COMMUNICATION STRATEGIES FOR THE MIMO INTERFERENCE CHANNEL

BY

CRAIG WILSON

THESIS

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Adviser:

Professor Venugopal Veeravalli

ABSTRACT

Managing interference for wireless networks is crucial for meeting future demand for higher mobile data rates. Interference can be viewed through the interference channel (IC) in which pairs of transmitters and receivers interfere with each other, so an interesting way to manage interference is to develop communication strategies for the IC. We consider the problem of designing signals to transmit over the multiple input and multiple output (MIMO) interference channel by extending the Max SINR algorithm. The Max SINR algorithm starts with arbitrary beamformers and then designs optimal receivers to maximize the SINR at each receiver. The Max SINR algorithm then alternates the direction of communication and repeats this process. This algorithm is known to perform well, but there is no proof that it converges. We propose a modification to Max SINR using a power control step to make a metric similar to sum rate converge. With successive interference cancellation (SIC), then the new metric is exactly the sum rate. Finally, simulations show that the performance of the modified Max SINR algorithm, unlike other convergent alternatives, is nearly identical to that of the original Max SINR algorithm.

To my parents, for their love and support

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I would like to thank my adviser, Prof. Venugopal Veeravalli, for guiding me while carrying out this work. My thanks also go out my group members for useful discussions and advice. Finally, I would like to thank my parents for the support they have always provided.

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CHAPTER 1

INTRODUCTION

Traditionally interference has been managed in cellular networks through cell planning and treating any remaining interference as noise. Now, with many cellular networks interference limited, there has been an increased interest in more sophisticated schemes to manage interference through base station cooperation. These questions are especially pertinent as mobile data demand is expected to grow explosively as demonstrated in Figure 1.1.

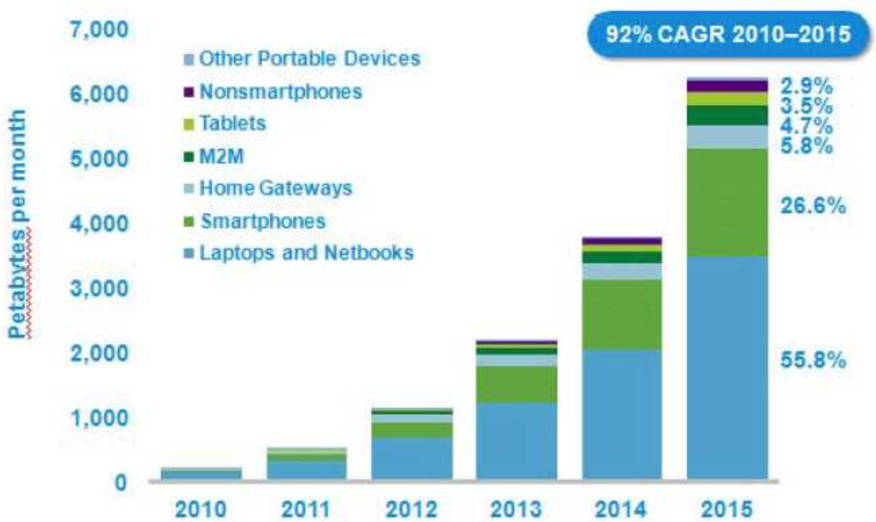


Figure 1.1: Cisco Mobile Data Demand Predictions [1]

The natural setting to consider managing interference is the interference channel in which each transmitter communicates with a paired receiver in the presence of interference from other pairs. As a result there is a great deal of interest in understanding the capacity of the interference channel (IC). Unfortunately, while some achievable rate regions are known from [2], the actual capacity region of the channel is unknown. In response to this difficulty several techniques have been developed to approximate the capacity region of the IC as in [3] with interference alignment and in [4] with capacity within a constant number of bits. These methods have provided some insight into how to communicate on the IC and suggest that there is significant value in more advanced ways to manage interference.

Based on insights gained from capacity analyses, several convergent iterative algorithms that seek to provide a high sum rate have been proposed using interference alignment (IA) from [3] as a motivation. In [5] an algorithm was proposed in which transmitters and receivers take turns adjusting their beamforming vectors to reduce interference leakage under the assumption of channel reciprocity. This algorithm monotonically reduces the total interference leakage power and converges. In [6] a similar algorithm without the reciprocity assumption was introduced. The work of [7] also extends [6] to give better sum rate performance.

Another iterative algorithm, Max SINR, was proposed in [5] that gives better sum rate performance. The Max SINR algorithm starts with arbitrary transmit beamforming vectors and then designs receivers to optimize the signal-to-interference-plus-noise ratio (SINR) at each receiver. Next, the algorithm alternates the direction of communication and repeatedly optimizes the SINR at each receiver. This algorithm outperforms the other algorithms mentioned before, but there is no proof that it actually converges.

In this thesis, we propose a convergent version of Max SINR with nearly identical sum rate performance. First, a power control step performed in each iteration insures that the same SINRs can be achieved in both directions of communication. With this observation, a performance metric similar to sum rate converges. In addition, using successive interference cancellation (SIC) makes the sum rate a monotonically increasing function of the SINRs, and so the sum rate converges. Since the sum rate for the interference channel is a non-convex function, the new algorithm, like the other proposed algorithms, converges to a local maximum and not necessarily a global maximum. Finally, simulations verify that the Modified Max SINR algorithm indeed has performance nearly identical to that of the original Max SINR algorithm.

1.1 Overview

In this thesis, we first give an overview of the interference channel model. Next, we describe several techniques to approximate the capacity of the interference channel. This is followed by a discussion of transmit strategies for the interference channel inspired by previously discussed approximate capacity techniques including the Max SINR algorithm. Finally, we introduce a convergent version of the Max SINR algorithm and analyze its performance through simulations.

1.2 Notation

Notation: Scalars are lower case, vectors are lower case bold, and matrices are upper case bold. Furthermore, $\boldsymbol{A}_{*d}$ is the d^{th} column of the matrix $\boldsymbol{A}$, $\boldsymbol{x}_\ell$ is the ℓ^{th} element of the vector $\boldsymbol{x}$, $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{A}^\dagger$ is the conjugate transpose, and $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of all users.

CHAPTER 2

INTERFERENCE CHANNEL MODEL

Consider a K-user multiple input and multiple output (MIMO) interference channel with the k^{th} user having $N_t^{[k]}$ inputs and $N_r^{[k]}$ outputs. The MIMO interference channel model here can arise from multiple antennas, from symbol extensions, or from OFDM subcarriers. Assuming each user sends $d^{[k]}$ streams the effective channel for user k is then given by Equation 2.1. We omit the channel use index as we generally consider communicating over one channel use, so only the constant MIMO interference channel is relevant. If we need to consider multiple channel uses, then we will use the channel use index n.

$$\boldsymbol{y}^{[k]} = \sum_{j=1}^{K} \boldsymbol{H}^{[kj]} \boldsymbol{x}^{[j]} + \boldsymbol{z}^{[k]} \tag{2.1}$$

The quantity $\boldsymbol{y}^{[k]}$ is the $N_r^{[k]} \times 1$ receive vector, $\boldsymbol{z}^{[k]}$ is the $N_r^{[k]} \times 1$ additive white Gaussian noise (AWGN) vector with distribution $\mathcal{CN}(0, \boldsymbol{I})$, $\boldsymbol{x}^{[k]}$ is the signal sent by user k satisfying the power constraint P_k:

$$\mathrm{tr}\{\mathbb{E}[\boldsymbol{x}^{[k]} \boldsymbol{x}^{[k]\dagger}]\} \leq P_k$$

$\boldsymbol{H}^{[kj]}$ is the $N_r^{[k]} \times N_t^{[j]}$ full rank matrix of channel coefficients between transmitter j and receiver k.

2.1 Capacity Region

Since the interference channel is a useful model for managing interference, it is of interest to quantify the performance in terms of rates achieved. This problem can be attacked using the same framework Shannon created for the simple point-to-point channel. The idea is to show exactly which rate tuples

$(R_1, \ldots, R_K) \in \mathbb{R}_+^K$ can be achieved by some type of coding scheme.

First, for a fixed rate tuple $(R_1, \ldots, R_K) \in \mathbb{R}_+^K$ consider coding over a block of length n with each message W_i taking values in the set $\{1, 2, \ldots, \lceil 2^{nR_i} \rceil\}$. Then a code consists of an encoder

$$\boldsymbol{x}_i^n : W_i \to \mathbb{C}^{N_t \times n}$$

and a decoder

$$\hat{W}_i : \mathbb{C}^{N_r \times n} \to \{1, 2, \ldots, \lceil 2^{nR_i} \rceil\}$$

Assuming the message is iid distributed, then the probability of decoding error is

$$e_n = \max_{1 \le i \le K} P\left(\hat{W}_i(\boldsymbol{y}_i^n) \neq W_i\right)$$

Then a rate tuple $(R_1, \ldots, R_K)$ is said to be *achievable* if there is a code such that $e_n \to 0$ as $n \to \infty$ that also satisfies an average power constraint

$$\mathbb{E}\left[\frac{1}{n}\sum_{j=1}^{n} \|X_i(t)\|^2\right] \le P_i \quad \forall i, 1 \le i \le K$$

Now the capacity region is defined to be the convex closure of all achievable rates.

$$\mathcal{C} = \overline{\bigcup_{\substack{\text{achievable} \\ (R_1,\ldots,R_K)}} (R_1, \ldots, R_K)}$$

The goal of an information theoretic analysis is to find the capacity region $\mathcal{C}$. The general approach to finding the capacity region is to find an achievable rate region $\mathcal{A}$ and an outer bound region $\mathcal{B}$ such that

$$\mathcal{A} \subset \mathcal{C} \subset B$$

and then show that in fact $\mathcal{A} = \mathcal{B}$, so that

$$\mathcal{C} = \mathcal{A} = \mathcal{B}$$

Then the capacity region is known and the possible performance of the IC is fully characterized.

2.2 Channel Knowledge

Throughout this thesis we will assume global channel knowledge. This means that the channel coefficients are fixed and known at all transmitters and receivers. The actual process used to find the channels is to have receiver i estimate the channels $\boldsymbol{H}^{[ij]}$ for all $i, 1 \leq i \leq K$. Then the estimated channels are fed back to the corresponding transmitter and exchanged with all other transmitters. For a cell network this exchange would occur through a fiber optic backhaul.

CHAPTER 3

APPROXIMATE CAPACITY OF THE INTERFERENCE CHANNEL

As discussed before, the problem of interference management can be formulated in terms of the interference channel introduced in the system model section, making the capacity region of the IC of great interest. Unfortunately, finding the capacity region of the interference channel has remained an open problem for over thirty years, so several techniques to describe the capacity region approximately have been developed. The two major techniques of interest are

1. **Capacity within a constant number of bits** - As the name suggests, try to find the capacity region within a constant number of bits for all signal-to-noise ratios (SNRs) and interference-to-noise ratios (INRs).

