DYNAMIC RETAIL LOCATION MODEL WITH MARKET LEARNING

BY

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Industrial Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2011

Urbana, Illinois

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ABSTRACT

In this thesis, we study the optimal store location decisions for a firm entering a new market where the market adoption rate can be learned over time. In the presence of market learning, the firm faces a trade-off between *active learning* and *deferred commitment*. To illustrate this trade-off, we introduce a two-stage retail location problem in which the market learning time (length of the first stage) is endogenously determined by the firm's first stage action. To solve the problem, we develop an efficient solution method which provides a framework to achieve a desired error rate of accuracy in the optimal solution. The proposed algorithm is tested on the network constructed using census data from the city of Chicago. Using the model, we first show that the lack of foresight results in lower profit with over-commitment in facility investment and that the difference increases with market uncertainty. We further show that the firm should prefer active learning over deferred commitment as consumers in the market become more conservative in making product adoption decisions.

To my parents, for their love and support.

ACKNOWLEDGMENTS

In the Name of Allah, the Most Beneficent, the Most Merciful. All praises and thanks are for Allah (God) and may His blessings, peace and favors descend in perpetuity on the beloved Prophet Muhammad (PBUH) who is mercy for all the worlds. First of all, I thank Allah Almighty for His help, support and blessings during the entire period of my studies and beyond.

I would like to express my gratitude to have Dr. Xin Chen as my academic adviser. During our weekly meetings, he always stressed on understanding the underlying mathematical structure of the problem, which was very helpful for me to achieve interesting results. I feel very fortunate and blessed to also have Dr. Michael Lim as my adviser. I would like to convey my special thanks to him for mentoring and guiding me during my MS. During the discussions, he dedicated special attention on inferring the results from mathematical structures which enabled me to arrive at useful managerial insights. Due to his special emphasis on formal writing, I significantly improved my writing skills and enhanced the quality of my thesis. I am also grateful to Dr. Ho-Yin Mak for his valuable suggestions and contribution to this research.

I am fortunate to have dedicated and devoted professors at UIUC such as Angelia Nedich, Uday Shanbhag, Enlu Zhou, Sheldon Jacobson, Limeng Feng, Rahim Benekohel, Jong-Shi Pang and many others. Furthermore, I met some of the brilliant students and learned from them during group projects and research discussions. I am very lucky to meet friends such as Rasoul

Etesami, Jayash Koshal, Limeng Pan, Farzad Yousfain, Alok Tiwari, Fan Ye, Fuyuan Wang, Yerkin Abdildin, Behrouz Touri, Jalal Etesami, Xiangqian Meng, Rui Yang, Peng Hu, Ehsan Salimi, Tao Zhu, and many others.

Finally, I am immensely indebted to my parents for their tireless efforts and numerous sacrifices that have made me the person I am today.

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LIST OF ABBREVIATIONS

MCLP	Maximal Coverage Location Problem
GCLP	Gradual Covering Location Problem
SRLP	Single-stage Retail Location Problem
SRLP-U	Single-stage Retail Location Problem with Uncertain Adoption Rate
UFLP	Uncapacitated Fixed Charge Location Problem
TRLP	Two-stage Retail Location Problem
TRLP-L	Two-stage Retail Location Problem with Learning

CHAPTER 1

INTRODUCTION

1.1. Motivation and Philosophy

Store location decisions are a crucial part of retail operations. Locations are one of the most influential considerations in consumers' choice of stores as they usually go to the closest or most conveniently located stores. If one firm occupies the most attractive location in the neighborhood, the competitors are relegated to the next-best locations. As a result, firms are often compelled to develop a sustainable competitiveness based on their store locations. Although store location decisions can create such strategic advantages, they also represent risk as they involve a significant commitment of resources for a long period of time. When a firm decides to enter a new market and selects a set of locations for its retail stores, it has to make a substantial investment to buy and develop the real estates or commit to a long-term lease ranging typically from 5 to 20 years (Levy and Weitz (2008)). Unlike poor pricing or inventory decisions, poor store location decisions negatively affect the firm for a longer period of time.

The risk of commitment in retail industry magnifies when it comes to entering a new market. When a firm is entering an emerging foreign market, for example, it faces high level of uncertainties in many aspects (such as uncertainty in local consumers' purchasing behavior or their product adoption decisions). As a result, it is difficult to anticipate how the products (or services) offered by the firm will be received by the local consumers. Under such uncertain business environment, firms dynamically deploy retail stores as they *learn* the market over time. For example, Apple is cautiously expanding in China due to high market uncertainty resulting from other established competitors (such as Lenovo and HP) and many local copycat manufacturers (Chow (2011)). Since opening its first store in 2008, it only recently added two new stores in Hong Kong and Shanghai. On the other hand, CVS Pharmacy decided to expand fast in the Puerto Rico market. In 2010, it opened the first 9 stores (Providence Business News, 2010). Due to aggressive entry, CVS Pharmacy quickly learned the market and recently announced to open 13 additional stores in the region by 2012 (Peurto Rico Daily Sun, 2011). Clearly, different firms take different market entry approaches and thus make different initial store deployment decisions. As a result, firms learn the market (consumers also learn the product/service offered by the firm) at a different rate and the resulting store location configurations may differ significantly.

In this thesis, we consider the retail store location problem in the presence of market learning. In particular, we study the optimal store location decisions for entering a new market when the market adoption rate can be learned over time. With the option of learning the uncertain market environment, a firm faces a trade-off between "*active learning*" and "*deferred commitment*" as described in the above examples. That is, a firm may want to actively learn the market through greater initial investment since demand data is collected at a faster rate. On the other hand, a firm may want to defer the commitment since overly-aggressive investment often results in sub-optimal store locations adversely affecting the firm in the long run. The primary objective of this study is to understand the impact of learning in retail store location decisions and to derive relevant managerial insights.

To this end, we introduce a two-stage retail location model which captures the market learning effect. We consider the consumer adoption rate of the market to be uncertain, but can be learned during the first stage. After the first stage, we assume that the adoption rate is fully learned and the firm has the option to locate extra facilities in the second stage. To reflect the trade-off between active learning and deferred commitment, we assume that the duration of the first stage (market learning time) is *endogenously* determined as a function of firm's first stage action. Under this setting, the firm chooses to either shorten (active learning) or lengthen (deferred commitment) the market learning time by changing the initial stage action. The main contribution of this research is two-folds. First, we develop an efficient and effective solution method for solving the two-stage retail location problem with market learning. This solution method is applicable to any location problems with endogenous learning time. We apply the algorithm to network based on data from Chicago and illustrate the performance of the proposed algorithm. Second, we provide insights on learning in retail store location decisions. Using a myopic decision maker (who does not take into account the effect of "learning") as a benchmark, we show that firms should prefer deferred commitment over active learning as market uncertainty increases. We also show that lack of learning typically results in over-commitment in facility investments with lower expected profit. By contrasting markets with different consumer characteristics, we show that the firm should prefer active learning over deferred commitment as consumers in the market become more conservative in making product adoption decisions.

1.2. Literature Review

Facility Location in retail setting has generally been formulated as a coverage problem. Church and ReVelle (1974) introduced the Maximal Coverage Location Problem [MCLP] which finds the locations of a given number of facilities to maximize the *customer coverage*, the total number of customers served by the set of opened facilities. The problem assumes a binary coverage scheme; i.e. service is accounted adequate if the customer is within a given distance and is considered inadequate if the distance exceeds some critical value. Daskin (1983) extends the problem to the "expected" covering case by taking into account possible facility congestions. More recently, Berman and Krass (2002) consider a generalized version of MCLP which allows partial coverage of customers instead of a binary coverage. Berman et al. (2003) and Drezner et al. (2004) discuss the Gradual Covering Location Problem [GCLP] in which the degree of customer coverage is defined as a function of traveling distance. In particular, they consider lower and upper thresholds in traveling distance; customers who have a traveling distance less than the lower threshold are fully covered whereas customers residing farther than the upper threshold are not covered at all. The coverage for customers located in between gradually decays as a function of the traveling distance. Drezner et al. (2010) extend GCLP to stochastic case when upper and lower distances are random variables. We consider a similar coverage scheme in this study. We assume the coverage function to be a non-increasing function of the distance between a demand node and its closest facility. For more details on the coverage location problems, please see Jacobson (1990) (for discrete models) and Plastria (2002) (for continuous models).

