THREE ESSAYS IN APPLIED MICROECONOMIC THEORY

BY

MEENAKSHI GHOSH

DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Economics
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2014

Urbana, Illinois

Doctoral Committee:

Professor Mattias Polborn, Chair
Professor Dan Bernhardt, Director of Research
Professor George Deltas
Professor Steven Williams

Abstract

The first chapter of my dissertation characterizes how the strategic choices of product variety, quality and pricing by a brick and mortar store evolve in the face of stiffening competition from online retailers in the market. For many consumers, online purchases are imperfect substitutes for store purchases due to delayed delivery, the need for unfamiliar self-installation, etc. As online products become better substitutes and online competition intensifies, the store moves from socially-efficient provision of product qualities and varieties, to reducing the quality of its low quality products, next, to selective provision of high quality products, before returning to socially-efficient provision of product quality and product varieties. All the while, the physical store's profits decline, and it finally exits the market when the internet competition becomes so stiff that it drives operating profits below its fixed costs of operation. An extension explores outcomes when the store has an online portal, characterizing when the physical store appears to act as a loss leader.

Chapter 2 analyzes the dynamics of positive and negative campaigning in primary and general elections, characterizing the strategic considerations that influence campaigning choices. Candidates devote resources to positive campaigning, which builds their reputation stocks, and to negative campaigning, which damages a rival's. An explanation is provided for why general campaigns are more negative than primary campaigns: in the general election, winning primary candidates benefit only from positive primary campaigning; and negative campaigning by a primary loser, impairs his party rival's chances. More generally, the impact of the (a) relative strengths of candidates (initial resources and reputations), (b) how much candidates care about winning vs just having their party win, and (c) the campaigning technology (effectiveness, decay in the effects of primary campaigning before the general election) on the magnitudes and composition of campaigning in both elections is characterized.

In Chapter 1, consumers are assumed to have either a high or a low valuation of quality. Chapter 3 looks at how outcomes change when consumer valuation types are continuous. A numerical analysis reveals that there exists a threshold of consumer valuation of quality such that consumers with valuations below the threshold buy the low quality product while those with valuations above the threshold buy the

high quality product. As online competition increases, the store initially lowers its prices in order to retain consumers who would otherwise switch to online purchases and this move initially allows it to attract more consumers than before. However, the store is unable to match the competitive offers available online for long; when the spread of consumer valuations is not too low, as online products become better substitutes, consumers with relatively low valuations increasingly switch to online products. The low quality product of the store now caters to the tastes of consumers with higher valuations than before: the store begins to increase the quality and price of its low quality products. To prevent its high valuation consumers from switching over to the low quality product, the store raises the quality of its high quality product too. All the while, its profits decline and the store eventually exits the market when it is unable to meet its fixed operating costs. The impact of a smaller spread of consumer valuations, lower marginal costs and higher travel costs on the strategic choices of the brick and mortar store is also explored.

To Ma Baba, to whom I owe everything.

Acknowledgments

I am indebted to my advisor Dan Bernhardt for his continuous guidance, support and encouragement over the past years. I would also like to express my gratitude to George Deltas, Mattias Polborn and Steven Williams for their constructive advice on all chapters.

Thanks are due to my friends at Urbana Champaign for many happy times shared at the department, and outside over meals and other activities; they kept me sane and in good humor. This dissertation would also never have been possible without the love and support of my mother and sister whose belief in me helped me through till the very end. Lastly, I would like to thank my husband; his quiet and unstinted support saw me through many moments of stress and panic.

Table of Contents

Chapter 1

Quality Provision and Pricing in the Face of Online Competition

1.1 Introduction

Recent years have seen a dramatic surge in online shopping as the ease and reliability of online purchases has improved.[1] So, too, recent years have seen the exit of many previously-profitable brick and mortar stores—Circuit City, Eatons, KMart, Linens N Things, Borders Books, Blockbuster Video and so on—in large part due to intensified online competition.

In this paper, we paint a portrait of a physical store flailing to find a successful strategic plan in the face of increased online competition. For many consumers, online purchases are imperfect substitutes for store purchases due to delayed delivery, the need for unfamiliar self-installation, etc. We characterize how the strategic choices by the physical store of product variety, quality and pricing evolve as online products become better substitutes and online competition intensifies. As online competition grows, the physical store appears to abandon one strategic plan for another and then for a third before returning to its original plan. All the while, the physical store's profits decline, and it finally exits the market when the internet competition becomes so stiff that it drives operating profits below its fixed costs of operation.

In our model, consumers are distinguished by (1) their costs of traveling to the brick and mortar store, (2) their utility loss associated with online purchases, which we capture by a multiplicative discount factor[2], and (3) their preferences for quality. Consumers know their travel costs and the utility/inconvenience costs incurred from online purchases, but do not learn their product valuations (high or low) unless they physically inspect products: as in Loginova (2009), online venues do not allow a sufficient hands-on inspection to permit consumers to learn their valuations. As a result, consumers may visit a physical store to learn their quality valuations, and hence which products they want to purchase. However, once at the

[1] A 2010 Nielsen survey, *Global Trends in Online Shopping, A Nielsen Global Consumer Report* (June 2010), found that 84% of consumers shop online, and online shopping accounted for more than 5 percent of total monthly spending for 56 percent of the respondents.

[2] Qualitative results are unaffected if online purchases instead incur a service cost that enters additively. Such additive costs only affect the range of goods provided online.

store, consumers can opt instead to buy online from a competitive e-retailer.[3] Given this online competition, the brick and mortar store chooses (a) the extent of product variety (one quality or two), (b) the levels of qualities offered, and (c) prices.

As a benchmark, we consider the case where the utility costs from online purchases are prohibitive, so the physical store is a monopolist. We show that the store provides the socially optimal product quality for each consumer valuation type, fully internalizing the surplus associated with efficient quality provision. More subtly, prices are only imperfectly pinned down. Because consumers do not learn their valuations unless they visit the store, decisions about whether to go to the store to make a purchase only hinge on the *expected* price paid. Once at the store, consumers are captured since travel costs are sunk. As a result, the range of equilibrium prices for the low quality good (and hence the high quality good) is only pinned down by the ex post individual rationality and incentive compatibility constraints.

Once the utility costs of online purchases fall by enough, e-retailers operate. We consider populations of consumers with representative high, medium and low utility costs for online purchases. Low utility cost consumers are comfortable with online purchases. In equilibrium, they either purchase directly from an online e-retailer without learning their quality valuation (if located far from the physical store), or they first visit the physical store (if close) to learn their valuations and then purchase online the product that they identify for themselves as right. In this way, the physical store provides these consumers a positive externality, but fails to profit from it. In contrast, online purchases are not a viable alternative for high utility cost consumers, who buy at the store, if they buy at all.

Our primary focus is on the intermediate population of consumers with medium online discount factors who may plausibly buy from either source, and on what happens as online purchases become more attractive. As long as the value of this medium online discount factor remains low enough, the physical store continues to provide socially optimal product qualities, although the set of possible equilibrium price combinations shrinks once online competition limits the extent to which it can set a high price for a quality. Once online discount factors rise by enough to make online purchases a viable alternative for consumers who want to buy at the outset without learning their valuations, the physical store's profits begin to fall, even though, at first it continues to provide socially optimal qualities, albeit at lower prices.

Online competition is more intense for products that appeal to low valuation consumers, as their

[3] A recent research survey revealed that 45 percent of customers shopping at brick and mortar locations will walk out and complete their purchase online for a discount as low as 2.5 percent. For a 5 percent savings, the survey showed 60 percent would leave to shop online. Labeled as "showrooming", this shopping trend is driven by the ease with which consumers now access information on prices online via mobile phones. ['Showrooming' shopping poses challenges for retailers, BusinessNews Daily]

utility loss from online purchases is less. Hence, the prices of lower quality products are more sensitive to online competition. In the interesting case where quality valuations do not differ vastly for different consumer types—so that consumers can be reasonably uncertain about which product is best for them—as online discount factors rise, eventually the decline in the price of the low quality good makes it attractive to high valuation consumers.

Now, as online discount factors rise further, the physical store appears to experiment with its strategic plan. First, in the face of its own product-line competition, it distorts the quality targeted at low valuation consumers downward as long as quality valuation differences are small enough. Next, provided enough consumers have high valuations, the own product-line competition from its low quality product for its high quality product becomes severe enough that the store does away with the lower quality product, switching to specializing in the provision of the high quality product.

This phase does not last. As online products become ever better substitutes, eventually direct online competition of high quality products begins to bite. Once the internet provides sufficiently good, competitively-priced high quality products, the physical store need no longer worry about own product-line competition—high valuation consumers are no longer attracted to lower quality in-store products, preferring higher quality online products, instead. In turn, this induces the physical store to return to its original strategic plan of providing a full range of product qualities, offering once more the socially optimal quality levels, and prices that just induce consumers with medium online discount factors to purchase from it. All the while its profits decrease and in the final stage, the physical store goes bankrupt and exits, as these prices fail to cover the fixed costs of its brick and mortar presence.

This process of repeated "experimentation" with different strategic plans—from reduced prices of a broad array of (socially optimal) qualities, to experiments with reduced low quality products, which are then dropped completely, before a return to an original plan of a broad array of socially optimal qualities sold at low prices, and then finally bankruptcy—together with ever-dropping prices and profits has the appearance of a physical store making strategic error after error, floundering in the face of stiffening online competition. In fact, the store always acts optimally, but it is unable to capitalize on the positive externalities it provides consumers who exploit the information conveyed by hands-on inspection, but then either shop elsewhere, or demand punishingly low prices to stay.

We then extend the model to explore the case where the physical store also has an online portal. We seek to capture the phenomena that physical stores often provide product installation support or servicing facilities that some consumers derive value from when buying at its online portal. Maintaining an online portal can benefit the store when diverting sales to its online outlet provides cost savings. Indeed, the store survives even if its physical store's sales cannot cover its fixed costs of operation by drawing on its profitable online sales—its online outlet appears to profit from prices that are "less competitive" than those of other online e-tailers, with the physical store appearing to act as a loss leader. However, it is only the physical store's presence that allows its online portal to generate profits.

Related literature. Our paper builds on work dating back to Mussa and Rosen (1978), Goldman et al. (1980), Spence (1980), and Maskin and Riley (1984) that explores price discrimination via quantity discounts and pricing of products of different qualities by monopolists facing heterogeneous consumers with different private valuations. In these models, in contrast to our benchmark setting, quality is distored downward for all but those with the highest willingness to pay, because downward incentive compatibility constraints bind. Rochet and Stole (1997) study duopoly nonlinear pricing in a model with horizontal and vertical product differentiation. When the degree of horizontal differentiation is so large that each firm is a local monopolist, perfect sorting arises, with quality distortions for all types but at the top and the bottom. In contrast, with little horizontal differentiation, the market is fully covered on both vertical and horizontal dimensions, and firms offer a cost-plus-fee pricing schedule with efficient quality provision. A more general analysis in Rochet and Stole (2002) covers both monopoly and duopoly, and allows for general distributions.

The most closely-related paper is Loginova (2009). She models competition between electronic retailers and brick and mortar stores that sell a homogeneous product to consumers with heterogeneous valuations (high or low) who only learn their valuations after visiting the store. Paradoxically, the presence of electronic retailers causes brick and mortar stores to raise prices. Rather than compete for low valuation consumers against e-retailers, stores target only high valuation consumers, whose demands are less price elastic. Low valuation consumers return home and buy online. Our paper extends the analysis by studying quality provision and pricing by stores and how they evolve in the face of increasing online competition.

The idea that consumers must learn by visiting a store also appears in the price search literature that dates back to Stahl (1979), and continues on in research such as Ellison's (2005) model of add-on pricing in which firms advertise a base price for a product and try to induce customers with a high willingness to pay to buy high-priced add-ons at the point of sale.

There is a burgeoning literature on consumer and firm behavior in online environments. Alba et al (1997), Danaher et al. (2003), Peterson et al. (1997), and Ratchford et al. (2001) conduct empirical analyses of the differences between online and offline purchase experiences, focusing on assessing the impact of prices, brand names and product attributes on consumer choice. These indicate that online shopping is well suited for functional products about which online stores can provide detailed information. However, online stores are less suited for products with sensory "touch and feel" attributes. Brown and Goolsbee (2002) use data on individual insurance policies to analyze the impact of comparison shopping on offline prices. They find that the introduction of the insurance-oriented web sites was at first associated with high price dispersion. As the use of these sites became more widespread, prices and dispersion fall. Sengupta and Wiggins (2006) find that increased online sales of airline tickets are associated with reduced online and offline prices. Lal and Sarvary (1999) distinguish between digital and non-digital product attributes. Digital attributes can be conveyed via the internet, while non-digital attributes can only be judged in person at a retail store. The introduction of online shopping may induce consumers not to search, but instead to order familiar products online. In turn, this increased consumer loyalty can induce firms to raise prices.

The paper is structured as follows. We next present the formal model. In Section 1.3, we study the case where the brick and mortar store exists as a monopoly and offers multiple qualities of the product. Section 1.4 analyzes our central internet setting where both e-retailers and stores operate in the market. Section 1.5 explores how outcomes are affected when the brick and mortar store also operates an online portal. Proofs are collected in Appendix A.

1.2 The Model

Our economy features a single brick and mortar (b*&*m) store that has a physical establishment, and many competitive electronic retailers that sell products on the internet. These stores differ in three ways: (1) it is costly for consumers to travel to the b*&*m store, but internet "transportation" costs are zero; (2) online purchases are imperfect and inferior substitutes for purchases from the b*&*m store reflecting that online deliveries take time and may require inconvenient and possibly flawed self-installation; and (3) there is a fixed cost $k > 0$ of having a physical establishment, but maintaining an internet presence is costless.

There is a measure one of consumers. Consumers differ from each other in three ways: (1) their distances x from the *b&m* store; (2) their valuations $\theta \in \{\theta_l, \theta_h\}$ of product quality; and (3) their online utility discount factors, $\delta \in \{0, \delta_m, 1\}$, where δ describes how good a substitute online purchases are for purchases at a b*&*m store. Consumer location, quality valuation and utility costs from online purchases are independently distributed in the population.

In the population of consumers, travel distances to the physical store are uniformly distributed on

$[0, 0.5]$. A consumer who travels distance x to the b&m store incurs costs tx, where $t > 0$. A consumer with quality valuation θ and online discount factor δ derives utility $\theta q - \frac{1}{2}q^2$ from purchasing a product of quality q from the b&m store, but he only derives utility $\delta(\theta q - \frac{1}{2}q^2)$, if he buys online. Fraction λ of consumers value quality at a high level, θ_h; the remaining consumers only value quality by $\theta_l < \theta_h$. Fraction μ_0 of consumers have prohibitively high utility losses from online purchases ($\delta = 0$), while fraction μ_1 of consumers are patient, and facile at self-installation ($\delta = 1$). The remaining measure $\mu_m = 1 - \mu_0 - \mu_1$ of consumers incur intermediate utility losses from online purchases ($\delta = \delta_m$). Thus, type $\delta = 0$ consumers never purchase online, while type $\delta = 1$ consumers always purchase online as long as e-retailers offer lower prices than the physical store. It is only type δ_m consumers who are potentially open to purchasing both online or at the b&m store.

A consumer knows his distance x from the b&m store, and the utility costs associated with his online purchases, but he does not learn his valuation of quality unless he goes to a store and inspects the products. A consumer can (1) buy a product online at the outset without knowing his valuation, or (2) visit a store, learn his valuation, and then decide which product, if any, to buy at the b&m store or online. Firms know the distribution of (x, δ, θ) in the economy, but do not observe these attributes in consumers.

It costs cq to produce a good of quality q, where $c > 0$. The b&m store chooses the level and variety of its product qualities, and their prices. We are interested in the setting where a consumer's possible valuations of quality, θ_l and θ_h, are not too different so that a consumer could plausibly be uncertain about his or her valuations. As a result, $\theta_l >> \max\{c, \theta_h - c\}$. Accordingly, it is socially efficient for the b&m store to provide quality $q_i^* = \theta_i - c$ to a consumer with quality valuation θ_i. The fact that θ_h and θ_l do not differ too substantially means that if the b&m store sells both qualities q_h^* and q_l^*, then it has to concern itself about the possibility that a high quality valuation customer may buy the low quality good whenever the price difference $p_h - p_l$ is large. That is, the b&m store has to worry about the competition provided by its own product quality line. Electronic retailers are perfectly competitive, implying that a consumer can buy online a good of any quality q at a price of cq.

1.2.1 Monopoly

We first consider a benchmark setting where $\mu_0 = 1$, so that online purchases are not a viable alternative for consumers, making the b&m store a monopolist. When the b&m store sells two product qualities q_l and q_h, $q_l < q_h$ at associated prices $p_l < p_h$, the expected utility of a consumer who visits the b&m store, inspects the products and decides which, if any, to buy is

$$\lambda \max\big(\theta_h q_h - \frac{1}{2}q_h^2 - p_h, \theta_h q_l - \frac{1}{2}q_l^2 - p_l, 0\big) \\ + (1-\lambda)\max\big(\theta_l q_h - \frac{1}{2}q_h^2 - p_h, \theta_l q_l - \frac{1}{2}q_l^2 - p_l, 0\big) - tx.$$

Because type θ_h consumers value quality by more than type θ_l consumers, product quality choices rise in θ. For type θ_h consumers to purchase q_h and type θ_l consumers to purchase q_l, the following incentive compatibility constraints must hold:

$$\theta_h(q_h - q_l) - \frac{1}{2}(q_h^2 - q_l^2) \geq (p_h - p_l)$$
$$\theta_l(q_h - q_l) - \frac{1}{2}(q_h^2 - q_l^2) \leq (p_h - p_l).$$

Further, the individual rationality constraint for low valuation types, $\theta_l q_l - \frac{1}{2}q_l^2 - p_l \geq 0$, must hold for type θ_l consumers to purchase at the store. The expected utility that a consumer located at x expects to derive from visiting the b&m store, prior to learning his valuation is

$$\lambda(\theta_h q_h - \frac{1}{2}q_h^2 - p_h) + (1-\lambda)(\theta_l q_l - \frac{1}{2}q_l^2 - p_l) - tx,$$

implying that the b&m store draws consumers over a distance

$$x = \frac{\lambda}{t}\big(\theta_h q_h - \frac{1}{2}q_h^2 - p_h\big) + \big(\frac{1-\lambda}{t}\big)\big(\theta_l q_l - \frac{1}{2}q_l^2 - p_l\big).$$

The b&m store's expected profits, under monopoly, are

$$\pi_M = \big[\frac{2\lambda}{t}\big(\theta_h q_h - \frac{1}{2}q_h^2 - p_h\big) + \frac{2(1-\lambda)}{t}\big(\theta_l q_l - \frac{1}{2}q_l^2 - p_l\big)\big]\big(\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l)\big) - k.$$

Proposition 1 characterizes the associated equilibria. All proofs are collected in an appendix.

Proposition 1. *Suppose the b&m store is a monopolist. Then, in equilibrium,*

1. *The b&m store always provides the socially efficient levels of quality, $\theta_j - c, j = h, l$.*

2. *There exists a continuum of supporting equilibrium prices, $\{p_l, p_h(p_l)\}$ indexed by $p_l \in [\underline{p_l}, \bar{p}_l]$ that induce the same participation decisions by consumers of whether to visit the b&m store and buy a product, and deliver the same expected profits to the monopolist.*

3. For $p_l \in (\underline{p_l}, \bar{p}_l)$, all incentive constraints hold as strict inequalities.

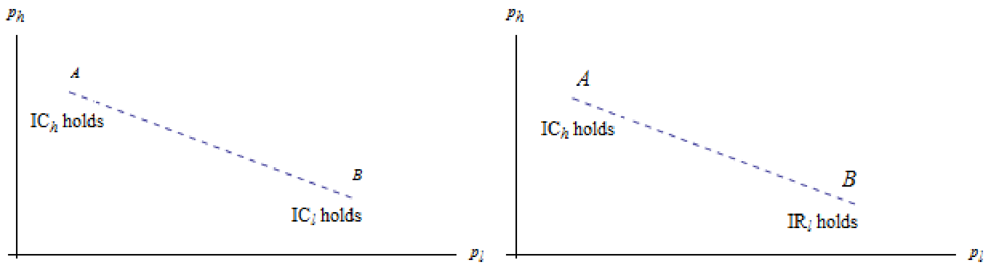

Figure 1.1: Continuum of equilibrium price offers when λ is relatively low and high

Figure 1.1 depicts the continuum of supporting equilibrium prices. The pricing equivalence result reflects that when consumers are deciding whether to travel to the b*&*m store, they do not know whether their valuations are high or low. As a result, only the expected price enters their calculations. Thus, consumers are indifferent to increases in the price of the high quality good when accompanied by a suitable decrease in the price of the low quality good (so that $\lambda dp_h + (1-\lambda)dp_l = 0$). Once at the b*&*m store, everyone purchases a product, as incentive constraints are slack.[4] At $\underline{p_l}$, the incentive compatibility constraint for the high valuation type holds at equality. Whether the upper bound on p_l is determined by the incentive compatibility constraint for the low valuation consumer or his individual rationality constraint depends on how differently consumers may value quality. Fixing θ_l, there exists a $\rho(\lambda, c)$ such that if the types are sufficiently close in quality, $\theta_h - \theta_l \leq \rho(\lambda, c)$, the incentive compatibility constraint holds at $\bar{p}_l$; and if the types are less close, $\theta_h - \theta_l > \rho(\lambda, c)$, the individual rationality constraint holds. Further, $\rho(\lambda, c)$ decreases in the fraction λ of consumers with high valuations. Our premise that consumers are uncertain about their valuations suggests that the relevant case is that where the products are relatively close substitutes, i.e., the incentive compatibility constraint becomes the relevant constraint.

The result that it is optimal for the b*&*m store to provide the socially efficient level of quality to each valuation type reflects the ex-post slackness of the incentive constraints. Therefore, the b*&*m store can extract all surplus associated with quality provision.

[4]Perturbing the economy, for example, by having a vanishingly small but positive measure of consumers who know their valuation, ex ante, will deliver a unique equilibrium pricing outcome (here, it would be the price combination pair in the set of equilibrium price offers that maximizes profits from these consumers).