2. **Degrees of freedom** - Study the scaling of the capacity region at high SNR.

3.1 Capacity within a Constant Number of Bits

The idea of this technique is to find an achievable region contained inside the capacity region and an outer bound that contains the capacity region such that the gap between the achievable region and the outer bound is never more than a fixed number of bits regardless of channel parameters or transmit powers. This idea is illustrated in Figure 3.1.

The most prominent example of this idea is given in [4]. In this paper the authors find the capacity region of the two-user ($K = 2$), single input and single output (SISO) ($N_t^{[k]} = N_r^{[k]} = 1$) interference channel to within one bit. They use a simplified version of the Han-Kobayashi scheme proposed in [2] as the achievability scheme and develop several new upper bounds to better characterize the capacity region.

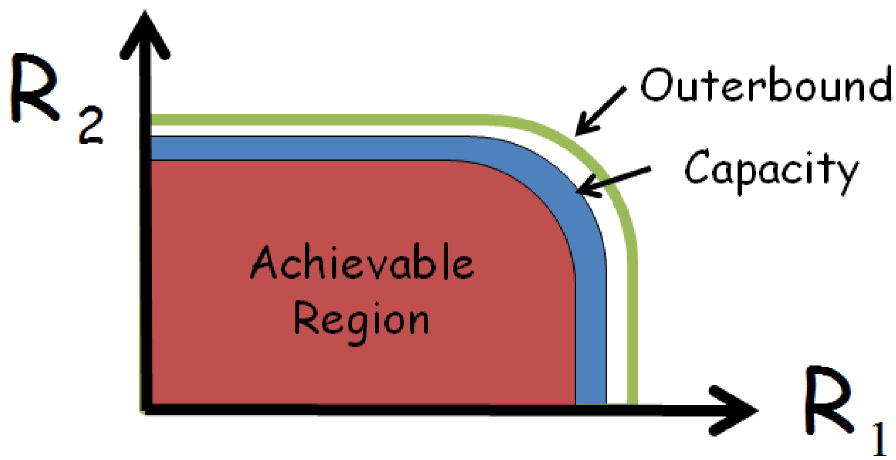


Figure 3.1: Capacity within a Constant Number of Bits

3.2 Degrees of Freedom

The technique that will be of more interest for this thesis is degrees of freedom analysis, which studies the shape of the capacity region at high SNR. Consider how the sum capacity scales as a function of log(SNR) as SNR $\rightarrow \infty$. Then the sum degrees of freedom of the interference channel is

$$\eta = \lim_{\mathrm{SNR}\rightarrow\infty} \frac{C_{\Sigma}(\mathrm{SNR})}{\log(\mathrm{SNR})}$$

where $C_{\Sigma}(\mathrm{SNR})$ is the sum capacity as function of the SNR or equivalently

$$C_{\Sigma}(\mathrm{SNR}) = \eta \log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$$

This approach can be extended to study other facets of the capacity region, but for this work only the sum degrees of freedom is of interest.

3.3 Sum Degrees of Freedom of the K-User SISO Interference Channel

In this section we summarize results related to the degrees of freedom of the K-user interference channel (IC) with only a single antenna at each transmitter and receiver, meaning $N_t^{[k]} = N_r^{[k]} = 1$.

A basic strategy for communicating on the IC is to use time division multiple access (TDMA). In this approach each unit of time is divided into K equal slots and each user is allowed to communicate during one slot. As a result each user achieves $\frac{1}{K}$ degrees of freedom, since

$$\eta_i = \lim_{P\to\infty} \frac{R_i}{\log P} = \lim_{P\to\infty} \frac{\frac{1}{K}\log(1+KP)}{\log(P)} = \frac{1}{K}$$

So the sum degrees of freedom equal to one can be achieved by TDMA. Now the natural question to ask is whether one sum degree of freedom is the best possible.

The first part of the answer to this question came from an upper bound on the sum degrees of freedom in [8]. This work demonstrated that for the two-user IC, the maximum possible sum degrees of freedom is one. So for the two-user IC the sum degrees of freedom is exactly one. With this result, by considering each pair of users in a K-user IC with $K > 2$ it can be seen that the sum degrees of freedom can be no more than $\frac{K}{2}$. This follows since for any pair of users $i \neq j$

$$\eta_i + \eta_j <= 1$$

by [8]. Then summing over all such equations gives

$$\sum_{i=1}^{K} \sum_{\substack{j=1 \\ j\neq i}}^{K} (\eta_i + \eta_j) \leq K(K-1) \tag{3.1}$$

Each transmitter/receiver appears in exactly $2(K-1)$ pairs, so Equation 3.1 is equivalent to

$$2(K-1)\eta \leq K(K-1)$$

Then it follows that

$$\eta \leq \frac{K}{2}$$

So it is clear that one sum degree of freedom is achievable and at most $\frac{K}{2}$ can be achieved. At the time of these results it was conjectured in [8] that the upper bound was not tight and that in fact the sum degrees of freedom equals one. It seemed likely that the $\frac{K}{2}$ bound would be loose, since it only considers pairs of users and looking at more than two users at a time would lead to tighter bounds. Surprisingly, though, it was proved in [3] that the upper bound is in fact tight, and so $\frac{K}{2}$ is the sum degrees of freedom of the interference channel. This result relied on the new ideas of *interference alignment* and *symbol extensions*. This was an exciting result because it showed that the capacity region of the IC is much larger than was believed. This result showed that at high SNRs managing interference in the context of the interference channel can provide large gains over orthogonalizing users through TDMA.

To understand the ideas from [3] consider the three-user IC. The idea behind symbol extensions is to communicate over α different channel uses and let $\alpha \to \infty$ as in Shannon's random block coding argument. With α symbol extensions the end-to-end system model can be written as

$$\begin{bmatrix} \boldsymbol{U}^{[1]} & 0 & 0 \\ 0 & \boldsymbol{U}^{[2]} & 0 \\ 0 & 0 & \boldsymbol{U}^{[3]} \end{bmatrix}^{\dagger} \begin{bmatrix} \boldsymbol{H}^{[11]} & \boldsymbol{H}^{[12]} & \boldsymbol{H}^{[13]} \\ \boldsymbol{H}^{[21]} & \boldsymbol{H}^{[22]} & \boldsymbol{H}^{[23]} \\ \boldsymbol{H}^{[31]} & \boldsymbol{H}^{[32]} & \boldsymbol{H}^{[33]} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}^{[1]} & 0 & 0 \\ 0 & \boldsymbol{V}^{[2]} & 0 \\ 0 & 0 & \boldsymbol{V}^{[3]} \end{bmatrix} \tag{3.2}$$

where $\boldsymbol{H}^{[kj]}$ is an $\alpha \times \alpha$ diagonal matrix of the form

$$\boldsymbol{H}^{[kj]} = \begin{bmatrix} \boldsymbol{H}^{[kj]}(1) & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{H}^{[kj]}(\alpha) \end{bmatrix}$$

Then the idea behind interference alignment is to find $\boldsymbol{V}^{[k]}$ and $\boldsymbol{U}^{[k]}$ such that the end-to-end model is diagonal. If the end-to-end matrix is diagonal, then at each receiver the desired signals lie in a subspace separate from the interfering signals. That is to say, the interfering signals are *simultaneously* aligned at each receiver in an interference subspace. So if it is possible to diagonalize Equation 3.2, then it is possible to achieve

$$\eta = \frac{1}{\alpha} \sum_{k=1}^{K} d^{[k]}$$

Using these ideas, it was proved in [3] that for the K-user IC as $\alpha \to \infty$ interference alignment is possible and results in $\frac{K}{2}$ degrees of freedom. The following simple example shows the general idea of the proof by showing how to achieve $\eta = \frac{4}{3}$ for the three-user IC with three symbol extensions ($\alpha = 3$).

1. The second transmitter selects any transmit vector $\boldsymbol{v}^{[2]}$. This is represented in Figure 3.2(a).

2. The third transmitter selects a transmit vector $\boldsymbol{v}^{[3]}$ such that $\boldsymbol{H}^{[12]}\boldsymbol{v}^{[2]}$ is in the subspace spanned by $\boldsymbol{H}^{[13]}\boldsymbol{v}^{[3]}$. When this condition is satisfied, then $\boldsymbol{v}^{[2]}$ is said to be aligned with $\boldsymbol{v}^{[3]}$ at receiver 1. This is always possible, since the channel matrices are drawn iid. Then, since the channel matrices are drawn iid, it will follow that $\boldsymbol{H}^{[22]}\boldsymbol{v}^{[2]}$ is not in the subspace spanned by $\boldsymbol{H}^{[23]}\boldsymbol{v}^{[3]}$. Then zero forcing can be used to separate out the signal associated with user 2 from the interference and decode it. Up to this point we have used up one dimension of receiver 1 and two dimensions of receivers 2 and 3. This is represented in Figure 3.2(b).

3. The first transmitter selects a receive vector $\boldsymbol{v}^{[11]}$ that is aligned with $\boldsymbol{v}^{[3]}$ at receiver 2. Up to this point we have used two dimensions at receiver 1 and 2, and three dimensions at receiver 3. Each transmitter now achieves one degree of freedom over three channel uses, so $\eta = 1$. This is represented in Figure 3.2(c).

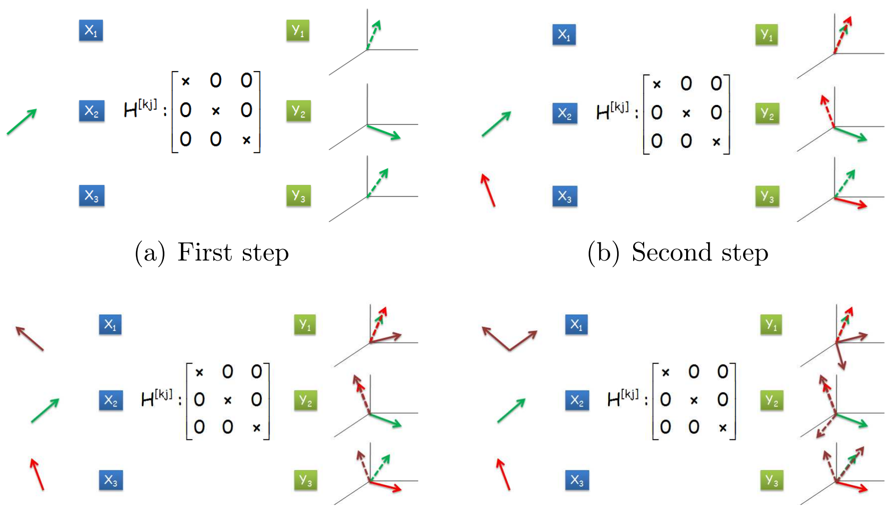
4. To complete the proof, transmitter 1 will design a second transmit vector $\boldsymbol{v}^{[12]}$. Align $\boldsymbol{v}^{[12]}$ with $\boldsymbol{v}^{[2]}$ at receiver 3. Then all three dimensions are used at all receivers with user 1 achieving two degrees of freedom and users 2 and 3 achieving one degree of freedom. This is represented in Figure 3.2(d).