Facilities typically function for an extended period of time, during which a certain aspect of market environment may be learned (Snyder (2006)). For this reason, many facility location problems involve an extended planning horizon where firms make a set of *dynamic* decisions over time. Pioneering work on the dynamic facility location problem has been done by Ballou (1968) and Wesolowsky (1973). Dynamic Location Problems provide a set of plans which involve expanding facilities or/and relocating existing facilities as uncertain information such as demand, travel cost, competition unveil over time. Van Roy and Erlenkotter (1982) and Baron et al. (2010) consider a facility location problem on a dynamic setting where demand changes over time. Campbell (1990) studies the dynamic location of transportation terminals where demand, transportation cost and facility cost alter over time. Hakimi (1990) and Fischer (2002) consider sequential location problems in which multiple firms (typically identified as leader and followers) compete for fixed demand. In this thesis, we assume consumer adoption rate to be the uncertain factor and a monopoly firm dynamically deploys facilities to maximize its expected profit.

We limit the problem to a two-stage setting (with infinite time horizon) since it suffices to study the trade-off between active learning and deferred commitment and the value of market learning. Similar to our setting, Current et al. (1997) consider two versions of two-stage facility location problems where the total number of facilities to open varies depending on the future scenario. Berman and Drezner (2008) also study a two-stage problem with the fixed number of facilities opened in the first stage. They seek to minimize the total cost of serving all the demand keeping in view that additional facilities can be opened in the future stage. While these papers share similar feature in demand uncertainty on a two-stage setting, they do not capture the market learning effect. In particular, the length of the first stage is exogenously given and the firm passively makes decisions under given setting. We explicitly incorporate the market learning effect by assuming the market learning time (length of the first stage) to be endogenously determined as a function of firm's first stage actions.

Hiller and Shapiro (1986) and Rob (1991) are the first ones to consider *learning* in firm's capacity expansion. Learning has also been studied in various fields of operations management including retail industry. Fisher and Raman (1996) introduce the market learning to improve the forecast accuracy of the demand of high-end fashion products. The firm initially commits to relatively low production in the first stage before the sales start and then further production decisions are made as more demand information arrives. Caro and Gallien (2007) apply learning for dynamic assortment of products in fashion retail industry. Araman and Caldentey (2009) take into account learning for dynamically updating the price of a product. However, market learning has not been studied in the retail store location setting, to the best of our knowledge. In this thesis, we study how the presence of learning affects the firm's decision in retail store location.

1.3. Structure of the Thesis

The remainder of this thesis is structured as follows. In Chapter 2, we introduce the two-stage retail location problem with market learning. We also provide interesting structural properties of the problem. In Chapter 3, we propose a solution method for the proposed problem and present the algorithm's performance. In Chapter 4, we first study the value of foresight by contrasting the optimal decision maker to a myopic decision maker. We then study the impact of learning by comparing the optimal policy under different market characteristics. Finally, we conclude the thesis by summarizing managerial insights and proposing directions for future research in Chapter 5.

CHAPTER 2

DYNAMIC RETAIL LOCATION MODEL

2.1. Single-stage Model

We first present a single-stage retail location problem without market learning. Consider a set of demand points $I = \{1, \cdots, m\}$ where consumers reside in and a set of sites $J = \{1, \cdots, n\}$ where the stores can be located. The underlying network is $G(V, A)$ where $V = \{I \cup J\}$ is the set of nodes and A is the set of arcs. In each demand node, h_i potential consumers live and only $0 \leq \theta \leq 1$ fraction of those consumers actually adopt (purchase) firm's product/sevice. We refer to this random variable θ as the consumer adoption rate. We consider the consumer demand at node $i \in I$ is partially covered by the coverage function $g_i(d) \in [0, 1]$ where d is the distance to its closest opened facility. The coverage function $g_i(d)$ is a non-increasing convex function of d with $g_i(0) = 1$ for all $i \in I$. Hence, the effective demand covered at node i with opened facility set X can be expressed as $\theta h_i g_i(d_i(X))$ where $d_i(X) = \min_{j \in X} d(i, j)$. Denoting revenue per unit demand per unit time by r, total revenue per unit time is $\sum_{i \in I} r\theta h_i g_i(d_i(X))$. We denote the fixed cost for operating facilities X per unit time by $f(X)$ where $f()$ is a modular function. With the discount rate α on an infinite time horizon, the single-stage retail location problem [SRLP] with given adoption rate that maximizes firm's profit can be formulated as follows:

$$
\begin{aligned}
\textbf{[SRLP]} \quad & \max_{X \subset J} \left\{ \int_0^\infty e^{-\alpha t} \Big(\sum_{i \in I} r\theta h_i g_i(d_i(X)) - f(X) \Big) dt \right\} \\
&= \max_{X \subset J} \left\{ \frac{1}{\alpha} \Big(\sum_{i \in I} r\theta h_i g_i(d_i(X)) - f(X) \Big) \right\}. \qquad (2.1)
\end{aligned}
$$

Now we consider the case in which the consumer adoption rate θ is unknown. Given the distribution of θ, one can then consider a problem of maximizing the *expected* profit as follows:

$$
\textbf{[SRLP-U]} \quad \max_{X \subset J} \left\{ \mathbb{E}_\theta \left[\frac{1}{\alpha} \Big(\sum_{i \in I} r\theta h_i g_i(d_i(X)) - f(X) \Big) \right] \right\}. \qquad (2.2)
$$

For simplicity, we reduce the facility candidate sites to the nodes of the network in this study. In the following proposition, we show that both problems indeed satisfy the nodal optimality property, i.e., at least one optimal solution exists with all the facilities located only on nodes, if the fixed cost for a facility on the edge joining two nodes is a convex combination of those two facility costs.

Proposition 2.1. *Consider a facility j_0 which is located on the edge joining the two nodes j_1 and j_2; i.e., $j_0 = \lambda j_1 + (1-\lambda) j_2$ where $0 \le \lambda \le 1$. If $f(j_0) = \lambda f(j_1) + (1-\lambda) f(j_2)$, then there exists at least one optimal solution which corresponds to locating only on the nodes of a network.*

Proof. Consider a solution X in which at least one facility is located on the edge joining nodes j_1 and j_2 where the distance between the two nodes is $\bar{d} = d(j_1, j_2)$. Take one facility on the edge, j_0, and denote the set of demand nodes covered by j_0 as I_0. At the moment, we fix the positions of the remaining facilities (X^{-j_0}). For a given θ, objective function can then be expressed as $\frac{1}{\alpha}\big\{\sum_{i\in I} r\theta h_i g_i(d_i(X)) - f(X)\big\} = \frac{1}{\alpha}\big\{\sum_{i\in I_0} r\theta h_i g_i(d_i(j_0)) - f(j_0) + \sum_{i\in I\setminus I_0} r\theta h_i g_i(d_i(X^{-j_0})) - f(X^{-j_0})\big\}$. Note that the minimum distance between j_0 and demand node $i \in I_0$ is $d_i(X^{-j_0}) = \min\big[d(i, j_1 + \lambda\bar{d}), d(i, j_2 + (1-\lambda)\bar{d})\big]$. Since the coverage function $g_i(d)$ is decreasing in d for each i, the first term in the objective function is $\sum_{i\in I_0} \max\big[r\theta h_i g_i(d(i, j_1 + \lambda\bar{d})), r\theta h_i g_i(d(i, j_2 + (1-\lambda)\bar{d}))\big]$. This is convex in λ since the maximum of two convex functions is convex, and the sum of convex functions is convex. Further, the second term, $f(j_0) = \lambda f(j_1) + (1-\lambda) f(j_2)$, is linear in λ and the remaining terms are constants. Hence, the objective function is convex in λ and is maximized at least at either $\lambda = 0$ or 1. Applying the same logic to other facilities on the edge, it follows that the node optimality condition holds for the [SRLP]. Since taking expectation of convex function is also convex, the node optimality condition also holds for the [SRLP-U]. $\square$

Finally, we note that the single-stage retail location problem can be transformed to the uncapacitated fixed charge location problem [UFLP].