1.2.2 Internet Competition

We now assume that online purchases are plausible alternatives for some consumers, i.e., μ_m and μ_1 are strictly positive, so that electronic retailers operate. We focus on the interesting case where μ_0 is not so high that the b&m store only wants to serve consumers for whom the internet is not an option (as then the monopoly product provision and pricing obtains).

A consumer located distance x from the b&m store can buy online at the outset without knowing his quality valuation, or he can visit the b&m store where he learns his valuation. Once he has learnt his valuation, he can opt to buy a product at the b&m store, or return and buy it online, if at all. If he buys online at the outset, marginal cost pricing on the part of electronic retailers implies that his quality choice solves

$$\max_q \delta\lambda\left(\theta_h q - \frac{1}{2}q^2\right) + (1-\lambda)\delta\left(\theta_l q - \frac{1}{2}q^2\right) - cq.$$

Hence, he purchases online a good of quality

$$q = \frac{\delta E[\theta] - c}{\delta},$$

provided that $\delta \geq \frac{c}{E[\theta]}$, obtaining an expected payoff of $(\delta E[\theta] - c)^2/2\delta$. A consumer who buys online after learning θ purchases a good of quality

$$q(\theta) = \frac{\delta\theta - c}{\delta},$$

provided that $\delta \geq \frac{c}{\theta}$, obtaining a payoff of $(\delta\theta - c)^2/2\delta$. Consumers with $\delta = 0$ never buy online. Consumers with $\delta = \delta_m$ may or may not derive a positive payoff from purchasing online. For instance, if $\delta_m \in \left(\frac{c}{\theta_h}, \frac{c}{E[\theta]}\right)$, a consumer would buy online if he learns that his valuation is high, but not at the outset or if he is a low type.

We omit the analysis of patient, computer-savvy consumers with $\delta = 1$ who are skilled at self-installation. Since the b&m store always prices above marginal cost, these consumers always make any purchases online. Only consumers with online discount factors $\delta \in \{0, \delta_m\}$ ever buy at the store. As before, individual rationality constraints imply that the maximum distances traveled by consumers with $\delta = 0$ and $\delta = \delta_m$ are

$$x_0^* = \frac{1}{t}\Big(\theta_h q_h - \frac{q_h^2}{2} - p_h\Big) + (1-\lambda)\Big(\theta_l q_l - \frac{q_l^2}{2} - p_l\Big)$$

and

$$x_m^* = \frac{\lambda}{t}\Big(\theta_h q_h - \frac{q_h^2}{2} - p_h\Big) + \Big(\frac{1-\lambda}{t}\Big)\Big(\theta_l q_l - \frac{q_l^2}{2} - p_l\Big) - \Big(\mathbf{1}_{\delta_m > \frac{c}{E[\theta]}} \frac{(\delta_m E[\theta] - c)^2}{2t\delta_m}\Big).$$

Hence, the *b&m* store's profits are

$$\begin{aligned}\pi_I &= \frac{2}{t}\Big((\mu_0+\mu_m)\Big(\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l)\Big) - \mu_m \mathbf{1}_{\delta_m > \frac{c}{E[\theta]}} \frac{(\delta_m E[\theta]-c)^2}{2\delta_m}\Big) \\ &\times \ \big(\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l)\big) - k. \qquad (1.1)\end{aligned}$$

Thus, the b&m store's maximization problem becomes:

$$\max \ \pi_I \ \text{ subject to}$$

$$\begin{aligned}&\mathbf{IC_h} : \theta_h(q_h - q_l) - \frac{1}{2}(q_h^2 - q_l^2) - (p_h - p_l) \geq 0 \\ &\mathbf{IC_l} : \theta_l(q_h - q_l) - \frac{1}{2}(q_h^2 - q_l^2) - (p_h - p_l) \leq 0 \\ &\mathbf{IR_h} : \theta_h q_h - \frac{1}{2}q_h^2 - p_h \geq \mathbf{1}_{\delta_m > \frac{c}{\theta_h}} \frac{(\delta_m\theta_h - c)^2}{2\delta_m} \\ &\mathbf{IR_l} : \theta_l q_l - \frac{1}{2}q_l^2 - p_l \geq \mathbf{1}_{\delta_m > \frac{c}{\theta_l}} \frac{(\delta_m\theta_l - c)^2}{2\delta_m}.\end{aligned}$$

We begin by characterizing equilibrium outcomes when internet options represent poor substitutes for in-store purchases for consumers with medium online discount factors.

Proposition 2. *Fixing other parameters, there exists a* $\underline{\delta}_m(\theta_l) \in [\frac{c}{\theta_l}, 1]$ *such that*

1. *For* $\delta_m < \underline{\delta}_m$, *in equilibrium, the b&m store provides the unconstrained optimal levels of quality,* $\theta_j - c$, $j = h, l$, *with supporting prices* $p_l^* \in [\underline{p_l}(\delta_m), \bar{p}_l(\delta_m)]$ *and* $p_h^* = h(p_l^*)$, $h' < 0$, *where* $\bar{p}_l(\delta_m) - \underline{p}_l(\delta_m)$ *is strictly decreasing in* δ_m *once* δ_m *is sufficiently large. For all* $p_l^* \in (\underline{p_l}(\delta_m), \bar{p}_l(\delta_m))$ *all incentive constraints hold as strict inequalities.*

2. *There exists a* $\delta_m^* < \underline{\delta}_m$ *such that for* $\delta_m \leq \delta_m^*$, *the b&m store's profits are fraction* $\mu_0 + \mu_m$ *of their monopoly level. Once* $\delta_m > \delta_m^*$, *its profits decline monotonically in* δ_m.

3. *At* $\underline{\delta}_m$, *the individual rationality constraints hold at equality if valuations are sufficiently different, i.e.,* $\theta_l < \bar{\theta}_l(\theta_h)$; *but if* $\theta_l > \bar{\theta}_l(\theta_h)$ *the individual rationality constraint for the low type and the incentive compatibility constraint for the high type hold at equality.*

The intuition is clean. When δ_m is low, the competitive pressure exerted by online stores is minimal.

Consequently, as when the *b&m* store is a monopolist, quality provision is efficient and its profits are a fraction $\mu_0 + \mu_m$ of monopoly profits (consumers with $\delta = 1$ buy online). As δ_m increases further, online purchases become viable alternatives for consumers at the outset, and at even higher discount factors, after realizations of low quality valuations. As a result, the *b&m* store's profits begin to fall with increases in δ_m as improved online alternatives curtail its ability to charge high prices. Still, the incentive compatibility constraints do not bind, implying efficient quality provision is optimal. The intuition reflects the fact that the exante incentives of consumers to visit the b&m store as well as the b&m store's profits from a representative consumer remain unchanged if the price of the high quality good is lowered and that of the low quality good is raised in a manner such that $\lambda dp_h + (1-\lambda)dp_l = 0$, while keeping qualities, $q_h^* = \theta_h - c, q_l^* = \theta_l - c$, unchanged. At $\underline{\delta}_m$, multiple constraints hold at equality—the individual rationality constraint for the low valuation type, and one of the high valuation consumer's constraints, where it is incentive compatibility constraint that holds if and only if consumer valuations of quality are not too different. Hence, supporting equilibrium prices are unique.

The next proposition provides our key characterization result. It describes the *b&m* store's optimal price quality offers, as δ_m rises past $\underline{\delta}_m$ and online goods become increasingly good substitutes for in-store purchases.

Proposition 3. *If consumer quality valuations are sufficiently close, $\theta_l > \hat{\theta}_l$ and enough consumers have high quality valuations, $\lambda > \bar{\lambda}(\theta_l)$, then there exists a range of online discount factors $(\hat{\delta}_m^1(\theta_l, \lambda), \hat{\delta}_m^2(\theta_l))$ such that if online purchases are:*

- **Less good substitutes**, *$\delta_m \in (\underline{\delta}_m, \hat{\delta}_m^1(\theta_l, \lambda))$, then the b&m store sells both product qualities, but distorts quality provision for the low type below the socially-efficient level.*
- **Better substitutes**, *$\delta_m \in (\hat{\delta}_m^1(\theta_l, \lambda), \hat{\delta}_m^2(\theta_l))$, then the b&m store only sells to consumers with high valuations of quality.*
- **Very good substitutes**, *$\delta_m > \hat{\delta}_m^2(\theta_l)$, then the b&m store offers both product qualities and quality provision is efficient.*

The higher is $\delta_m > \underline{\delta}_m$ the more attractive are online purchases, especially for low valuation types. This is because products available online are offered at competitive prices and the utility cost of online purchases, $(1-\delta_m)(\theta_l q - \frac{q^2}{2})$, for a consumer with a low valuation for quality is less than that for a high valuation type. Consequently, a smaller price differential between the b&m store's price and online offers is enough to trigger a switch to online purchases for low valuation consumers, once δ_m is high enough. In turn, this means that the *b&m*'s own product-line competition intensifies as online competition rises.

When quality valuations are close, a binding incentive compatibility constraint for the high type consumers impinges on the b&m store's ability to extract consumer surplus from high types: charging too much for the high quality product may induce high valuation consumers to switch to the low quality product. This is exacerbated by internet competition: once the individual rationality constraint for low valuation consumers binds, the store must increase the surplus of its low valuation consumers, $\theta_l q_l - \frac{q_l^2}{2} - p_l$, as internet products become better substitutes, else these consumers switch to online purchases. In such situations, doing away with this product line competition by offering a single product to high valuation consumers allows the store to charge a high price, albeit at the loss of the business of low valuation consumers. All in all, this benefits the store as long as enough of its customer base values quality highly.

However, as the online discount factor rises still further, online products become increasingly good substitutes. As a result, eventually the preferred alternative of high valuation consumers becomes a slightly discounted high quality online product rather than an undiscounted, inferior in-store product: online competition swamps the store's product line competition. High valuation consumers now view online high quality substitutes as more attractive than the store's low quality product—the incentive compatibility constraint for the high valuation consumers becomes slack, and the store reverts to offering efficient qualities to both valuation types.

This analysis has ignored the fact that the *b&m* store has fixed costs of operation that its operating profits must cover for it to survive. As online competition intensifies, certainly as $\delta_m \to 1$ (when μ_0 is small enough), the b&m store will be driven out of the market. The timing of its exit will, of course, depend on the level of its fixed costs:

Corollary 1. *Depending on the level of fixed costs, various exit patterns emerge. The b&m store transits from an offer of two qualities to none if fixed costs are relatively high; from two qualities to a single one to none if fixed costs are only moderately large; and from two qualities to one to two to none if fixed costs are small.*

Remark. Even if μ_0 is tiny, so there are few computer-illiterate consumers who rely on the *b&m* store for purchases, its exit can sharply impair consumer welfare due to the loss of informational externalities: even computer-savvy consumers value visiting the *b&m* store in order to freely inspect products, learn which is best for them, and then purchase accordingly.

1.3 Online portal

In this section we investigate how outcomes are changed when the b&m store also operates an online portal, and offers technical support that reduces the installation costs that some consumers would otherwise incur from online purchases. Concretely, we model the phenomena that a consumer can go to Best Buy or Apple to get computer service help. We model this by assuming that a fraction α of consumers who would otherwise only receive a discounted online payoff (δ_m) receive, instead, the full undiscounted utility payoff if they buy from the b&m's online portal because they can exploit the physical store's technical support. Other independent electronic retailers cannot offer this support to online purchasers.

The *b&m* store may gain in three ways from its online portal. First, the online portal, like any other e-retailer, has no fixed costs. Second, together with its portal, the *b&m* store may be able to capture the rents associated with providing technical support to consumers. Third, by shifting sales to its online portal, the *b&m* store may be able to reduce its physical footprint and hence its costs. We model this with a slightly more sophisticated cost structure. We assume that the b&m store incurs fixed costs of production, k and marginal costs, $s > 0$ of handling consumers who opt to buy at the b&m store, possibly because it can reduce the size of its expensive store front space. This creates an incentive for the b&m store to divert its sales to its online outlet. All other aspects of the model remain the same.

Now, of those consumers with $\delta = \delta_m$, a fraction α may buy at the b&m's online portal, while the remainder buy at its physical store. Computer savvy consumers buy from an independent electronic retailer, as they derive no additional utility from the services offered by the b&m store, and hence are unwilling to pay higher prices.

Once more, computer-illiterate consumers with $\delta = 0$ are drawn to the b&m store over distance,

$$x_0^* = \frac{1}{t}\left(\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l)\right)$$

Consumers with online discount factor δ_m who do not derive value from the b&m's online portal, purchase at the physical store as before. Recognizing that such consumers have the alternative to buy the product online at the outset, they are drawn over a distance,

$$x_m^* = \frac{1}{t}\left(\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l) - \frac{(\delta_m E[\theta] - c)^2}{2\delta_m}\right)$$

Finally, consider consumers who derive value from the b&m store's services even if they opt to buy at its online outlet. In equilibrium, the prices charged at the b&m store or its online outlet leave these consumers indifferent between the two: the b&m store extracts the entire surplus from the service it provides these consumers. Then, such consumers travel over the same distance, x_m^*.

Thus, the expected profit of the b&m store when it operates an online outlet is

$$\begin{aligned}\pi_I' = \frac{2}{t}\Big[\big(\mu_0+\mu_m(1-\alpha)\big)\Big(\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l)\Big) - \mu_m(1-\alpha)\frac{(\delta_m E[\theta]-c)^2}{2\delta_m}\Big] \\ \times\big(\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l) - s\big) \\ + \frac{2\mu_m\alpha}{t}\Big[\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l) - \frac{(\delta_m E[\theta]-c)^2}{2\delta_m}\Big] \\ \times\big(\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l)\big) - k.\end{aligned}$$

Hence, it maximizes profits subject to the same constraints as when it does not operate an online portal and faces internet competition.

The relevant benchmark for comparison is a setting where the physical store incurs a marginal cost of handling customers but has no online presence. It is straightforward to show that with the online portal, the physical store sets lower prices. This is because the portal increases its profit margin, as the ability to divert consumers to the online portal reduces its marginal costs, encouraging it to draw more consumers. The following is almost immediate:

Corollary 2. *Fixing other parameters, there exist bounds, k_1, k_2 on the b&m store's fixed costs of operation k, such that if $k \in (k_1, k_2)$, the physical store's operating profits do not cover its fixed costs, but its online portal's profits more than cover those losses. If $k > k_2$ the b&m store exits the market.*

If $s = 0$, total profits are the same with the portal as without—it does not matter where goods are sold from the store's perspective—but share $\frac{\alpha\mu_m}{\mu_0+\mu_m}$ of its operating profits appear to accrue to its portal. When $s > 0$, the portal increases the total profits. In both cases, whenever total net profits are positive, but the share $\frac{(1-\alpha)\mu_m+\mu_0}{\mu_0+\mu_m}$ of operating profits fail to cover the fixed costs, k, then the physical store appears to be losing money, acting as a loss leader, as it were. However, it is only the presence of the physical store and the services it provides portal users that allow the portal to price above marginal cost and make money. From the perspective of consumers who are comfortable with self-installation, the portal appears to be overcharging, offering uncompetitive prices. In fact, the physical store and portal provide a joint product that appeals to those customers who benefit from the installation services that the physical store provides.

1.4 Conclusion

This paper examines the impact of electronic retailers on quality provision by a brick and mortar store. We analyze how the b&m store's strategic pricing, product selection and product quality provision evolve in the face of increasing online competition from e-retailers. In our model some consumers do not know which product they want until they physically visit a store. We characterize how increased online competition due to improved online product provision affects the physical store's strategic choices. As online utility costs fall, the store moves from socially-efficient provision of product qualities and varieties, to reducing the quality of its low quality products, next, to selective provision of high quality products, before returning to socially-efficient provision of product quality and product varieties. Eventually, it exits the market when the heightened price competition drives operating profits below the fixed costs of operation. Thus, we paint the portrait of a physical store flailing to find a successful strategic plan in the face of increased online competition. We conclude by exploring how outcomes are affected when the physical store also has an online portal. We provide an explanation for why online portals of physical stores (a) win a share of the market despite charging higher prices than other e-retailers; and (b) appear to be the profit driver for the firm.

Chapter 2

Positive and Negative Campaigns in Primary and General Elections

2.1 Introduction

The literature on political campaigning has highlighted the extent to which negative campaigning has come to dominate the political debate between candidates prior to an election, and the consequences of that campaigning for electoral outcomes. For example, 85 percent of the $404 million spent by President Obama on advertising was negative in nature, while challenger Mitt Romney devoted 91 percent of his $492 million budget to negative ads.[1] What has received vastly less attention are the sharp differences in the nature of campaigning in primary versus general elections. Campaigning has been far less negative in primaries than in general elections, especially when the primary winner faces an established incumbent candidate, and primary underdogs run more positive campaigns than front runners. For example, in the 2004 Democratic presidential primary, 38% of ads were negative, whereas in the general election 61% were (CMAG). So, too, by mid-February in the 2012 Republican primary, the presumptive favorite Mitt Romney ran 93% more negative ads than positive ads, while Gingrich, Paul and Santorum collectively ran 27% more positive ads than negative ones.[2]

Our paper builds a model that delivers these dynamics of positive and negative campaigning. We consider a setting in which two challengers compete against each other in a primary, with the winner advancing to face an established incumbent from the opposing party in a general election.[3] Candidates only care about who wins the general election. Obviously, a challenging candidate hopes to win the general election; failing that, he prefers that his party primary opponent win; and his least preferred outcome is for the incumbent to be re-elected. The incumbent seeks to be re-elected.

In addition to his preferences over who wins the general election, each candidate is described by his initial reputational stock, and a resource budget that can be devoted both to developing his own repu-

[1]Kantar Media's Campaign Media Analysis Group (CMAG).

[2]http://www.washingtonpost.com/politics/study-negative-campaign-ads-much-more-frequent-vicious-than-in-primaries-past/2012/02/14/gIQAR7ifPR_story.html

[3]The "incumbent" could alternatively be a winner of a primary for the opposing party.

tational stock via positive campaigning and to damaging an opponent's reputational stock via negative campaigning. Election outcomes are determined by a success function, where the probability a candidate wins depends on his post-campaigning reputational stock and that of his opponent.[4] A candidate chooses both how extensively to campaign in each election, and the positive and negative composition of that campaign. Negative and positive campaigning can have different impacts on reputations—the preponderance of negative ads suggest that they have bigger impacts on reputations than positive ads, i.e., it is easier to damage a reputation than to build one up. We also allow the effects of primary campaigning on a winning challenger's reputation to decay prior to a general election. This decay reflects that between the primary and general election, voters may forget some of a primary campaign; and that the moderate voters who determine who wins a general election may pay less attention to primary campaigns than party partisans.

Our paper characterizes how the primitives of the economy—(a) relative candidate strengths (resources and reputations); (b) the technology of campaigning (effectiveness and decay); and (c) a candidate's preferences over electoral outcomes (personally winning vs. having the party's nominee win)—affect the strategic calculus of campaigning. We first analyze campaigning in the general election. In the general election, campaigning choices only reflect their relative effectiveness in influencing who *wins* the election. As a result, in the general election, a candidate with sufficient resources equates marginal benefits by allocating more resources to negative campaigning than to positive campaigning; and a candidate with very limited resources only campaigns negatively.

In contrast, as long as the effects of primary campaigning do not fully decay prior to the general election, a challenger campaigns relatively more positively (less negatively) in a primary than in a general election—a greater share of primary expenditures are devoted toward positive campaigning. The heightened focus on positive campaigning in primaries reflects two forces. First, when a candidate wins a primary election, the benefits of positive campaigning in the primary enhance his reputational stock in the general election, but the effects of negative campaigning against a primary opponent do not carry over. Thus, a candidate benefits from a negative primary campaign only to the extent that it raises his chance of winning the primary, giving him the opportunity to compete in the general election. Second, when a candidate loses a primary, a positive primary campaign does not tar a primary opponent in the general election, but negative campaigning does. That is, when a candidate loses the primary, any negative campaigning adversely reduces his primary rival's reputational stock, reducing the probability that he wins

[4]Contest functions have been used to model positive and negative campaigning in a static game between two candidates (Soubeyran (2009)).

the general election. Since challengers always want the incumbent to lose the general election regardless of who wins the primary, this force causes them to reduce negative campaigning in primaries.

The less decay there is in the effects of primary campaigning, the relatively less challengers devote to negative campaigning, and the more they devote to positive campaigning. Indeed, we establish that *if* the effects of primary campaigning do not decay at all, and primary candidates are ex-ante symmetric (with identical reputations, budgets and preferences) then primary campaigns feature more positive than negative campaigning—the exact *opposite* of what happens in general elections. Thus, our theory can reconcile the empirical observation that primary campaigns are less negative than general elections, and it provides testable predictions regarding the composition of campaigning in primaries and general elections as a function of the 'importance' of an election, which proxies for the extent of decay in the effects of primary campaigning.

The shift toward positive campaigning and away from negative campaigning in a primary when more of the effects of primary campaigning persist to the general election leads naturally to a conjecture that reduced decay *must* raise the probability that the primary winner defeats the incumbent. In fact, this need not be so: when negative campaigning is only marginally more effective than positive campaigning, reduced decay also causes challengers to spend more in the primary. The reduction in the resources that a primary winner has available for the general election can swamp the beneficial effects of reduced negative primary campaigning in determining general election outcomes, when decay is substantial. Phrased differently, the probability that the incumbent loses can be a hump-shaped function of the extent to which the effects of primary campaigning carry over to the general election.

The ways in which challengers tradeoff between positive and negative campaigning in a primary hinge on the strength of their general election opponent. If a primary winner will face a strong incumbent who is well-regarded and has extensive resources, a bruising primary battle will weaken both challengers, making them unlikely to win the general election. As a result, primary campaigns are more limited and less negative when an incumbent is stronger. Facing a stronger incumbent, a challenger's focus shifts from bettering his own chances of being elected, to raising the probability that whoever wins the primary also wins the general election. That is, facing a strong incumbent, it is a Pyrrhic victory for a challenger to win a primary by expending resources in a negative campaign, if he, unlike the incumbent, then has a small reputation stock and little way to enhance it.