The above process occurs over three channel uses, and therefore $\eta = \frac{2+1+1}{3} = \frac{4}{3}$. The proof that $\eta = \frac{K}{2}$ follows from extending this procedure.

3.4 Degrees of Freedom of MIMO Interference Channel

Now consider the case where $N_t^{[k]}$ and $N_r^{[k]}$ are not restricted to one. There has been some progress on some cases but the problem is not completely solved.

For $N_t^{[k]} = N_r^{[k]} = N$, Cadambe and Jafar in [3] proved that the degrees of freedom for this channel equals $\frac{KN}{2}$ by treating each antenna as a different



(a) First step (b) Second step

(c) Third step (d) Fourth step

Figure 3.2: Achieving $\eta = \frac{4}{3}$

user and using the result from the SISO case to derive a new upper bound. The two-user case has been fully solved in [9] and degrees of freedom is known to be

$$\eta = \min \left\{ N_t^{[1]} + N_t^{[2]}, N_r^{[1]} + N_r^{[2]}, \max \left\{ N_t^{[1]}, N_r^{[2]} \right\}, \max \left\{ N_r^{[1]}, N_t^{[2]} \right\} \right\}$$

The general case with different numbers of transmit and receive antennas at different users is difficult and there has been little progress.

3.5 Constant Channel

Interference alignment as presented in [3] depends on letting the number of symbol extensions go to infinity. The raises the question of how many degrees of freedom can be achieved without symbol extensions or only a fixed, finite number of symbol extensions. Clearly any achievable scheme for the constant channel can be used for the IC with an arbitrary number of symbol extensions, so the degrees of freedom of the constant IC can be no more than the case with symbol extensions and maybe less.

3.5.1 Constant SISO Channel

In [10], a new type of alignment technique called *real interference alignment* was presented based on number theory and the problem of Diophantine approximation - approximating an irrational number by a real number. This technique is complicated and impractical, so we omit a discussion of the particulars here and simply cite the result that the degrees of freedom of the constant SISO channel equals $\frac{K}{2}$. If we restrict ourselves to considering only vector space based interference alignment, then the degrees of freedom has not been fully characterized.

3.5.2 Constant MIMO Channel

For the constant MIMO interference channel there has been some progress using tools from algebraic geometry to prove when a diagonalization like in Equation 3.2 is possible. The diagonalization problem is equivalent to

$$\begin{aligned} \operatorname{rank}\left(\boldsymbol{U}^{[k]\dagger}\boldsymbol{H}^{[kk]}\boldsymbol{V}^{[k]}\right) &= d^{[k]} \qquad (3.3) \\ \boldsymbol{U}^{[k]\dagger}\boldsymbol{H}^{[kj]}\boldsymbol{V}^{[j]} &= \mathbf{0} \quad \forall j \neq k \qquad (3.4) \end{aligned}$$

If the channel matrices are drawn iid from a continuous distribution, then Equation 3.3 is satisfied almost surely, so only Equation 3.4 needs to be checked.

The goal of algebraic geometry is to answer when a system of polynomials has solutions and to characterize the solutions. Expanding Equation 3.4 yields a system of polynomials in the transmit vectors $\boldsymbol{V}^{[i]}$ and receive vectors $\boldsymbol{U}^{[i]}$. We want these polynomials to be zero to yield an interference alignment solution; algebraic geometry can answer when this is possible.

One of the first algebraic geometry results to show the feasibility of interference alignment for the MIMO K-user IC with $d^{[k]} = 1$, $N_t^{[k]} = N_t$, and $N_r^{[k]} = N_r$ was given in [11] and relied on a basic observation: a system of polynomial equations is solvable iff the number of equations is no greater than the number of variables. The number of equations is simple to count and given by

$$N_e = K(K-1)$$

and the number of variables is given by

$$N_v = K(N_t + N_r - 2)$$

The number of variables is not simply $K(N_t + N_r)$, since for interference alignment what matters is the subspace generated by a given beamforming vector at a receiver and not the particular vector. Each transmit vector and receive vector can be scaled without affecting whether Equation 3.4 is satisfied, so there are two fewer free variables for each pair of transmit and receive vectors, yielding $K(N_t + N_r - 2)$ total variables. Then the necessary condition for alignment is given by $N_v \geq N_e$ which is equivalent to

$$N_t + N_r \geq K + 1$$

The converse of this result is given by using a basic result of algebraic geometry, Bernshtein's theorem, to show that alignment is impossible almost surely as long as $N_v > N_e$. Bernshtein's theorem gives a way to count the number of solutions to a system of polynomial equations under certain conditions.

A major limitation of the work from [11] is that it breaks down for the multibeam case($d^{[k]} > 1$). However, other algebraic geometry based work has been carried out by Bresler and Tse in [12] to show when interference alignment is possible for some special multibeam cases including the square symmetric case and the three user case. For the square symmetric case where $N_t^{[k]} = N_r^{[k]} = N$ and $d^{[k]} = d$, alignment is possible iff

$$2N \geq d(K + 1)$$

CHAPTER 4

COMMUNICATION STRATEGIES FOR THE INTERFERENCE CHANNEL

The goal of the algorithms presented in this chapter is to design receive and transmit strategies for the MIMO IC to achieve a good sum rate. From the previous section we know that interference alignment is a good way to achieve a good sum rate at high SNR, but a major practical issue is to actually find IA solutions. It is therefore of interest to find approximate or iterative methods to find IA solutions. There are two key ideas from interference alignment that drive the development of algorithms for the IC:

1. Linear transmit strategies are optimal for achieving the number of degrees of freedom at high SNR, so it may be a good idea to try linear transmit strategies that mimic IA at any finite SNR.

2. The sum degrees of freedom of the IC may provide a guideline for choosing the number of transmit streams at finite SNRs

So in the context of Equation 3.2 we simply want to design good transmit and receive vectors, but we may no longer exactly diagonalize the end-to-end system model.

4.1 Specific Channel Model

Since we only consider linear transmit strategies we can specialize the interference channel system model to this case. Assuming each user sends $d^{[k]}$ streams, the effective channel for stream ℓ of user k is then given by Equation 4.1. We omit the channel use index as we only consider communicating over one channel use, so only the constant MIMO interference channel is relevant.

$$\begin{aligned}\boldsymbol{y}_\ell^{[k]} &= \sum_{s=1}^{d^{[k]}} \sqrt{\rho^{[ks]}} \boldsymbol{H}^{[kk]} \boldsymbol{V}_{*s}^{[k]} \tilde{\boldsymbol{x}}_s^{[k]} \\ &\quad + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{s=1}^{d^{[j]}} \sqrt{\rho^{[js]}} \boldsymbol{H}^{[kj]} \boldsymbol{V}_{*s}^{[j]} \tilde{\boldsymbol{x}}_s^{[j]} + \boldsymbol{z}^{[k]} \end{aligned} \tag{4.1}$$

The quantity $\boldsymbol{y}_\ell^{[k]}$ is the $N_r^{[k]} \times 1$ receive vector, $\boldsymbol{z}^{[k]}$ is the $N_r^{[k]} \times 1$ additive white Gaussian noise (AWGN) vector with distribution $\mathcal{CN}(0, \boldsymbol{I})$, $\tilde{\boldsymbol{x}}^{[k]}$ is the $d^{[k]} \times 1$ vector of symbols sent by user k with $x^{[k]} \sim \mathcal{CN}(0, \boldsymbol{I}_{d^{[k]}})$, $\boldsymbol{V}^{[k]}$ is the matrix of beamforming vectors with $\boldsymbol{V}_{*s}^{[k]}$ the beamformer used by transmitter k for stream s, and $\boldsymbol{H}^{[kj]}$ is the $N_r^{[k]} \times N_t^{[j]}$ full rank matrix of channel coefficients between transmitter j and receiver k.

The quantity $\rho^{[ks]}$ is the power allocated to stream s of user k. Define a $\sum_{k=1}^{K} d^{[k]} \times 1$ vector $\boldsymbol{\rho} = [\rho^{[11]}, \ldots, \rho^{[kd^{[k]}]}]^\top$. For all the iterative algorithms in this section we impose a per user power constraint of P.

4.2 Reciprocal Channel

For the K-user interference channel defined above we define a reciprocal channel corresponding to reversing the direction of communication. From now on an arrow above a quantity will indicate the direction of communication. The channel model for the reciprocal channel is given by

$$\begin{aligned}\overleftarrow{\boldsymbol{y}}_\ell^{[k]} &= \sum_{s=1}^{d^{[k]}} \sqrt{\overleftarrow{\rho}^{[ks]}} \overleftarrow{\boldsymbol{H}}^{[kk]} \overleftarrow{\boldsymbol{V}}_{*s}^{[k]} \overleftarrow{x}_s^{[k]} \\ &\quad + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{s=1}^{d^{[j]}} \sqrt{\overleftarrow{\rho}^{[js]}} \overleftarrow{\boldsymbol{H}}^{[kj]} \overleftarrow{\boldsymbol{V}}_{*s}^{[j]} \overleftarrow{x}_s^{[j]} + \overleftarrow{\boldsymbol{z}}^{[k]} \end{aligned} \tag{4.2}$$

where $\overleftarrow{\boldsymbol{H}}^{[kj]} = \overrightarrow{\boldsymbol{H}}^{[jk]\dagger}$. For the next four definitions the arrow to indicate direction of communication is omitted as the expressions are identical otherwise. Define $\boldsymbol{U}_{*\ell}^{[k]}$ to be the receive vector for stream ℓ of user k. Define the stream-to-stream link gains as

$$G_{k,\ell}^{j,s} = \left| \frac{\boldsymbol{U}_{*\ell}^{[k]\dagger}}{\|\boldsymbol{U}_{*\ell}^{[k]}\|_2} \boldsymbol{H}^{[kj]} \boldsymbol{V}_{*s}^{[j]} \right|^2$$

The noise-plus-interference covariance matrices are

$$\begin{aligned} \boldsymbol{B}^{[k\ell]} &= \boldsymbol{I}_{N_r[k]} + \sum_{s=1}^{d^{[j]}} \rho^{[ks]} \boldsymbol{H}^{[kk]} \boldsymbol{V}_{*s}^{[k]} \boldsymbol{V}_{*s}^{[k]\dagger} \boldsymbol{H}^{[kk]\dagger} \\ &\quad + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{s=1}^{d^{[j]}} \rho^{[js]} \boldsymbol{H}^{[kj]} \boldsymbol{V}_{*s}^{[j]} \boldsymbol{V}_{*s}^{[j]\dagger} \boldsymbol{H}^{[kj]\dagger} \end{aligned}$$

Then the signal to interference and noise ratio (SINR) for a stream on the forward or reciprocal network is given by

$$\mathrm{SINR}^{[k\ell]} = \frac{\rho^{[k\ell]} G_{k,\ell}^{k,\ell}}{1 + \sum_{\substack{s=1 \\ s \neq \ell}}^{d^{[j]}} \rho^{[ks]} G_{k,\ell}^{k,s} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{s=1}^{d^{[j]}} \rho^{[js]} G_{k,\ell}^{j,s}}$$

Eventually we will consider using SIC, which will result in minor changes to these expressions. Depending on the order of cancellation, some of the terms associated with the streams of a given user will not be present. For example, assuming cancellation in lexicographic order, the SINR expression would become

$$\mathrm{SINR}^{[k\ell]} = \frac{\rho^{[k\ell]} G_{k,\ell}^{k,\ell}}{1 + \sum_{s=\ell+1}^{d^{[j]}} \rho^{[ks]} G_{k,\ell}^{k,s} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{s=1}^{d^{[j]}} \rho^{[js]} G_{k,\ell}^{j,s}}$$

As these modifications are straightforward due to the limited space they are omitted. We will assume it is clear from the context which definition is being used.

