsuccessful, another attempt is made at power level M. If this attempt is unsuccessful, another attempt is made at power level H. If the last attempt is also unsuccessful, then the packet is dropped.

To find the equilibrium we need to find the equations for the rates going into each of the three power levels (g_L, g_M, g_H) and the probabilities of success of each power level ($P_{SUC,L}, P_{SUC,M}, P_{SUC,H}$). The first equation can be easily found because the rate going into the first power level is the same as the flow of packets into the system (λ).

$$g_L = \lambda \tag{4.1}$$

The flow of packets between the low level and medium level, g_m , is equal the flow of low packets multiplied by the failure rate of packets transmitted at the low power level:

$$g_M = g_L(1 - P_{SUC,L}) \tag{4.2}$$

Similarly the flow of packets into the higher power level, g_h , is equal the flow of low packets multiplied by the failure rate of packets transmitted at the

medium power level:

$$g_H = g_M(1 - P_{SUC,M}) \quad (4.3)$$

Next we need to find the equations for the probabilities of success for transmission at each power level, $P_{SUC,M}$, $P_{SUC,M}$, $P_{SUC,H}$. To find the probability of success we need to sum the probability of each possible scenario occurring that may result in a successful transmission. In this calculation we also use the assumption that the flows g_l, g_m and g_h are independent because they have infinite users and small transmission rates. We also use the indicator functions $I_{SUC,L}$, $I_{SUC,M}$ and $I_{SUC,H}$. An indicator function is one if the transmission is successful given the specified number of additional packets of power levels low, medium and high. The number of packets of power levels low, medium and high will be denoted by l, m and h sequentially. Another assumption used is that a packet in a system with Poisson arrival rate λ will expect to collide with λ other packets. Therefore we can find the following equations to model the system's success rate for every power level:

$$P_{SUC,L} = e^{-(g_L+g_M+g_H)} \sum_{l=1}^{\infty} \sum_{m=0}^{\infty} \sum_{h=0}^{\infty} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{L,SUC} \quad (4.4)$$

$$P_{SUC,M} = e^{-(g_L+g_M+g_H)} \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \sum_{h=0}^{\infty} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{M,SUC} \quad (4.5)$$

$$P_{SUC,H} = e^{-(g_L+g_M+g_H)} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{h=1}^{\infty} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{H,SUC} \quad (4.6)$$

The limits for the summations can be made finite because in each case there is a limit beyond which there can be no successful transmissions. For example, when calculating $P_{SUC,L}$ there can never be more than $\frac{1}{\beta}$ packets at any power level or else no packets transmitted at a low power level can be received successfully. But when we are calculating $P_{H,SUC}$, a total of $\frac{P_H}{P_L} \frac{1}{\beta}$ low power packets can be tolerated before all high power packets are not received successfully.

Using the same logic we can find the following summation limits for the

equations:

$$P_{SUC,L} = e^{-(g_L+g_M+g_H)} \sum_{l=1}^{\frac{1}{\beta}} \sum_{m=0}^{\frac{1}{\beta}} \sum_{h=0}^{\frac{1}{\beta}} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{L,SUC} \quad (4.7)$$

$$P_{SUC,M} = e^{-(g_L+g_M+g_H)} \sum_{l=0}^{\frac{P_M}{P_L} \frac{1}{\beta}} \sum_{m=1}^{\frac{1}{\beta}} \sum_{h=0}^{\frac{1}{\beta}} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{M,SUC} \quad (4.8)$$

$$P_{SUC,H} = e^{-(g_L+g_M+g_H)} \sum_{l=0}^{\frac{P_H}{P_L} \frac{1}{\beta}} \sum_{m=1}^{\frac{P_H}{P_M} \frac{1}{\beta}} \sum_{h=0}^{\frac{1}{\beta}} \frac{g_L^l}{l!} \frac{g_M^m}{m!} \frac{g_H^h}{h!} I_{H,SUC} \quad (4.9)$$

$$g_L = \lambda \quad (4.10)$$

$$g_M = g_L(1 - P_{SUC,L}) \quad (4.11)$$

$$g_H = g_M(1 - P_{SUC,M}) \quad (4.12)$$

Using the above equations, we can solve for an equilibrium point using a fixed point iteration.

4.3 Variable number of power levels (n power levels)

In this section we generalize the results we reached in the previous section to any number of power levels and any nondecreasing power level pattern. Here, the term *sequence* is used to describe the order of power levels for transmissions used after each successive failed transmission. For example the transmission pattern [1,1,2,2,3] will start by transmitting at power level 1. If the transmission is unsuccessful, the second transmission will be at power level 1. If it is unsuccessful again, it will retransmit at power level 2, then again at power level 2 and finally at power level 3. We will use the notation P_k to denote the power level at phase k of the transmission pattern.

The power levels themselves are defined in another vector. For example the power level vector [1, 3, 9] means that power level 1 transmits with power of 1 W, while power level 2 transmits with power of 3 W, and power level 3 transmits with power 9 W.