Proposition 2.2. *The single-stage retail location problem [SRLP] is reducible to the uncapacitated fixed charge location problem [UFLP].*

Proof. Let H represent the maximum total demand of the market, $H = \sum_{i\in I} h_i$. Define a new distance metric as $\widetilde{d}_i(X) = \max_{j\in X}[1 - g_i(d(i,j))]$. Then, it follows that $\widetilde{d}_i(X) = \max_{j\in X}[1-g_i(d(i,j))] = 1-\min_{j\in X} g_i(d(i,j)) = 1 - g_i(\min_{j\in X} d(i,j)) = 1 - g_i(d_i(X))$. Thus, for any $X \subset J$,

$$\begin{aligned}
\frac{1}{\alpha}\Big(\sum_{i\in I} r\theta h_i g_i(d_i(X)) - f(X)\Big) &= \frac{1}{\alpha}\Big(\sum_{i\in I} r h_i(1 - (1 - g_i(d_i(X)))) - f(X)\Big) \\
&= \frac{1}{\alpha}\Big(rH - r\sum_{i\in I} h_i(1 - g_i(d_i(X))) - f(X)\Big) \\
&= \frac{1}{\alpha}\Big(rH - r\sum_{i\in I} h_i \widetilde{d}_i(X) - f(X)\Big).
\end{aligned}$$

Since H and r are constants, maximizing this problem is equivalent to the following UFLP:

$$\min_{X \subset J} \left\{ f(X) + k \sum_{i \in I} h_i \widetilde{d}_i(X) \right\}$$

where k is a constant. $\square$

This is an interesting and useful result since UFLP, while NP-Hard, has many practical solution methods available (Daskin (1995)).

2.2. Two-stage Model with Learning

Now, we present a two-stage retail location problem incorporating *market learning*. The subject of learning is the consumer adoption rate of the local market which, in turn, determines the market demand for each node. In the first stage, the adoption rate, θ, is uncertain, but its distribution is known (as in [SRLP-U]). Based on its distribution, the firm must decide in advance where and how many stores to open, X^1, taking into account the next stage. In stage 2, the adoption rate is fully learned, i.e., the precise value of θ is known. Upon the realization of θ, the firm deploys additional facilities accordingly, X^2, (as in [SRLP]) to maximize its total profit. For simplicity, we assume the facilities opened at the previous stage cannot be closed or relocated. Relaxing this assumption, however, does not change the key insights of our results if the facility closing cost is moderately high. Denoting the market learning time to T, the two-stage retail location problem for given T [TRLP] is formulated as follows:

$$\max_{X^1 \subset J} \left\{ \mathbb{E}_\theta \left[\int_0^T e^{-\alpha t} \Big(\sum_{i \in I} r\theta h_i g_i(d_i(X^1)) - f(X^1) \Big) dt + e^{-\alpha T} V(X^2; X^1, \theta) \right] \right\} \tag{2.3}$$

where $V(X^2; X^1, \theta)$ is the optimal objective value of:

$$\max_{X^2 \subset J \setminus X^1} \left\{ \int_T^\infty e^{-\alpha t} \Big(\sum_{i \in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^1 \cup X^2) \Big) dt \right\}. \tag{2.4}$$

With some algebraic work, the problem can be reexpressed as:

$$\textbf{[TRLP]} \max_{\substack{X^1 \subset J \\ X^2 \subset J \setminus X^1}} \frac{1}{\alpha} \left\{ \mathbb{E}_\theta \left[(1 - e^{-\alpha T}) \sum_{i \in I} r\theta h_i g_i(d_i(X^1)) - f(X^1) + \right.\right.$$
$$\left.\left. e^{-\alpha T} \left(\sum_{i \in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^2) \right) \right] \right\}. \tag{2.5}$$

As for the single-stage problem, the two-stage problem also satisfies the node optimality condition if the facility fixed cost on an edge is a convex combination of the facility costs of the two linked nodes.

Proposition 2.3. *Consider the facility fixed cost on an edge connecting two nodes is a convex combination of those two facility costs. Then, there exists at least one optimal solution for the [TRLP] which corresponds to locating only on the nodes of a network for both stages.*

Proof. We prove this holds for each stage in backwards. Given X^1 and θ, consider a second stage solution in which at least one facility is located on the edge. By fixing the location of the remaining facilities, we can show that the objective function of the second stage, (2.4), improves by relocating the facility to either one side of the node (using the same logic for proving the node optimality for the [SRLP] from Proposition 2.1). This holds for any X^1 and θ, thus the node optimality holds for the second stage problem. For every possible θ, consider a first stage solution in which at least one facility is located on the edge. Similar to the [SRLP-U], we can show that the objective function of the first stage, (2.3), can be improved by relocating the facility on the edge to either one side of the node while fixing the others. Therefore, there exists at least one optimal solution for the [TRLP] in which all facilities are located only at the nodes of the network. □

By extending the problem to two-stage, we note that the firm now has incentive to deploy less facilities in the first stage since it has an option to deploy more in the second stage. This dynamic nature of the problem leads the firm to prefer "deferred commitment." We characterize the relationship between the market learning time and firm's optimal solution in the following proposition. It follows that when the market learning time takes extreme values, the solution of the [TRLP] reduces to one of the single stage problems.

Proposition 2.4. *(i) There exists a threshold in learning time $\bar{\tau}$ such that if $\bar{\tau} \leq T$, the optimal first stage solution of the [TRLP] coincides with the optimal solution of the [SRLP-U].*

(ii) There exists a threshold in learning time $\underline{\tau}$ such that if $\underline{\tau} \geq T$, the optimal second stage solution of the [TRLP] coincides with the optimal solution of the [SRLP] for given θ.

Proof. Let us denote the first-stage expected profit per unit time by $\pi_1(X^1) = \mathbb{E}_\theta\big[\sum_{i\in I} r\theta h_i g_i(d_i(X^1)) - f(X^1)\big]$ and similarly the second-stage expected profit per unit time as $\pi_2(X^2;X^1) = \sum_{i\in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^1 \cup X^2)$. Then, the objective function can be written as follows: $Z_{[TRLP]} = \frac{1}{\alpha}\big\{(1 - e^{-\alpha T})\pi_1(X^1) + e^{-\alpha T}\pi_2(X^2;X^1)\big\}$.

(i) Let X^* be the optimal solution for the [SRLP-U]. We show that there exists $\bar{\tau}$ such that, if $\bar{\tau} \le T$, then

$$\begin{aligned} & Z_{[TRLP]}(X^*, X^2) \ \ge \ Z_{[TRLP]}(\tilde{X}^1, \tilde{X}^2) \\ \Longleftrightarrow \quad & \frac{1}{\alpha}\Big\{(1-e^{-\alpha T})\pi_1(X^*) + e^{-\alpha T}\pi_2(X^2; X^*)\Big\} \\ & \ge \frac{1}{\alpha}\Big\{(1-e^{-\alpha T})\pi_1(\tilde{X}^1) + e^{-\alpha T}\pi_2(\tilde{X}^2; \tilde{X}^1)\Big\} \\ \Longleftrightarrow \quad & \pi_1(X^*) \ge \frac{e^{-\alpha T}}{(1-e^{-\alpha T})}\Big[\pi_2(\tilde{X}^2; \tilde{X}^1) - \pi_2(X^2; X^*)\Big] + \pi_1(\tilde{X}^1) \end{aligned} \tag{2.6}$$

holds for any $(\tilde{X}^1, \tilde{X}^2)$. Since $\pi_1(X^*) \ge \pi_1(\tilde{X}^1)$ and $\frac{e^{-\alpha T}}{(1-e^{-\alpha T})}$ approaches to 0 as T increases, there exists $\bar{\tau}$ such that it satisfies (2.6) if $\bar{\tau} \le T$.