Remark on Definition for Stream Link Gains: In subsequent sections we will consider the situation where we use transmit and receive vectors $\overrightarrow{\boldsymbol{V}}^{[k]}$ and $\overrightarrow{\boldsymbol{U}}^{[k]}$ in the forward direction and then switch to use $\overleftarrow{\boldsymbol{V}}^{[k]} = \overrightarrow{\boldsymbol{U}}_{*\ell}^{[k]} / \|\overrightarrow{\boldsymbol{U}}_{*\ell}^{[k]}\|_2$ and $\overleftarrow{\boldsymbol{U}}^{[k]} = \overrightarrow{\boldsymbol{V}}^{[k]}$ in the reverse direction with the reciprocal beamformers scaled to meet the power constraint. Then simple computation shows that $\overleftarrow{G}_{k,\ell}^{j,s} = \overrightarrow{G}_{j,s}^{k,\ell}$. This property will be important in the following sections.

4.3 Min Leakage Algorithm

One of the first algorithms inspired by IA, Min Leakage, was presented in [5]. The idea behind this algorithm is to minimize the sum leaked interference power at all the receivers. There are two key ideas to this algorithm:

1. Design receive vectors $\boldsymbol{U}^{[k]}$ to minimize the leaked interference power.

2. Repeatedly reverse the direction of communication and design receive vectors in this manner for both directions.

The first ingredient for this algorithm is to find the receive vectors that minimize the power leaked by interference. To solve this problem we first need the interference covariance matrix at each receiver denoted by $\boldsymbol{Q}^{[k]}$:

$$\boldsymbol{Q}^{[k]} = \sum_{\substack{j=1 \\ j \neq k}}^{K} \frac{P}{d^{[k]}} \boldsymbol{H}^{[kj]} \boldsymbol{V}^{[k]} \boldsymbol{V}^{[k]\dagger} \boldsymbol{H}^{[kj]\dagger}$$

Then for receive vectors $\boldsymbol{U}^{[k]}$ the leaked interference power is $\mathrm{Tr}\left(\boldsymbol{U}^{[k]\dagger}\boldsymbol{Q}^{[k]}\boldsymbol{U}^{[k]}\right)$. We want to minimize this quantity and it can be easily seen that the solution to this problem is to choose the receive vectors to be the $d^{[k]}$ least dominant eigenvectors of $\boldsymbol{Q}^{[k]}$.

The second ingredient is to communicate in both directions using the reciprocal channel. What this means is we first work with the normal channel given in the system model section. Next, we switch the role of $\boldsymbol{U}^{[k]}$ and $\boldsymbol{V}^{[k]}$ and transmit in the reverse direction. Figure 4.1 describes how the forward and reciprocal channels work for Min Leakage.

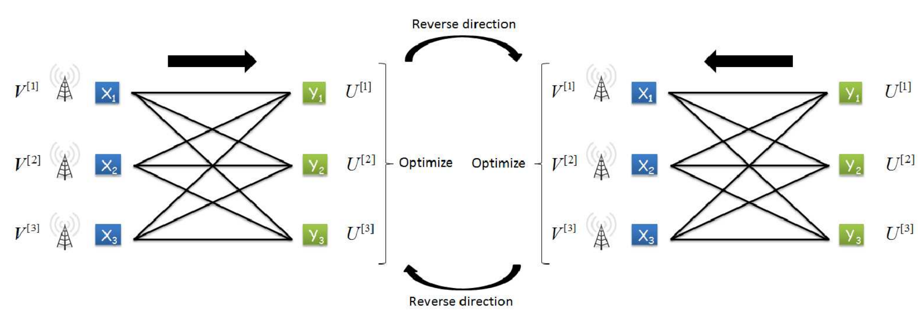


Figure 4.1: Operation of Min Leakage

With these two ingredients the Min Leakage algorithm proceeds as described in Algorithm 1.

Algorithm 1 Min Leakage Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}^{[k]}(1)\}$ such that

$$\|\overrightarrow{\boldsymbol{V}}^{[k]}_{*\ell}(1)\|_2 = 1$$

and set

$$\rho^{[kl]} = \frac{P}{d^{[k]}}$$

2: Choose receive vectors to minimize leakage. That is compute $\overrightarrow{\boldsymbol{Q}}^{[k]}$ at each receiver and take $\overrightarrow{\boldsymbol{U}}^{[k]}$ to be the $d^{[k]}$ least dominant eigenvectors.

3: Reverse the direction of communication. Set

$$\overleftarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n) = \overrightarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)$$

and

$$\overleftarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n) = \overrightarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n)$$

4: Choose receive vectors to minimize leakage. That is compute $\overleftarrow{\boldsymbol{Q}}^{[k]}$ at each receiver and take $\overleftarrow{\boldsymbol{U}}^{[k]}$ to be the $d^{[k]}$ least dominant eigenvectors.

5: Reverse the direction of communication. Set

$$\overrightarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n+1) = \overleftarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)$$

and

$$\overrightarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n+1) = \overleftarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n)$$

6: Repeat steps 2 through 5 until convergence.

4.3.1 Convergence of Min Leakage

The Min Leakage Algorithm makes a metric known as weighted leakage interference (WLI) converge with WLI given by

$$I_w = \sum_{k=1}^{K} \sum_{\substack{j=1 \\ j \neq K}}^{K} \frac{P}{d^{[k]}} \frac{P}{d^{[j]}} \mathrm{Tr}\left(U^{[k]\dagger} \boldsymbol{H}^{[kj]} \boldsymbol{V}^{[j]} \boldsymbol{V}^{[j]\dagger} \boldsymbol{H}^{[kj]\dagger} U^{[k]}\right)$$

It converges monotonically since each step of this algorithm makes this quantity decrease and it is bounded below by zero. If this quantity converges to zero, then the resulting transmit and receive vectors constitute an alignment solution, but this need not occur. There are no known conditions to guarantee that WLI goes to zero.

4.4 Other IA Inspired Algorithms

Several other algorithms based on the idea of trying to mimic interference alignment have been proposed. One other example is the Peters-Heath algorithm from [6] which works in a similar way to Min Leakage without the requirement of reciprocity. Other proposed work includes a modification of the Peters-Heath algorithm to achieve a better sum rate in [7] and several other similar algorithms for interference alignment including [13] and [14].

4.5 Max SINR

The proposed algorithms based on interference alignment all have a few problems. To understand the problems, consider the case where $N_t^{[k]} = N_r^{[k]} = N$. Since we are aiming for an interference alignment solution to get a good sum rate, at best we can get

$$C_\Sigma(\mathrm{SNR}) = \frac{KN}{2}\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$$

But at any low or moderate SNR the terms in $o(\log(\mathrm{SNR}))$ matter, so we cannot just aim for any alignment solution. Different alignment solutions may have drastically different sum rate performance. This problem is even worse since there may be many alignment solutions. As discussed before, [11] proved that for the case when $K < 2N - 1$ and $d^{[k]} = 1$ there are an infinite number of alignment solutions all of which have different sum rate performance, so finding a good interference alignment solution may not be easy.

One way to try to tackle these issues is to copy the basic structure of Min Leakage but use a different metric than interference leakage. The Max SINR Algorithm proposed in [5] replaces interference leakage with the SINR of each stream. Max SINR designs receive vectors for each stream to maximize the SINR of each stream, which are simply minimum mean squared error (MMSE) receive vectors. It is known through simulation that this algorithm produces a high sum rate for all SNRs. The Max SINR algorithm is summarized in Algorithm 2.

4.5.1 MMSE Receiver

As discussed above at each step we want to choose a receive vector to maximize the SINR of each stream, which as discussed above is a MMSE receive vector. To describe the MMSE receive vectors, first define $\tilde{\boldsymbol{H}}_{*\ell}^{[kj]} = \boldsymbol{H}^{[kj]}\boldsymbol{V}_{*\ell}^{[j]}$. Then the MMSE receiver is given by

$$\boldsymbol{U}_{\ell}^{[k]} = \frac{\rho^{[k\ell]}}{1 + \tilde{\boldsymbol{H}}_{*\ell}^{[kk]\dagger}(\boldsymbol{B}^{[kl]})^{-1}\tilde{\boldsymbol{H}}_{*\ell}^{[kk]}}(\boldsymbol{B}^{[k\ell]})^{-1}\tilde{\boldsymbol{H}}_{*\ell}^{[kk]\dagger}$$

Algorithm 2 Max SINR Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}^{[k]}(1)\}$ such that

$$\|\overrightarrow{\boldsymbol{V}}_{*\ell}^{[k]}(1)\|_2 = 1$$

and set

$$\rho^{[kl]} = \frac{P}{d^{[k]}}$$

2: Compute MMSE RX vectors $\{\overrightarrow{\boldsymbol{U}}^{[k]}(n)\}$ and then $\overrightarrow{R}_{\mathrm{sum}}(n)$.

3: Reverse the direction of communication. Set

$$\overleftarrow{\boldsymbol{V}}_{*\ell}^{[k]}(n) = \frac{\overrightarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n)}{\|\overrightarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n)\|_{\mathrm{F}}}$$

and

$$\overleftarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n) = \overrightarrow{\boldsymbol{V}}_{*\ell}^{[k]}(n)$$

Calculate the sum rate on the reciprocal network $\overleftarrow{R}_{\mathrm{sum}}^{(\mathrm{switch})}(n)$.

4: Compute MMSE RX vectors $\{\overleftarrow{\boldsymbol{U}}^{[k]}(n)\}$ and then $\overleftarrow{R}_{\mathrm{sum}}(n)$.

5: Reverse the direction of communication. Set

$$\overrightarrow{\boldsymbol{V}}_{*\ell}^{[k]}(n+1) = \frac{\overleftarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n)}{\|\overleftarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n)\|_{\mathrm{F}}}$$

and

$$\overrightarrow{\boldsymbol{U}}_{*\ell}^{[k]}(n+1) = \overleftarrow{\boldsymbol{V}}_{*\ell}^{[k]}(n)$$

Now calculate the sum rate on the forward network $\overrightarrow{R}_{\mathrm{sum}}^{(\mathrm{switch})}(n)$.