We will use the same notations we used previously for the rates and probabilities of success. The notation g_k denotes the rate of flow of packets into phase k of the transmission pattern, where k is an integer representing the phase of the transmission pattern. The notation for the probability of success is $PSUC_k$, where k is an integer representing the phase of the transmission pattern. We will use $n_1, n_2, \dots, n_k$ to denote the number of packets received in phase k of the transmission.

Using the same logic we used for the three-power-level model, we arrive at the following equations:

$$g_1 = \lambda \quad (4.13)$$

$$g_2 = g_1(1 - PSUC_{,1}) \quad (4.14)$$

$$\vdots$$

$$g_n = g_{n-1}(1 - PSUC_{,n-1}) \quad (4.15)$$

$$P_{SUC,1} = e^{-(g_1+g_2+\dots+g_n)} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_{max}=0}^{\infty} \frac{g_1^{n_1}}{n_1!} \frac{g_2^{n_2}}{n_2!} \frac{g_{n_{max}}^{n_{max}}}{n_{max}!} I_{SUC,1} \quad (4.16)$$

$$P_{SUC,2} = e^{-(g_1+g_2+\dots+g_{n_{max}})} \sum_{n_1=0}^{\infty} \sum_{n_2=1}^{\infty} \dots \sum_{n_{max}=0}^{\infty} \frac{g_1^{n_1}}{n_1!} \frac{g_2^{n_2}}{n_2!} \frac{g_{n_{max}}^{n_{max}}}{n_{max}!} I_{SUC,2} \quad (4.17)$$

$$\vdots$$

$$P_{SUC,n_{max}} = e^{-(g_1+g_2+\dots+g_{n_{max}})} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_{max}=1}^{\infty} \frac{g_1^{n_1}}{n_1!} \frac{g_2^{n_2}}{n_2!} \frac{g_{n_{max}}^{n_{max}}}{n_{max}!} I_{SUC,n_{max}} \quad (4.18)$$

Finite limits can be found for the summations. As a general rule, one can use $\frac{P_{n_{max}}}{P_k}$ as the limit for packets in phase k of the transmission sequence. The equilibrium point for the above equations can be found using the fixed point analysis.

Figure 4.2 shows the result of increasing the power ratio between the levels. The maximum throughput of the system increases as the power ratio increases. This is because the packets in the lower power levels interfere less

with packets at higher power levels. Therefore the system has more successful transmissions at higher levels, leading to more successes at lower power levels.

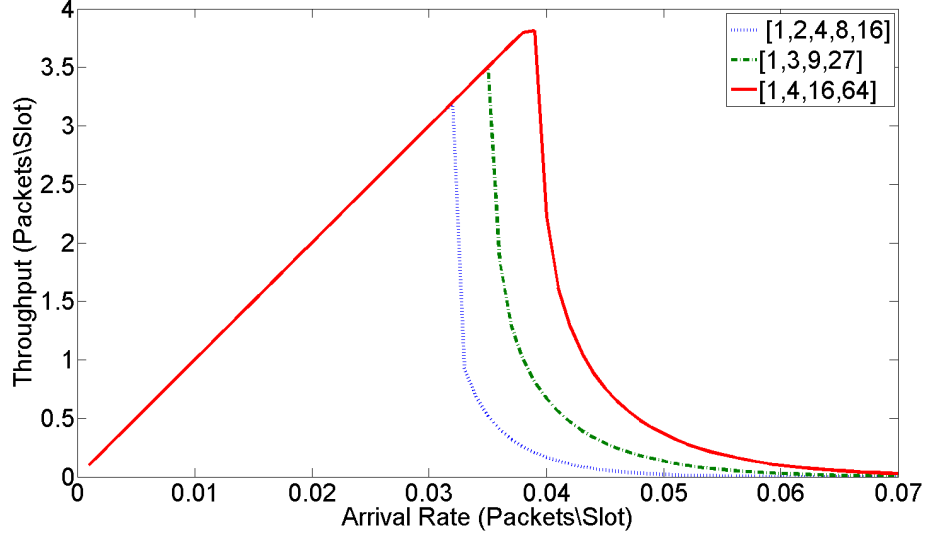


Figure 4.2: The effect of increasing the power ratio between levels on the system throughput

The increase in power ratio also has the effect of reducing the rapid decrease in throughput after the maximum throughput point. Figure 4.2 shows the tails of the throughput curves becoming thicker as the power ratio increases.

4.4 Variable number of power levels of very large power ratios

In this section we look at the special case where the power levels satisfy $P_0 \ll P_1 \ll \dots \ll P_{n_{max}}$. In this case, a higher power level is so large that packets of a lower power level do not interfere because they are insignificant. Therefore, the probability of success of a power level depends only on the number of packets in the same phase of the transmission pattern and the success of transmissions at higher levels of the pattern. In this scenario a transmission will be successful if there are fewer than $\frac{1}{\beta}$ other packets transmitted in

the same phase of the transmission sequence and all transmissions at higher power level are successful.