(ii) Let X^* be the optimal solution for the [SRLP] for given θ. Similar to (i), we show that there exists $\underline{\tau}$ such that, if $\underline{\tau} \ge T$, then

$$\begin{aligned} & Z_{[TRLP]}(X^1, X^*) \ge Z_{[TRLP]}(\tilde{X}^1, \tilde{X}^2) \\ \Longleftrightarrow \quad & \pi_2(X^*; X^1) \ge \frac{(1-e^{-\alpha T})}{e^{-\alpha T}}\Big[\pi_1(\tilde{X}^1) - \pi_1(X^1)\Big] + \pi_2(\tilde{X}^2; \tilde{X}^1) \end{aligned} \tag{2.7}$$

holds for any $(\tilde{X}^1, \tilde{X}^2)$. Since $\frac{(1-e^{-\alpha T})}{e^{-\alpha T}}$ is increasing in T, the threshold value $\underline{\tau}$ which satisfies (2.7) can be obtained when $X^1 = \emptyset$. That is,

$$T \le \underline{\tau} = -\frac{1}{\alpha}\ln\left[\max\left[\frac{\pi_1(\tilde{X}^1)}{\pi_2(X^*;\emptyset) - \pi_2(\tilde{X}^2;\tilde{X}^1) + \pi_1(\tilde{X}^1)}\right]\right].$$

Note $\pi_2(X^*;\emptyset) \ge \pi_2(\tilde{X}^2;\tilde{X}^1)$ for any θ, thus $0 \le \frac{\pi_1(\tilde{X}^1)}{\pi_2(X^*;\emptyset)-\pi_2(\tilde{X}^2;\tilde{X}^1)+\pi_1(\tilde{X}^1)} \le 1$. Therefore, there exists $\underline{\tau}$ such that it satisfies (2.7) if $T \le \underline{\tau}$. □

Proposition 2.4 (i) implies that if the market learning time is long enough, the second stage profit will be small enough not to affect the first stage decision. Hence, the first-stage action will be identical to the single-stage case. On the other hand, Proposition 2.4 (ii) implies that if the intrinsic market learning time is fast enough, there is no incentive to take action in the first stage. The firm can rather maximize the profit by forgoing the first stage and deploy facilities on the second stage with full market information. To summarize, this proposition suggests that the firm always benefits by deploying facilities in the second stage (thus less in the first stage) unless T takes an extremely large value. In fact, as the market learning T decreases, the firm is more likely to be less aggressive in the first stage. This captures the deferred commitment feature in the retail store location problem.

We now consider that the market learning time, T, is endogenously determined as a function of first stage action. While T can be a function of X^1 in any form, we assume it depends on the first-stage "coverage" defined as $c(X^1) = \sum_{i \in I} h_i g_i(d(X^1))$. More specifically, we assume $T = \phi(c(X^1)) > 0$ is a decreasing function in coverage with some finite intrinsic learning time $\phi(\emptyset)$. Hence, the more consumers covered in the first stage, the faster the consumer adoption rate is learned. Finally, the two-stage retail location problem with learning [TRLP-L] is:

$$\textbf{[TRLP-L]} \max_{\substack{X^1 \subset J \\ X^2 \subset J \setminus X^1}} \frac{1}{\alpha} \Bigg\{ \mathbb{E}_\theta \Bigg[(1 - e^{-\alpha T}) \sum_{i \in I} r\theta h_i g_i(d_i(X^1)) - f(X^1) + e^{-\alpha T} \Bigg(\sum_{i \in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^2) \Bigg) \Bigg] \Bigg\} \tag{2.8}$$

where the learning time is $T = \phi(c(X^1))$.

We note that endogenous learning time does not affect the nodal optimality that we showed for [TRLP] in Proposition 2.3. That is, if the facility operations cost on the edge connecting two nodes is a convex combination of those two facility costs, then there exists at least one optimal solution for this problem which corresponds to locating only on the nodes of a network for both stages.

The endogenous learning time introduces incentive for the firm to be aggressive in the first stage since it shortens the market learning time. Thus the option of market learning leads the firm to prefer "active learning." We illustrate this effect by contrasting the optimal solutions of the endogenous and exogenous learning models in the following proposition.

Proposition 2.5. *Let* (X^{1*}, X^{2*}) *be the optimal solution of the [TRLP-L] and* $c(X^{1*})$*,* $T^* = \phi(c(X^{1*}))$ *be the corresponding first stage coverage and the learning time. For the exogenous learning time* $T = T^*$*, let* (X_T^{1*}, X_T^{2*}) *be the optimal solution of the [TRLP] and* $c(X_T^{1*})$ *be the corresponding first stage coverage. Then,* $c(X_T^{1*}) \leq c(X^{1*})$*.*

Proof.

Lemma 2.1. *The optimal objective value* $Z_{[TRLP]}$ *is a decreasing function in* T*.*

Proof. As in the proof of Proposition 2.4, we express the objective function of the [TRLP] as $Z_{[TRLP]} = \frac{1}{\alpha}\big\{(1 - e^{-\alpha T})\pi_1(X^1) + e^{-\alpha T}\pi_2(X^2; X^1)\big\}$ where $\pi_1(X^1)$ and $\pi_2(X^2; X^1)$ are the first-stage and second-stage expected profit per unit time, respectively. Thus, $Z_{[TRLP]} = \frac{1}{\alpha}\big\{\pi_1(X^1) + e^{-\alpha T}(\pi_2(X^2; X^1) - \pi_1(X^1))\big\}$. Since $\pi_1(X^1) \leq \pi_2(X^2; X^1)$, we know that $Z_{[TRLP]}$ is decreasing in T. $\square$

First, we know $Z_{[TRLP-L]}(X^{1*}, X^{2*}) \leq Z_{[TRLP]}(X_T^{1*}, X_T^{2*})$ if $T = T^*$ is given for the [TRLP] (because [TRLP] has less constraint than [TRLP-L]). Now, suppose $c(X_T^{1*}) > c(X^{1*})$. This implies $T^* > T = \phi(c(X_T^{1*}))$ since $\phi(c)$ is decreasing in coverage c. From Lemma 2.1, we know that $\frac{1}{\alpha}\big\{(1 - e^{-\alpha T^*})\pi_1(X_T^{1*}) + e^{-\alpha T^*}\pi_2(X_T^{2*})\big\} < \frac{1}{\alpha}\big\{(1 - e^{-\alpha T})\pi_1(X_T^{1*}) + e^{-\alpha T^*}\pi_2(X_T^{2*})\big\}$.

Thus, it follows that

$$\begin{aligned} Z_{[TRLP-L]}(X^{1*}, X^{2*}) &= \frac{1}{\alpha}\Big\{(1-e^{-\alpha T^*})\pi_1(X^{1*}) + e^{-\alpha T^*}\pi_2(X^{2*})\Big\} \\ &\leq \frac{1}{\alpha}\Big\{(1-e^{-\alpha T^*})\pi_1(X_T^{1*}) + e^{-\alpha T^*}\pi_2(X_T^{2*})\Big\} \\ &< \frac{1}{\alpha}\Big\{(1-e^{-\alpha T})\pi_1(X_T^{1*}) + e^{-\alpha T}\pi_2(X_T^{2*})\Big\} \\ &= Z_{[TRLP-L]}(X_T^{1*}, X_T^{2*}). \end{aligned}$$

This is a contradiction since $Z_{TRLP-L}(X^{1*}, X^{2*})$ is the optimal objective value for the [TRLP-L]. □

The proposition shows that the presence of endogenous learning promotes the firm to cover more consumers in the first stage and learn the market faster. This is because the decision maker for the exogenously determined market learning time model does not have any incentive to aggressively deploy facilities given the same amount of learning time. This captures active learning feature in the retail store location problem.

As shown in Proposition 2.4 and 2.5, the proposed model clearly preserves the trade-off between "active learning" and "deferred commitment." While the firm always benefits from a short learning time, first stage decision is irreversible in the future. Hence, short-sighted initial decisions may lead to sub-optimal facility locations and harm the firm in the long run. In the next chapters, we develop a solution method for the proposed model in Chapter 4 and derive relevant managerial insights in Chapter 5.