6: Repeat steps 2 through 5.

4.5.2 Convergence Issues

In simulations of Max SINR, the sum rate appears to converge monotonically, but there is no proof that this actually occurs. Experimentally it is sometimes the case that $\overrightarrow{R}_{\text{sum}}(n) > \overleftarrow{R}^{(\text{switch})}_{\text{sum}}(n)$, but after optimization $\overrightarrow{R}_{\text{sum}}(n) \leq \overleftarrow{R}_{\text{sum}}(n)$. In the forward channel a similar process occurs and so $\overrightarrow{R}_{\text{sum}}(n) \leq \overrightarrow{R}_{\text{sum}}(n+1)$. Figure 4.2 shows an example of this phenomenon. In addition, convergence may not be monotone as seen in Figure 4.3.

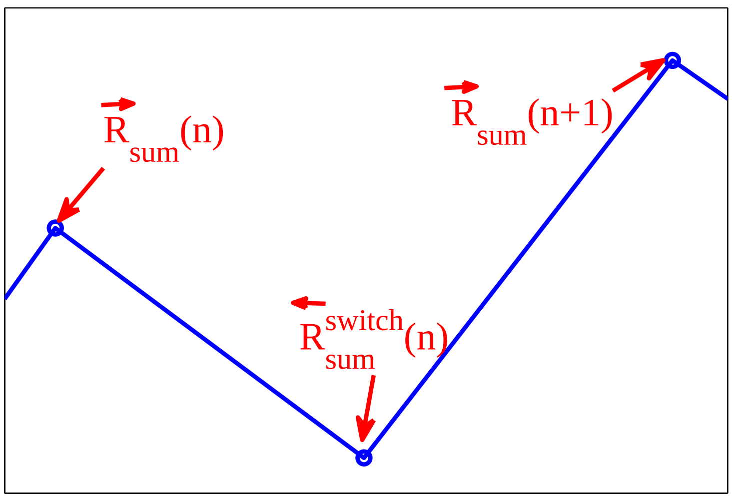


Figure 4.2: Max SINR Convergence Behavior

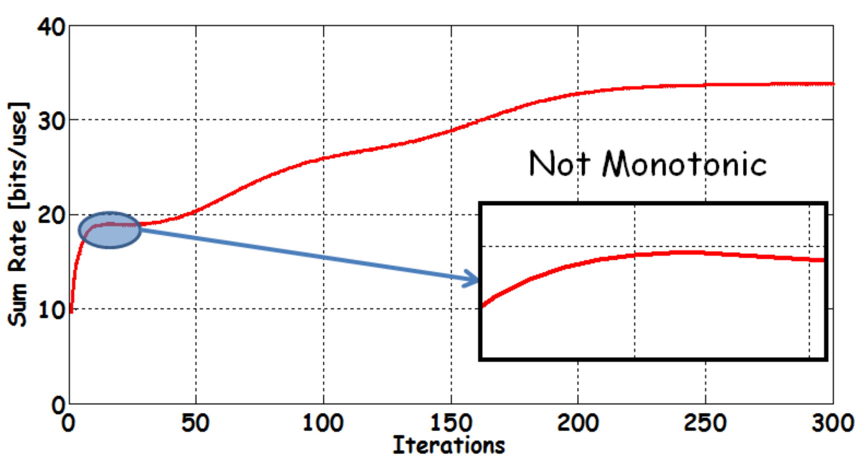


Figure 4.3: Max SINR Convergence Not Monotone

However, due to the complexity of the sum rate and MMSE receiver expressions, it is difficult to prove that the sum rate eventually converges. The rest of this thesis shows a way to overcome this difficulty.

4.6 Comparison of Algorithms

Figure 4.4 gives a representative example of the performance of Max SINR against Min Leakage and Peters-Heath. As the plot shows, Max SINR outperforms the Min Leakage and Peters-Heath algorithms at low SNRs, but the performance gap disappears at high SNRs. This makes sense since interference alignment is sum rate optimal at high SNRs and Min Leakage and Peters-Heath aim for IA solutions. The performance for other system parameters is similar, so this plot is representative of the difference between these algorithms.

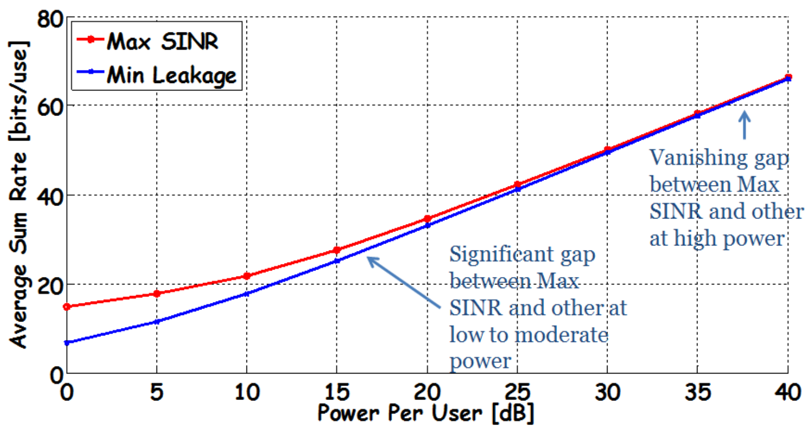


Figure 4.4: Sum Rate vs. P for $K = 5$, $N_t = N_r = 3$, $d^{[k]} = 1$

CHAPTER 5

A CONVERGENT VERSION OF THE MAX SINR ALGORITHM

In this section we demonstrate a way to modify the Max SINR algorithm to guarantee sum rate convergence. To do this we consider a slightly different metric than sum rate: sum stream rate. Next, we show that adjusting the power allocation appropriately when switching the direction of communication will guarantee that the same SINR is achieved on the forward and reciprocal networks. By combining these two ideas we can show that the sum stream rate monotonically increases and therefore converges. Finally, we show that using SIC leads to a direct expression for the sum rate in terms of the SINRs. With SIC, the sum rate also converges and equals the sum stream rate

5.1 Sum Power Constraint

In developing a convergent version of the Max SINR algorithm, we will require a power control step in which we impose a sum power constraint. Imposing a sum constraint can be justified in some cases in which the main issue is to manage interference and in which each individual transmitter can transmit at high power. Instead of imposing an individual power constraint to manage interference we can impose a sum constraint and manage interference more intelligently with beamforming.

For example, consider the femtocell/macrocell interference model from [15] in which several femotocells and an umbrella macrocell interfere with each other. In this case the femtocells can transmit at high power if needed, so there is a very high individual power constraint on the femtocells. One approach to managing interference is to impose a much smaller individual power constraint to control the coverage of the femtocells. Another approach is to impose a sum power constraint over all the femtocells set at the maximum power at which any given femtocell can transmit at, and use beamforming to manage interference. In this chapter we consider the second approach and use a sum power constraint.

5.2 Sum Stream Rate

The sum stream rate is similar to the sum rate and is given by

$$R_{sum-stream} = \sum_{k=1}^{K} \sum_{\ell=1}^{d^{[k]}} \log(1 + SINR^{[k\ell]})$$

This is the sum of the Shannon rates achieved for each stream and is the same form for the forward and reciprocal channel.

Using SIC leads to the following result:

Lemma 1. *In either direction of communication, for any $\boldsymbol{V}^{[k]}$ and any channel realization with MMSE-SIC receivers,*

$$\mathrm{R}_{\mathrm{sum}} \quad = \quad \sum_{k=1}^{K} \sum_{l=1}^{d^{[k]}} \log\left(1 + \mathrm{SINR}^{[k\ell]}\right) = \mathrm{R}_{\mathrm{sum-stream}}$$

Proof. This proof is just an application of the chain rule for mutual information and is almost the same as the one for the point-to-point MIMO channel in [16]. □

5.3 Generalization of Network Duality

In this section we show that by selecting the power appropriately we can achieve the same SINRs on the forward and reciprocal channels. This section extends the ideas from [17] to allow for SIC. As all the proofs and development are similar to [17], they are placed in the appendix

Let $\gamma^{[k\ell]}$ be the target SINR for stream ℓ of user k. Following [17], define

$$\boldsymbol{D} = \mathrm{diag}\{\frac{\gamma[11]}{\overrightarrow{G}_{1,1}^{1,1}}, \ldots, \frac{\gamma[Kd^{[K]}]}{\overrightarrow{G}_{K,d^{[K]}}^{K,d^{[K]}}}\}$$

and $\boldsymbol{G}$ such that

$$\boldsymbol{G}_{\sum_{m=1}^{k-1} d^{[m]}+\ell, \sum_{n=1}^{j-1} d^{[n]}+s} = \begin{cases} 0, & \text{if } k = j \text{ and } \ell < s, \\ \overrightarrow{G}_{k,\ell}^{j,s}, & \text{else} \end{cases}$$

Set

$$\boldsymbol{A} = \boldsymbol{D}^{-1} - \boldsymbol{G}$$

For the forward and reciprocal directions use the power allocations

$$\overrightarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-1}\mathbf{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-1}\right)\mathbf{1} \tag{5.1}$$

$$\overleftarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-\top}\mathbf{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-\top}\right)\mathbf{1} \tag{5.2}$$

The following lemma shows that it is possible to achieve the same SINRs on the forward and reciprocal channels. Also, the following lemma shows that, as long as we meet the power constraint initially, $\overrightarrow{\boldsymbol{\rho}}^*$ and $\overleftarrow{\boldsymbol{\rho}}^*$ will meet it too. The proof of this lemma follows from [17] and is developed in the appendix.

Lemma 2. *Suppose we choose a power allocation $\boldsymbol{\rho}$ such that $\mathbf{1}^\top\boldsymbol{\rho} \leq KP$ and transmit and receive vectors $\{\boldsymbol{V}^{[k]}\}$ and $\{\boldsymbol{U}^{[k]}\}$. Let $\gamma^{[k\ell]}$ be the resulting SINRs. Then*

$$\mathbf{1}^\top\overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top\overleftarrow{\boldsymbol{\rho}}^* \leq KP$$

and

$$\overrightarrow{\mathrm{SINR}}^{[k\ell]}(\overrightarrow{\boldsymbol{\rho}}^*) = \overleftarrow{\mathrm{SINR}}^{[k\ell]}(\overleftarrow{\boldsymbol{\rho}}^*) = \gamma^{[k\ell]}$$

Remark on SIC: For this results to hold, $\boldsymbol{A}^\top$ must describe the reciprocal network, so equivalently $\boldsymbol{G}^\top$ must describe the interference. This means that if we use SIC we must reverse the order of cancellation in the reciprocal network.