$$g_1 = \lambda \quad (4.19)$$

$$g_2 = g_1(1 - P_{SUC,1}) \quad (4.20)$$

$$\vdots$$

$$g_n = g_{n-1}(1 - P_{SUC,n-1}) \quad (4.21)$$

$$P_{SUC,1} = \sum_{n_1=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} e^{-(g_1)} P_{SUC,2} P_{SUC,3} P_{SUC,n_{max}} \quad (4.22)$$

$$P_{SUC,2} = \sum_{n_2=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} P_{SUC,3} P_{SUC,n_{max}} \quad (4.23)$$

$$\vdots$$

$$P_{SUC,n_{max}} = \sum_{n_{max}=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} e^{-(g_{n_{max}})} \frac{g_{n_{max}}^{n_{max}}}{n_{max}!} \quad (4.24)$$

It can be observed that on average at each level n , $(g_n)(P_{SUC,n})$ packets are successfully transmitted each time slot, where $(g_n)(1 - P_{SUC,n})$ packets collide and are transferred to the next higher power level. Therefore we can find the throughput of the system by summing the successfully transmitted packets at each power level using the formula

$$\text{Throughput (TP)} = \sum_{n=1}^{n_{max}} g_n P_{SUC,n} \quad (4.25)$$

Figure 4.3 shows the effect of increasing the number of power levels in a SIC system on the throughput of the system. The results in the graph are from a system with $\beta = \frac{1}{3}$ and the system arrival rate $\lambda = 3$. The number of power levels was increased and the throughput calculated for each number of power levels.

It is noted that the largest throughput increase always occurs when the first additional power level is added. The increase in throughput decreases

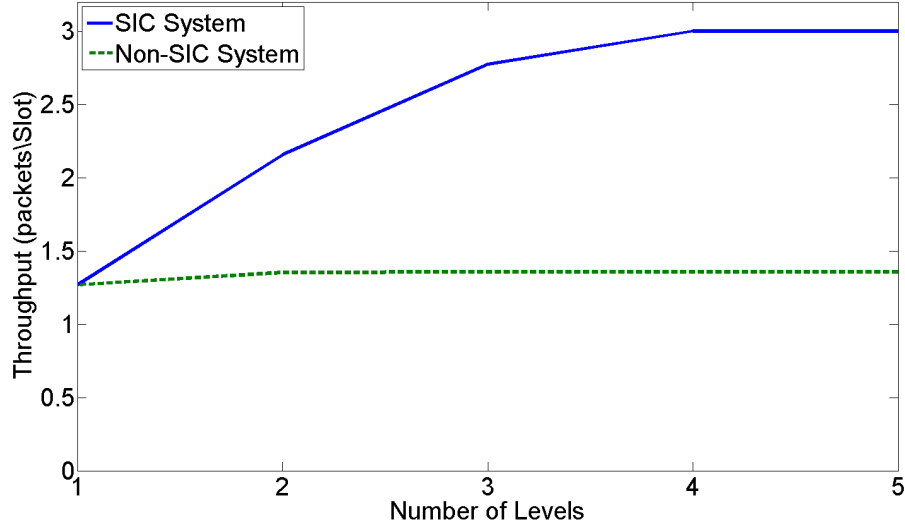


Figure 4.3: The effect of increasing the number of power levels on throughput for a system with SIC and a system without SIC

as subsequent power levels are added. In most scenarios that were tested, any increase in power levels beyond five produces insignificant increase in throughput.

Figure 4.3 also depicts the best case scenario in throughput increase for all possible arrival rates. This is because the throughput increases for cases with lower or higher arrival rates (λ) are not as substantial. At lower arrival rates, the system reaches a throughput equal to the arrival rate with fewer power levels. A higher arrival rate λ results in more collisions; therefore, the throughput actually decreases.

The green (lower) line in Figure 4.3 represents the throughput of a system without SIC. The throughput of a system using SIC is larger than that of a system not using SIC. This is because the probabilities of success of a multiple power system with SIC are larger than those of a multiple power system without SIC: the latter requires that no packets arrive at higher power levels, whereas the former only requires that packets at higher power levels are successful. This allows up to $\lfloor \frac{1}{\beta} \rfloor$ packets at any higher power level. This is clearly more likely to happen than no arrivals (P_{Zero}) because the possible cases of no arrivals in higher layers are a subset of the possible cases of no

collision in higher levels. Therefore the probability of no arrivals at higher levels will be less than the probability of success at all higher levels.

The fixed point equations used to calculate the throughput are as follows:

$$g_1 = \lambda \quad (4.26)$$

$$g_2 = g_1(1 - P_{SUC,1}) \quad (4.27)$$

$$\vdots$$

$$g_n = g_{n-1}(1 - P_{SUC,n-1}) \quad (4.28)$$

$$P_{SUC,1} = \sum_{n_1=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} e^{-(g_1)} P_{Zero,2} P_{Zero,2} P_{Zero,n_{max}} \quad (4.29)$$

$$P_{SUC,2} = \sum_{n_2=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} P_{Zero,3} P_{Zero,n_{max}} \quad (4.30)$$

$$\vdots$$

$$P_{SUC,n_{max}} = \sum_{n_{max}=0}^{\lfloor \frac{1}{\beta} \rfloor - 1} e^{-(g_{n_{max}})} \frac{g_{n_{max}}^{n_{max}}}{n_{max}!} \quad (4.31)$$

Figure 4.4 compares the throughputs of systems with and without SIC with respect to the arrival rate of packets. The graph shows the scenario with $\beta = \frac{1}{3}$. The arrival rate is increased by 0.1 each iteration. The throughput is then calculated. The blue line represents the throughput of the system employing SIC while the green graph represents the throughput of the system without SIC. From the graph we can see that both systems can keep up with the arrival rate until around 1.4 packets per slot. At that point the system without SIC starts experiencing collisions between the higher and lower power levels, which decreases the throughput of the system. In contrast, the SIC systems continues to process all the packets because the higher levels are removed before processing the lower levels.