Conversely, if the challengers have good reputations and extensive resources, or the incumbent is weak and scandal-ridden, then the challengers understand that the primary winner is likely to win the general election. This causes them to shift their focus toward improving their own chances of getting elected, encouraging them to campaign more aggressively (spending more, and being more negative) in the primary. Symmetric increases in challenger reputations cause them to campaign more aggressively in primaries, but less aggressively in the general election. In contrast, symmetric increases in their resources cause them to campaign more aggressively and more negatively in both elections. This difference reflects the fungibility of budgets, but not reputations.

How challengers campaign also depends on the extent to which their interests are aligned—if a challenger's payoff when his primary opponent wins the general election rises, he cares less about who wins the primary, and more about having the party's nominee win the general election. As a result, challengers conserve their resources for the general election, and engage in relatively less negative primary campaigning.

The effectiveness of campaign advertising interacts subtly with a challenger's resources to affect campaigning choices. When reputations are more sensitive to campaigning, primary campaigning is reduced and becomes more positive if challengers have limited budgets: a challenger's resource disadvantage against a strong incumbent is aggravated when reputations are more sensitive to campaigning advertising, causing a challenger to hoard resources for the general election. Conversely, when a primary winner will have a resource advantage in the general election, increasing the sensitivity of reputations has the opposite effect on how challengers allocate resources.

Differences between challengers affect how they campaign against each other. Perhaps counterintuitively, when challengers are similar, increasing *one* challenger's resources encourages *both* challengers to campaign more aggressively in the primary, as long as each cares enough more about personally winning versus having the incumbent lose. The effect on challenger i of giving him a little more resources than his rival is straightforward—he campaigns more aggressively in the primary both because he has more to spend in both elections, and because he will be the party's stronger candidate in the general election. The effect on his primary rival j is less clear: (1) challenger i is more likely to defeat the incumbent in the general election, so j has an incentive to campaign less aggressively in the primary; but (2) because challenger i campaigns more aggressively in the primary, j is more likely to lose, and because j wants to be the party representative, j has an incentive to campaign more aggressively in the primary. When challengers care enough more about winning rather than just have their party winning, it dominates, causing the weaker rival to increase his primary campaigning.

For identical reasons, increasing one challenger's reputation causes his rival to campaign more aggressively in the primary, when they are similarly situated. However, paradoxically, improving i's reputation causes him to campaign *less* aggressively in the primary whenever the incumbent is strong. This is because i's better reputation helps him in the primary, encouraging him to conserve resources to raise his chances of defeating the strong incumbent in the general election. Improving i's reputation only causes him to campaign more aggressively in the primary when he faces a weak incumbent (with a poor reputation or a limited budget)[5]

The effects of increased asymmetries are very different when one challenger is far stronger than the other. Then, making a strong challenger even stronger causes his rival to reduce his primary campaigning, especially his negative campaigning. The weak rival internalizes that his strong primary opponent is more likely than he to win the general election, that negative primary campaigning reduces that probability, and that greater primary campaigning reduces his already low chances of winning the general election. As a result, a desire for the party's nominee to defeat the incumbent causes the weak rival to campaign less aggressively–spending less, and campaigning more positively. This, in turn, induces the stronger challenger to lower his primary campaigning too, because he is already likely to win the primary due to his greater resources and reputation, and his primary rival's less aggressive campaign. The stronger challenger still spends far more, and is far more negative than his weaker rival in order to ensure victory. Thus, our model delivers the pattern of primary campaigning in the 2012 Republican primary—Romney had a stronger reputation and far greater resources, and hence he spent far more, and was much more negative than his primary rivals.

The literature on positive and negative campaigns dates back to Skaperdas and Groffman (1995) and Harrington and Hess (1996). Skaperdas and Groffman (1995) study the choice of negative and positive campaigning by candidates in the context of two or three-candidate plurality elections. They predict that when two candidates compete in an election, the front-runner engages in more positive and less negative campaigning than his opponent. In a three-candidate contest, no candidate engages in negative campaigning against the weakest opponent, so that to the extent there is negative campaigning, it is either directed against the front-runner or it comes from the front-runner himself. Harrington and Hess (1996) explore negative and positive campaigning in a spatial setting in which (a) agents begin with initial locations but can engage in costly relocation, and (b) an agent's relocation is affected by her rival's actions as well. They predict that the candidate who is perceived as having less attractive personal attributes runs a more negative campaign.

[5]The opposing "incumbent" could also be the winner of the rival party's primary—such an "incumbent" could be weak if he has limited resources, or a low initial reputation.

Chakrabarti (2007) extends Harrington and Hess (1995) by introducing a valence dimension that captures personal traits such as integrity. Candidates can now influence both ideological and valance factors via negative advertising: ideological spending shifts an opponent's policy position away from the median and valence spending reduces the opponent's valence index. Candidates campaign more negatively on the issue in which they have an advantage.

Polborn and Yi (2006) develop a model of negative and positive campaigning wherein each candidate can reveal a (good) attribute about himself, or a (bad) attribute about a competitor, and voters update rationally about the information that is not transmitted. They predict that positive and negative campaigning are equally likely.

Brueckner and Kangoh (2013) explore negative campaigning in a probabilistic voting model, wherein individual vote outcomes are stochastic due to the presence of a random, idiosyncratic valence effect along with other shocks that affect all voters in common. A relatively centrist candidates campaign more negatively than a relatively extreme candidate.

Peterson and Djupe (2005) empirically study the timing and the electoral context in which primary races are likely to become negative. Using a content analysis of newspaper coverage of contested Senate primaries, they find that negativity is an interdependent function of the timing during the race, the status of the Senate seat (whether the seat is open, whether the incumbent is in the primary, etc.), and the number and quality of the challengers in the primary (based on whether challengers previously held office).

The paper is structured as follows. We next present the model and central results. Section 2.3 analyzes outcomes when most of the effects of primary campaigns decay before the general election and challengers have similar or identical preferences, reputations and budgets. Section 2.4 concludes the paper. Proofs are collected in Appendix B.

2.2 Model

There are three candidates, i, j and I. Candidates i and j belong to the same party, while I belongs to a different party and is presently in office. Challengers i and j first compete in a primary election, with the winner advancing to face the incumbent I in a general election. Candidates only care about who wins the general election—challenger $k \in \{i, j\}$ receives a payoff U_k from winning and a payoff V_k if his primary opponent wins the general election, and a normalized payoff of 0 if the incumbent wins, where $U_k > V_k > 0$. The incumbent receives a positive payoff if he is re-elected, and none if he loses.

Electoral outcomes are determined by a contest, where the probability a candidate wins rises with his reputation, and declines with his opponent's reputation. Specifically, in a primary election, if $\bar{Z}_{i0}$ and $\bar{Z}_{j0}$ are the candidates' respective reputational stocks just prior to the election, then candidate i wins with probability $\frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$, and candidate j wins with residual probability. An analogous contest determines the winner of the general election.

Candidate $k \in \{i, j, I\}$ starts out with an initial reputational stock of $\bar{X}_k$, and a resource budget $\bar{B}_k$. A candidate can devote his resources both to developing his own reputational stock via positive campaigning and to destroying his opponent's reputational stock via negative campaigning. In the primary election, if candidate $k \in \{i, j\}$ invests p_{k0} into positive campaigning to boost his own reputation, and his rival $\tilde{k}$ spends n_{k0} on negatively campaigning to reduce k's reputation, then candidate k's reputational stock in the primary becomes $\bar{Z}_{k0} = \bar{X}_k \frac{(1+p_{k0})^{\alpha}}{(1+\rho n_{\tilde{k}0})^{\alpha}}$. Here $\alpha > 0$ captures the sensitivity of a candidate's reputational stock to campaigning and ρ captures the effectiveness of negative campaigning relative to positive campaigning. In most of our analysis, we presume that negative campaigning has a greater influence on candidate reputations than positive campaigning, i.e., that $\rho > 1$. This structure allows us to reconcile the preponderance of negative campaigning in political campaigns.

If candidate k wins the primary, then he enters the general election with a reputational stock of $\bar{Z}_{k1} = \bar{X}_k \frac{(1+\beta p_{k0})^{\alpha}}{(1+\beta\rho n_{\tilde{k}0})^{\alpha}}$. Here, $\beta \in [0, 1]$ captures the observation that the effects of primary campaigns on candidate reputations decay prior to a general election. A small β, i.e., extensive decay, can reflect both that voters largely forget primary campaigns by the time of the general election and that primary campaigns appeal more narrowly to party partisans, and the more moderate voters who determine the general election winner pay less attention to primary campaigns. The extent of decay may vary with the electoral context, reflecting that, for example, only Republicans/Democrats follow developments in their respective party primary campaigns for the House, but more voters follow presidential campaigns closely. In the general election, challenger k's final reputational stock is $\bar{Z}_{k2} = \bar{Z}_{k1} \frac{(1+p_{k1})^{\alpha}}{(1+\rho n_{I1})^{\alpha}}$, and the incumbent's final reputational stock is $\bar{Z}_{I2} = \bar{X}_I \frac{(1+p_{I1})^{\alpha}}{(1+\rho n_{k1})^{\alpha}}$, $k \in \{i, j\}$.

Challenger k's total electoral resource constraint is $\sum_{t=0}^{1} p_{kt} + n_{kt} \leq \bar{B}_k$ (where $p_{kt}, n_{kt} \geq 0$). Thus, when challenger k wins the primary, he has funds $\bar{B}_k - (p^*_{k0} + n^*_{k0}) \equiv \bar{B}_{k1}$ at his disposal in the general election. The incumbent's resource constraint is $p_{I1} + n_{I1} \leq \bar{B}_I \equiv \bar{B}_{I1}$.

Without loss of generality, we write challenger $k \in \{i, j\}$'s ex ante expected payoff as

$$\pi_k = (M_k Pr_{k1} - Pr_{\tilde{k}1}) Pr_{k0} + Pr_{\tilde{k}1},$$

where $M_k = \frac{U_k}{V_k}$ captures the relative payoff challenger k receives from winning the general election versus having his primary opponent win, and Pr_{kt} is the probability that k wins the primary ($t = 0$) or general ($t = 1$) election. When we compare how differences in challengers affect how they campaign, it eases characterizations to assume that $M_k \geq 3$, i.e., challengers strongly prefer personally winning the general election to having a primary rival win.[6]

We pose our analysis in a setting where two challengers face off in a primary with the winner facing an incumbent in the general election. However, it follows directly that our analysis describes equilibrium outcomes when, rather than facing an incumbent in the general election, the two possibly heterogeneous challengers will face the winner of a primary in the rival party, and the other party's candidates are symmetric in all regards. In this setting, the challengers care only about the equilibrium reputational stock and resource budget that their general election rival will have, and not about who wins the rival party's primary.

We begin by characterizing equilibrium campaigning in a general election.

Proposition 4. *In equilibrium, in the general election, as long as candidate $k \in \{i, j, I\}$ has sufficient resources at his disposal so that $B_{k1} > \frac{\rho-1}{\rho}$, then candidate k engages in both positive and negative campaigning, albeit allocating more resources to negative campaigning:*

$$n^*_{k1} = \frac{B_{k1}}{2} + \frac{\rho - 1}{2\rho} \quad \textit{and} \quad p^*_{k1} = \frac{B_{k1}}{2} - \frac{\rho - 1}{2\rho}.$$

If, instead, candidate $k \in \{i, j, I\}$ has only modest funds at his disposal, $B_{k1} < \frac{\rho-1}{\rho}$, then k only campaigns negatively, $n^*_{k1} = B_{k1}$ and $p^*_{k1} = 0$.

The proof follows directly. The probability challenger $k \in i, j$ defeats the incumbent is

$$Pr_{k1} = \frac{\bar{Z}_{k2}}{\bar{Z}_{k2} + \bar{Z}_{I2}} = \frac{\bar{Z}_{k1} \frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha}}{\bar{Z}_{k1} \frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha} + \bar{X}_I \frac{(1+p_{I1})^\alpha}{(1+\rho n_{k1})^\alpha}},$$

which, written as a function of what k controls in the general election, takes the form

$$Pr_{k1} = \frac{a(1 + p_{k1})^\alpha}{a(1 + p_{k1})^\alpha + b/(1 + \rho n_{k1})^\alpha},$$

where a and b are positive constants. Multiplying numerator and denominator by $(1 + \rho n_{k1})/b$, and

[6] We also make the implicit premise that when one challenger is far stronger than his rival, the weak rival cares enough about personally winning that he prefers to win the primary, to having the stronger challenger win, even though this means that the incumbent is more likely to win re-election.

simplifying yields

$$Pr_{k1} = \frac{a[(1+p_{k1})^{\alpha}(1+\rho n_{k1})^{\alpha}]/b}{a[(1+p_{k1})^{\alpha}(1+\rho n_{k1})^{\alpha}]/b+1},$$

implying that challenger k maximizes $(1+p_{k1})^{\alpha}(1+\rho n_{k1})^{\alpha} \equiv [P_{k1}N_{k1}]^{\alpha}$ in the general election.

In the general election, only the relative effectiveness of each form of campaigning in influencing election outcomes matters for how candidates allocate their resources. Because $\rho > 1$ means that negative campaigning is more effective than positive campaigning—it is easier to tear down a reputation than build one up—candidates spend more on negative campaigns than positive ones, and whenever their budgets are sufficiently limited, they only negatively campaign. Thus, we predict that especially weak/underfunded challengers only campaign "against" an incumbent in the general election, and that the greater is a candidate's budget, the greater is the share the candidate devotes to positive campaigning.

Proposition 5 establishes that candidates campaign relatively more positively in the primary than in the general election.

Proposition 5. *In equilibrium, provided candidate $k \in \{i, j\}$ devotes any resources to positive campaigning, he campaigns relatively more positively in the primary than in the general election: $n^*_{k0} - p^*_{k0} \leq n^*_{k1} - p^*_{k1} = \frac{\rho-1}{\rho}$, where the inequality is strict unless $\beta = 0$.*

As long as the effects of primary campaigns do not fully decay before the general election, a candidate campaigns relatively more positively in the primary than in the general election for two reasons: (1) there is a lingering beneficial effect of positive campaigning in the primary on the challenger's reputation in the general election too, should he survive the primary election; and (2) the adverse effects of negative campaigning in the primary against his party rival also carry over to the general election, should the latter emerge from the primary victoriously. Both effects induce a challenger to allocate relatively more resources in a primary toward positive campaigns, and away from negative campaigns.

We next characterize how candidates' primary campaigns are affected when more of the effects of primary campaigning on reputation persist to the general election.

Proposition 6. *Suppose that challengers are symmetric, with identical reputations, budgets and preferences. Then there exists a $\hat{\beta} \in (0, 1]$, such that for all $\beta \leq \hat{\beta}$, increases in β:*

1. *Reduce negative campaigning in the primary election.*

2. *Increase positive campaigning in the primary provided a challenger campaigns positively, as long as*

the resources of challengers are not too great.

3. *When ρ is close to one, so that negative campaigning is not much more effective than positive campaigning, the increase in positive primary campaigning exceeds the decrease in negative campaigning implying that total primary campaigning expenditures rise.*

That is, when the beneficial effects of primary campaigns persist more strongly, candidates campaign relatively more positively and less negatively in the primary. In fact, if enough of the effects of primary campaigning persist to the general election, challengers campaign more positively than negatively in the primary:

Proposition 7. *Suppose that challengers are symmetric, with identical reputations, budgets and preferences. Then when enough of the effects of primary campaigns persist, and challengers have enough resources that they campaign positively in both elections, they campaign more positively than negatively in the primary. That is, there exists a $\beta^* \in (0, 1]$ such that if $\beta \geq \beta^*$, then $p^*_{k0} > n^*_{k0}$.*

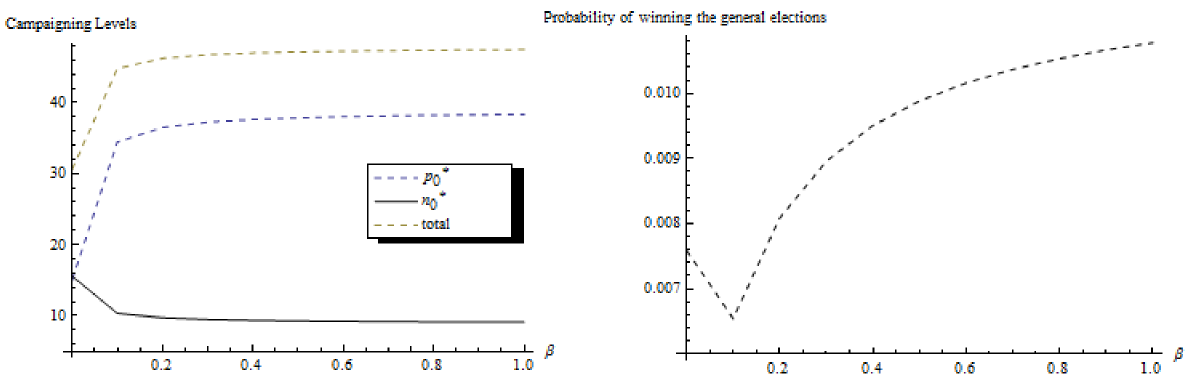

Figure 2.1: Primary campaigning and probability that a challenger wins in the general election as a function of the extent to which primary campaigning decays when challengers have limited resources. Parameters: $\bar{X}_I = 100, \bar{B}_I = 200, \bar{X}_k = 50, \bar{B}_k = 100, M_k = 10, \rho = 2, \alpha = 2, k \in \{i, j\}$.

The analytical characterizations in Propositions 6 and 7 only obtain when β is sufficiently small or large. However, a numerical analysis reveals that these results extend to intermediate levels of β. That is, as more of the effects of primary campaigning persist to affect general election reputations, positive campaigning in the primary rises while negative campaigning falls. Further, consistent with Proposition 6, Figure 2.1 illustrates that when there is nearly complete decay in effects of primary campaigning prior to the general election, total primary campaign expenditures rise with β. A consequence is that with less resources remaining for challengers to devote to the general election (and modest persistence of the effects of primary campaigning), the probability a challenger actually wins the general election initially decreases as decay first falls. It is only when enough of the effects of primary campaigning persist that the probability a challenger wins rises with further increases in β. Figure 2.1 illustrates a scenario where the incumbent is far stronger than the challengers; but the same qualitative patterns hold when challengers are stronger.

2.3 Comparative statics

We next derive how changes in the primitives describing the economy affect campaigning when challengers are symmetric, with identical resources, reputations and preferences, and β is small enough that primary campaigning has only modest effects on a candidate's reputation in the general election. The latter may be expected to hold in elections for the House or Senate where only party partisans follow developments in their party primaries, but moderate voters follow general election campaigning closely.

We first note that when β is close to zero, the direction of the impact of changes in primitives is the same for n^*_{i0} and p^*_{i0} at an interior optimum, since $n^*_{i0} - p^*_{i0} \to \frac{\rho-1}{\rho}$ (save possibly for a change in ρ). Further, one can derive the effect on the direction of changes in n^*_{i1} and p^*_{i1} via the impact on the resource constraint. At an equilibrium $(n^*_{i0}, p^*_{i0}, n^*_{j0}, p^*_{j0})$,

$$\underbrace{\frac{\partial^2 \pi^i}{\partial \theta \partial n_{i0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^i}{\partial {n_{i0}}^2}\frac{dn_{i0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial p_{i0} \partial n_{i0}}\frac{dp_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^i}{\partial p_{j0} \partial n_{i0}}\frac{dp_{j0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial n_{j0} \partial n_{i0}}\frac{dn_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0.$$

for $\theta \in \bar{X}_I, \bar{B}_I, \alpha$. Also, when challengers are symmetric, a change in their budgets, preferences or reputations involves equal changes in both $\bar{B}_i, \bar{B}_j$, or M_i, M_j, or $\bar{X}_i, \bar{X}_j$ so that

$$\underbrace{\frac{\partial^2 \pi^i}{\partial \theta_i \partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial \theta_j \partial n_{i0}}}_{\text{direct effect of a change in } \theta_i, \theta_j} + \overbrace{\frac{\partial^2 \pi^i}{\partial {n_{i0}}^2}\left(\frac{dn_{i0}}{d\theta_i} + \frac{dn_{i0}}{d\theta_j}\right) + \frac{\partial^2 \pi^i}{\partial p_{i0} \partial n_{i0}}\left(\frac{dp_{i0}}{d\theta_i} + \frac{dp_{i0}}{d\theta_j}\right)}^{\text{indirect effect via change in } i\text{'s actions}}$$
$$+ \underbrace{\frac{\partial^2 \pi^i}{\partial p_{j0} \partial n_{i0}}\left(\frac{dp_{j0}}{d\theta_i} + \frac{dp_{j0}}{d\theta_j}\right) + \frac{\partial^2 \pi^i}{\partial n_{j0} \partial n_{i0}}\left(\frac{dn_{j0}}{d\theta_i} + \frac{dn_{j0}}{d\theta_j}\right)}_{\text{indirect effect via change in } j\text{'s actions}} = 0.$$

Lemma 1. *Suppose that challengers have identical endowments, budgets and preferences. Then when*

$\beta = 0$, $sign\{\frac{dn_{i0}}{d\theta}\} = sign\{\frac{\partial^2 \pi^i}{\partial\theta\partial n_{i0}}\}$ and $sign\{\frac{dn_{i0}}{d\theta_i} + \frac{dn_{i0}}{d\theta_j}\} = sign\{\frac{\partial^2 \pi^i}{\partial\theta_i\partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial\theta_j\partial n_{i0}}\}$.

The lemma states that the indirect effect of a change in a parameter on n^*_{i0}, p^*_{i0} in a symmetric equilibrium, reinforces, or if in the opposite direction, is outweighed by the direct effect. This means that we can derive the sign of the changes in n^*_{i0} and p^*_{i0} from the sign of the partial derivatives, $\frac{\partial^2 \pi}{\partial\theta\partial n_{i0}}$, $\frac{\partial^2 \pi^i}{\partial\theta_i\partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial\theta_j\partial n_{i0}}$, alone. Proposition 8 derives the impacts of changes in the challengers' budgets, reputations and preferences on primary campaigning when challengers are symmetric.