5.3.1 Computing Power Required for Duality

Equations 5.1 and 5.2 give a centralized way to compute the required powers to achieve equal SINRs in both the forward and reciprocal directions. In this section we discuss how to use the framework from [18] to compute the required powers in a distributed way. For either direction of communication, define the vector valued function $\boldsymbol{I}(\boldsymbol{\rho})$ by

$$\boldsymbol{I}^{[k\ell]}(\boldsymbol{\rho}) = \frac{\gamma^{[k\ell]}\boldsymbol{\rho}^{[k\ell]}}{\mathrm{SINR}^{[k\ell]}(\boldsymbol{\rho})}$$

Then this is a standard interference function, so we can use the well-known totally asynchronous power control algorithm from [18] in Algorithm 3, which is guaranteed to converge to $\boldsymbol{\rho}^*$ by Theorem 4 of [18].

Algorithm 3 Distributed Power Control Algorithm

1: Choose initial power vector $\boldsymbol{\rho}(0)$ such that

$$\sum_{k=1}^{K}\sum_{\ell=1}^{d^{[k]}} \rho^{[k\ell]}(0) \leq KP$$

2: Compute the iterate

$$\boldsymbol{\rho}(n+1) = \boldsymbol{I}(\boldsymbol{\rho}(\boldsymbol{n}))$$

3: Repeat step 2 until convergence.

So we can either solve a centralized version of the power control or solve a distributed version with the totally asynchronous power control algorithm.

5.4 Modified Max SINR

In this section we describe a modified version of Max SINR in which the sum rate converges. Modified Max SINR follows the same basic principle as Max SINR with two key differences. First, Modified Max SINR uses the sum stream metric as a convergence criterion. Second, Modified Max SINR changes the powers for each stream when it reverses the direction of communication to achieve SINR duality. The Modified Max SINR Algorithm is summarized in Algorithm 4.

Computing Power Allocation to Achieve SINR Duality: In order to achieve SINR duality we need to compute the appropriate power allocation every time we change the direction of communication using either (5.1) and (5.2) or a distributed power control algorithm. For the Equation (5.1) and (5.2) approach, assuming the users estimate their own row of $\boldsymbol{A}$ and exchange their estimates, each user can solve $\boldsymbol{A}\overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}$ to find its power allocation. In

Algorithm 4 Modified Max SINR Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}^{[k]}(1)\}$ and $\overrightarrow{\boldsymbol{\rho}}(1)$ satisfying the power constraint.

2: Compute MMSE RX vectors $\{\overrightarrow{\boldsymbol{U}}^{[k]}(n)\}$ and then $\overrightarrow{R}_{\text{sum-stream}}(n)$.

3: Reverse the direction of communication. Calculate the powers $\overleftarrow{\boldsymbol{\rho}}(n)$ to achieve SINR duality. Set

$$\begin{aligned}\overleftarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n) &= \frac{\overrightarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)}{\|\overrightarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)\|_F} \\ \overleftarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n) &= \overrightarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n)\end{aligned}$$

Now calculate the sum rate on the reciprocal network $\overleftarrow{R}^{\text{(switch)}}_{\text{sum-stream}}(n)$.

4: Compute MMSE RX vectors $\{\overleftarrow{\boldsymbol{U}}^{[k]}(n)\}$ and then $\overleftarrow{R}_{\text{sum-stream}}(n)$.

5: Reverse the direction of communication. Calculate the powers $\overrightarrow{\boldsymbol{\rho}}(n+1)$ to achieve SINR duality. Set

$$\begin{aligned}\overrightarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n+1) &= \frac{\overleftarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)}{\|\overleftarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n)\|_F} \\ \overrightarrow{\boldsymbol{U}}^{[k]}_{*\ell}(n+1) &= \overleftarrow{\boldsymbol{V}}^{[k]}_{*\ell}(n)\end{aligned}$$

Now calculate the sum rate on the forward network $\overrightarrow{R}^{\text{(switch)}}_{\text{sum-stream}}(n)$.

6: Repeat steps 2 through 5 until convergence of $\overrightarrow{R}_{\text{sum-stream}}(n)$.

this case Modified Max SINR converges but is no longer fully decentralized.

The other option is to use the distributed algorithm from [18]. Then Modified Max SINR is convergent and distributed but the iterative power control algorithm must be run at every step, which greatly increases the number of transmissions from each user needed. In many cases it may be better to solve the centralized problem instead of running the iterative power control algorithm.

Theorem 1. *With Modified Max SINR, $\overrightarrow{R}_{\text{sum-stream}}(n)$ converges. Also, Modified Max SINR meets the power constraint at every step.*

Proof. The initial power allocation $\{\overrightarrow{\boldsymbol{\rho}}(1)\}$ meets the power constraint, so the constraint will continue to be met due to Lemma 2. Now

$$\begin{aligned}
\overrightarrow{R}_{\text{sum-stream}}(n) &\overset{(a)}{=} \overleftarrow{R}^{\text{(switch)}}_{\text{sum-stream}}(n) \\
&\overset{(b)}{\leq} \overleftarrow{R}_{\text{sum-stream}}(n) \\
&\overset{(c)}{=} \overrightarrow{R}^{\text{(switch)}}_{\text{sum-stream}}(n) \\
&\overset{(d)}{\leq} \overrightarrow{R}_{\text{sum-stream}}(n+1)
\end{aligned}$$

where (a) and (c) follow by SINR duality and (b) and (d) follow since MMSE receive vectors maximize SINR. So the sum rate increases monotonically and therefore converges. □

Although we know that $R_{\text{sum-stream}}(n)$ converges, since $R_{\text{sum-stream}}(n)$ is a non-convex function, we cannot show that $R_{\text{sum-stream}}(n)$ converges to a global maximum.

5.5 Using SIC

If in addition we use SIC, then we get a more powerful result.

5.5.1 MMSE - SIC for MIMO Interference Channel

In this section we show that using SIC leads to a direct expression for the sum rate in terms of the SINR.

Lemma 3. *For any* $\{\overrightarrow{\boldsymbol{V}}^{[k]}\}$ *and* $\{\overleftarrow{\boldsymbol{V}}^{[k]}\}$ *and any channel realization with MMSE-SIC receivers*

$$\begin{aligned}
\overrightarrow{\mathrm{R}}_{\text{sum}} &= \sum_{k=1}^{K}\sum_{l=1}^{d^{[k]}} \log\left(1+\overrightarrow{\mathrm{SINR}}^{[k\ell]}\right) \\
\overleftarrow{\mathrm{R}}_{\text{sum}} &= \sum_{k=1}^{K}\sum_{l=1}^{d^{[k]}} \log\left(1+\overleftarrow{\mathrm{SINR}}^{[k\ell]}\right)
\end{aligned}$$

Proof. This proof is essentially the same as Lemma 1. □

5.5.2 Convergence of Max SINR without SIC

We can modify Modified Max SINR to make use of SIC. The algorithm is exactly the same except we compute MMSE-SIC receive vectors instead of using MMSE. If we use Max SINR with the same power control as Modified Max SINR and MSSE-SIC, then by Lemma 3 it follows that R_{sum} converges by using the same proof as Theorem 1.

CHAPTER 6

PERFORMANCE OF MODIFIED MAX SINR ALGORITHM

This section provides simulation results that compare the performance of Modified Max SINR to the original and that show the performance of the distributed power control.

6.1 Sum Rate Performance

Figure 6.1 shows a comparison of Max SINR, Modified Max SINR, and the algorithms from [5] and [6] for the case of $K = 4$ users each with $N_r^{[k]} = N_t^{[k]} = 4$ antennas and $d^{[k]} = 2$. As the plots shows, there is not much of a difference between the three Max SINR methods which all outperform the other two. So Modified Max SINR has performance comparable to Max SINR but with guaranteed convergence.

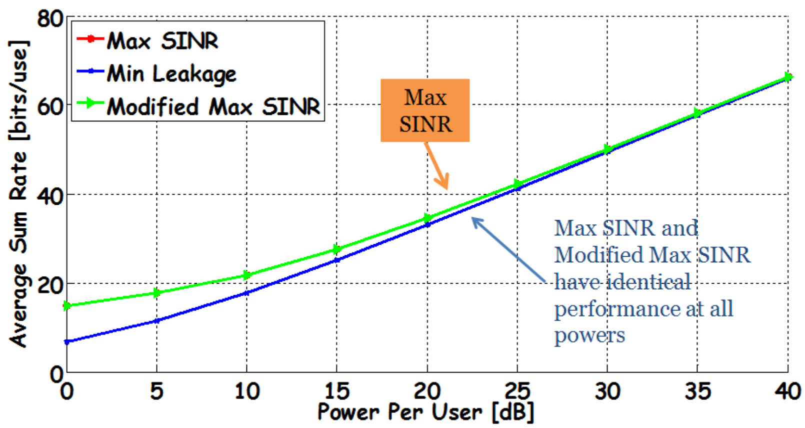


Figure 6.1: Performance of Max SINR for $K = 4$,$N_r^{[k]} = N_t^{[k]} = 4$, and $d^{[k]} = 2$

Figure 6.2 shows another comparison of the four algorithms with $K = 5$ users, each with $N_r^{[k]} = N_t^{[k]} = 3$ antennas and $d^{[k]} = 1$.

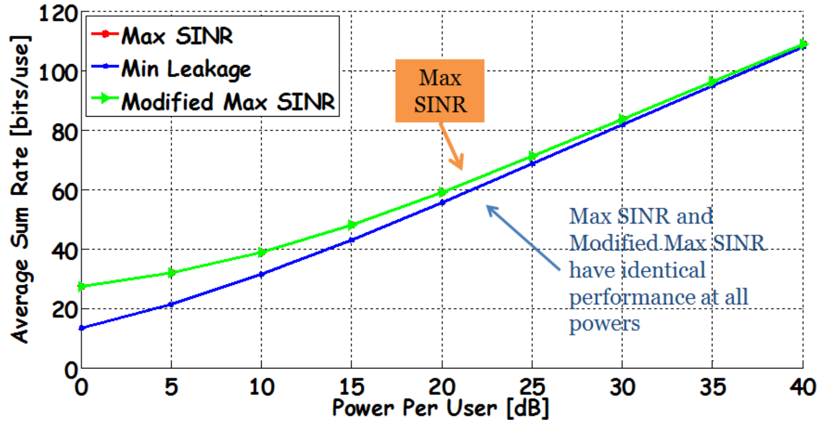


Figure 6.2: Performance of Max SINR for $K = 5$,$N_r^{[k]} = N_t^{[k]} = 3$, and $d^{[k]} = 1$

6.2 Convergence Behavior

For the first case the convergence behavior is demonstrated in Figure 6.3

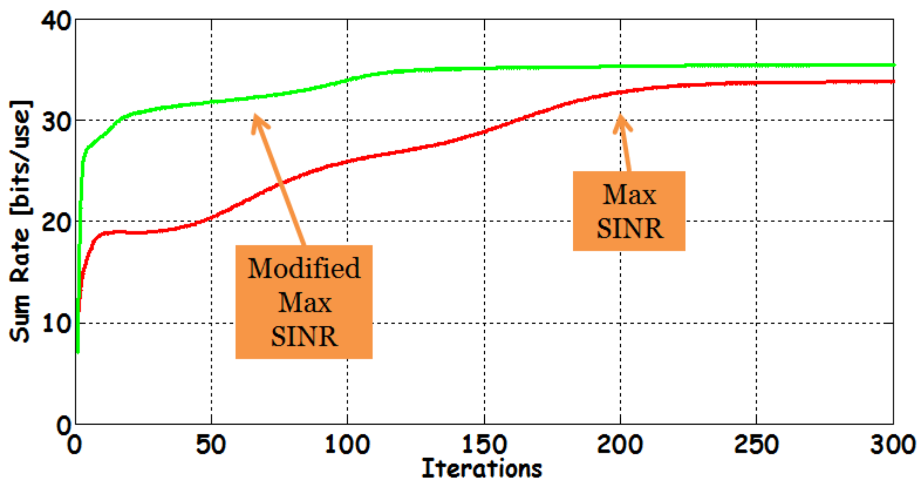


Figure 6.3: Convergence of Max SINR for $K = 4$,$N_r^{[k]} = N_t^{[k]} = 4$, and $d^{[k]} = 2$

For the second case the convergence behavior is demonstrated in Figure 6.4. These two cases show that Modified Max SINR and Max SINR have different convergence behavior. Modified Max SINR may converge faster or slower than Max SINR depending on the system parameters and channel coefficients.