The SIC system reaches its peak at an arrival rate of around 3.1 pack-

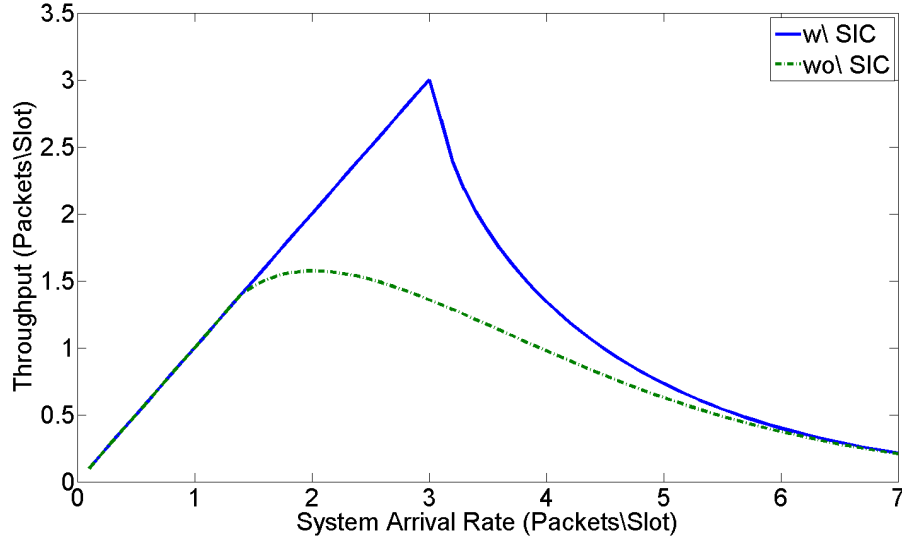


Figure 4.4: Comparison of maximum throughput for a system with SIC and a system without SIC with very large power ratios

ets/slot. Afterward the throughput decreases rapidly and eventually converges with the throughput of a system without SIC. The steep decrease in throughput happens because SIC optimizes the system so that most packets at lower levels are successful as long as the highest power level is also successful. As the load on the system increases and the highest level becomes congested, collisions become more likely. When the highest level results in a collision, all the packets in the lower levels are also unsuccessful. Therefore, the decline in throughput is very steep compared to a less optimized system such as the multiple power system without SIC.

From the point of view of the users of the system, it is desirable to maintain the throughput at its maximum level (at the peak of the curve). This can be done by requesting that users back off randomly in order to reduce the arrival rate. A simple method of implementing this is to send a number between zero and one to all users. This number represents the percentage of users that the system desires to retain. Every user then generates a random number between zero and one independently and compares it to the received number. If the generated number is less than the number sent by the hub, then the user continues transmitting. If it is greater than the number sent by the hub, then the user backs off and drops all pending packets until the

hub signals that it can accept additional users. This would artificially push the arrival rate back to the maximum throughput point. The throughput versus rate graph would look similar to Figure 4.5.

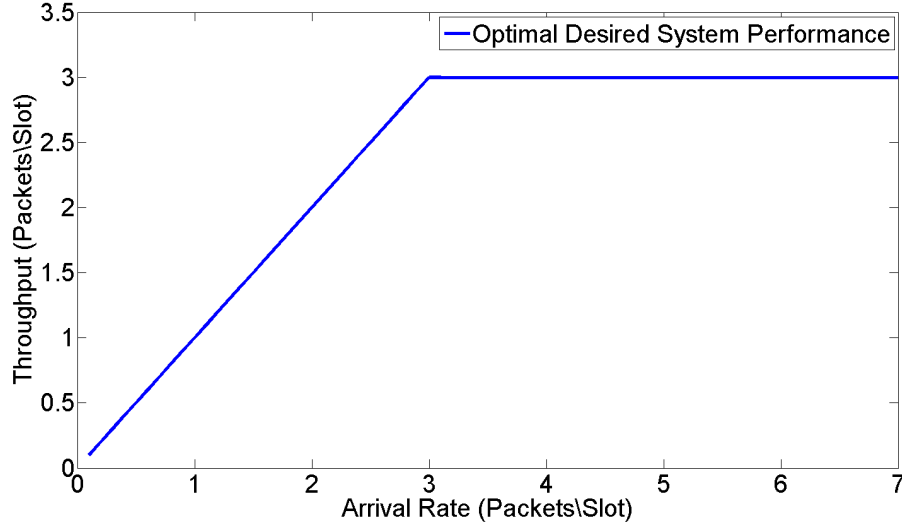


Figure 4.5: The desired throughput graph of a SIC system

Another interesting variable to look at when studying the system is the effect of the number of power levels on the system. Figure 4.6 shows the changes to the throughput graph as new levels are added. The graph was constructed by using a SIC system with $\beta = \frac{1}{3}$ and with a fixed number of power levels. The arrival rate of the system was increased from 0.1 to 7 packets per slot. The resulting throughputs for all the different arrival rates with the different numbers of power levels were then plotted on the same graph.