Proposition 8. *Suppose challengers have identical endowments, budgets and preferences, and that they have enough resources that they engage in both positive and negative campaigning in both elections. Then for all β sufficiently small,*

1. *Improving challenger reputations, $\bar{X}_i = \bar{X}_j \equiv \bar{X}_C$, causes challengers to increase both positive and negative primary campaigning. Their campaigning expenditures in the general election fall, but the probability they defeat the incumbent rises.*

2. *Increasing challenger resources, $\bar{B}_i = \bar{B}_j \equiv \bar{B}_C$, causes challengers to increase positive and negative campaigning in both elections. The probability a challenger defeats the incumbent rises.*

3. *Increasing challenger payoffs, $U_i = U_j \equiv U_C$, from winning office, or reducing payoffs, $V_i = V_j \equiv V_C$, when a party rival wins office, causes challengers to raise both positive and negative primary campaigning. The probability that a challenger defeats the incumbent falls.*

When challengers have better reputations, whoever wins the party primary is more likely to defeat the incumbent. In turn, challengers campaign more aggressively in the primary. Reinforcing the direct effect, when i campaigns more negatively in the primary, this causes j to campaign more negatively. This is because with less funds for the general election, i's chances of defeating the incumbent fall, making j more worried about losing the primary to i (j wants to be assured of a party win, at the least).

Greater resources allow challengers to spend more in both elections, improving their chances of winning both races. The effects of greater resources on general election campaigning differ from the effects of better reputations: improving challenger reputations raises primary campaigning expenditures, but reduces general election expenditures; in contrast, the fungibility of resources means that increased resources are spread across both elections.

Finally, increases in M_i, M_j mean that challengers care more about winning than just about ousting the incumbent from office. As a result, the challengers campaign more aggressively in the primary, reducing the chances that the party's nominee wins the general election.

A numerical analysis reveals that the comparative statics in Proposition 8 mostly extend regardless of the extent of persistence β in the effects of primary campaigning on reputations in the general election. Finally, we observe that the analytical results in Proposition 8 could have been posed on the impacts of identical increases in both challenger's reputations or resources or payoff parameters when challengers are *close* enough to being identical.

Proposition 9. *Suppose that challengers have sufficiently similar reputations, resources and preferences, and that they have enough resources that they positively and negatively campaign in both elections. Then for all sufficiently small* β*, increasing an incumbent's resources,* $\bar{B}_I$*, or improving his reputation,* $\bar{X}_I$*, causes both challengers to reduce both positive and negative campaigning in the primary. Their campaigning expenditures in the general election rise, but the probability they defeat the stronger incumbent falls.*

When the incumbent is stronger, primary campaigning falls — facing a stronger incumbent, challengers limit their spending in the primary to save more for the general election. Further, the strategic complementarities in campaigning mean that when i spends less on campaigning in the primary, j also lowers his primary campaigning. Again this reflects that since i saves more funds for the general election, i is more likely to defeat the incumbent. As a result, j is less worried about losing the primary to i. A numerical study reveals that these results hold regardless of the extent β of decay in the effects of primary campaigning.

Proposition 10. *Suppose that challengers have sufficiently similar reputations, resources and preferences, and that they have enough resources that they positively and negatively campaign in both elections. Then for all sufficiently small* β*, there exists a* $\bar{B}_i^*(\beta)$*, such that if challenger resources exceed* $\bar{B}_i^*(\beta)$*, i.e.,* $\bar{B}_i > \bar{B}_i^*(\beta)$*, increasing the sensitivity,* α*, of reputations to campaigns causes challengers to increase their campaigning in the primary. Their campaigning expenditures in the general election decrease, but the probability that they defeat the incumbent rises. If, instead,* $\bar{B}_i < \bar{B}_i^*(\beta)$*, the opposite holds.*

Increases in the sensitivity of reputations to campaigning, α, reduce negative campaigning in the primary if and only if challengers have sufficiently limited resources that they will have less funds in the general election than the incumbent. This is because the challengers, when crippled by low budgets, are unable to campaign as intensely as the incumbent in the general election. Increasing the sensitivity of reputations to campaigning aggravates this disadvantage, causing them to reduce their primary campaigning expenditures. Even though the challengers devote more of their resources to the general election, they still fail to match the incumbent's spending, so their chances of defeating the incumbent fall as α rises. The opposite occurs if the challengers will have more resources at their disposal in the general election than the incumbent; and if $\bar{B}_i = \bar{B}_i^*$, increasing the sensitivity of reputations to campaigning has no effect on outcomes. A numerical study reveals that these results hold for all values of β.

We conclude by investigating how differences between the two challengers affect how they campaign. We first consider how *small* differences affect challenger choices.

Proposition 11. *Suppose that challengers have sufficiently similar reputations, resources and preferences, and enough resources that they campaign both positively and negatively in both elections. Then for all β sufficiently small,*

1. *Improving challenger i's reputation, $\bar{X}_i$, causes challenger j to increase his primary campaigning, reducing his chances of winning the general election. Improving challenger i's reputation causes i to campaign more aggressively in the primary if and only if the incumbent is sufficiently weak.*
2. *Increasing challenger i's resources, $\bar{B}_i$, causes both challengers to increase both positive and negative primary campaigning.*
3. *Raising challenger i's payoff, U_i, from being elected to office, or reducing his payoff, V_i, if his party opponent wins office, causes both challengers to spend more on primary campaigns, reducing their chances of winning the general election.*

When challenger i's reputation is better, challenger j devotes *more* resources to primary campaigning. There are offsetting considerations. On the one hand, challenger i's better reputation improves the party's chances in the general election should i survive the primary, providing challenger j an incentive to campaign less aggressively in the primary. On the other hand, challenger i's stronger reputation also helps him in the primary, and challenger j prefers to personally win the general election than just to have his primary rival defeat the incumbent from the other party. As long as $M_i = U_i/V_i \geq 3$, the desire to personally win the general election dominates, causing challenger j to step up primary campaigning in response to a stronger party rival.

One might think that improving challenger i's reputation always encourages him to coast on his better reputation in the primary by reducing his primary expenditures, to conserve more resources to defeat the incumbent in the general election. In fact, i does this only if the incumbent is sufficiently strong. When the incumbent is relatively weak, i.e., when i is likely to win the general election if he manages to win the primary, then improving i's reputation causes him to devote *more* resources to the primary. With a weak incumbent (e.g., with a scandal-ridden incumbent, or a weak challenger from a rival party primary), challenger i's toughest battle becomes the primary, and he responds to the increased campaigning intensity by challenger j by raising his primary campaigning.

Figures 2.2 and 2.3 illustrate how challenger i's reputation affects the primary campaigning choices of both challengers when they face a strong and weak incumbent, respectively, and $\beta = 0$ so that all effects of primary campaigning decay prior to the general election, while Figure 2.4 shows the effects of

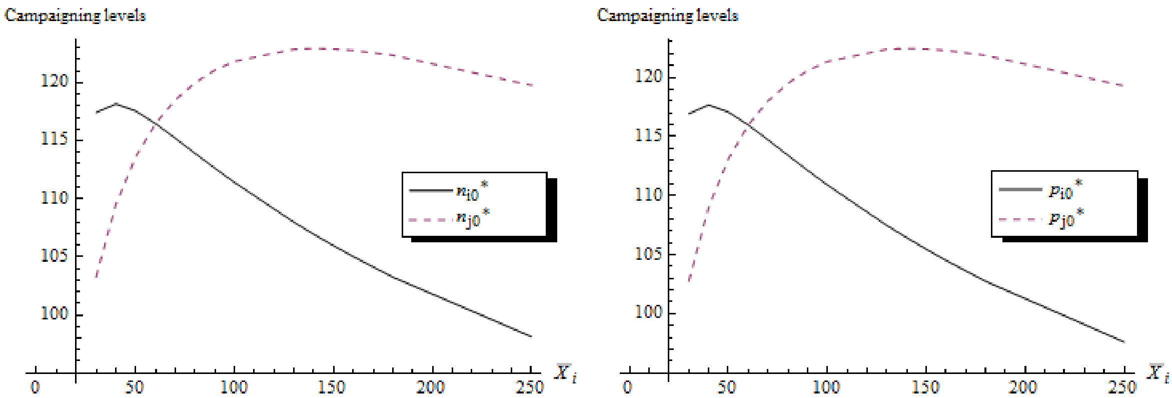

Figure 2.2: Primary campaigning as a function of challenger i's reputation when the incumbent is strong and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{B}_I = 900, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

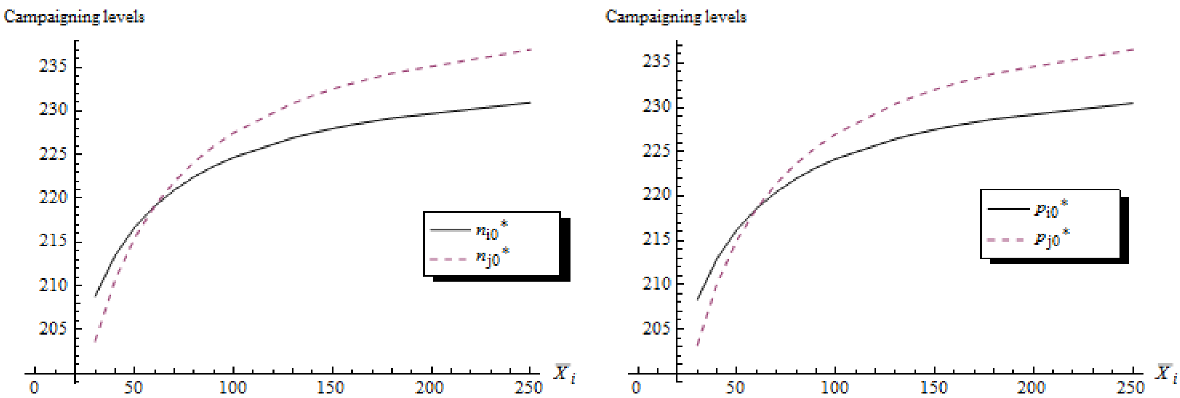

Figure 2.3: Primary campaigning as a function of challenger i's reputation when the incumbent is weak and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{B}_I = 100, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

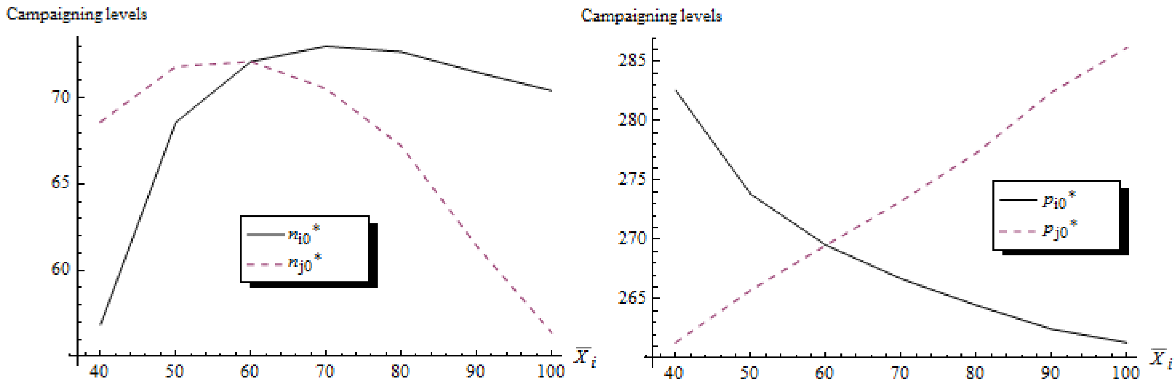

Figure 2.4: Primary campaigning as a function of challenger i's reputation when the incumbent is strong and effects of primary campaigning persist. Parameters: $\beta = 1, \bar{B}_I = 900, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

challenger i's reputation when $\beta = 1$. This analysis sheds light on what happens when one challenger is much stronger than the other. Consistent with Proposition 11, the figures reveal that when challengers are close to symmetric (when $\bar{X}_i$ is close to 60) and $\beta = 0$, improvements in i's reputation always induce his primary challenger j to campaign more aggressively. In contrast, improvements in i's own reputation cause i to campaign more aggressively in the primary when the incumbent is weak, but not when he is strong.

The effects of improving challenger i's reputation are very different when one challenger has a far stronger reputation than the other. For instance, if the incumbent is strong and i has a much stronger reputation than j, then i would rather devote the bulk of his resources to the general election, and his stronger reputation allows him to do so, while remaining confident of winning the primary. Now, as challenger i's reputation advantage grows further, challenger j responds not by increasing his primary campaign expenditures, but by *reducing* them. This is because challenger j internalizes the fact that i is far more likely than he to win the general election, that negative primary campaigning reduces that probability and that greater primary campaigning reduces his own already low chances of winning the general election. In contrast, when i is far weaker than j, slight improvements in i's reputation cause j to campaign more aggressively in the primary, as j now worries more that i may win the primary.

Contrasting Figures 2.2 and 2.4 reveal qualitatively similar shape features for the most part. However, when the effects of primary campaigning do not decay, i.e., when $\beta = 1$, candidates campaign less negatively in the primary, but *far* more positively. When $\beta = 1$, and challenger i's reputation is low, initial increases in his reputational stock cause him to increase his negative primary campaigning (which raises his chances of winning the primary), accompanied by a decline in positive and total campaigning expenditures in the primary. This allows him to devote more resources to the general election where he faces a strong incumbent. However, once i's reputation is sufficiently higher than j's, further increases in i's reputation cause him to reduce his negative campaigning in the primary, as his strong reputation is now likely to be enough to secure a primary win. The effects of further increases in i's reputation on j's campaigning are very different, when i is far stronger than j: j responds by *sharply* reducing his negative campaigning so as to not adversely affect his party opponent's chances in the general election; but to compensate j steps up his positive primary campaigning to improve his own chances of winning.

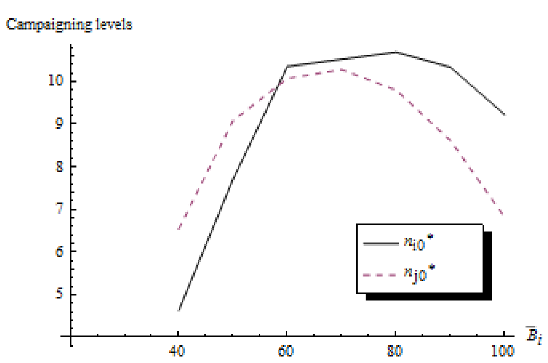

Figure 2.5: Primary campaigning as a function of challenger i's resources, when challenger j's resources are not "high enough" and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{X}_j = 40, \bar{B}_j = 65, \bar{X}_i = 50, M_j = M_i = 10, \bar{B}_I = 110, \bar{X}_I = 100, \alpha = 2, \rho = 2$.

Proposition 11 also establishes that when the challengers are similar, increasing challenger i's resources causes *both* challengers to campaign more aggressively in the primary. That challenger j raises his primary campaigning expenditures reflects the same considerations that drive him when his primary opponent's reputation improves—as long as j cares enough more about winning than about just having his party's nominee win, he devotes more resources to defeating his stronger primary rival. In contrast to the effects of increasing challenger i's reputation on his primary campaigning (which hinge on the relative strength of the incumbent), giving i more resources always induces him to campaign more aggressively in both the primary and the general election. This reflects that campaign resources are fungible, and can be allocated at will across both elections, but reputations are not.

Figure 2.5 and 2.6 show how primary campaigning by the two challengers varies with challenger i's resources when $\beta = 0$, while Figure 2.7 does so for $\beta = 1$[7]. The qualitative impacts of a stronger challenger are roughly similar regardless of whether a challenger is stronger due to having more resources or a better

[7]The parameter values used to construct Figure 2.5 make the incumbent a strong opponent in the general election. We vary multiple parameters vis a vis the other figures to better illustrate the qualitative features (i.e., the hump-shaped relationship between challenger i's resources and both negative and positive primary campaigning by *both* challengers). Thus, as challenger i's budget increases beyond a point, both lower their spending in the primary and devote more to the general election where they face a strong adversary.

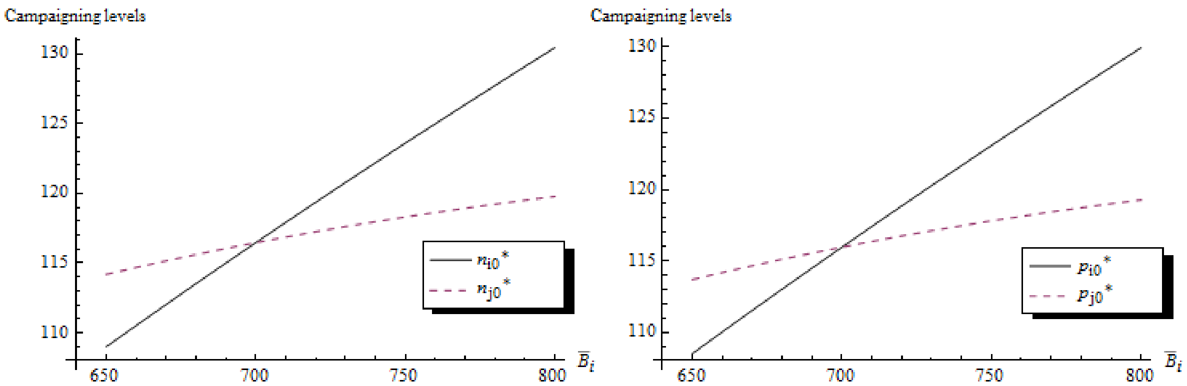

Figure 2.6: Primary campaigning as a function of challenger i's resources, when challenger j's resources are "high enough" and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{X}_I = 150, \bar{B}_I = 900, \bar{B}_j = 700, \bar{X}_j = \bar{X}_i = 60, M_j = M_i = 10, \alpha = 1, \rho = 2$.

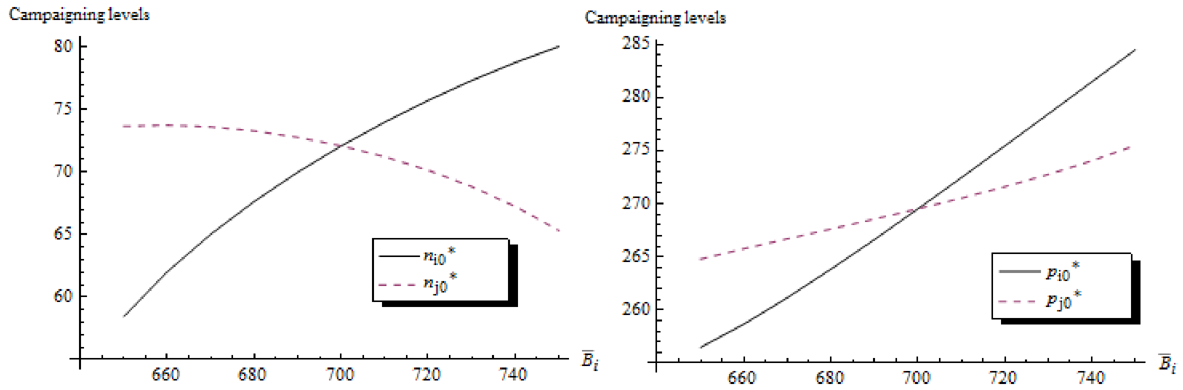

Figure 2.7: Primary campaigning as a function of challenger i's resources when the effects of primary campaigning persist. Parameters: $\beta = 1, \bar{X}_I = 150, \bar{B}_I = 900, \bar{B}_j = 700, \bar{X}_j = \bar{X}_i = 60, M_j = M_i = 10, \alpha = 1, \rho = 2$.

reputation, subject to the caveat regarding the fungibility of resources. This reflects that the key strategic force is the relative strength of one challenger relative to the other challenger and to the incumbent, and not the source of the strength. Concretely, when one challenger is far stronger than the other, further increases in that challenger's strength causes his rival to sharply reduce his negative campaigning in the primary and to raise his positive campaigning. Thus, our model can reconcile the composition of campaigning in the 2012 Republican primary by Romney and his rivals—the far stronger Romney campaigned far more negatively than positively, whereas his weaker rivals did the opposite. Moreover, when Gingrich appeared to gain in reputation following the South Carolina primary, Romney massively increased his campaigning, particularly his negative campaigning against Gingrich in the next primaries.