CHAPTER 3

SOLUTION APPROACH

3.1. Nonlinear Integer Programming Formulation

In this section, we present the solution approach to the two-stage retail location problem with market learning. We first formulate the problem as an integer programming problem. Let θ be a discrete random variable with $|S|$ possible outcomes (scenarios) such that the probability of a scenario s is $P(\theta = \theta^s) = p^s$ and $\sum_{s \in S} p^s = 1$. Thus, the problem can be formulated as follows:

$$
\begin{aligned}
\text{[P1]} \quad \max_{\mathbf{X},\mathbf{Y},T} \sum_{s \in S} \frac{p^s}{\alpha} \Bigg[& \left(1 - e^{-\alpha T}\right) \sum_{i \in I} \sum_{j \in J} r \theta^s h_i g_i(d_{ij}) Y_{ij}^1 - \sum_{j \in J} f_j X_j^1 \\
& + e^{-\alpha T} \left\{ \sum_{i \in I} \sum_{j \in J} r \theta^s h_i g_i(d_{ij}) Y_{ijs}^2 - \sum_{j \in J} f_j X_{js}^2 \right\} \Bigg] \qquad (3.1)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & Y_{ij}^1 \le X_j^1, \quad Y_{ijs}^2 \le X_j^1 + X_{js}^2 \quad \forall i \in I, \forall j \in J, \forall s \in S, && (3.2) \\
& \sum_{j \in J} Y_{ij}^1 = 1, \quad \sum_{j \in J} Y_{ijs}^2 = 1 \quad \forall i \in I, \forall s \in S, && (3.3) \\
& X_j^1 + X_{js}^2 \le 1 \quad \forall j \in J, \forall s \in S, && (3.4) \\
& T \ge \phi\left(\sum_{i \in I} \sum_{j \in J} h_i g_i(d_{ij}) Y_{ij}^1 \right), && (3.5) \\
& T \ge 0, \quad X_j^1, X_{js}^2, Y_{ij}^1, Y_{ijs}^2 \in \{0, 1\} \quad \forall i \in I, \forall j \in J, \forall s \in S. && (3.6)
\end{aligned}
$$

The objective function (3.1) consists of four terms. The first term represents the expected present value discounting revenue over T periods of time for stage 1 and second term accounts for total cost of the operating facilities in the first stage over the two stages. The third term represents the discounted value of the second stage revenue and the last term represents the total cost for the operating facilities opened in the second stage discounted at present value from time T to ∞. Constraints (3.2) state that each demand can only be covered by an open facility for each stage. Constraints (3.3) ensure that each demand is covered by at least one facility. Constraints (3.4) state that we cannot locate another facility if one already exist. We refer constraint (3.5) as the Coverage constraint, since it expresses the relationship between learning time T and the coverage $c(X^1, Y^1) = \sum_{i \in I} \sum_{j \in J} h_i g_i(d_{ij}) Y^1_{ij}$. Constraints (3.6) represent non-negativity and integrality of decision variables.

The above problem is a mixed integer program with X^1, Y^1, X^2 and Y^2 as decision variables and T as an auxiliary decision variable. The learning time T is endogenously determined by the first stage demand coverage defined as $c(X^1, Y^1)$. For convenience, we will use c to represent the coverage in the first stage and also use $T = \phi(c)$. We obtain the upper bound in coverage, $\overline{c}$, by opening all the facilities in the first stage. The lower bound in coverage, $\underline{c}$, can be obtained by finding the commonly opened facilities for solving the single-stage problem with the adoption rate in each scenario. Since the endogenous characteristic of learning time T brings nonlinearity to the objective function (3.1), the currently proposed formulation [P1] is very challenging to solve. In the following section, we provide an approximation algorithm for the problem.

3.2. Solution Method

In this section, we develop an efficient solution method to solve the nonlinear integer program. Note that the constraint set of problem [P1] may not necessarily be a convex set due to (3.5). Mahajan and Munson (2010) proposed to solve a class of nonlinear programming problems involving nonconvex constraint sets by decomposing the constraint set into several convex sets. Similar to this approach, we decompose the constraint set into several subproblems with convex constraint sets and then use standard convex optimization technique to solve the individual subproblems.

To remove the exponential terms in the objective function in [P1], we first introduce a new decision variable $W = e^{-\alpha T}$. Then, we know the following:

Lemma 3.1. *$W = e^{-\alpha T}$ is an increasing function of the coverage $c(X^1, Y^1)$.*

Proof. Since $T = \phi(X^1, Y^1) = \phi(c(X^1, Y^1))$ is a decreasing function of coverage $c(X^1, Y^1)$, so T decreases as we increase the coverage and $W = e^{-\alpha T}$ being a decreasing function of T increases with a decrease in T. Hence $W(c) = e^{\phi(c)} = e^{-\alpha T}$ is an increasing function of coverage. □

Using W, the Constraint (3.5) can be revised as $W \leq e^{-\alpha\phi(c)}$ eliminating the decision variable T. Since this constraint may create non-convexity in the constraint set, we approximate $W = e^{-\alpha\phi(c)}$ to $\widehat{W}$ using piece-wise linear functions of c. More specifically, we divide the range of first stage coverage into a number of intervals such that the linear approximation of W in each interval satisfies $0 \leq \frac{W - \widehat{W}}{W} \leq \epsilon$. The error rate ϵ determines the precision level of the proposed approximation. Denoting the resulting intervals by $k \in K$, we represent the lower and upper bound of coverage for each interval by $\underline{c}_k$ and $\overline{c}_k$, and the corresponding bounds of W by $\underline{\omega}_k$ and $\overline{\omega}_k$, respectively. The linear approximation in the k^{th} interval can be expressed as $\widehat{W} = a_k + b_k c_k$, where a_k and b_k are constants, and the coverage $c_k \in [\underline{c}_k, \overline{c}_k]$. Note that the piece-wise linear approximation of $W = e^{-\alpha\phi(c)}$ should be an increasing function of c to avoid overlapping intervals $[\underline{\omega}_k, \overline{\omega}_k]$. This satisfies Lemma 3.1 and thus learning time T is decreasing function of coverage.

We approximate the Coverage constraint by $|K|$ linear functions with the domain restricted to $[\underline{c}_k, \overline{c}_k]$ for the k^{th} approximation. So, the problem [P1] is decomposed into $|K|$ subproblems. The optimal solution of problem [P1] then corresponds to the maximum of the optimal solutions of these subproblems. Hence the problem [P1] assumes the following form.

$$[\text{P2}] \quad \max_{k \in K} \ \Pi_k,$$

$$\text{where } \Pi_k = \max_{\mathbf{X},\mathbf{Y},\widehat{W}} \sum_{s\in S} \frac{p^s}{\alpha} \Bigg[\big(1-\widehat{W}\big) \sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij}) Y^1_{ij} - \sum_{j\in J} f_j X^1_j + \widehat{W} \left\{ \sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij}) Y^2_{ijs} - \sum_{j\in J} f_j X^2_{js} \right\} \Bigg]$$

$$\begin{aligned}
\text{s.t.} \quad & Y^1_{ij} \le X^1_j, \quad Y^2_{ijs} \le X^1_j + X^2_{js} \quad \forall i \in I, \forall j \in J, \forall s \in S,\\
& \sum_{j\in J} Y^1_{ij} = 1, \quad \sum_{j\in J} Y^2_{ijs} = 1 \quad \forall i \in I, \forall s \in S,\\
& X^1_j + X^2_{js} \le 1 \quad \forall j \in J, \forall s \in S,\\
& \widehat{W} = a_k + b_k c_k,\\
& \widehat{W} \in [\underline{\omega}_k, \overline{\omega}_k], \quad X^1_j, X^2_{js}, Y^1_{ij}, Y^2_{ijs} \in \{0,1\} \quad \forall i \in I, \forall j \in J, \forall s \in S.
\end{aligned}$$

The coverage constraint is changed from an inequality to an equality constraint to impose the bounds on coverage, since each interval of coverage is associated with the respective interval $[\underline{\omega}_k, \overline{\omega}_k]$. The problem [P2] is an approximation of the problem [P1], however we can find the bound on the relative error that accumulates in approximating the total profit over the two stages.

Proposition 3.1. *Let* $\Pi(W)$ *and* $\Pi(\widehat{W})$ *be the optimal profits of the problems [P1] and [P2] respectively. Then,* $\Pi(\widehat{W})$ *is a lower bound for* $\Pi(W)$ *and the relative error between* $\Pi(\widehat{W})$ *and* $\Pi(W)$ *is bounded by the relative error rate in linear approximation* ϵ*; i.e.,* $\frac{\Pi(W)-\Pi(\widehat{W})}{\Pi(W)} \leq \epsilon$.

Proof. Let us denote the first stage expected profit per unit period (terms in the objective function corresponding to the first stage) by $\pi_1(X^1, Y^1) = \sum_{s\in S} p^s \left[\sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij})Y^1_{ij} - \sum_{j\in J} f_j X^1_j\right]$ and the second-stage expected profit per unit period by $\pi_2(X^2, Y^2; X^1, Y^1) = \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} rp^s\theta^s h_i g_i(d_{ij})Y^2_{ijs} - \sum_{s\in S}\sum_{j\in J} p^s f_j X^2_{js} - \sum_{j\in J} f_j X^1_j$. Therefore, the objective function can be expressed as

$$\begin{aligned}\Pi(W) &= \frac{1}{\alpha}\left[(1-W)\pi_1(X^1, Y^1) + W\pi_2(X^2, Y^2; X^1, Y^1)\right] \\ &= \frac{1}{\alpha}\left[\pi_1(X^1, Y^1) + W\left\{\pi_2(X^2, Y^2; X^1, Y^1) - \pi_1(X^1, Y^1)\right\}\right].\end{aligned}$$

Here we note that $\pi_1(X^1, Y^1) \leq \pi_2(X^2, Y^2; X^1, Y^1)$ holds for any solution (X^1, Y^1, X^2, Y^2) since one can only improve the expected unit profit in the second stage by deploying additional facilities (otherwise, one can preserve the first stage expected unit profit by choosing not to open new facilities).