In most scenarios, the system showed a notable increase in throughput up to the fifth level. After the fifth level, the increase was negligible. In Figure 4.6 we have plotted the throughput of systems with 1, 2, 3, 4 and 20 levels. The plots of the system with 4 levels and 20 levels overlap almost perfectly. As a benchmark, the throughputs of systems that do not use SIC are plotted in Figure 4.7.

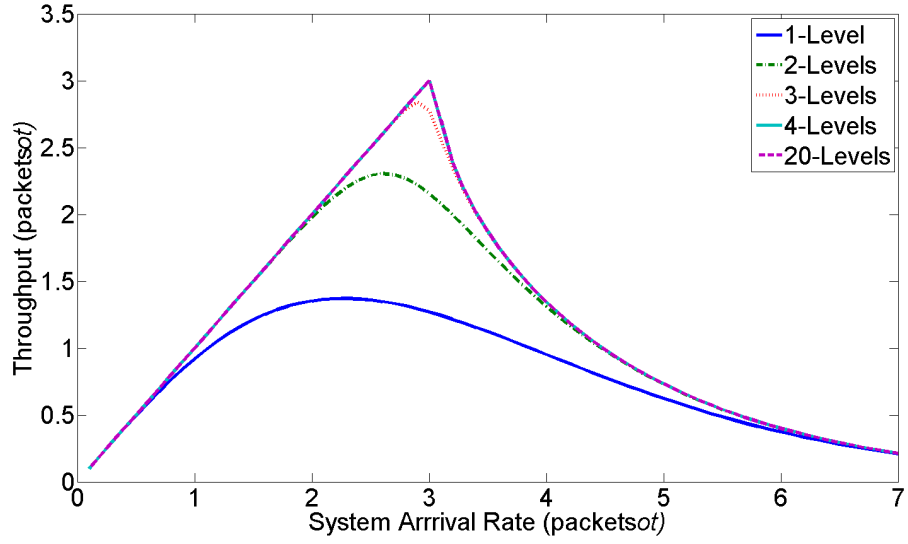


Figure 4.6: The effect of increasing the number of levels of a SIC system on throughput

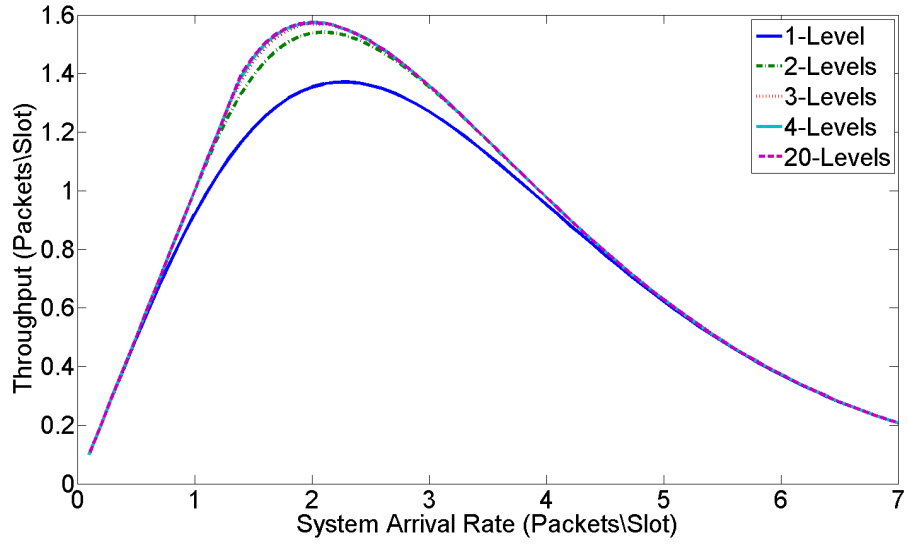


Figure 4.7: The effect of increasing the number of levels of a system without SIC systems on throughput

Proposition: If a system is operating under the assumptions of the fluid model with n power levels of very large power ratios and a maximum throughput of $TP^{*(n)}$ corresponding to arrival rate $\lambda^{*(n)}$, then increasing the number of power level will result in a larger maximum throughput $TP^{*(n+1)}$ for some arrival rate $\lambda^{*(n+1)}$ such that $TP^{*(n+1)} > TP^{*(n)}$ and $\lambda^{*(n+1)} > \lambda^{*(n)}$. Figure 4.8 shows the effect of adding an extra power level to the system.