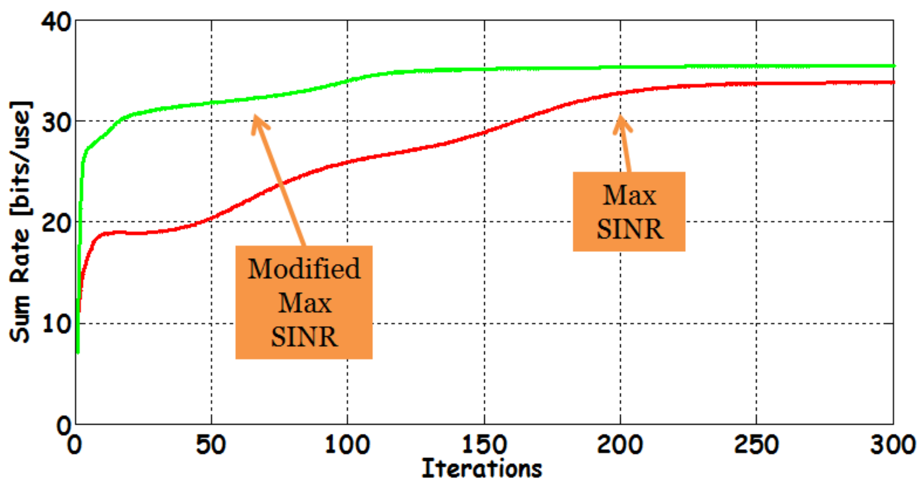


Figure 6.4: Convergence of Max SINR for $K = 5, N_r^{[k]} = N_t^{[k]} = 3$, and $d^{[k]} = 1$

CHAPTER 7

CONCLUSION

Interference is a key issue to overcome in order to provide good data rates to meet future data rate demands. One of the most promising ways to deal with interference is to view users as communicating on an interference channel. This model allows one to perform an information theoretic analysis to determine what is theoretically possible and to design communication strategies to mitigate interference using insights gained from a theoretical analysis.

An exact information theoretic analysis of the interference channel is difficult, so several techniques have been developed to give the approximate capacity region such as finding the capacity within a constant number of bits and degrees of freedom. The degrees of freedom analysis in particular suggests that at high SNRs the capacity region is large because the sum rate scales like $\frac{K}{2}\log(SNR) + o(\log(SNR))$. This suggests that studying the interference channel can provide substantial gains. The idea of interference alignment used to derive this result also suggests that linear transmit strategies are sufficient to achieve high rates at high SNRs and may be a good idea in general.

Several algorithms have been proposed to take advantage of these observations, such as the Min Leakage algorithm and Peters-Heath algorithm. Both of these algorithms rely on the idea of an alternating minimization of transmit and receive vectors to find good transmit and receive vectors. These algorithms are outperformed by the Max SINR algorithm, but there is no proof that this algorithm converges.

In this thesis, a modification to Max SINR is presented in which the sum rate converges. The key idea to make Max SINR converge is to choose the power allocation appropriately every time we reverse the direction of communication to equate the SINR in both directions of communication. With this modification we show that the sum stream rate converges. If we also use SIC, then the sum rate converges.

7.1 Future Directions

Good directions for future work include finding simpler modifications to guarantee that the Max SINR algorithm converges and finding conditions under which the unaltered Max SINR algorithm converges. Also of interest is to develop other algorithms to find good interference alignment solutions and to work on cooperative versions of these iterative algorithms.

Another option is to introduce the idea of asymmetric complex signaling from [19] into the iterative algorithms discussed in this thesis. The idea of asymmetric complex signaling is to treat the real and complex parts of the input signal $\boldsymbol{x}^{[j]} = \boldsymbol{x}_R^{[j]} + \mathrm{j}\boldsymbol{x}_I^{[j]}$ separately. This means it is necessary to design separate transmit vectors for the real and complex part, so the input signal will look like

$$\boldsymbol{x}^{[j]} = \boldsymbol{V}_R^{[j]}\tilde{\boldsymbol{x}}_R^{[j]} + \mathrm{j}\boldsymbol{V}_I^{[j]}\tilde{\boldsymbol{x}}_I^{[j]}$$

From [19] it is known that for the three-user constant SISO channel it is possible to achieve $\eta = \frac{5}{4} > 1$ using asymmetric complex signaling. This implies that at high SNR, asymmetric complex signaling is useful; so, as before, it is interesting to examine the low to moderate SNR impact of asymmetric signaling.

APPENDIX A

POWER CONTROL FOR MORE GENERAL NETWORKS

In this section we show that by selecting the power appropriately we can achieve the same SINRs on the forward and reciprocal channels. In this section, we extend the ideas from [17] to an interference network with SIC.

A.1 Forward Channel

Consider the power optimization problem (POP) given by

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}} \\ \text{subject to} & \overrightarrow{\mathrm{SINR}}^{[k\ell]} \geq \gamma^{[k\ell]} \\ & \overrightarrow{\boldsymbol{\rho}} \geq \mathbf{0} \end{array}$$

which corresponds to trying to transmit with the least sum power to meet SINR constraints. The next two lemmas show that if this problem is feasible, then the SINR constraints uniquely determine the solution.

Lemma 4. *For $\alpha > 1$, $\mathrm{SINR}^{[k\ell]}(\overrightarrow{\boldsymbol{\rho}}) < \mathrm{SINR}^{[k\ell]}(\alpha\overrightarrow{\boldsymbol{\rho}})$.*

Proof.

$$\begin{aligned} \mathrm{SINR}^{[k\ell]}(\alpha\overrightarrow{\boldsymbol{\rho}}) &= \frac{\alpha\overrightarrow{\rho}^{[k\ell]}\overrightarrow{G}^{k,\ell}_{k,\ell}}{1+\sum_{s=\ell+1}^{d^{[k]}}\alpha\overrightarrow{\rho}^{[ks]}\overrightarrow{G}^{k,s}_{k,\ell}+\sum_{j=1}^{K}\sum_{s=1}^{d^{[j]}}\alpha\overrightarrow{\rho}^{[js]}\overrightarrow{G}^{j,s}_{k,\ell}} \\ &> \frac{\alpha\overrightarrow{\rho}^{[k\ell]}\overrightarrow{G}^{k,\ell}_{k,\ell}}{\alpha+\sum_{j=1}^{K}\sum_{s=1}^{d^{[j]}}\alpha\overrightarrow{\rho}^{[js]}\overrightarrow{G}^{j,s}_{k,\ell}} \\ &= \frac{\overrightarrow{\rho}^{[k\ell]}\overrightarrow{G}^{k,\ell}_{k,\ell}}{1+\sum_{s=\ell+1}^{d^{[k]}}\overrightarrow{\rho}^{[ks]}\overrightarrow{G}^{k,s}_{k,\ell}+\sum_{j=1}^{K}\sum_{s=1}^{d^{[j]}}\overrightarrow{\rho}^{[js]}\overrightarrow{G}^{j,s}_{k,\ell}} \\ &= \mathrm{SINR}^{[k\ell]}(\overrightarrow{\boldsymbol{\rho}}) \end{aligned}$$

$\square$

Lemma 5. *If POP is feasible, then all the inequality constraints are active and the solution is given by the unique solution to the SINR constraints with equality.*

Proof. Let $\boldsymbol{\rho}^*$ be optimal and suppose for some (k,l) that $\mathrm{SINR}^{[k\ell]}(\boldsymbol{\rho}^*) > \gamma^{[k\ell]}$. Define

$$\hat{\rho}^{[js]} = \begin{cases} \rho^{*[js]}, & \text{if } (j,s) \neq (k,l) \\ \frac{\gamma^{[k\ell]}}{\mathrm{SINR}^{[k\ell]}} \rho^{*[k\ell]}, & \text{else} \end{cases}$$

Then $\hat{\boldsymbol{\rho}}$ is clearly feasible, since $\mathrm{SINR}^{[k\ell]} = \gamma^{[k\ell]}$ and all other SINRs are increased. Also, $\mathbf{1}^\top \boldsymbol{\rho}^* > \mathbf{1}^\top \hat{\boldsymbol{\rho}}$, which is a contradiction.