Now from the error rate inequality $\frac{W-\widehat{W}}{W} \leq \epsilon$, it follows that $(1-\epsilon)W \leq \widehat{W}$

and thus

$$\begin{aligned}(1-\epsilon)\Pi(W) &= (1-\epsilon)\frac{1}{\alpha}\big[\pi_1 + W(\pi_2-\pi_1)\big] \\ &\leq \frac{1}{\alpha}\left[\pi_1 + (1-\epsilon)W(\pi_2-\pi_1)\right] \\ &\leq \frac{1}{\alpha}\left[\pi_1 + \widehat{W}(\pi_2-\pi_1)\right] = \Pi(\widehat{W}).\end{aligned}$$

Hence, we have $\frac{\Pi(W)-\Pi(\widehat{W})}{\Pi(W)} \leq \epsilon$. □

Proposition 3.1 states that the approximate profit obtained by the solving problem [P2] provides a good lower bound on the true optimal profit which is the objective value of the problem [P1]. This enables us to bound the relative error rate in the approximate and the true profit by ϵ. Since this error rate is the same as the error rate of linear approximation, therefore the network designer can achieve the desired precision in the profit approximation by appropriately choosing the error rate ϵ. These results are based on the fact that we intend to under-approximate the profit and thus it provides us with a least profit that can be obtained following the approximation. In other words, the decision maker can be conservative while approximating the value of W, so that the approximated profit provides a lower bound for the exact solution.

Although the constraint sets in problem [P2] are linear, the objective functions Π_k's still contain nonlinearity because they involves product of decision variables namely of W with Y^1, Y^2 and X^2. We linearize the objective function by exploiting the fact that Y^1, Y^2 and X^2 are binary variables. We introduce a constraint for each variable which allows the decision variable W to get into effect if the corresponding binary variable is 1 and forces the value of the product to be 0 otherwise. But to achieve this, we first introduce continuous decision variable ζ^{Y1}, ζ^{Y2} and ζ^{X2} corresponding to each binary variable. We define the coefficients of the binary variables as $D_{ij}^{Y1}(\widehat{W}) = (\frac{1}{\alpha}) r h_i g_i(d_{ij}) \sum_{s\in S} p^s \theta^s \widehat{W}$, $D_{ijs}^{Y2}(\widehat{W}) = (\frac{1}{\alpha}) r p^s \theta^s h_i g_i(d_{ij}) \widehat{W}$ and $D_{js}^{X2}(\widehat{W}) = (\frac{1}{\alpha}) p^s f_j \widehat{W}$, $\forall i \in I, \forall j \in J, \forall s \in S$, so that we can express the objective function Π_k as

$$\max_{\mathbf{X},\mathbf{Y},\widehat{W}} \sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(1) Y_{ij}^1 - \sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(\widehat{W}) Y_{ij}^1 - \frac{1}{\alpha}\sum_{s\in S} p^s \sum_{j\in J} f_j X_j^1$$
$$+ \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} D_{ijs}^{Y2}(\widehat{W}) Y_{ijs}^2 - \sum_{s\in S}\sum_{j\in J} D_{js}^{X2}(\widehat{W}) X_{js}^2 \tag{3.7}$$

Now provided that we have the following bounds for the values of $D's$ subject to the constraints in Problem [P2]

$$\underline{D}_{ij}^{Y1} \le D_{ij}^{Y1}(\widehat{W}) \le \overline{D}_{ij}^{Y1}$$
$$\underline{D}_{ijs}^{Y2} \le D_{ijs}^{Y2}(\widehat{W}) \le \overline{D}_{ijs}^{Y2}$$
$$\underline{D}_{js}^{X2} \le D_{js}^{X2}(\widehat{W}) \le \overline{D}_{js}^{X2}$$

then by following the technique of Oral and Kettani (1992) we express problem [P2] as:

[P3] $\max_{k\in K} \Pi_k$,

where $\Pi_k = \max_{\mathbf{X},\mathbf{Y},\widehat{W},\zeta_{ij}^{Y1},\zeta_{ijs}^{Y2},\zeta_{js}^{X2}} \sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(1) Y_{ij}^1 - \sum_{i\in I}\sum_{j\in J} (\underline{D}_{ij}^{Y1} Y_{ij}^1 + \zeta_{ij}^{Y1})$

$$- \frac{1}{\alpha}\sum_{s\in S} p^s \sum_{j\in J} f_j X_j^1 + \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} (\overline{D}_{ijs}^{Y2} Y_{ijs}^2 - \zeta_{ijs}^{Y2})$$
$$- \sum_{s\in S}\sum_{j\in J} (\underline{D}_{js}^{X2} X_{js}^2 + \zeta_{js}^{X2})$$

$$
\begin{aligned}
\text{s.t.}\quad & \zeta_{ij}^{Y1} \geq D_{ij}^{Y1}(\widehat{W}) + (\overline{D}_{ij}^{Y1} - \underline{D}_{ij}^{Y1})Y_{ij}^1 - \overline{D}_{ij}^{Y1} \quad \forall i \in I, \forall j \in J \\
& \zeta_{ijs}^{Y2} \geq -D_{ijs}^{Y2}(\widehat{W}) + (\overline{D}_{ijs}^{Y2} - \underline{D}_{ijs}^{Y2})Y_{ij}^2 + \underline{D}_{ijs}^{Y2} \quad \forall i \in I, \forall j \in J, \forall s \in S \\
& \zeta_{js}^{X2} \geq D_{js}^{X2}(\widehat{W}) + (\overline{D}_{js}^{X2} - \underline{D}_{js}^{X2})X_{js}^2 - \overline{D}_{js}^{X2} \quad \forall j \in J, \forall s \in S \\
& Y_{ij}^1 \leq X_j^1, \quad Y_{ijs}^2 \leq X_j^1 + X_{js}^2 \quad \forall i \in I, \forall j \in J, \forall s \in S, \\
& \sum_{j \in J} Y_{ij}^1 = 1, \quad \sum_{j \in J} Y_{ijs}^2 = 1 \quad \forall i \in I, \forall s \in S, \\
& X_j^1 + X_{js}^2 \leq 1 \quad \forall j \in J, \forall s \in S, \\
& \widehat{W} = a_k + b_k c_k, \\
& X_j^1, X_{js}^2, Y_{ij}^1, Y_{ijs}^2, Z_\tau \in \{0, 1\} \quad \forall i \in I, \forall j \in J, \forall s \in S, \\
& \zeta_{ij}^{Y1}, \zeta_{ijs}^{Y2}, \zeta_{js}^{X2} \geq 0, \quad \widehat{W} \in [\underline{\omega}_k, \overline{\omega}_k] \quad \forall i \in I, \forall j \in J, \forall s \in S.
\end{aligned}
$$

In order to calculate the bounds on $D's$, we utilize the fact that the functions $D's$ are increasing functions of $\widehat{W}$ and consequently achieve the lower and upper bounds at the the respective bounds of $\widehat{W}$.

3.3. Computational Performance

We performed a set of numerical experiments to illustrate the performance of the proposed solution method. The algorithm is coded in C++ integrating ILOG CPLEX 12.2 and run on an HP Z400 desktop with 2.93GHz CPU and 16GB of RAM. The proposed algorithm was applied to the networks generated from the networks constructed using 2000 census data from the city of Chicago. The data includes the distance matrix with each element representing the distance between the nodes, average income of the household and population at various nodes. We generate facility cost f_i proportional to the income and potential consumers h_i proportional to the population of the nodes $i \in I$.