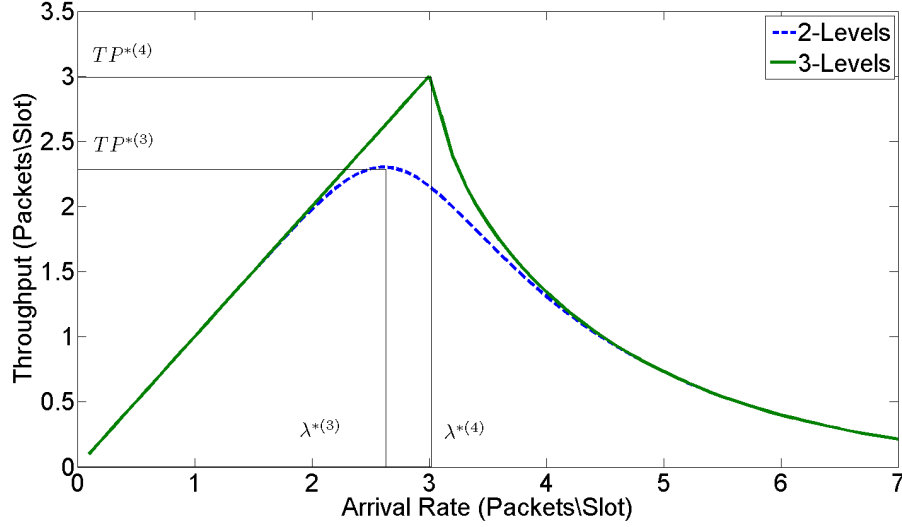


Figure 4.8: Increasing number of levels increases maximum throughput

The proposition can be proved using Figures 4.9 and 4.10. Figure 4.9 shows a system with n power levels. The system accepts an arrival rate of λ_1 and produces a throughput of TP_1 , which is the aggregate of the successful transmissions of all the individual power levels.

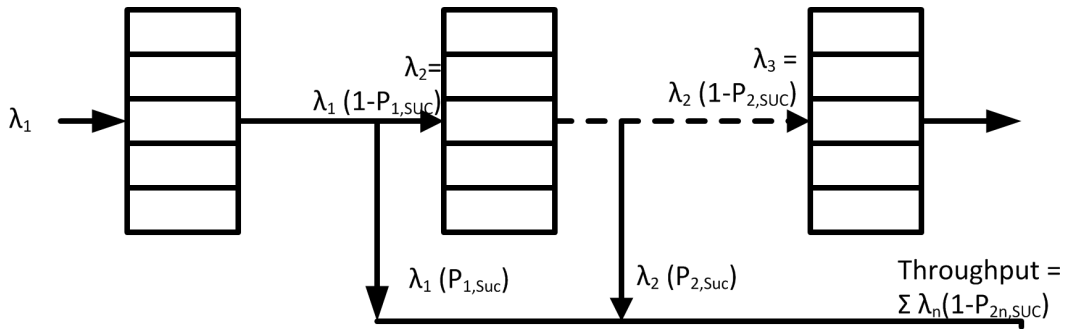


Figure 4.9: Arrival rate and throughput of a system before adding level

Figure 4.10 shows a system with $n + 1$ power levels, where the new level is added at the front of the system. Therefore, we can choose a new arrival rate $\lambda_0 > \lambda_1$ for the system such that the input to the second phase of the system is λ_1 . This results in a throughput of TP_1 plus the added throughput from the successful transmissions in the first phase of the system. Therefore, the total throughput of the system with an additional power level TP_0 will be larger than the throughput of the previous system TP_1 . This completes the proof of the proposition.

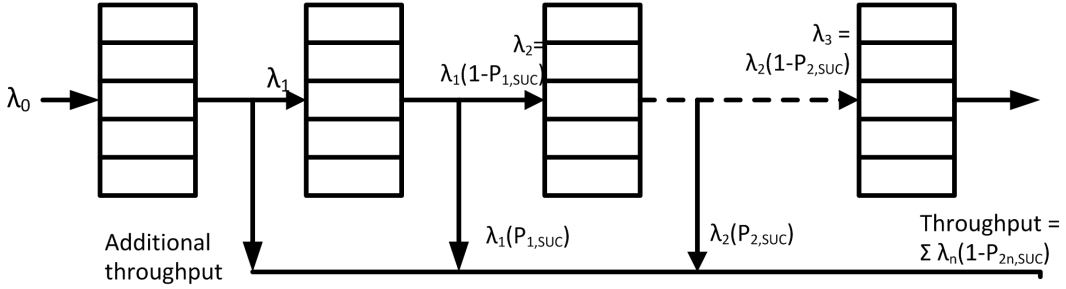


Figure 4.10: Additional throughput gained by adding a level

Conjecture: For a multiple power Aloha system using SIC, very large power ratio, infinite number of users and a fixed arrival rate λ , if the number of power levels in the system is increasing, then the throughput is a nondecreasing function.

While it is difficult to prove the above conjecture analytically, it can be seen from Figure 4.6 that even though there seems to be a limit to the throughput, it always increases as the number of levels increases.

CHAPTER 5

SIMULATION RESULTS

The SIC system is modeled using Matlab. The number of users in the system, represented by variable M , is fixed at 100. Each user simulates random arrivals using a Bernoulli random variable with probability of arrival Q_a . Therefore the arrival of the system can be approximated with a Poisson distribution. The transmission of packets is randomized using a Bernoulli random variable. Each user with a packet transmits in a timeslot with probability Q_x .