Now suppose both $\boldsymbol{\rho}^*$ and $\hat{\boldsymbol{\rho}}$ are optimal. Then there exist (k,ℓ) and $\alpha > 1$ such that $\alpha\boldsymbol{\rho}^* \geq \hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\rho}}^{[k\ell]} = \alpha\boldsymbol{\rho}^{*[k\ell]}$. Then,

$$\begin{aligned}
\gamma^{[k\ell]} &\overset{(a)}{=} \overrightarrow{\mathrm{SINR}}^{[k\ell]}(\boldsymbol{\rho}^*) \\
&\overset{(b)}{<} \overrightarrow{\mathrm{SINR}}^{[k\ell]}(\alpha\boldsymbol{\rho}^*) \\
&= \frac{\hat{\boldsymbol{\rho}}^{[k\ell]} \overrightarrow{G}^{k,\ell}_{k,\ell}}{1 + \sum_{s=\ell+1}^{d^{[k]}} \alpha\boldsymbol{\rho}^{*[ks]} \overrightarrow{G}^{k,s}_{k,\ell} + \sum_{j=1}^{K} \sum_{s=1}^{d^{[j]}} \alpha\boldsymbol{\rho}^{*[js]} \overrightarrow{G}^{j,s}_{k,\ell}} \\
&\overset{(c)}{\leq} \frac{\hat{\boldsymbol{\rho}}^{[k\ell]} \overrightarrow{G}^{k,\ell}_{k,\ell}}{1 + \sum_{s=\ell+1}^{d^{[k]}} \hat{\boldsymbol{\rho}}^{[ks]} \overrightarrow{G}^{k,s}_{k,\ell} + \sum_{j=1}^{K} \sum_{s=1}^{d^{[j]}} \hat{\boldsymbol{\rho}}^{[js]} \overrightarrow{G}^{j,s}_{k,\ell}} \\
&= \overrightarrow{\mathrm{SINR}}^{[k\ell]}(\hat{\boldsymbol{\rho}}) \\
&\overset{(d)}{=} \gamma^{[k\ell]}
\end{aligned}$$

where (a) follows from the first part of this lemma, (b) follows from Lemma 4, (c) follows since $\alpha\boldsymbol{\rho}^* \geq \hat{\boldsymbol{\rho}}$, and (d) from the first part of this lemma. This is a contradiction. $\square$

We next consider the power maximization problem (PMP) given by

$$\begin{aligned}
\text{maximize} \quad & \mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}} \\
\text{subject to} \quad & \overrightarrow{\mathrm{SINR}}^{[k\ell]} \leq \gamma^{[k\ell]} \\
& \overrightarrow{\boldsymbol{\rho}} \geq \mathbf{0}
\end{aligned}$$

Trying to use as much power as possible without exceeding an SINR constraint does not seem usefull; however, we will be interested in the dual of

POP which has the same form as PMP. The following lemma shows that POP and PMP actually have the same solution.

Lemma 6. *If PMP is feasible, then all the inequality constraints are active and the solution is given by the unique solution to the SINR constraints with equality. Therefore, POP and PMP have the same solution.*

Proof. Follows same proof from Lemma 5. $\square$

A.2 Reciprocal Channel and Dual of POP

Next, we examine the dual of POP. It will turn out the dual of POP corresponds to reversing the direction of communication and switching the roles of $\boldsymbol{U}^{[k]}$ and $\boldsymbol{V}^{[k]}$. To find the dual of the POP problem, we need to put the SINR constraints in standard form. By expanding any given constraint, $\overrightarrow{\mathrm{SINR}}^{[k\ell]} \geq \gamma^{[k\ell]}$, we get

$$\overrightarrow{\rho}^{[k\ell]}\overrightarrow{G}^{k,\ell}_{k,\ell} \geq \gamma^{[k\ell]} + \gamma^{[k\ell]}\sum_{s=\ell+1}^{d^{[k]}}\overrightarrow{\rho}^{[ks]}\overrightarrow{G}^{k,s}_{k,\ell} + \gamma^{[k\ell]}\sum_{j=1}^{K}\sum_{s=1}^{d^{[j]}}\overrightarrow{\rho}^{[js]}\overrightarrow{G}^{j,s}_{k,\ell}$$

which can be simplified to

$$\overrightarrow{\rho}^{[k\ell]}\frac{\overrightarrow{G}^{k,\ell}_{k,\ell}}{\gamma^{[kl]}} - \sum_{s=\ell+1}^{d^{[k]}}\overrightarrow{\rho}^{[ks]}\overrightarrow{G}^{k,s}_{k,\ell} - \sum_{j=1}^{K}\sum_{s=1}^{d^{[j]}}\overrightarrow{\rho}^{[js]}\overrightarrow{G}^{j,s}_{k,\ell} \geq 1$$

Define

$$\boldsymbol{D} = \mathrm{diag}\{\frac{\gamma[11]}{\overrightarrow{G}^{1,1}_{1,1}}, \ldots, \frac{\gamma[Kd^{[K]}]}{\overrightarrow{G}^{K,d^{[K]}}_{K,d^{[K]}}}\}$$

and $\boldsymbol{G}$ such that

$$\boldsymbol{G}_{\sum_{m=1}^{k-1} d^{[m]}+\ell, \sum_{n=1}^{j-1} d^{[n]}+s} = \begin{cases} 0, & \text{if } k=j \text{ and } \ell < s, \\ \overrightarrow{G}^{j,s}_{k,\ell}, & \text{else} \end{cases}$$

Setting

$$\boldsymbol{A} = \boldsymbol{D}^{-1} - \boldsymbol{G}$$

allows us to express the SINR constraints as $\boldsymbol{A}\overrightarrow{\boldsymbol{\rho}} \geq \mathbf{1}$, so POP is given by:

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \boldsymbol{A}\overrightarrow{\boldsymbol{\rho}} \geq \mathbf{1} \\ & \overrightarrow{\boldsymbol{\rho}} \geq \mathbf{0} \end{aligned}$$

In matrix form the solution to POP and PMP can be expressed as

$$\overrightarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-1}\mathbf{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-1}\right)\mathbf{1} \tag{A.1}$$

The dual of POP using [20] is

$$\begin{aligned} \text{maximize} \quad & \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \boldsymbol{A}^\top\overleftarrow{\boldsymbol{\rho}} \leq \mathbf{1} \\ & \overleftarrow{\boldsymbol{p}} \geq \mathbf{0} \end{aligned}$$

which is of the same form as PMP and denoted DPMP. Now $A^\top = D^{-1} - G^\top$, so for fixed (k, ℓ) the constraint $\left(A^\top \overrightarrow{\boldsymbol{\rho}}\right)_{\sum_{i=1}^{k-1} d^{[i]}+\ell} \leq 1$ is given by

$$\overleftarrow{\rho}^{[k\ell]} \frac{\overrightarrow{G}^{k,\ell}_{k,\ell}}{\gamma^{[k\ell]}} - \sum_{s=1}^{\ell-1} \overleftarrow{\rho}^{[ks]} \overleftarrow{G}^{k,s}_{k,\ell} - \sum_{j=1}^{K} \sum_{s=1}^{d^{[j]}} \overleftarrow{\rho}^{[js]} \overrightarrow{G}^{k,\ell}_{j,s} \leq 1$$

which can be expressed as $\overleftarrow{\mathrm{SINR}}^{[k\ell]} \leq \gamma^{[k\ell]}$. Then DPMP is given by

$$\begin{aligned} \text{maximize} \quad & \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \overleftarrow{\mathrm{SINR}}^{[k\ell]} \leq \gamma^{[k\ell]} \\ & \overleftarrow{\boldsymbol{p}} \geq \mathbf{0} \end{aligned}$$

Remark on Deriving the Dual: To derive the dual, $\boldsymbol{A}^\top$ must describe the dual network. Now $\boldsymbol{A}^\top = \boldsymbol{D}^{-1} - \boldsymbol{G}^\top$, so $\boldsymbol{G}^\top$ must describe the interference. This means that if we use SIC we must reverse the order of cancellation to ensure that the dual corresponds to the reciprocal network. As discussed in the section on link gains, since $\boldsymbol{G}^\top$ describes the dual network, the dual network corresponds to reversing the direction of communication and switching the roles of $\boldsymbol{V}^{[k]}$ and $\boldsymbol{U}^{[k]}$.

By a similar argument to Lemma 6, DPMP is equivalent to DPOP given by

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} & \overleftarrow{\text{SINR}}^{[k\ell]} \geq \gamma^{[k\ell]} \\ & \overleftarrow{\boldsymbol{p}} \geq \mathbf{0} \end{array}$$

Then the solution to DPOP and DPMP is given by

$$\overleftarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-\top}\mathbf{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-\top}\right)\mathbf{1} \tag{A.2}$$

The following theorem shows that it is possible to achieve the same SINRs on the forward and reciprocal channels:

Theorem 2. *POP is feasible iff DPOP is feasible. Furthermore,* $\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^*$ *for the optimal power vectors and* $\overrightarrow{\text{SINR}}^{[k\ell]} = \overleftarrow{\text{SINR}}^{[k\ell]} = \gamma^{[k\ell]}$.

Proof. The first part follows from strong duality. The SINR result follows from Lemma 5. ☐

When changing powers, we need to be sure we still meet our sum power constraint. The following lemma shows that as long as we meet the power constraint for some $\boldsymbol{\rho}$ we will meet it for $\boldsymbol{\rho}^*$ too.

Lemma 7. *Suppose we choose a power allocation* $\boldsymbol{\rho}$ *such that* $\mathbf{1}^\top \boldsymbol{\rho} \leq KP$ *and transmit and receive vectors* $\{\boldsymbol{V}^{[k]}\}$ *and* $\{\boldsymbol{U}^{[k]}\}$. *Let* $\gamma^{[k\ell]}$ *be the resulting SINRs. Then for the optimal power allocation for POP and DPOP,* $\overrightarrow{\boldsymbol{\rho}}^*$ *and* $\overleftarrow{\boldsymbol{\rho}}^*$ *respectively,*

$$\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^* \leq KP$$

Proof. Since $\boldsymbol{\rho}$ is feasible, we must have

$$\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* \leq \mathbf{1}^\top \boldsymbol{\rho} \leq KP$$

By Theorem 2,

$$\mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* \leq KP$$

☐

The relation between all of the problems introduced here is summarized in Figure A.1.

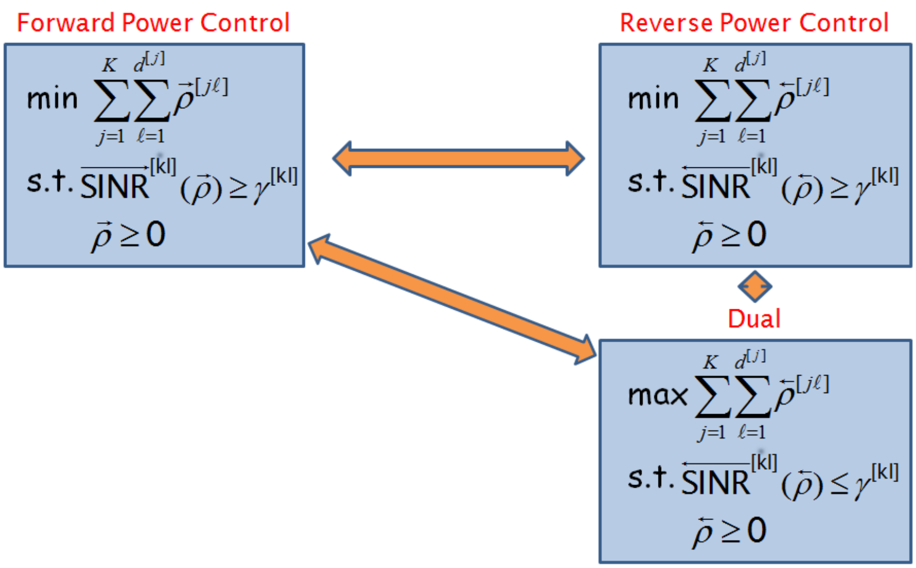


Figure A.1: Relation between Power Control Problems

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