The computational performance of the algorithm (solution time measured in seconds) is illustrated in Table 3.1. We use two different networks of size 43 (downtown Chicago) and 102 (greater downtown Chicago) with three different sets of candidate sites to evaluate the performance of the proposed algorithm. For each of these settings, we report the average solution time of 10 instances (generated by varying the standard deviation of the adoption rate) for both 25 and 50 scenarios. The standard deviation is varied by changing the support of the uniform distribution where the mean adoption rate is 0.5. The market learning time is determined by $T = \gamma e^{-\beta c(X^1, Y^1)}$ with $\gamma = 30$ and $\beta = 3.5/\overline{c}$, where $\overline{c}$ is lower bound of the first stage coverage for each problem. Other parameters are set to $\alpha = 0.05, \lambda = 0.01$ and $r = 190$. We present the solution times for two different levels of error rate, $\epsilon = 0.1\%$ and 0.01%.

Demand ($\|I\|$)	Candidate Sites ($\|J\|$)	Scenarios ($\|S\|$)			
		$\epsilon = 0.1\%$		$\epsilon = 0.01\%$	
		25	50	25	50
43	10	19	22	40	44
	20	31	44	58	83
	30	190	311	380	594
102	30	216	681	402	1,239
	50	823	2,991	1,541	5,429
	70	1,507	9,523	2,703	15,023

Table 3.1: Computational performance of the proposed algorithm

As shown in Table 3.1, the proposed algorithm solves the problem quite efficiently. The largest problem in the study with 100 demand nodes and 70 candidate locations in less that 10,000 seconds on average with an error rate of 0.1%. Note that there is a significant difference between the solution times of 25 and 50 scenarios indicating that the solution time is substantially affected by the number of scenarios. As anticipated, we observe that a higher level of accuracy in the optimal solution increases the solution time.

CHAPTER 4

NUMERICAL RESULTS AND INSIGHTS

In this chapter, we numerically study the impact of considering the market learning effect in retail store location decisions. We first study the value of foresight in the presence of market learning effect by contrasting the optimal planner to a myopic planner who does not have foresight. We further analyze the trade-off between active learning and deferred commitment under markets with different consumer learning characteristics. We use an example of 102 demand nodes with 70 candidate store locations from the previous chapter. The number of scenarios is set to 25 and the parameters are set identical as in Section 3.3 unless stated otherwise.

4.1. Value of Foresight with Market Learning Effect

We consider a myopic planner who maximizes the profit for the current stage only without considering recourse option. Myopic planner represents a firm without foresight: a firm who decides to abandon the long term plan with a pressure to perform immediately. This exercise illustrates how firms may react differently under different market circumstances depending on their degree of foresight. Similar to the optimal planner, the myopic planner only knows the distribution of the adaption rate and deploys the facilities based on the expected adoption rate in the first stage. While making the first stage decision, however, she does not take into account future decisions although market learning process is still in place based on the first stage coverage. After the learning is completed, the myopic planner will then deploy additional facilities to maximize the profit of her second stage. The formulation for the two-stage myopic retail location problem is as follows:

$$\int_0^T e^{-\alpha t}\overline{V}(X^1)dt + \int_T^\infty e^{-\alpha t}\overline{V}(X^2; X^1, \theta)dt, \tag{4.1}$$

$$\begin{aligned}\text{where}\quad \overline{V}(X^1) &= \max_{X^1 \subset J}\left\{\mathbb{E}_\theta\left[\sum_{i\in I} r\theta h_i g_i(d_i(X^1)) - f(X^1)\right]\right\},\\ \overline{V}(X^2; X^1, \theta) &= \max_{X^2 \subset J\setminus X^1}\left\{\sum_{i\in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^1 \cup X^2)\right\}\end{aligned}$$

and the learning time is $T = \phi(c(X^1))$. The objective function of the myopic planner's problem (4.1) can be rewritten as

$$\frac{1}{\alpha}\Big\{(1 - e^{-\alpha T})\overline{V}(X^1) + e^{-\alpha T}\overline{V}(X^2; X^1, \theta)\Big\}. \tag{4.2}$$

To contrast the behavior of two planners, we use the numerical example introduced in the previous section with $\beta = 3.5/\bar{c}$. We compare their respective decisions as the market variability changes, where market variability represents coefficient of variation of the adoption rate. The adoption rate follows a uniform distribution with mean of 0.5. For convenience, we let Π_O, f_O, r_O and N_O, and Π_M, f_M, r_M and N_M be the expected total profit, facility operation cost, total revenue and average number of opened facilities corresponding to the optimal and the myopic planner respectively. Also, let π_O^i and π_M^i denote the expected profit per unit time in stage i for the optimal and the myopic planner respectively.

Since the myopic planner seeks to maximize the current stage's expected profit only, her first stage expected unit profit is always equal or greater than that of the optimal planner ($\pi_O^1 \leq \pi_M^1$) as shown in Figure 4.1. The first stage expected unit profit for the myopic planner remains flat regardless of market uncertainty because it does not take into account the second stage. On the other hand, the optimal planner attempts to hedge against market uncertainty, thus the first stage profit decreases as market variability increases. In the second stage, however, the optimal planner gains a greater expected unit profit ($\pi_O^2 \leq \pi_M^2$). In fact, we observe that the difference between the two planners' second stage expected unit profit increases with market uncertainty. This explains the optimal planner's decreasing profit in the first stage pays off in the second stage as market uncertainty increases.

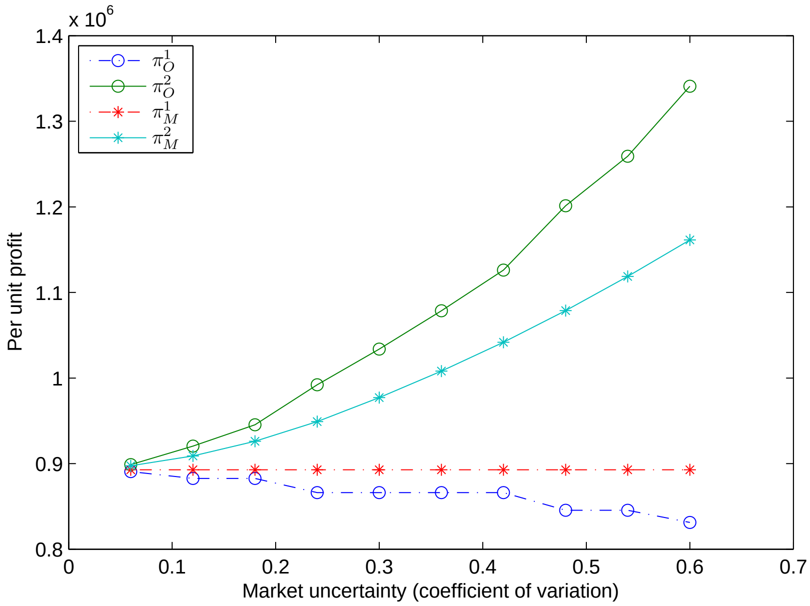

Figure 4.1: Profit comparison for the optimal and the myopic planner at each stage

Table 4.1 compares the two planners' optimal profits and the corresponding market learning time for different levels of market variability. The expected total profit for both planners increase with market variability. This means the

Market Variability	Optimal Planner				Myopic Planner			
	Profit			T_O	Profit			T_M
	π_O^1	π_O^2	Π_O		π_M^1	π_M^2	Π_M	
0.06	891	899	17,930	6.50	893	898	17,925	6.22
0.12	883	920	18,177	7.31	893	909	18,093	6.22
0.18	883	945	18,523	7.31	893	926	18,343	6.22
0.24	866	992	18,990	8.23	893	949	18,681	6.22
0.30	866	1,034	19,542	8.23	893	977	19,092	6.22
0.36	866	1,077	20,134	8.23	893	1,008	19,545	6.22
0.42	866	1,126	20,765	8.23	893	1,042	20,040	6.22
0.48	845	1,201	21,449	8.97	893	1,079	20,581	6.22
0.54	845	1,259	22,188	8.97	893	1,119	21,1667	6.22
0.60	831	1,340	22,963	9.48	893	1,161	21,792	6.22

Table 4.1: Comparison of profits (in thousand \$) and learning time (in unit time) for the optimal and the myopic planner

firm can gain higher expected total profit when entering a market with higher uncertainty since it can take advantage of high adoption rates in a highly uncertain market. This insight coincides with the real options literature (Dixit (1992)): the value of real option increases as the market variability increases (the decision maker can exercise the option when market turns out to be good; otherwise, simply stay put). However, the difference between the total profit of the optimal and the myopic planner increases as market uncertainty increases. This reveals that lack of foresight harms the firm more as the market variability increases. Myopic planner does not account for the value of this real option in her first stage decisions, although she still benefits from the option when the second stage comes.