Each user can only process one packet at a time. If a user receives a new arrival as it is processing a packet, then the new packet is buffered. The user buffer is modeled as a counter that increases whenever a new arrival is received and decreased when a buffered packet is processed.

The simulation approximates frequency division multiplexing by using time division multiplexing because randomizing the over time is a close approximation to approximating over frequency. This is true because the transmission time is small compared to the RTT.

Figure 5.1 shows the effect of increasing the number of levels on the throughput of a finite user system. The system in the figure has a power ratio $\frac{P_{n+1}}{P_n}=3$ and $\beta = 3$. The first observation is that the the throughput graphs cross as the number of levels increases. This did not happen in the case of the fixed point analysis where throughput always increased at every point as the number of levels increased.

From Figure 5.1 we see that the maximum throughput increases as the number of levels increases. But because we are using a finite power ratio, the packets at the lower levels start to cause significant interference for higher

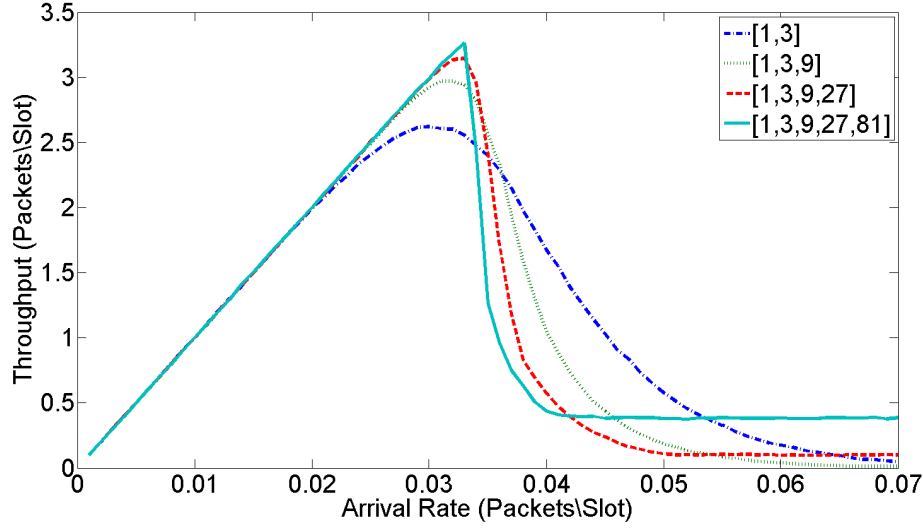


Figure 5.1: Additional throughput gained by adding a level

levels. Therefore, the systems with more levels have a sharper decrease in throughput after reaching the maximum throughput point.

We also notice that as the number of levels increases, the minimum throughput of the system decreases as the limit of the system throughput does not approach zero. Instead it remains constant at a point. This is because of the effect of the buffer and because the traffic begins to thin out as the number of levels increases due to the assumption that a node processes only one packet at a time. Another factor is the randomization over time and frequency.

Figure 5.2 shows how power ratio can also affect the minimum throughput. The system in the graph had 100 users with $\beta = 3$ and $Q_a=0.2$. When the arrival rate of the system exceeds four packets per slot, all of the users become backlogged. Therefore the new arriving packets are buffered and the actual transmission rate of packets remains constant. In the example in Figure 5.2, when all of the users are backlogged a maximum of 100 users are waiting to transmit. These users will be spread out over the five power levels, meaning there will be around 20 users waiting to transmit at each power level. The packets at each level are randomized over either time or frequency, resulting in an average of four packets per slot. Therefore, the system will be successful with a certain probability. As the power ratio increases, the probability

of success also increases because the lower power packets do not interfere as much with the higher power packets.

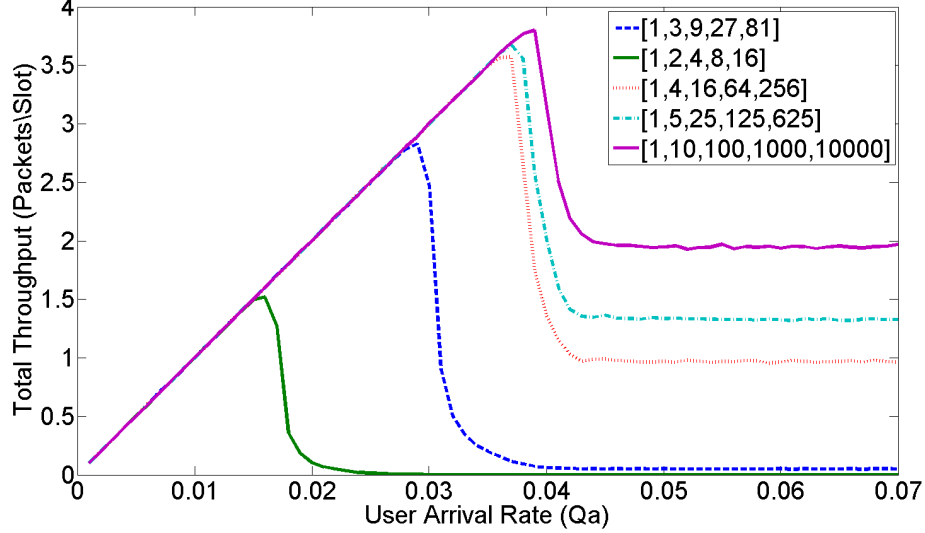


Figure 5.2: Additional throughput gained by increasing power ratio

Another observation is that repeating the same power level multiple times increases the throughput slightly in the region where the arrival rate is less than the point of maximum throughput. But repeating a power level greatly reduces the throughput in the points afterward. This can be seen in the results shown in Figure 5.3.