The final result in Proposition 11 shows that increasing challenger i's payoff when he is elected to

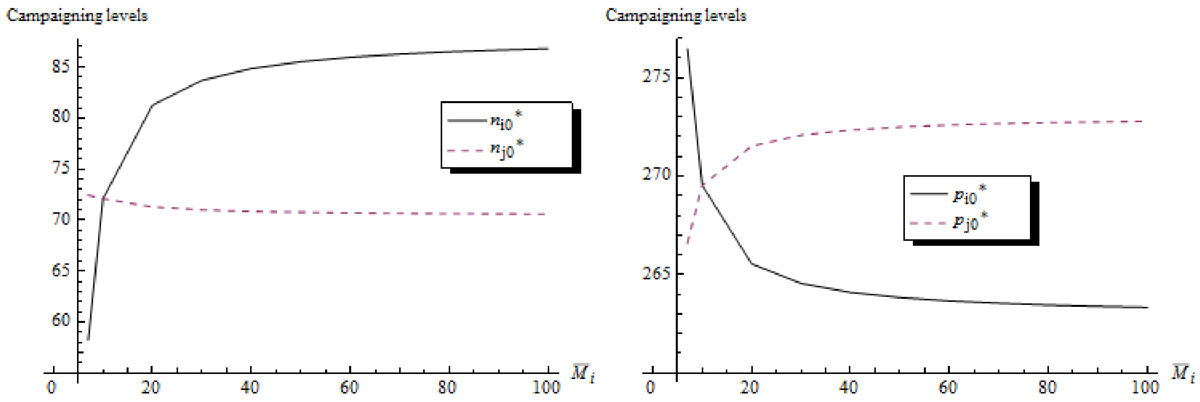

Figure 2.8: Primary campaigning as a function of challenger i's preferences when the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{X}_I = 150, \bar{B}_I = 900, M_j = 10, \bar{X}_j = \bar{X}_i = 60, \bar{B}_j = \bar{B}_i = 700, \alpha = 1, \rho = 2$.

office so that he cares more about winning himself rather than just ensuring a party win causes i to campaign more aggressively in the primary. But this reduces the funds available for campaigning in the general election, owing to which his chances of ousting the incumbent are lowered. This induces challenger j to campaign aggressively in the primary too as it is now more imperative that he win the primary election. Figure 2.8 depicts this relationship when $\beta = 0$ while Figure 2.9 does so for $\beta = 1$. Consistent

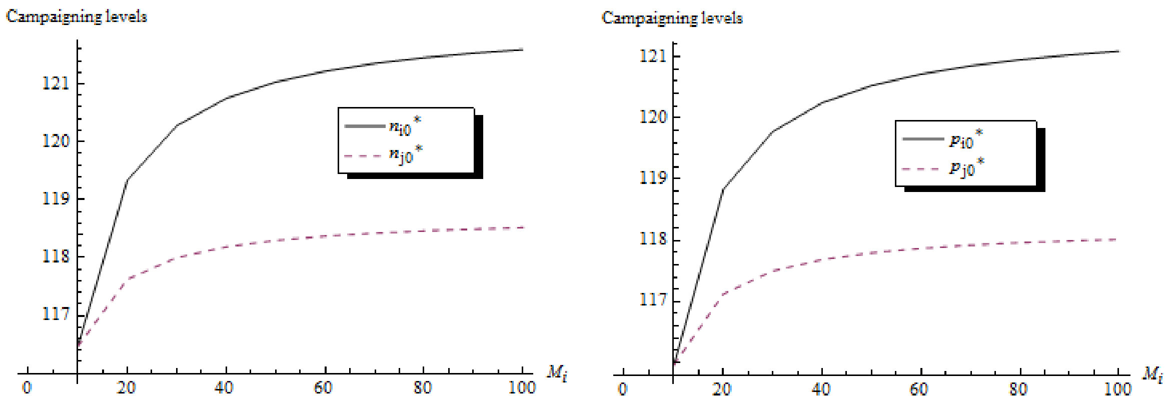

Figure 2.9: Primary campaigning as a function of challenger i's preferences when the effects of primary campaigning persist. Parameters: $\beta = 1, \bar{X}_I = 150, \bar{B}_I = 900, M_j = 10, \bar{X}_j = \bar{X}_i = 60, \bar{B}_j = \bar{B}_i = 700, \alpha = 1, \rho = 2$.

with the proposition, we see that when challenger i cares more about personally winning, he steps up his primary campaigning in order to ensure a win for himself. This lowers his chances of a win in the general election so that in response challenger j increases his primary campaigning, too. Figure 2.9 highlights the consequences for the composition of campaigns. When challenger i cares more about personally winning, he increasingly opts for the more effective form of campaigning, i.e. a negative campaign to ensure a win. Positive campaigning levels decrease but total campaigning expenditures in the primary increase so that fewer resources are left over for the general election. Apprehensive of a party defeat in the general election should i survive the primary election, the other challenger steps up his positive and total primary campaigning expenditures in order to improve his chances of a win in the primary[8].

2.4 Conclusion

In recent years, negative campaigning has come to dominate the airwaves in general elections throughout the entire United States. What has received far less attention is that much of the campaigning in primaries is positive in nature. Our paper analyzes such campaigning, providing an explanation for the preponderance of negative campaigning in general elections, but positive campaigning in primaries. The negative campaigning in general elections reflects that candidates only care about winning the general election—and it is easier to damage an opponent's reputation via negative campaigning than to build up one's own reputation via positive campaigning. Two considerations drive why primary campaigns are less negative: candidates internalize both that a primary winner benefits in the general election only from his positive primary campaigning; and a primary loser, impairs his party rival's chances in the general election by campaigning negatively. More generally, we derive how (a) relative strengths of candidates (initial resources and reputations), (b) how much candidates care about winning vs just having their party win, and (c) the campaigning technology (effectiveness, decay in the effects of primary campaigning before the general election) affect the magnitudes and composition of campaigning in both elections.

Our paper characterizes how the primitives of the economy—(a) relative candidate strengths (resources and reputations); (b) the technology of campaigning (effectiveness and decay); and (c) a candidate's preferences over electoral outcomes (personally winning vs. having the party's nominee win)—affect the strategic calculus of campaigning. We show, for example, that if challengers are similarly situated, with comparable resources and reputations, then improving one challenger's reputation causes his primary rival to respond by campaigning more aggressively; but the challenger with the enhanced reputation only campaigns more aggressively if the incumbent is sufficiently weak, as then the primary winner is also the likely general election winner. Increases in one challenger's resources, however, causes both challengers to

[8] When challenger j is somewhat stronger and cares more about personally winning, and the incumbent is somewhat weaker than in the scenario illustrated in Figure 2.9 (e.g., when $\bar{X}_j = 70, \bar{X}_I = 100$ and $M_j = 20$ rather than $\bar{X}_j = 60, \bar{X}_I = 150$ and $M_j = 10$) then challenger j's negative primary campaigning rises with M_i rather than falling as in Figure 2.9. This reflects the twin effects of caring more about personally winning, and the increased probability that if challenger j wins the primary he will also win the general election.

campaign more aggressively in the primary—the challenger with the increased resources exploits their fungibility, splitting them across both elections, while his rival increases his spending to offset this advantage.

The impact of primitives are very different when one challenger is far stronger than the other. Making a strong challenger even stronger causes his rival to reduce his negative campaigning. The weak rival internalizes that his strong primary opponent is more likely than he to win the general election. As a result, a desire for the party's nominee to defeat the incumbent causes the weak rival to campaign less aggressively—spending less, and campaigning more positively. This, in turn, induces the stronger challenger to lower his primary campaigning too, because he is already likely to win the primary due to his greater resources and reputation, and his primary rival's less aggressive campaign. The stronger challenger still spends far more, and is far more negative than his weaker rival in order to ensure victory. Thus, our model delivers the pattern of primary campaigning in the 2012 Republican primary—Romney had a stronger reputation and far greater resources, and hence he spent far more, and was much more negative than his primary rivals.

Chapter 3

Quality Provision and Pricing when Consumer Valuations are Continuous

3.1 Introduction

Recent years have seen a dramatic surge in online shopping as the ease and reliability of online purchases has improved.[1] In this paper, we study the strategic choices of product variety, quality and pricing of products by a brick and mortar store in the face of online competition by electronic retailers. We find that as online competition increases, the store initially lowers its prices in order to retain consumers who would otherwise switch to online purchases and this move initially allows it to attract more consumers than before. However, the store is unable to match the competitive offers available online for long; when the spread of consumer valuations is high, as online products become better substitutes, consumers with relatively low valuations increasingly switch to online products. The low quality product of the store now caters to the tastes of consumers with higher valuations than before: the store begins to increase the quality and price of its low quality products. To prevent its high valuation consumers from switching over to the low quality product, the store raises the quality of its high quality product too; the latter is now targeted at consumers with higher valuations. All the while, its profits decline and the store eventually exits the market when its unable to meet its fixed operating costs.

The model is largely similar to that in Bernhardt and Ghosh (2014) with two distinctions: consumer valuation types are uniformly distributed over an interval and consumers have a common discount factor. Thus, in our model, consumers are only distinguished by (1) their costs of traveling to the brick and mortar store and (2) their preferences for quality. As a benchmark, we consider the case where the utility costs from online purchases are prohibitive, so the physical store is a monopolist. We show numerically that there exists a threshold of consumer valuation of quality such that consumers with valuations below the threshold buy the low quality product while those with valuations above the threshold buy the high

[1] A 2010 Nielsen survey, *Global Trends in Online Shopping, A Nielsen Global Consumer Report* (June 2010), found that 84% of consumers shop online, and online shopping accounted for more than 5 percent of total monthly spending for 56 percent of the respondents.

quality product. Further, we lose the price equivalence result of Bernhardt and Ghosh (2014): an increase in the price of the high quality products when suitably matched by a decline in the price of the low quality product (and vice versa) does not deliver the same expected return to consumers from visiting the store and the same expected payoff to the store, so that prices are perfectly pinned down to those that maximize profits for the monopolist.

Once the utility costs of online purchases fall by enough, e-retailers operate. As expected, the price and quality levels in the benchmark setting are nearly identical to the equilibrium qualities and prices that prevail when online retailers operate in the market and the discount factor δ is very low. This is because when the discounted payoff from online purchases is very low, the store effectively operates as a monopoly. However, as the discount factor increases, online competition begins to impinge on the ability of the store to freely set its product variety, quality and pricing. When the support of consumer valuations of quality is high, the store initially offers products of nearly the same quality at increasingly lower prices as the discount factor increases. This move initially lures consumers with very low valuations to buy at the store when online competition is low. However, the store is unable to match the competitive offers online for long and quits catering to those with very low valuations eventually. Instead, its products begin to cater to consumers with relatively higher valuations than before and consequently the quality of its products increases. All the while, the store witnesses a steady decline in profits as the number of consumers who visit and buy at the store decreases along with the prices it charges for some of its products.

A decrease in marginal costs is found to increase the quality and prices of products while an increase in the travel costs is found to decrease the number of consumers visiting and buy at the store and its profits only, while leaving other strategy choices of product variety, quality and pricing unchanged. Lastly, when the support of consumer valuations is very small, the store offers the same quality to the same set of consumer types at increasingly lower prices. This is because when the spread of consumer valuations is low, consumers who buy either of the two products have relatively similar valuations (than when the spread is high). Hence, the products are a better match than those available online for all consumers who buy it as long as the price difference is not too high. Consequently, even when online competition intensifies, the store continues to cater to the same set of consumer valuation types and offers the same qualities at steadily declining prices.

Related literature. Our paper builds on work dating back to Mussa and Rosen (1978), Goldman et al. (1980), Spence (1980), and Maskin and Riley (1984) that explores price discrimination via quantity discounts and pricing of products of different qualities by monopolists facing heterogeneous consumers with different private valuations. In these models, in contrast to our benchmark setting, the optimal mechanism distorts quality downward for all but those with the highest willingness to pay, because downward incentive compatibility constraints bind. Rochet and Stole (1997) study duopoly nonlinear pricing in a model with horizontal and vertical product differentiation. When the degree of horizontal differentiation is so large that each firm is a local monopolist, perfect sorting arises, with quality distortions for all types but at the top and the bottom. In contrast, with little horizontal differentiation, the market is fully covered on both vertical and horizontal dimensions, and firms offer a cost-plus-fee pricing schedule with efficient quality provision. A more general analysis in Rochet and Stole (2002) covers both monopoly and duopoly, and allows for general distributions.

The most closely-related paper is Bernhardt and Ghosh (2014). The distinctions in the model are two fold: it assumes that consumer valuation types are either high or low and the consumers' discount factor δ may take any of the three values $\{0, \delta_m, 1\}$. It finds that if online competition is modest, the physical store provides the socially-efficient product qualities and varieties, and earns monopoly profits. As online competition intensifies, it begins to bite for consumers who have low quality valuations, as the utility loss from online purchases is lower for them. In response, the physical store reduces the price of its low quality good, eventually creating own product line competition, which impinges on its ability to extract the consumer surplus of those with the highest willingness to pay. To continue to extract rents from high valuation consumers, the store first distorts the quality of its low quality product. Next, provided there are enough high valuation consumers, and product valuations are not so different, the store stops providing the low quality product, focusing exclusively on selling high quality products. However, this stage does not last. As online competition intensifies further, the best alternative for high valuation consumers is now not the physical store's low quality product, but instead a high quality product available online. As a result, the store returns to producing a full line of products at the socially-optimal quality levels. Of course, the store's profits decline throughout, and eventually it cannot cover its operating costs and exits. It then extends this analysis to show how outcomes are affected when the physical store also has an online portal.

Another closely related paper is Loginova (2009). It models competition between electronic retailers and brick and mortar stores that sell a homogeneous product to consumers with heterogeneous valuations (high or low) who only learn their valuations after visiting the store. Paradoxically, the presence of electronic retailers causes brick and mortar stores to raise prices. Rather than compete for low valuation consumers against e-retailers, stores target only high valuation consumers, whose demands are less price elastic. Low valuation consumers return home and buy online. Our paper extends the analysis by studying quality provision and pricing by stores and how they evolve in the face of increasing online competition.

The idea that consumers must learn by visiting a store also appears in the price search literature that dates back to Stahl (1979), and continues on in research such as Ellison's (2005) model of add-on pricing in which firms advertise a base price for a product and try to induce customers with a high willingness to pay to buy high-priced add-ons at the point of sale.

There is a burgeoning literature on consumer and firm behavior in online environments. Alba et al (1997), Danaher et al. (2003), Peterson et al. (1997), and Ratchford et al. (2001) conduct empirical analyses of the differences between online and offline purchase experiences, focusing on assessing the impact of prices, brand names and product attributes on consumer choice. These indicate that online shopping is well suited for functional products about which online stores can provide detailed information. However, online stores are less suited for products with sensory "touch and feel" attributes. Brown and Goolsbee (2002) use data on individual insurance policies to analyze the impact of comparison shopping on offline prices. They find that the introduction of the insurance-oriented web sites was at first associated with high price dispersion. As the use of these sites became more widespread, prices and dispersion fall. Sengupta and Wiggins (2006) find that increased online sales of airline tickets are associated with reduced online and offline prices. Lal and Sarvary (1999) distinguish between digital and non-digital product attributes. Digital attributes can be conveyed via the internet, while non-digital attributes can only be judged in person at a retail store. The introduction of online shopping may induce consumers not to search, but instead to order familiar products online. In turn, this increased consumer loyalty can induce firms to raise prices.

The paper is structured as follows. We next present the formal model. In Section 3.3, we present results of a numerical analysis that allows us to explore the equilibrium product variety, quality and pricing strategy of the store along with the changes in its customer base and profits that ensue as it adapts to increasing competition from online retailers. The last section concludes the paper.

3.2 The Model

Our economy features a single brick and mortar (b*&*m) store that has a physical establishment, and many competitive electronic retailers that sell products on the internet. These stores differ in three ways: (1) it is costly for consumers to travel to the b*&*m store, but internet "transportation" costs are zero; (2) online purchases are imperfect and inferior substitutes for purchases from the b*&*m store reflecting that online deliveries take time and may require inconvenient and possibly flawed self-installation; and (3) there is a fixed cost $k > 0$ of having a physical establishment, but maintaining an internet presence is costless.

Consumers differ from each other in two ways: (1) their distances x from the *b&m* store; and (2) their valuations θ of product quality which are uniformly distributed over $[a, b]$. Consumer location and quality valuation are independently distributed in the population. In addition, all consumers incur utility losses of $(1 - \delta)\theta q$ from online purchases.

A consumer who travels distance x to the b&m store incurs costs tx, where $t > 0$. A consumer with quality valuation θ and online discount factor δ derives utility θq from purchasing a product of quality q from the b&m store, but he only derives utility $\delta\theta q$, if he buys online. In this paper we primarily focus on how the degree $\delta_m \in (0, 1)$ with which online purchase retain their value relative to in-store purchases for these consumers influences the price quality offers at the b&m store.

A consumer knows his distance x from the b&m store, and the utility costs associated with his online purchases, but he does not learn his valuation of quality unless he goes to a store and inspects the products. A consumer can (1) buy a product online at the outset without knowing his valuation, or (2) visit a store, learn his valuation, and then decide which product, if any, to buy at the b&m store or online.

It costs cq^2 to produce a good of quality q, where $c > 0$. The b&m store chooses the level and variety of its product qualities, and their prices. Note that if the b&m store sells both qualities q_h^* and q_l^*, then it has to concern itself about the possibility that a high quality valuation customer may buy the low quality good whenever the price difference $p_h - p_l$ is large. That is, the b&m store has to worry about the competition provided by its own product quality line. Electronic retailers are perfectly competitive, implying that a consumer can buy online a good of any quality q at a price of cq.

3.2.1 Monopoly

We first consider a benchmark setting where online purchases are not a viable alternative for consumers so that the b&m store operates as a monopolist. Now, suppose the b&m store sells two product qualities q_l and q_h, $q_l < q_h$ at associated prices $p_l < p_h$. Further, suppose consumers with valuations $\theta \in [g, h]$ buy the low quality product while consumers with valuations $\theta \in [k, l]$ buy the high quality product. Since consumers with a higher valuation value quality more, the quality choices made by any two valuation types is non decreasing in θ. To see this, note that given a choice between q_i and q_j, $q_i > q_j$, a consumer of type θ purchases q_j only if $\theta(q_i - q_j) - \frac{1}{2}(q_i^2 - q_j^2) < p_i - p_j$. The larger is θ, the greater is the reduction in price required to induce a consumer to purchase the lower quality. Hence, it is impossible to induce a higher type consumer to purchase a lower quality than a lower type. Further, since a consumer's payoff is increasing in θ, it follows that $a \leq g < h = k < l = b$.

Self selection by consumers means that for type $\theta \in [h, b]$ consumers to purchase q_h and type $\theta \in [g, h]$ consumers to purchase q_l, the following incentive compatibility constraints must hold:

$$\theta(q_h - q_l) - (p_h - p_l) \leq 0 \quad \forall\, \theta \in [g, h]$$
$$\theta(q_h - q_l) - (p_h - p_l) \geq 0 \quad \forall\, \theta \in [h, b].$$

Further, the individual rationality constraint for consumers who buy the low quality product, $\theta q_l - p_l \geq 0 \quad \forall\, \theta \in [g, h]$, must hold.

Then, the expected utility of a consumer who visits the b&m store, inspects the products and decides which, if any, to buy is

$$\pi_M = \frac{1}{t}\left[\int_g^h (\theta q_l - p_l) f(\theta)\, \mathrm{d}\theta + \int_h^b (\theta q_h - p_h) f(\theta)\, \mathrm{d}\theta\right] \times \left[\int_g^h (p_l - c q_l^2) f(\theta)\, \mathrm{d}\theta + \int_h^b (p_h - c q_h^2) f(\theta)\, \mathrm{d}\theta\right] - k.$$

Since consumer valuations are uniformly distributed over $[a, b]$, the above reduces to

$$\pi_M = \frac{1}{(b-a)^2 t}\left[\int_g^h (\theta q_l - p_l)\, \mathrm{d}\theta + \int_h^b (\theta q_h - p_h)\, \mathrm{d}\theta\right] \times \left[(h-g)(p_l - c q_l^2) + (b-h)(p_h - c q_h^2)\right] - k.$$

3.2.2 Internet Competition

We now assume that online purchases are plausible alternatives for some consumers so that electronic retailers operate.

A consumer located at a distance x from the b&m store can buy the product online at the outset without knowing his quality valuation, or he can visit the b&m store where he learns his valuation. Once he has learnt his valuation, he can opt to buy the product at the b&m store, or return and buy it online, if at all. If he buys online at the outset, marginal cost pricing on the part of electronic retailers implies that his quality choice solves

$$\max_q \delta \int_a^b \theta q f(\theta)\,\mathrm{d}\theta - cq^2.$$

Hence, he purchases online a good of quality

$$q = \frac{\delta(b+a)}{4c}$$

obtaining an expected payoff of $\frac{\delta^2(b+a)^2}{16c}$. A consumer who buys online after learning θ purchases a good of quality

$$q(\theta) = \frac{\delta\theta}{2c},$$

obtaining a payoff of $\frac{\delta^2\theta^2}{4c}$.

Now, suppose, as before that consumers with valuations $\theta \in [g,h]$ buy the low quality product, those with valuations $\theta \in [j,k]$ buy the high quality product while the rest buy the product online. Then, a consumer's expected payoff from visiting a store is

$$\int_g^h (\theta q_l - p_l) f(\theta)\,\mathrm{d}\theta + \int_j^k (\theta q_h - p_h) f(\theta)\,\mathrm{d}\theta + \int_a^g \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta + \int_h^j \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta \\ + \int_k^b \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta - \frac{\delta^2(a+b)^2}{16c} - tx$$

so that the store draws consumers over a distance

$$\frac{1}{t}\left[\int_g^h (\theta q_l - p_l) f(\theta)\,\mathrm{d}\theta + \int_j^k (\theta q_h - p_h) f(\theta)\,\mathrm{d}\theta + \int_a^g \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta + \int_h^j \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta \right. \\ \left. + \int_k^b \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta - \frac{\delta^2(a+b)^2}{16c}\right].$$

Thus, the expected profits of the store are

$$\begin{aligned}\pi^I = \frac{1}{t}&\left[\int_g^h (\theta q_l - p_l) f(\theta)\,\mathrm{d}\theta + \int_j^k (\theta q_h - p_h) f(\theta)\,\mathrm{d}\theta + \int_a^g \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta \right. \\ &\left. + \int_h^j \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta + \int_k^b \frac{\delta^2\theta^2}{4c} f(\theta)\,\mathrm{d}\theta - \frac{\delta^2(a+b)^2}{16c}\right] \\ &\times \left[\left(\frac{h-g}{b-a}\right)(p_l - cq_l^2) + \left(\frac{k-j}{b-a}\right)(p_h - cq_h^2)\right] - k\end{aligned}$$

and it maximizes profits subject to incentive compatibility and individual rationality constraints. For consumers buying at the store, these are

$$\begin{aligned}
&\theta q_l - p_l - \frac{\delta^2\theta^2}{4c} \geq 0 \quad \forall\, \theta \in [g, h] \\
&\theta q_h - p_h - \frac{\delta^2\theta^2}{4c} \geq 0 \quad \forall\, \theta \in [j, k] \\
&\theta(q_h - q_l) - (p_h - p_l) \leq 0 \quad \forall\, \theta \in [g, h] \\
&\theta(q_h - q_l) - (p_h - p_l) \geq 0 \quad \forall\, \theta \in [j, k].
\end{aligned}$$

3.3 Numerical Analysis

Below, we present results from a numerical analysis of the model for different parametric specifications. We explore the price quality offer, the consumer valuation types targeted for each of its products, profits as well as the number of consumers visiting and buying at the store as a function of the discount factor. Figures 3.1 and 3.2 depict the price and quality of the high and low quality product offered at the store for different levels of the discount factor.