We also note that the market learning time of the optimal planner increases as the variability in the market increases. This suggests that the optimal planner takes a cautious approach, preferring deferred commitment, when entering markets with high uncertainty. The learning time for the myopic planner however does not change with market variability since its coverage remains the same regardless of market variability.

Table 4.2 shows that the myopic planner invests more on facility (both in terms of numbers and cost) relative to the optimal planner. The over-commitment in facility investment for the myopic planner is due to its lack of foresight while making decisions in the first stage. Since the myopic planner invests more on the facility costs, she typically gains higher revenue. However, the difference in revenue and facility cost (i.e. the expected total profit) of the optimal planner is always greater than that of the myopic planner and it increases with market variability.

Market Variability	Optimal Planner			Myopic Planner		
	N_O	f_O	r_O	N_M	f_M	r_M
0.06	17.4	42,707	60,638	17.76	43,635	61,561
0.12	17.64	41,666	59,843	18.48	45,389	63,482
0.18	18.16	42,780	61,303	19.00	46,694	65,037
0.24	18.20	41,217	60,208	19.52	48,122	66,803
0.30	18.68	42,275	61,818	20.04	49,416	68,508
0.36	19.00	42,924	63,058	20.36	50,169	69,714
0.42	19.52	44,087	64,852	20.92	51,548	71,588
0.48	19.40	42,507	63,955	21.24	52,339	72,920
0.54	19.80	43,215	65,402	21.68	53,274	74,440
0.60	20.52	42,658	65,621	22.24	54,454	76,245

Table 4.2: Comparison of facility cost and revenue for the optimal and the myopic planner (in thousand $)

To better understand the over-commitment in facility investment for the myopic planner, we compare the ratio of total facility operation cost to total revenue for both planners in Figure 4.2. This ratio represents the marginal rate of return in facility investment. We observe that the return on investment for the myopic planner is always lower than that of the optimal planner. Further, its difference increases as the market uncertainty increases. This illustrates that the over-commitment in facility deployment adversely effects the myopic planner and its magnitude increases with market uncertainty.

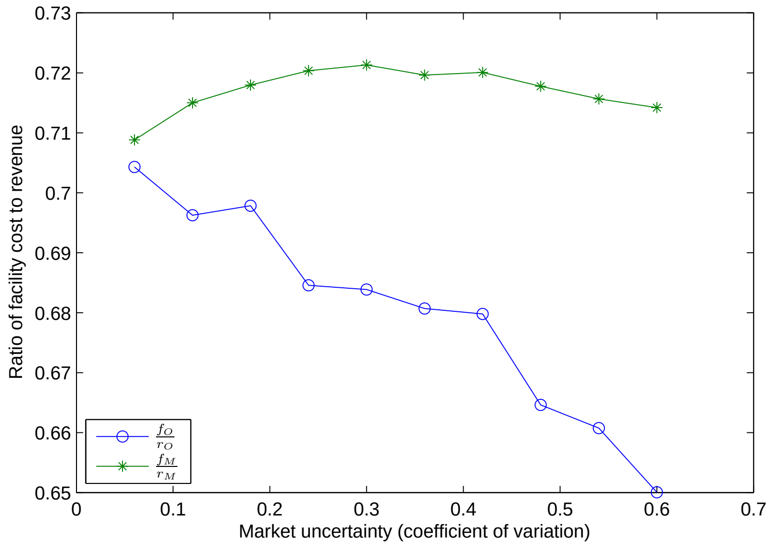

Figure 4.2: Over-commitment of the myopic planner

To summarize, the myopic planner's decision tends to differ from the optimal planner's decision with market uncertainty as evident from the above discussion. The myopic planner incurs larger facility cost but earns lower profits as compared to the optimal planner. On the other hand, the optimal planner becomes more prudent (deferred commitment) as uncertainty in the market increases and thus gains higher expected total profit over the two stages. Therefore, the firm should prefer deferred commitment over active learning as market uncertainty increases.

4.2. Active Learning vs. Deferred Commitment

We now construct four different markets where consumers take different amount of time on average before making the product adoption decision. Markets with many early adopters make faster product adoption decision on average and reveals the adoption rate quickly. For such market, the market learning time is shorter with same level of first stage demand coverage. On the other hand, markets with many late adopters are conservative to newly introduced products and make slower product adoption decision. We refer to this market behavior as sensitivity in new product introduction; i.e. the market learning time is shorter for the sensitive market since it reacts faster to the product exposure (first stage coverage). We define the market learning time as $T = \gamma e^{-\beta c(X^1, Y^1)}$ and capture different learning trends by varying γ as shown in Figure 4.3. Hence, a market with low γ represents sensitive market whereas a market with high γ represents insensitive market. Parameter β is set to $\beta = 3.0/\bar{c}$ for all markets. Rest of the parameters are set identical

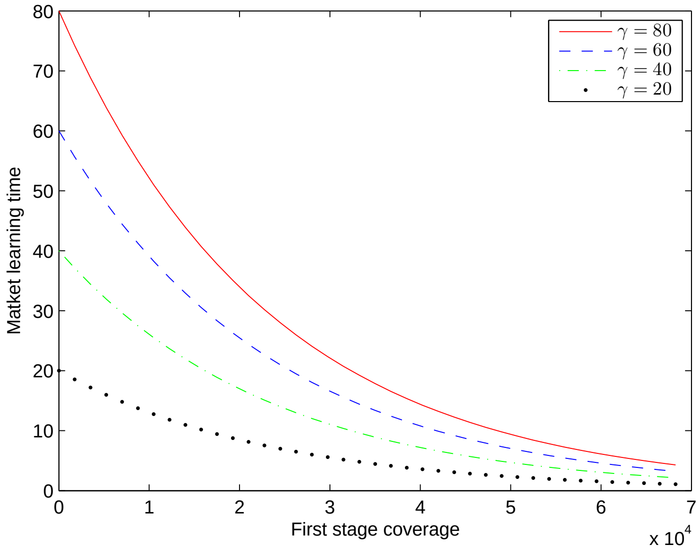

Figure 4.3: Market learning time with different market sensitivities

to previous section across all markets.

Table 4.3 compares the optimal solutions for markets with different sensitivities. We note that the firm increases its first stage coverage as market sensitivity decreases; i.e. the firm should increase the investment in facilities as the market becomes less sensitive to the product introduction. This suggests that the firm has incentive to learn the market faster as market sensitivity decreases and thus prefers active learning over deferred commitment. We further observe that the expected profit decreases as market sensitivity decreases. This is because the market takes longer time to reveal its adoption rate as market sensitivity decreases.

γ	20	40	60	80
Coverage	20,104	24,156	28,233	30,579
Profit (in thousand \$)	24,429	21,143	20,191	19,916

Table 4.3: Optimal profit and coverage comparison under different market sensitivities

CHAPTER 5

CONCLUSIONS

In this thesis, we study the two-stage retail location problem in the presence of endogenous market learning. In particular, a firm sequentially deploys facilities over two stages when the customer adoption rate is learned at the end of the first stage. We formulate the problem into a two-stage nonlinear integer program and propose an efficient and effective solution method. The proposed algorithm provides a framework to achieve a desired error rate of accuracy in the optimal solution.

Using the model, we first study the trade-off between active learning and deferred commitment under different consumer characteristics. We show that the firm should *prefer active learning over deferred commitment as market becomes insensitive to new product introduction.* In other words, the firm should be aggressive when the market has many late adopters while prudent when the market has many early adopters. Second, we investigate the value of foresight by contrasting the optimal planner to the myopic planner who does not take into account the effect of learning. We show that the firm should *prefer deferred commitment over active learning as market uncertainty increases* as the value of foresight increases with market uncertainty in the presence of market learning effect. We also show that lack of market learning typically leads the firm to over-commit in facility investments while earning lower expected profit.

This research can be extended in several ways. First, it would be interesting to consider site specific adoption rate θ_i. While adoption rates on the same market will be likely to be correlated, relaxing uniform adoption rate will certainly enrich the proposed model. Another possible research direction is to study a multi-stage version of the problem where the adoption rate (or its distribution) is partially learned over time. One may consider Bayesian learning scheme in implementing this research. Another potential extension is to incorporate firm's various risk attitudes in a two-stage model.

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