Figure 5.4 shows a comparison between the results of the simulation and the fixed point calculations for systems using the same power level sequences. We find that the fixed point analysis provides a higher throughput compared to the equivalent simulation. This is because of the effect of the buffer.

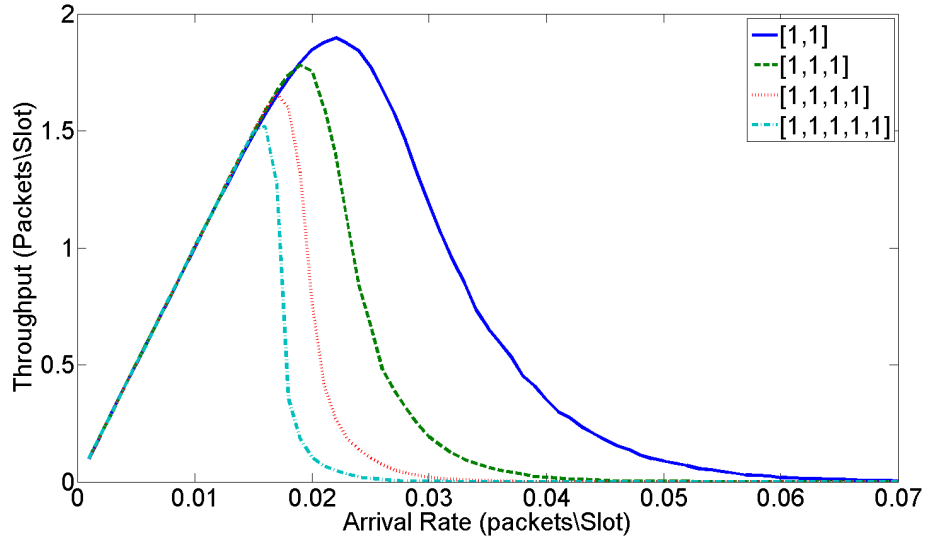


Figure 5.3: A plot showing the effect of repeating the same power level on the throughput of the system

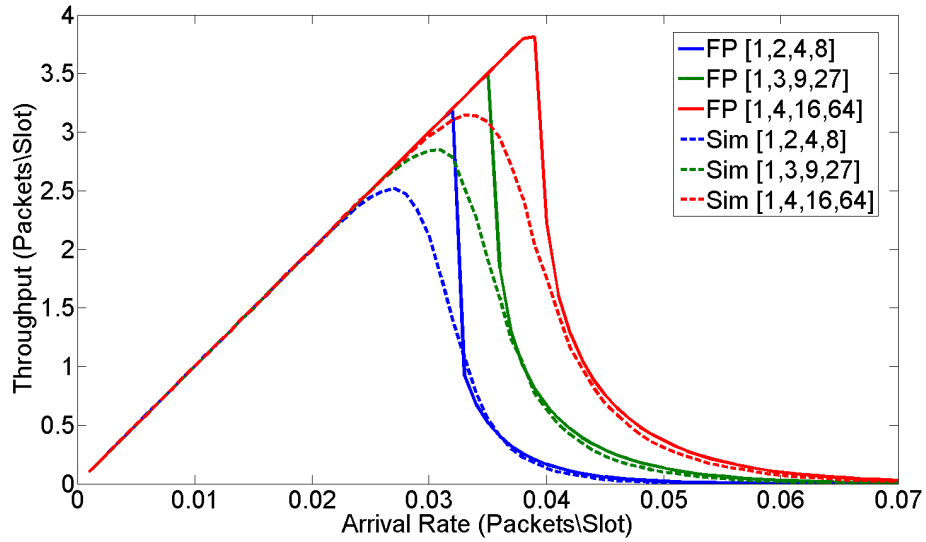


Figure 5.4: Comparison of the results of the fixed point calculations and simulation

CHAPTER 6

REMAINING WORK

The simulations presented in this thesis used time division multiplexing to approximate frequency division multiplexing. Therefore, the simulation could be enhanced to model the real system more closely. In addition, it does not take into account the long delay of the satellite network as the simulation was intended merely to provide intuition into the workings of the real system.

In our study we have focused on using predetermined power sequences to increase the throughput of the system. But we could use random transmission sequences to improve the throughput of the system. For example, instead of using an increasing sequence, a packet could randomly be assigned a power level at arrival. This would decrease the arrival rate of the system to $\lambda/3$ for each power level instead of λ to the first power level, reducing the chances of collision in the case of large bursts of traffic.

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