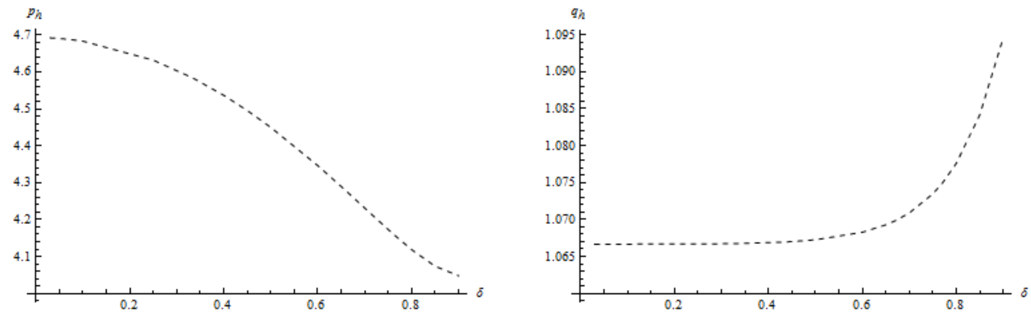

Figure 3.1: Price and Quality of the high quality product as a function of the discount factor. Parameters: $t = 1, c = 3, a = 2, b = 8$

As the discount factor increases, online products become increasingly better substitutes of those offered at the store as waiting costs decrease and prices offered online are more competitive. Hence, there is an incentive for consumers, especially for those with a low valuation of quality, to switch to online purchases. In order to prevent this move, the brick and mortar store lowers the prices while maintaining nearly the same quality of its products. When online competition is modest, this induces more and more consumers (with increasingly lower valuations) to buy at the store. This is depicted in the first panel of Figure 3.3 which shows the highest valuation type that buys the low and high quality product respectively

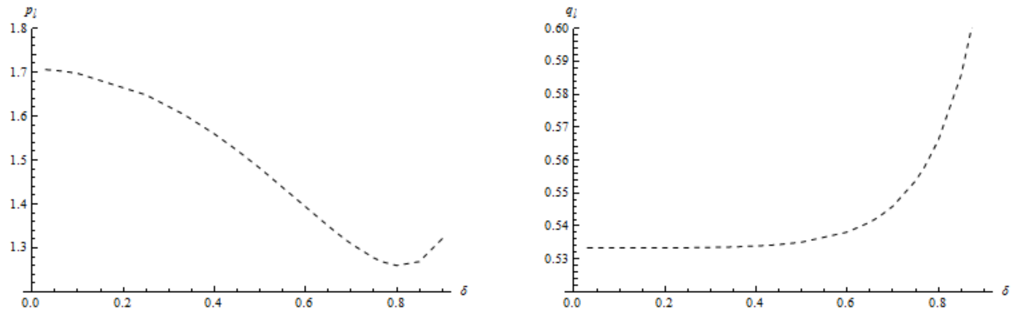

Figure 3.2: Price and Quality of the low quality product as a function of the discount factor. Parameters: $t = 1, c = 3, a = 2, b = 8$

as a function of the discount factor. For instance, as long as they are located near enough, when $\delta = 1/2$, consumers of valuation type $\theta \in [2, 3.15]$ buy the product online after visiting the store, while those of type $\theta \in [3.15, 5.58]$ buy the low quality product at the store. Consumers of type $\theta \in [5.58, 8]$ buy the high quality product at the store. Thus, we observe that as the discount factor rises and online competition intensifies, the store initially is able to offer its low quality product to consumers with increasingly low valuations. This, in turn, results in more and more consumers with relatively high valuations switching to the low quality product, as shown in the second panel of Figure 3.3.

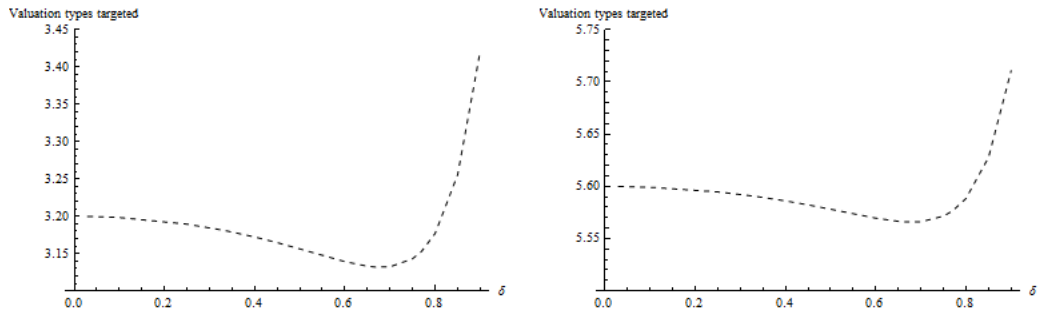

Figure 3.3: Valuation types targeted by the store for its low and high quality product as a function of the discount factor. Parameters: $t = 1, c = 3, a = 2, b = 8$

However, when the discount factor is relatively high, implying that online products are nearly perfect substitutes, the store is unable to match the offers online any further. Consequently, consumers with very low valuations switch to online products. The low quality product offered is now catered towards consumers with relatively higher valuations than before. Consequently, as the store is now able to extract the higher willingness to pay of these consumers with a higher valuation, the quality of the low quality product increases sharply as does its price. The incentive compatibility constraint for consumers with relatively high valuations dictates that in order to ensure that they continue to purchase the high quality product, the quality of the high quality product and its price increase (as does the lowest valuation type who purchases the high quality product).

The number of consumers visiting and buying at the store both decrease as the discount factor increase as shown in Figure 3.4; this likely reflects the increasing reservation payoff of consumers who now derive a higher payoff than before from buying the product online at the outset. Profits of the store decline too as its consumer base declines along with the prices it can charge for its products (Figure 3.5).

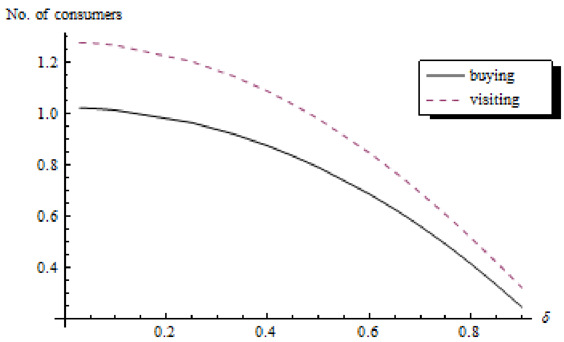

Figure 3.4: Consumers visiting and buying at the store as a function of the discount factor. Parameters: $t = 1, c = 3, a = 2, b = 8$.

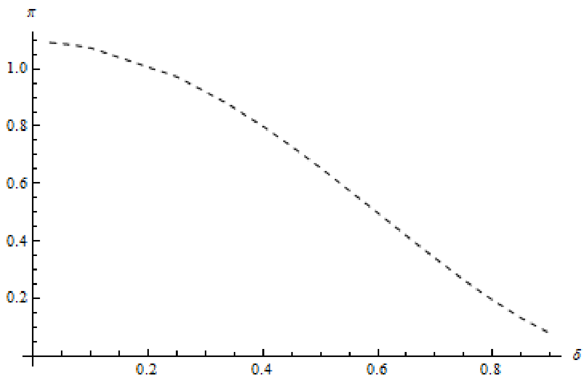

Figure 3.5: Profits of the store as a function of the discount factor. Parameters: $t = 1, c = 3, a = 2, b = 8$.

The impact of a lower cost parameter is primarily to increase the prices charged and the quality levels of the products offered by the store, as expected (Figures 3.6 and 3.7).

Figure 3.8 depicts the impact of an increase in the travel cost parameter t. We find that an increase in travel costs has no bearing on the prices and quality offered or the valuation types targeted; its only impact is to lower the number of consumers visiting (and buying) at the store as well as its profits.

When the support of consumer valuation types is small, i.e. consumers do not differ largely in their valuation of quality, we find that prices decline as δ increases with constant quality levels, for the following

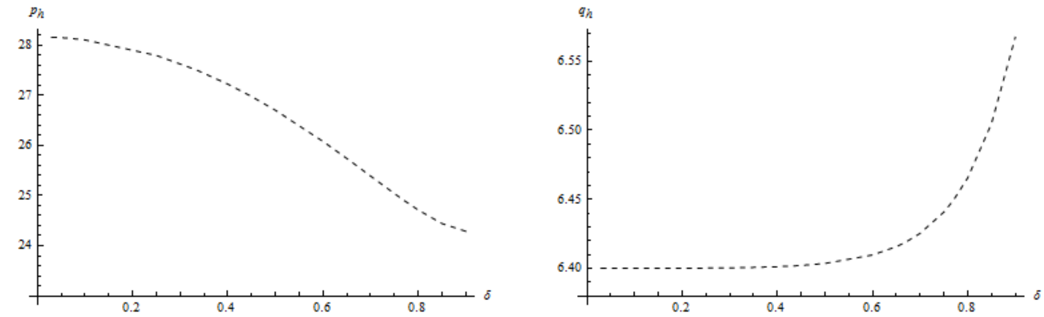

Figure 3.6: Price and Quality of the high quality product as a function of the discount factor when marginal costs are low. Parameters: $t = 1, c = \frac{1}{2}, a = 2, b = 8$.

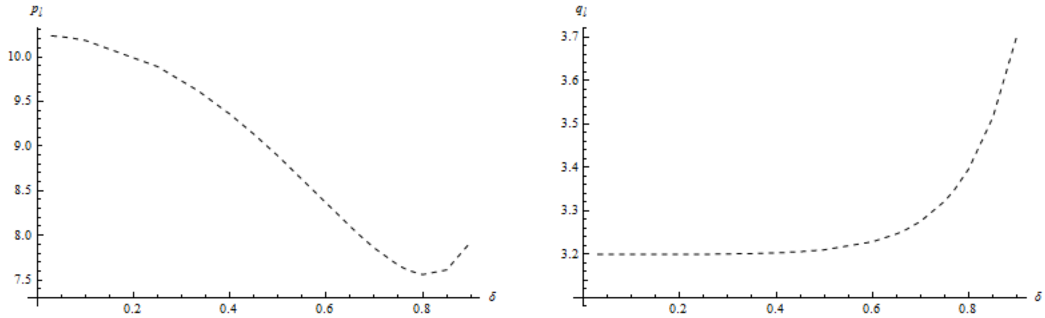

Figure 3.7: Price and Quality of the low quality product as a function of the discount factor when marginal costs are low. Parameters: $t = 1, c = \frac{1}{2}, a = 2, b = 8$.

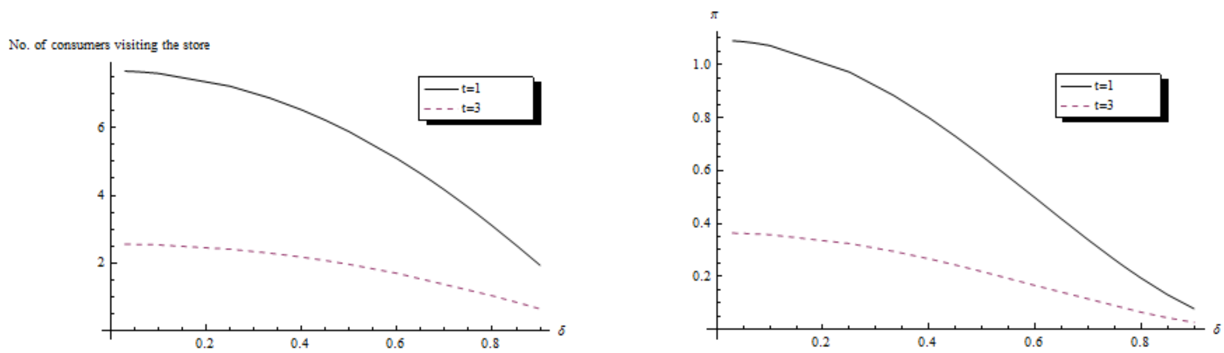

Figure 3.8: Consumer base and profits of the store as a function of the discount factor, for high and low travel costs. Parameters: $c = \frac{1}{2}, a = 2, b = 8$.

reason. Here, the spread of consumer valuations is low so that consumers who buy the low quality product have relatively similar valuations (than when the spread is high) and analogously for the high quality product. Hence, the store is able to cater more closely to the tastes of all its consumers; the products offered at the store are a better match than those available online for all consumers who buy it as long as the price difference is not too high. Consequently, even when online competition intensifies, the store continues to cater to the same set of consumer valuation types and offers the same qualities, albeit at steadily declining prices (see Figure 3.9, 3.10). Profits decline as δ increases, for similar reasons as in the above cases, as shown in Figure 3.11.

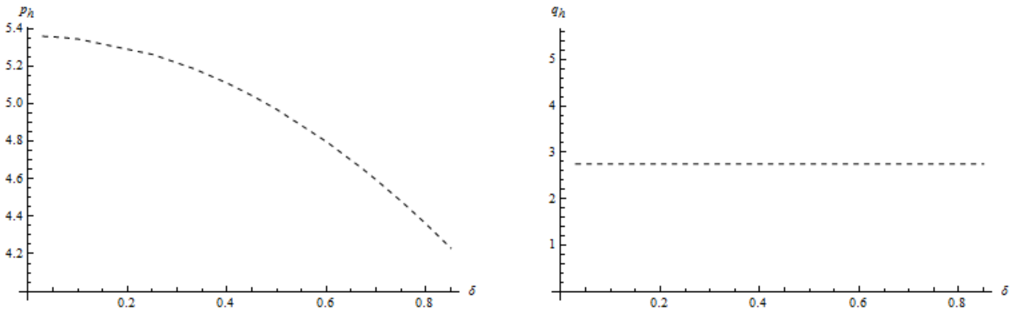

Figure 3.9: Price and quality of the high quality product when the support of consumer valuation types is small as a function of the discount factor. Parameters: $t = 1, c = \frac{1}{2}, a = 2, b = 3$.

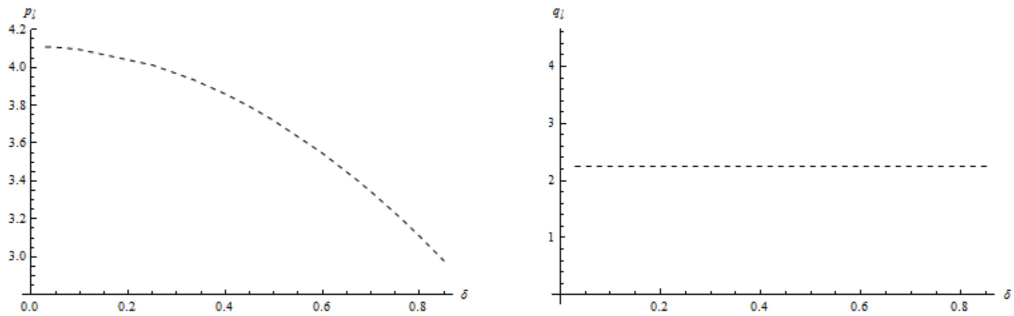

Figure 3.10: Price and quality of the low quality product when the support of consumer valuation types is small as a function of the discount factor. Parameters: $t = 1, c = \frac{1}{2}, a = 2, b = 3$.

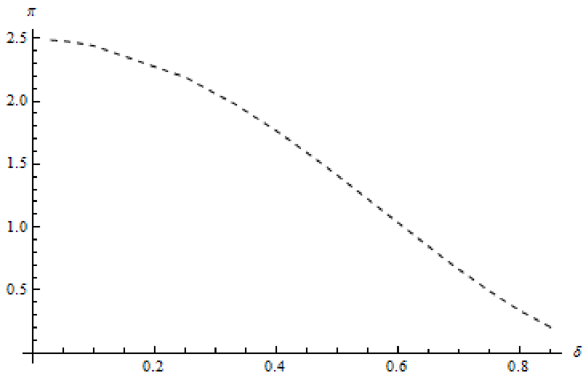

Figure 3.11: Profits of the store when the support of consumer valuation types is small as a function of the discount factor. Parameters: $t = 1, c = \frac{1}{2}, a = 2, b = 3$.

Finally, we study the behavior of the model under monopoly. The following table summarizes the results under different parametric specifications. As expected, the price and quality levels are nearly iden-

Parameter values	Prices	Qualities
$t = 1, a = 2, b = 3, c = \frac{1}{2}$	$p_h = 5.3593$, $p_l = 4.1093$	$q_h = 2.75$, $q_l = 2.25$
$t = 1, a = 2, b = 8, c = \frac{1}{2}$	$p_h = 28.16$, $p_l = 10.24$	$q_h = 6.4$, $q_l = 3.2$
$t = 1, a = 2, b = 8, c = 3$	$p_h = 4.693$, $p_l = 1.7066$	$q_h = 1.0667$, $q_l = 0.5333$

Table 3.1: Equilibrium Prices and Qualities under Monopoly

tical to the equilibrium qualities and prices in the internet setting when the discount factor δ is very low, implying that when the discounted payoff from online purchases is very low, the store effectively operates as a monopoly.

3.4 Conclusion

This paper explores numerically how the strategy choices of product variety, quality and pricing by a brick and mortar store are influenced by stiffening online competition when consumer valuation types are continuous. When the support of consumer valuations of quality is high, the store initially offers products of nearly the same quality at increasingly lower prices as the discount factor increases. This move initially lures consumers with very low valuations to buy at the store when online competition is low. However, as it intensifies, the store is unable to match the competitive offers online and quits catering to those with very low valuations; the latter switch to online purchases. Instead, its products now cater to consumers with relatively higher valuations than before, i.e. the quality of its products increases. All the while, the store witnesses a steady decline in profits as the number of consumers who visit and buy at the store decreases along with the prices it charges for some of its products.

A decrease in marginal costs is found to increase the quality and prices of products while an increase in the travel costs is found to decrease the number of consumers visiting and buy at the store and its profits only, while leaving other strategy choices of product variety, quality and pricing unchanged. Lastly, when the support of consumer valuations is very small, the store offers the same quality to the same set of consumer types at increasingly lower prices. This is because when the spread of consumer valuations is low, consumers who buy either of the two products have relatively similar valuations (than when the spread is high). Hence, the products are a better match than those available online for all consumers who buy it as long as the price difference is not too high. Consequently, even when online competition intensifies, the store continues to cater to the same set of consumer valuation types and offers the same qualities at steadily declining prices.

Appendix A

Proof of Proposition 1. The b&m store's expected profits are

$$\pi_M = \frac{2}{t}\left[\lambda\left(\theta_h q_h - \frac{1}{2}q_h^2 - p_h\right) + (1-\lambda)\left(\theta_l q_l - \frac{1}{2}q_l^2 - p_l\right)\right](\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l)) - k.$$

Differentiating with respect to p_h, p_l, q_h, q_l and solving for the optimal price-quality offers reveals that quality provision is always efficient, $q_l^* = \theta_l - c$ and $q_h^* = \theta_h - c$. However, there is a continuum of optimal price offers, where the high quality good's price is indexed by p_l^*:

$$p_h^* = \frac{\lambda(\theta_h + c)^2 + (1-\lambda)(\theta_l + c)^2 - 4c^2}{4\lambda} - \frac{(1-\lambda)p_l^*}{\lambda} \equiv f(p_l^*).$$

Each price pair, $(p_l^*, f(p_l^*))$ induces the same expected expenditures by consumers. It is this payment that enters travel decisions of the marginal consumer x^*, and the store's maximized profits:

$$\pi_M^* = \frac{(\lambda(\theta_h - c)^2 + (1-\lambda)(\theta_l - c)^2)^2}{8t} - k.$$

Note that p_h^* and p_l^* are inversely related, where the slope, $-\frac{(1-\lambda)}{\lambda}$, reflects the tradeoff between the two required to support constant consumer expenditure and profit.

At an optimum, all constraints must be slack. The incentive compatibility constraint for high valuation types bounds p_l^* from below:

$$p_l^* \geq \frac{\lambda(\theta_h - \theta_l)(3\theta_l - \theta_h - 2c) + (\theta_l + c)^2 - 4c^2}{4}.$$

So, too, the two constraints for low types place an upper bound:

$$p_l^* \leq \min\left(\frac{\lambda(\theta_h - \theta_l)(3\theta_h - \theta_l - 2c) + (\theta_l + c)^2 - 4c^2}{4}, \frac{\theta_l^2 - c^2}{2}\right).$$

These bounds are simply the prices of the low quality good corresponding to optima where *one* constraint holds at equality. When the incentive compatibility constraint for the high valuation type holds with equality, it is easy to solve for

$$\begin{aligned} p_l^{IC_h} &= \frac{\lambda(\theta_h-\theta_l)(3\theta_l-\theta_h-2c)+(\theta_l+c)^2-4c^2}{4}, \\ p_h^{IC_h} &= p_l^{IC_h}+\frac{(\theta_h-\theta_l)(2c+\theta_h-\theta_l)}{2}. \end{aligned}$$

The individual rationality constraint for the low type is satisfied, as

$$\begin{aligned} \theta_l q_l - \frac{1}{2}q_l^2 - p_l &= \theta_l(\theta_l-c)-\frac{1}{2}(\theta_l-c)^2-\frac{(\theta_l-c)(3c+\theta_l)+\lambda(\theta_h-\theta_l)(3\theta_l-\theta_h-2c)}{4} \\ &= \frac{1}{4}\left((3\lambda+1)\theta_l^2-\theta_l(2c+2c\lambda+4\lambda\theta_h)+c^2+2c\lambda\theta_h+\lambda\theta_h^2)\right)>0, \end{aligned}$$

since it is a positive quadratic in θ_l. Hence, $\underline{p_l} = p_l^{IC_h}$.

Now, define $\rho(\lambda, c)$ as the larger root of

$$3\lambda(\theta_h-\theta_l)^2+2\lambda(\theta_l-c)(\theta_h-\theta_l)-(\theta_l-c)^2=0,$$

where

$$\rho(\lambda,c)=(\theta_l-c)\left(\frac{-1+\sqrt{1+\frac{3}{\lambda}}}{3}\right).$$

Note that $\rho(\lambda, c)$ decreases in λ. It is easy to show that if $\theta_h - \theta_l \le \rho(\lambda, c)$, the incentive compatibility constraint for the low valuation type holds at the optimum, with associated prices:

$$\begin{aligned} p_l^{IC_l} &= \frac{\lambda(\theta_h-\theta_l)(3\theta_h-\theta_l-2c)+(\theta_l+c)^2-4c^2}{4}, \\ p_h^{IC_l} &= p_l^{IC_l}+\frac{(\theta_h-\theta_l)(2c-\theta_h+\theta_l)}{2}. \end{aligned}$$

The individual rationality constraint for low valuation types is satisfied if

$$\begin{aligned} \theta_l q_l - \frac{1}{2}q_l^2 - p_l &= \theta_l(\theta_l-c)-\frac{1}{2}(\theta_l-c)^2-\frac{(\theta_l-c)(3c+\theta_l)+\lambda(\theta_h-\theta_l)(3\theta_h-\theta_l-2c)}{4} \\ &= -3\lambda(\theta_h-\theta_l)^2-2\lambda(\theta_l-c)(\theta_h-\theta_l)+(\theta_l-c)^2>0, \end{aligned}$$

which always holds as long as $\theta_h - \theta_l \le \rho(\lambda, c)$. Here, $\bar{p_l} = p_l^{IC_l}$.

Finally, if $\theta_h - \theta_l > \rho(\lambda, c)$, the individual rationality constraint for the low type holds. When the individual rationality constraint holds at equality, one can solve for

$$p_l^{IR_l} = \frac{\theta_l^2 - c^2}{2},$$
$$p_h^{IR_l} = p_l^{IR_l} + \frac{\lambda(\theta_h - \theta_l)(\theta_h + \theta_l + 2c) - (\theta_l - c)^2}{4\lambda}.$$

The incentive compatibility constraint for the low type holds since

$$\theta_l(q_l - q_h) - \frac{1}{2}(q_l^2 - q_h^2) - (p_l - p_h) = 3\lambda(\theta_h - \theta_l)^2 + 2\lambda(\theta_l - c)(\theta_h - \theta_l) - (\theta_l - c)^2 > 0$$

for $\theta_h - \theta_l > \rho(\lambda, c)$. Here, $\bar{p_l} = p_l^{IR_l}$.

To see that no constraint *binds*, suppose (for example) by way of contradiction that a constraint for only the low type binds. Then one can reduce the price of the low quality good (and raise that of the high quality good) in such a manner that all constraints are now slack and revenues are unchanged, a contradiction of the premise that the constraint was binding. ■

Proof of Proposition 2. That equilibrium pricing and quality provision corresponds to the monopoly outcome whenever δ_m is sufficiently small (save for the loss of computer-literate consumers who reduce operating profits by μ_1%) is almost immediate. When $\delta_m \leq \frac{c}{E[\theta]}$, consumers do not buy online at the outset, so the objective function under internet competition is a linear transformation of that under monopoly, i.e., $\pi_I = (\mu_0 + \mu_m)\pi_M$, and the IR constraints are identical too, whenever $\delta_m \leq \frac{c}{\theta_h}$. Hence, for $\delta_m \leq \frac{c}{\theta_h}$, all properties extend.

However, once $\delta_m > \frac{c}{\theta_h}$, the right-hand side of IR_h is non-zero. As a result, the continuum of price offers that can be supported as internet equilibria *eventually* begins to shrink progressively with increases in δ_m. We now calculate explicitly when this occurs. Recall from Proposition 1 that IC_h holds at point A in Figure 1.1. The IR_h constraint evaluated at A,

$$\frac{1}{4}\left((\theta_h - c)^2 - (1-\lambda)(\theta_h - \theta_l)(2c + \theta_h - 3\theta_l)\right) - \frac{(\delta_m\theta_h - c)^2}{2\delta_m} \geq 0,$$

is satisfied for $\delta_m \in (\frac{c}{\theta_h}, \widetilde{\delta_m})$, where $\widetilde{\delta_m}$ is the larger root of

$$\frac{1}{4}\left((\theta_h - c)^2 - (1-\lambda)(\theta_h - \theta_l)(2c + \theta_h - 3\theta_l)\right) - \frac{(\delta_m\theta_h - c)^2}{2\delta_m} = 0.$$

Therefore, for $\delta_m \in (\frac{c}{\theta_h}, \widetilde{\delta_m})$, A can be supported in equilibrium. Further, since equilibrium monopoly quality provision is efficient everywhere on the continuum, $\theta_h q_h^* - \frac{q_h^{*2}}{2} - p_h^* - \frac{(\delta_m \theta_h - c)^2}{2\delta_m}$ rises as p_h^* falls along the continuum from A to B. Therefore, if the constraint holds at A for some δ_m, it holds at all points on the continuum. If $\widetilde{\delta_m} \geq \frac{c}{E[\theta]}$, then for all $\delta_m \leq \frac{c}{E[\theta]}$, the entire continuum can be supported as equilibria under internet competition. Here $\delta_m^* = \frac{c}{E[\theta]}$.

If instead, $\widetilde{\delta_m} < \frac{c}{E[\theta]}$, the subset of the continuum that can be supported as equilibria decreases in δ_m: as δ_m rises above $\widetilde{\delta_m}$, the individual rationality constraint is violated at A. In particular, at any $\delta_m > \widetilde{\delta_m}$, any $p_h > \underline{p_h}(\delta_m)$ and $p_l < \underline{p_l}(\delta_m) = f^{-1}(\underline{p_h}(\delta_m))$ on the continuum can no longer be supported, where $\underline{p_h}$ solves

$$\theta_h q_h^* - \frac{q_h^{*2}}{2} - p_h - \frac{(\delta_m \theta_h - c)^2}{2\delta_m} = 0.$$

Substituting for $q_h^* = \theta_h - c$ and solving the above equation for p_h yields $\underline{p_h} = \frac{\theta_h^2 - c^2}{2} - \frac{(\delta_m \theta_h - c)^2}{2\delta_m}$. Note that $\underline{p_l}(\delta_m)$ increases in δ_m. Thus, the continuum begins to shrink and collapses either to B, or to a range of offers $p_l \in [f^{-1}(\overline{p}_h), \bar{p}_l]$, where $\bar{p}_l$ is the price offer at point B and $\overline{p}_h = \frac{\theta_h^2 - c^2}{2} - \frac{c(\theta_h - \theta_l)^2(1-\lambda)^2}{2E[\theta]}$ solves the individual rationality constraint for the high type at $\delta_m = \frac{c}{E[\theta]}$:

$$\frac{\theta_h^2 - c^2}{2} - \frac{c(\theta_h - \theta_l)^2(1-\lambda)^2}{2E[\theta]} - p_h = 0.$$

In the more interesting case where quality valuations are not too different, i.e., when $\theta_h - \theta_l \leq \rho(\lambda, c)$, from Proposition 1, the incentive compatibility constraint for low types holds at B. Now, the individual rationality constraint for the high type evaluated at B,

$$\frac{1}{4}\big((\theta_h - c)^2 - (1-\lambda)(\theta_h - \theta_l)(2c - 3\theta_h + \theta_l)\big) - \frac{(\delta_m \theta_h - c)^2}{2\delta_m} \geq 0$$

is strictly positive for $\delta_m = \frac{c}{E[\theta]}$. Therefore, the continuum shrinks to a range of price offers, $p_l \in [f^{-1}(\overline{p}_h), p_l^{IC_l}]$ at $\delta_m = \frac{c}{E[\theta]}$. Here, $\delta_m^* = \frac{c}{E[\theta]}$.

Lastly, since these results hold for $\delta_m \leq \frac{c}{E[\theta]}$, the expected profits of the b*&*m store at the equilibrium, π_I^*, equal $(\mu_0 + \mu_m)\pi_M^*$, where π_M^* is its equilibrium profit under monopoly.

Next consider $\delta_m > \frac{c}{E[\theta]}$. Now, prior to consumers learning their valuations, online purchases at the outset have a positive value: their reservation utility rises to $\frac{(\delta_m E[\theta] - c)^2}{2\delta_m}$. This reduces how far consumers are willing to travel to the store. Equation (1.1) becomes:

$$\pi_I = \frac{2}{t}\left((\mu_0+\mu_m)\left(\lambda(\theta_h q_h - \frac{q_h^2}{2} - p_h) + (1-\lambda)(\theta_l q_l - \frac{q_l^2}{2} - p_l)\right) - \mu_m \frac{(\delta_m E[\theta]-c)^2}{2\delta_m}\right) \\ \times\left(\lambda(p_h - cq_h) + (1-\lambda)(p_l - cq_l)\right) - k. \tag{A.1}$$

Differentiating equation (A.1) with respect to p_h, p_l, q_h and q_l and solving the first-order conditions for the unconstrained optimal price quality offer gives $q_l^* = \theta_l - c$, $q_h^* = \theta_h - c$ and a continuum of inversely-related prices, $p_h^* = g(p_l^*)$, where $g' = -\frac{1-\lambda}{\lambda} < 0$. Profits decrease in δ_m as the number of consumers who visit the b&m store and the profits, P, from a representative consumer both fall:

$$\frac{dN}{d\delta_m} = -\frac{\mu_m\big((\delta_m\theta_l - c) + \delta_m\lambda(\theta_h-\theta_l)\big)\big(\delta_m\theta_l + c + \delta_m\lambda(\theta_h-\theta_l)\big)}{2t\delta_m^2} < 0,$$
$$\frac{dP}{d\delta_m} = -\frac{\mu_m\big((\delta_m\theta_l - c) + \delta_m\lambda(\theta_h-\theta_l)\big)\big(\delta_m\theta_l + c + \delta_m\lambda(\theta_h-\theta_l)\big)}{4\delta_m^2(\mu_0+\mu_m)} < 0.$$

At an interior optimum, all constraints must be slack. The incentive compatibility and individual rationality constraints for the low types remain slack if

$$(\theta_l - \theta_h + 2c)\left(\frac{\theta_h-\theta_l}{2}\right) - (p_h^* - p_l^*) < 0 \quad \text{and} \quad \frac{\theta_l^2 - c^2}{2} - \mathbf{1}_{\delta_m > \frac{c}{\theta_l}}\frac{(\delta_m\theta_l - c)^2}{2\delta_m} - p_l^* > 0.$$

Since p_h^* decreases in p_l^*, these two constraints set upper bounds for interior solutions on p_l^* of the form $p_l^* < \min(p_l^{*1}, p_l^{*2})$. The analogous constraints for high types

$$(\theta_h - \theta_l + 2c)\left(\frac{\theta_h-\theta_l}{2}\right) - (p_h^* - p_l^*) > 0 \quad \text{and} \quad \frac{\theta_h^2 - c^2}{2} - \frac{(\delta_m\theta_h - c)^2}{2\delta_m} - p_h^* > 0$$

set a lower bound on p_l^* of the form $p_l^* > \max(p_l^{*3}, p_l^{*4})$. The prices, $(p_l^{*1}, p_l^{*2}, p_l^{*3}, p_l^{*4})$, that constitute these bounds are the prices of the low quality good that prevail at an equilibrium when only *one* constraint holds with equality at the optimum. These bounds may be restated as $\underline{p_l}(\delta_m) = \max(p_l^{IC_h}, p_l^{IR_h}) < p_l^* < \min(p_l^{IC_l}, p_l^{IR_l}) = \bar{p}_l(\delta_m)$.

The inequality, $\max(p_l^{IC_h}, p_l^{IR_h}) < p_l^* < \min(p_l^{IC_l}, p_l^{IR_l})$, sheds light on which constraints hold at equality at any δ_m, and thus on how the continuum of equilibrium price pairs (with its "length" determined by the two bounds) evolves as online competition intensifies. First observe that $p_l^{IR_h} - p_l^{IC_h}$ rises in δ_m:

$$\frac{d}{d\delta_m}(p_l^{IR_h} - p_l^{IC_h}) > 0 \iff$$
$$= \delta_m^2 \mu_m (\theta_h - \theta_l) \Big(\theta_h(1-\lambda^2) + \theta_l(1-\lambda)^2\Big) + (\delta_m^2\theta_h^2 - c^2)(2\mu_0 + \mu_m) > 0.$$

Next note that when θ_l is close to θ_h and $\delta_m = \frac{c}{E[\theta]}$, $p_l^{IR_l} - p_l^{IC_l} \to \frac{(\theta_h - c)^2}{4} > 0$ so that $\min(p_l^{IC_l}, p_l^{IR_l}) = p_l^{IC_l}$, i.e., IC_l holds at equality. Further, as $\theta_l \to \theta_h$, $\frac{d}{d\delta_m}(p_l^{IR_l} - p_l^{IC_l}) < 0$ when $\delta_m > \frac{c}{\theta_l}$, i.e., when quality valuations are not too different and the discount factor is high enough, as δ_m rises, IR_l begins to hold at equality at a higher discount factor. Lastly, $p_l^{IR_l} - p_l^{IC_l}$, rises in θ_l when $\delta_m \to \frac{c}{E[\theta]}$: when valuations are relatively different, IR_l holds with equality at a lower θ_l. Thus, when the valuations are not too different and the discount factor is relatively low, the incentive compatibility constraint for high types holds with equality at the left end of the continuum, with the incentive compatibility constraint for low types holding at the other end, i.e. $\underline{p_l}(\delta_m) = p_l^{IC_h}$ and $\bar{p}_l(\delta_m) = p_l^{IC_l}$.

We now show that as δ_m rises past $\frac{c}{E(\theta)}$, the "length" of this continuum of supporting equilibrium price pairs eventually falls in δ_m. To see this, note that

$$\frac{dp_l^{IC_h}}{d\delta_m} = \frac{dp_l^{IC_l}}{d\delta_m} = \frac{dp_h^{IC_h}}{d\delta_m} = -\frac{\mu_m(\delta_m E(\theta) - c)(\delta_m E(\theta) + c)}{4\delta_m^2(\mu_0 + \mu_m)} < 0,$$

so the length of the continuum, given by $p_l^{IC_l} - p_l^{IC_h}$, is unaffected by increases in δ_m. Since the slope of the continuum, $g' = -\frac{1-\lambda}{\lambda}$, is independent of δ_m, the continuum undergoes a *parallel* shift (since the slope is unchanged) *below* (since $p_h^{IC_h}$ decreases) to the *left* (since $p_l^{IC_h}, p_l^{IC_l}$ both decrease) as δ_m rises. This continues until either:

Case I: IR_l begins to hold with equality before IR_h. Then this occurs at $\delta_m = \delta_m^1$ where δ_m^1 solves $p_l^{IC_l} - p_l^{IR_l} = 0$. Note that $\delta_m^1 > \frac{c}{\theta_l}$. This is because as long as $\delta_m < \frac{c}{\theta_l}$, then solving IR_L viz. $\theta_l q_l^* - \frac{q_l^{*2}}{2} - p_l = 0$ for $p_l^{IR_L}$ yields $p_l^{IR_L} = \frac{(\theta_l^2 - c^2)}{2}$. Since $p_l^{IR_L} - p_l^{IC_L} > 0$ and $p_l^{IR_L}$ does not vary with δ_m, but $p_l^{IC_L}$ decreases, it follows that $p_l^{IR_L} - p_l^{IC_L}$ increases in δ_m for $\delta_m \in (\frac{c}{E[\theta]}, \frac{c}{\theta_l}]$. However, once $\delta_m > \frac{c}{\theta_l}$, $p_l^{IR_l}$ also decreases in δ_m with derivative $-\frac{\delta_m^2\theta_l^2 - c^2}{2\delta_m^2}$.

It then follows that the continuum of equilibrium price pairs shrinks if and only if $\frac{d}{d\delta_m}(p_l^{IR_l} - p_l^{IC_h}) <$

0. To begin, one can show that

$$\begin{aligned}&\frac{dp_l^{IR_l}}{d\delta_m}-\frac{dp_l^{IC_h}}{d\delta_m}<0\\&\Longleftrightarrow 2c^2\mu_0+c^2\mu_m-\delta_m^2\left(\theta_l^2\left(2\mu_0+\mu_m\right)-\lambda\mu_m\left(\theta_h-\theta_l\right)\left(\lambda(\theta_h-\theta_l)+2\theta_l\right)\right)<0.\end{aligned} \tag{A.2}$$

Inspection of inequality (A.2) reveals that *if* it is negative at any δ_m, then it continues to be so with further increases in δ_m. The inequality holds at $\delta_m=\delta_m^1$ as $\frac{dp_l^{IR_l}}{d\delta_m}-\frac{dp_l^{IC_h}}{d\delta_m}=-\lambda\left(\frac{dp_h^{IR_l}}{d\delta_m}-\frac{dp_l^{IR_l}}{d\delta_m}\right)<0$. To see that $\frac{d}{d\delta_m}(p_h^{IR_l}-p_l^{IR_l})>0$ at $\delta_m=\delta_m^1$, recall that for $\delta_m=\delta_m^1$, the relevant constraint determining the upper bound on p_l switches from IC_l to IR_l. Since, $p_l^{IR_l}=p_l^{IC_l}$ and $p_h^{IR_l}=p_h^{IC_l}$ at $\delta_m=\delta_m^1$, it follows that $p_h^{IR_l}-p_l^{IR_l}=p_h^{IC_l}-p_l^{IC_l}=\theta_l(q_h^*-q_l^*)-\frac{1}{2}(q_h^{*2}-q_l^{*2})$. However, as δ_m rises further (and lies in the upper neighborhood of δ_m^1), $p_h^{IR_l}-p_l^{IR_l}>\theta_l(q_h^*-q_l^*)-\frac{1}{2}(q_h^{*2}-q_l^{*2})$ (a constant independent of δ_m). Thus, it must be that $\frac{dp_h^{IR_l}}{d\delta_m}-\frac{dp_l^{IR_l}}{d\delta_m}>0$ at $\delta_m=\delta_m^1$ and consequently $\frac{dp_l^{IR_l}}{d\delta_m}-\frac{dp_l^{IC_h}}{d\delta_m}<0$. But if so, then inequality (A.2) implies that this must hold for all $\delta_m>\delta_m^1$.

Thus, as δ_m increases, $p_l^{IR_l}-p_l^{IC_h}$ decreases. Now, let δ_m^2 solve $p_l^{IC_h}-p_l^{IR_h}=0$ and let $\grave{\delta}_m$ solve $p_l^{IC_h}-p_l^{IR_l}=0$. If $\grave{\delta}_m<\delta_m^2$, then $p_l^{IR_l}-p_l^{IC_h}$ shrinks to zero at $\grave{\delta}_m$ where the IR_l, IC_h *bind* (here, $\underline{\delta}_m(\theta_l)=\grave{\delta}_m$). However, if $\delta_m^2<\grave{\delta}_m$, then IC_h slackens and IR_h begins to hold at the left end, when $\delta_m=\delta_m^2$. Now,

$$\begin{aligned}\frac{dp_l^{IR_l}}{d\delta_m}-\frac{dp_l^{IR_h}}{d\delta_m}=&-\frac{2\mu_1\left(\delta^2\lambda\left(\theta_h^2-\theta_l^2\right)+\delta^2\theta_l^2-c^2\right)}{4\delta_m^2(1-\lambda)(\mu_0+\mu_m)}\\&-\frac{\mu_2\left(\delta^2\lambda\left(\theta_h-\theta_l\right)\left((2-\lambda)\theta_h+\lambda\theta_l\right)+\delta^2\theta_l^2-c^2\right)}{4\delta_m^2(1-\lambda)(\mu_0+\mu_m)}<0\end{aligned}$$

since $\delta_m>\frac{c}{\theta_l}$, so that the continuum shrinks with increases in δ_m until it collapses to a point at $\delta_m=\acute{\delta}_m$ where $\acute{\delta}_m$ solves $p_l^{IR_l}=p_l^{IR_h}$. The two individual rationality constraints now begin to *bind* (here $\underline{\delta}_m=\acute{\delta}_m$) and quality provision is efficient. Note that the incentive compatibility constraint for high types places a lower bound on δ_m at which an equilibrium where the two individual rationality constraints bind can be supported:

$$\theta_h q_h^*-\frac{1}{2}q_h^{*2}-p_h^*-(\theta_l q_l^*-\frac{1}{2}q_l^{*2}-p_l^*)=\frac{1}{2}(\theta_h-\theta_l)(2\theta_h-\delta_m\theta_h-\delta_m\theta_l)>0$$

as long as $\delta_m>\frac{2\theta_l}{\theta_h+\theta_l}$.

Case II: IR_h holds with equality before IR_l. Then, IR_h holds at equality at the left end of the continuum and IC_l holds at the other end. As above, one can establish that the length of the continuum,

$p_l^{IC_l} - p_l^{IR_h}$, decreases with δ_m. As δ_m rises, the continuum collapses to a point where either (1) IC_l and IR_h bind *or* (2) IR_l begins to hold with equality rather than IC_l (so that the relevant bounds on p_l now are $p_l^{IR_h}, p_l^{IR_l}$). We analyzed case (2) earlier. We now show that case (1) can never arise. Suppose, by way of contradiction, that the continuum indeed collapses where the IC_l and IR_h constraints bind. Then, first order conditions yield $q_l^* = \theta_l - c, q_h^* < \theta_h - c$. But by increasing q_h^*, the store can appropriate the entire increase in surplus, $\theta_h q_h^* - \frac{1}{2}q_h^{*2} - cq_h^*$, in the form of a higher $p_h^* - cq_h^*$, increasing profits, a contradiction.

Similarly, when the valuations are relatively different, IC_h and IR_l hold with equality at the two ends of the continuum when δ_m is low. One can show that the continuum begins to shrink once δ_m is high enough, and as before, the two cases discussed earlier obtain.

Thus, as δ_m rises, the continuum shrinks to a point at $\underline{\delta}_m(\theta_l)$, where either the two individual rationality constraints bind, or IC_h binds along with IR_l. Now, let $\bar{\theta}_l(\theta_h)$ solve $p_l^{IC_h} = p_l^{IR_h} = p_l^{IR_l}$. Recall that for the two individual rationality constraints to bind $\delta_m > \frac{2\theta_l}{\theta_h+\theta_l}$, or equivalently, $\theta_l < \frac{\delta_m\theta_h}{2-\delta_m}$. One can again show that IC_h and IR_l cannot bind when $\delta_m > \frac{2\theta_l}{\theta_h+\theta_l}$, or equivalently, $\theta_l < \frac{\delta_m\theta_h}{2-\delta_m}$. Define $\frac{\delta_m\theta_h}{2-\underline{\delta}_m} = \bar{\theta}_l$. Thus, IC_h and IR_l bind at the optimum at $\underline{\delta}_m(\theta_l)$ for $\theta_l > \bar{\theta}_l$, i.e., when the valuations are not too different. The two individual rationality constraints bind otherwise.

Once multiple constraints begin to bind, and tighten progressively with increases in δ_m, the store's expected profits keep declining: these new price quality offers were feasible at lower values of δ_m but were suboptimal (else they would have been offered). ■

Proof of Proposition 3. Recall that at $\delta_m = \underline{\delta}_m$, IC_h and IR_l bind when the valuations are relatively close. Solving the first-order conditions yields $q_h^* = \theta_h - c, q_l^* < \theta_l - c$.

Now, a slack individual rationality constraint for the high type implies

$$\theta_h q_h^* - \frac{1}{2}q_h^{*2} - \frac{(\delta_m\theta_h - c)^2}{2\delta_m} - p_h^* \;=\; \frac{1}{2}(\theta_h - \theta_l)(2c + 2q_l^* - \delta_m(\theta_h + \theta_l)) > 0.$$

This holds as long as $q_l^* > \frac{\delta_m}{2}(\theta_h + \theta_l) - c$. But then $\theta_l - c > q_l^* > \frac{\delta_m}{2}(\theta_h + \theta_l) - c \Rightarrow \delta_m < \frac{2\theta_l}{\theta_h+\theta_l}$ i.e., the constraint is violated if $\delta_m \geq \frac{2\theta_l}{\theta_h+\theta_l}$. Then, by continuity, there exists a $\bar{\delta}_m < \frac{2\theta_l}{\theta_h+\theta_l}$ such that for $\delta_m > \bar{\delta}_m$, at the optimum, IC_h and IR_l do not both bind.

In fact, for $\delta_m \in [\bar{\delta}_m, \frac{2\theta_l}{\theta_h+\theta_l}]$, if the store offers two qualities, the two rationality constraints and the incentive compatibility constraint for the high type bind, with $q_h^* = \theta_h - c, q_l^* = \frac{\delta_m(\theta_h+\theta_l)}{2} - c$, and prices pinned down by the two IR constraints. When $\lambda \to 1$, if the store offers two qualities with the above mentioned constraints binding, its equilibrium profits are

$$\pi = \frac{\mu_0(1-\delta_m)(\delta_m\theta_h - c)^2(\delta_m\theta_h^2 - c^2)}{2t\delta_m^2} - k.$$

In contrast, if the store offers a single product to the high type only, it sets $q^* = \theta_h - c$, and earns expected profits of

$$\pi = \frac{(\delta_m\mu_0(\theta_h - c)^2 + (1-\delta_m)\mu_m(\delta_m\theta_h^2 - c^2))^2}{8t\delta_m^2(\mu_0 + \mu_m)} - k$$

when $\lambda \to 1$. It is easy to show that the latter is strictly higher. Then, by continuity of the profit function in λ (and θ_l), there exists a $\bar{\lambda}(\theta_l) \in (0,1)$ such that for all $\lambda \in [\bar{\lambda}(\theta_l), 1]$, offering a single product is optimal. Further, one can show that profits decrease in δ_m.

By continuity of the profit function in δ_m, there exists a $\hat{\delta}_m^1(\theta_l, \lambda) \leq \bar{\delta}_m$, such that for $\lambda \in [\bar{\lambda}(\theta_l), 1]$, further increases in δ_m beyond $\hat{\delta}_m^1(\theta_l, \lambda)$ result in the b&m store switching to a single product offer. Similarly, note that as δ_m rises above $\frac{2\theta_l}{\theta_h+\theta_l}$, only the two individual rationality constraints bind. This is optimal for the b&m store as the surplus from either valuation type is maximized and there is no in-store product line competition. Then, by continuity, there exists a $\hat{\delta}_m^2(\theta_l) = \frac{2\theta_l}{\theta_h+\theta_l}, \hat{\delta}_m^{2\prime}(\theta_l) > 0$, such that for $\delta_m > \hat{\delta}_m^2$, the b&m store reverts to offering two socially optimal qualities.

Finally, we derive a lower bound on θ_l such that the result holds. Note that the two IR constraints bind for $\delta_m \in [\max(\underline{\delta}_m, \frac{2\theta_l}{\theta_h+\theta_l}), 1)$, or equivalently, for $\theta_l < \frac{\delta_m\theta_h}{2-\delta_m}$ when $\delta_m \geq \underline{\delta}_m$. IC_h binds with one or more individual rationality constraints when $\delta_m \geq \underline{\delta}_m$ and $\theta_l > \frac{\delta_m\theta_h}{2-\delta_m}$. Hence, there exists a $\hat{\theta}_l$, such that for $\delta_m > \underline{\delta}_m$ and $\theta_l \geq \hat{\theta}_l \geq \frac{\delta_m\theta_h}{2-\delta_m}$, the result holds. ■

Appendix B

Proof of Propositions 4 and 5. Suppose candidate i wins the primary. The probability he defeats the incumbent in the general election is

$$Pr_{i1} = \frac{\bar{Z}_{i2}}{\bar{Z}_{i2} + \bar{Z}_{I2}} = \frac{\bar{Z}_{i1}\frac{(1+p_{i1})^\alpha}{(1+\rho n_{I1})^\alpha}}{\bar{Z}_{i1}\frac{(1+p_{i1})^\alpha}{(1+\rho n_{I1})^\alpha} + \bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{i1})^\alpha}} = \frac{a[(1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha]/b}{a[(1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha]/b+1}.$$

where a and b are positive constants outside of candidate i's control at $t = 1$. Candidate i maximizes the probability of winning the general election by maximizing

$$\max_{p_{i1},n_{i1}} (1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha \text{ subject to } p_{i1}+n_{i1} \leq \bar{B}_i - (p_{i0}+n_{i0}) \equiv B_{i1},\ p_{i1}, n_{i1} \geq 0,$$

where B_{i1} denotes the funds at his disposal for campaigning in the general election. Since the probability of winning increases in both p_{i1}, n_{i1}, the constraint binds, i.e., $p_{i1} + n_{i1} = B_{i1}$. Thus, i's maximization problem reduces to

$$\max_{p_{i1},n_{i1}} (1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha \text{ subject to } p_{i1}+n_{i1} = B_{i1},\ p_{i1}, n_{i1} \geq 0.$$

Hence, the challenger's optimal choices of negative and positive campaigning are

$$p^*_{i1} = \begin{cases} \frac{B_{i1}}{2} - \frac{\rho-1}{2\rho}, & \text{if } \rho - 1 - \rho B_{i1} \leq 0 \\ 0, & \text{if } \rho - 1 - \rho B_{i1} > 0 \end{cases} \quad \text{and} \quad n^*_{i1} = \begin{cases} \frac{B_{i1}}{2} + \frac{\rho-1}{2\rho}, & \text{if } \rho - 1 - \rho B_{i1} \leq 0 \\ B_{i1}, & \text{if } \rho - 1 - \rho B_{i1} > 0. \end{cases}$$

Similarly, the incumbent's campaigning solves

$$\max_{p_{I1},n_{I1}} (1+p_{I1})^\alpha(1+\rho n_{I1})^\alpha \text{ subject to } p_{I1}+n_{I1} = \bar{B}_I,\ p_{I1}, n_{I1} \geq 0,$$

with solution

$$n_{I1}^* = \frac{\rho - 1}{2\rho} + \frac{\bar{B}_I}{2} \quad \text{and} \quad p_{I1}^* = \frac{\bar{B}_I}{2} - \frac{\rho - 1}{2\rho}$$

if $\bar{B}_I > \frac{\rho-1}{\rho}$ and $n_{I1}^* = \bar{B}_I$ if $\bar{B}_I \leq \frac{\rho-1}{\rho}$.

The probability candidate i wins the primary is $Pr_{i0} = \frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$, and since he only cares about winning the general election, he chooses p_{i0} and n_{i0} to maximize

$$\pi^i = (M_i Pr_{i1} - Pr_{j1}) Pr_{i0} + Pr_{j1}.$$

To ease presentation, define $P_{it} = 1+p_{it}$ and $N_{it} = 1+\rho n_{it}$ where $t \in \{0,1\}$, and let $Q_{i0}^* = \frac{\bar{X}_j (P_{j0}^* N_{j0}^*)^\alpha}{\bar{X}_i (P_{i0}^* N_{i0}^*)^\alpha}$, $Q_{i1}^* = \frac{\bar{X}_I (P_I^* N_I^* (1-\beta+\beta N_{j0}^*))^\alpha}{\bar{X}_i (P_{i1}^* N_{i1}^* (1-\beta+\beta P_{i0}^*))^\alpha}$, with analogous expressions for j. Suppose that the challenger's resources are high enough that he engages in both negative and positive campaigning in the general election. Then, at an interior optimum, his primary campaigning choices (N_{i0}^*, P_{i0}^*) satisfy the first-order conditions:

$$\begin{aligned}
\frac{\partial \pi^i}{\partial N_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial Pr_{i0}}{\partial N_{i0}} + Pr_{i0} \left(M_i \frac{\partial Pr_{i1}}{\partial N_{i0}} - \frac{\partial Pr_{j1}}{\partial N_{i0}} \right) + \frac{\partial Pr_{j1}}{\partial N_{i0}} \\
&= \left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*} \right) \frac{\alpha Q_{i0}^*}{N_{i0}^* (1+Q_{i0}^*)^2} - \frac{1}{1+Q_{i0}^*} \left(\frac{\alpha M_i Q_{i1}^*}{N_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha \beta Q_{j1}^*}{(1-\beta+\beta N_{i0}^*)(1+Q_{j1}^*)^2} \right) \\
&\qquad - \frac{\alpha \beta Q_{j1}^*}{(1-\beta+\beta N_{i0}^*)(1+Q_{j1}^*)^2} = 0, \\
\frac{\partial \pi^i}{\partial P_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial Pr_{i0}}{\partial P_{i0}} + M_i Pr_{i0} \frac{\partial Pr_{i1}}{\partial P_{i0}} \\
&= \left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*} \right) \frac{\alpha Q_{i0}^*}{P_{i0}^* (1+Q_{i0}^*)^2} - \frac{\alpha M_i Q_{i1}^*}{(1+Q_{i0}^*)(1+Q_{i1}^*)^2} \left(\frac{1}{P_{i1}^*} - \frac{\beta}{1-\beta+\beta P_{i0}^*} \right) = 0.
\end{aligned}$$

After canceling terms on both sides, these first-order conditions simplify to:

$$\left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*} \right) \frac{Q_{i0}^*}{N_{i0}^* (1+Q_{i0}^*)} = \frac{M_i Q_{i1}^*}{N_{i1}^* (1+Q_{i1}^*)^2} + \frac{\beta Q_{j1}^* Q_{i0}^*}{(1-\beta+\beta N_{i0}^*)(1+Q_{j1}^*)^2} \tag{B.1}$$

and

$$\left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*} \right) \frac{Q_{i0}^*}{P_{i0}^* (1+Q_{i0}^*)} = \frac{M_i Q_{i1}^*}{(1+Q_{i1}^*)^2} \left(\frac{1}{P_{i1}^*} - \frac{\beta}{1-\beta+\beta P_{i0}^*} \right). \tag{B.2}$$

Dividing equation (B.2) by (B.1), and then rearranging terms, yields

$$\frac{N_{i0}^*}{P_{i0}^*} \left(\frac{M_i Q_{i1}^*}{N_{i1}^* (1+Q_{i1}^*)^2} + \frac{\beta Q_{j1}^* Q_{i0}^*}{(1-\beta+\beta N_{i0}^*)(1+Q_{j1}^*)^2} \right) = \frac{M_i Q_{i1}^*}{P_{i1}^* (1+Q_{i1}^*)^2} - \frac{\beta M_i Q_{i1}^*}{(1-\beta+\beta P_{i0}^*)(1+Q_{i1}^*)^2}.$$

Multiplying both sides by P_{i0}^* yields

$$\frac{M_i Q_{i1}^*}{(1+Q_{i1}^*)^2}\left(\frac{N_{i0}^*}{N_{i1}^*}-\frac{P_{i0}^*}{P_{i1}^*}+\frac{\beta P_{i0}^*}{1-\beta+\beta P_{i0}^*}\right)+\frac{\beta N_{i0}^* Q_{j1}^* Q_{i0}^*}{(1-\beta+\beta N_{i0}^*)(1+Q_{j1}^*)^2}=0. \tag{B.3}$$

At an interior optimum, $n_{i0}^*, p_{i0}^* > 0$, or equivalently $N_{i0}^*, P_{i0}^* > 1$. The second term of equation (B.3) is positive, so the first grouped term must be negative, and hence

$$\frac{N_{i0}^*}{N_{i1}^*}-\frac{P_{i0}^*}{P_{i1}^*}<0 \Longleftrightarrow \frac{P_{i0}^*}{N_{i0}^*}>\frac{P_{i1}^*}{N_{i1}^*}=\frac{1}{\rho},$$

or equivalently,

$$n_{i0}^*-p_{i0}^*<n_{i1}^*-p_{i1}^*=\frac{\rho-1}{\rho}.$$

Note that when primary campaigning has no impact on reputations in the final election, i.e., when $\beta=0$, then equations (B.1) and (B.2) reduce to

$$\left(\frac{M_i}{1+Q_{i1}^*}-\frac{1}{1+Q_{j1}^*}\right)\frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)}-\frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)^2}=0 \tag{B.4}$$

and

$$\left(\frac{M_i}{1+Q_{i1}^*}-\frac{1}{1+Q_{j1}^*}\right)\frac{Q_{i0}^*}{P_{i0}^*(1+Q_{i0}^*)}-\frac{M_i Q_{i1}^*}{P_{i1}^*(1+Q_{i1}^*)^2}=0. \tag{B.5}$$

Then, as before, it follows that

$$\frac{P_{i0}^*}{P_{i1}^*}-\frac{N_{i0}^*}{N_{i1}^*}=0 \Longleftrightarrow \frac{P_{i0}^*}{N_{i0}^*}=\frac{P_{i1}^*}{N_{i1}^*}=\frac{1}{\rho} \Longleftrightarrow n_{i0}^*-p_{i0}^*=n_{i1}^*-p_{i1}^*=\frac{\rho-1}{\rho}. \tag{B.6}$$

One can show that even when challengers have less resources, so that they only campaign negatively in the general election, then in the primary, at an interior optimum, $\rho P_{i0}^*=N_{i0}^*$ or $n_{i0}^*-p_{i0}^*=\frac{\rho-1}{\rho}$. ■

Proof of Proposition 6. Omitting the i index on π, the following equations hold at a symmetric equilibrium since $\frac{dN_{i0}}{d\beta}=\frac{dN_{j0}}{d\beta}$ and $\frac{dP_{i0}}{d\beta}=\frac{dP_{j0}}{d\beta}$:

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}+\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\frac{dN_{i0}}{d\beta}+\left(\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}\right)\frac{dP_{i0}}{d\beta}=0 \tag{B.7}$$

and

$$\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}+\left(\frac{\partial^2\pi}{\partial N_{i0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}\right)\frac{dN_{i0}}{d\beta}+\left(\frac{\partial^2\pi}{\partial P_{i0}^2}+\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}\right)\frac{dP_{i0}}{d\beta}=0. \tag{B.8}$$

Then, from equations (B.7) and (B.8),

$$\frac{dN_{i0}}{d\beta} = \frac{\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)}{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right)}. \tag{B.9}$$

and

$$\frac{dP_{i0}}{d\beta} = \frac{\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial N_{i0}{}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right) - \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)}{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right)}. \tag{B.10}$$

The proof consists of the following steps.

Step 1. We show that as $\beta \to 0$, $\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} < 0$. To see this, note that

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} = \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\beta} - \frac{\partial Pr_{j1}}{\partial\beta}\right) + Pr_{i0}\left(M_i\frac{\partial^2 Pr_{i1}}{\partial\beta\partial N_{i0}} - \frac{\partial^2 Pr_{j1}}{\partial\beta\partial N_{i0}}\right) + \frac{\partial^2 Pr_{j1}}{\partial\beta\partial N_{i0}}.$$

Then, as $\beta \to 0$ (so that $N^*_{i0} \to \rho P^*_{i0}$ from equation (B.6)) and under symmetry (so that $\frac{P^*_{i0}}{P^*_{i1}} \to \frac{(M_i-1)(1+Q^*_{i1})}{2M_iQ^*_{i1}}$ — this follows from equation (B.5)) under symmetry by substituting $Q^*_{i0} = Q^*_{j0} = 1, Q^*_{i1} = Q^*_{j1}$) at an interior optimum, one can show that

$$\begin{aligned}\lim_{\beta\to 0}\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} &= \frac{\alpha Q^*_{i1}}{2(1+Q^*_{i1})^2}\left(-\frac{\alpha(M_i-1)(N^*_{i0}-P^*_{i0})}{2N^*_{i0}} - \frac{\alpha M_i(N^*_{i0}-P^*_{i0})(1-Q^*_{i1})}{N^*_{i1}(1+Q^*_{i1})} - 1\right)\\ &= -\frac{\alpha^2(M_i-1)(\rho-1)}{4\rho(1+Q^*_{i1})^2} - \frac{\alpha Q^*_{i1}}{2(1+Q^*_{i1})^2} < 0.\end{aligned} \tag{B.11}$$

Step 2. We compare the coefficients of $\frac{\partial P_{i0}}{\partial\beta}$ and $\frac{\partial N_{i0}}{\partial\beta}$ in equations (B.7) and (B.8) respectively. We show that the coefficient of $\frac{\partial P_{i0}}{\partial\beta}$ in equation (B.7) and that of $\frac{\partial N_{i0}}{\partial\beta}$ in equation (B.8) are equal as $\beta \to 0$. Note that this follows, if in the limit, $\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} = \frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}$. Now,

$$\begin{aligned}\lim_{\beta\to 0}\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} = (M_iPr_{i1} - Pr_{j1})\frac{\partial^2 Pr_{i0}}{\partial N_{j0}\partial P_{i0}} + \frac{\partial Pr_{i0}}{\partial P_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial N_{j0}} - \frac{\partial Pr_{j1}}{\partial N_{j0}}\right)\\ + M_i\left(\frac{\partial Pr_{i1}}{\partial P_{i0}}\frac{\partial Pr_{i0}}{\partial N_{j0}} + Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial N_{j0}\partial P_{i0}}\right).\end{aligned}$$

Since at a symmetric equilibrium, $\frac{\partial^2 Pr_{i0}}{\partial N_{j0}\partial P_{i0}} = \frac{\alpha^2 Q^*_{i0}(1-Q^*_{i0})}{N^*_{i0}P^*_{i0}(1+Q^*_{i0})^3} = 0$ while $\frac{\partial Pr_{i1}}{\partial N_{j0}}, \frac{\partial Pr_{i1}}{\partial N_{j0}\partial P_{i0}} = 0$ as $\beta \to 0$, it follows that

$$\lim_{\beta\to 0}\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} = \frac{\alpha^2 Q^*_{i1}(M_i+1)}{4\rho P^*_{i0}P^*_{i1}(1+Q^*_{i1})^2}. \tag{B.12}$$

Similarly, as $\beta \to 0$,

$$\frac{\partial^2 \pi}{\partial P_{j0} N_{i0}} = (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial P_{j0} \partial N_{i0}} - \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{j1}}{\partial P_{j0}} + M_i \frac{\partial Pr_{i0}}{\partial P_{j0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} = \frac{(M_i+1)\alpha^2 Q_{i1}^*}{4\rho P_{i1}^* P_{i0}^* (1+Q_{i1})^2}. \quad \text{(B.13)}$$

Further, we show that the coefficients $\frac{\partial P_{i0}}{\partial \beta}$ and $\frac{\partial N_{i0}}{\partial \beta}$ are negative. To see this, note that

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{i0} \partial P_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial N_{i0} \partial P_{i0}} + M_i \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{i1}}{\partial P_{i0}} + M_i \frac{\partial Pr_{i0}}{\partial P_{i0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial N_{i0} \partial P_{i0}} \\
&= -\frac{\alpha^2 M_i Q_{i1}^*}{4P_{i0}^* N_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha^2 M_i Q_{i1}^*}{4N_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^* (1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4N_{i1}^* P_{i1}^* (1+Q_{i1}^*)^3} \\
&= -\frac{\alpha^2 M_i Q_{i1}^*}{2\rho P_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^* (1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4\rho P_{i1}^{*2} (1+Q_{i1}^*)^3},
\end{aligned} \quad \text{(B.14)}$$

as $\frac{\partial^2 Pr_{i0}}{\partial N_{i0} \partial P_{i0}} = -\frac{\alpha^2 Q_{i0}^* (1-Q_{i0}^*)}{N_{i0}^* P_{i0}^* (1+Q_{i0}^*)^3} = 0$ under symmetry. Therefore, as $\beta \to 0$ from equations (B.12), (B.13) and (B.14),

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} &= \frac{\partial^2 \pi^i}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi^i}{\partial P_{i0} \partial N_{i0}} \\
&= \frac{-(M_i-1)\alpha^2 Q_{i1}^*}{4\rho P_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^* (1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4\rho P_{i1}^{*2} (1+Q_{i1}^*)^3} \\
&= -\frac{(M_i-1)\alpha(1+2\alpha+Q_{i1}^*)}{8\rho P_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} < 0.
\end{aligned} \quad \text{(B.15)}$$

Similarly, we compare the coefficient of $\frac{\partial N_{i0}}{\partial \beta}$ in equation (B.7) with that of $\frac{\partial P_{i0}}{\partial \beta}$ in equation (B.8). To do this, we note that

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{i0}^2} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial N_{i0}^2} + 2M_i \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial N_{i0}^2} \\
&= \left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*} \right) \left[\frac{-\alpha Q_{i0}^* (1+\alpha-(\alpha-1)Q_{i0}^*)}{N_{i0}^{2*} (1+Q_{i0}^*)^3} \right] - 2M_i \left(\frac{\alpha Q_{i0}^*}{N_{i0}^* (1+Q_{i0}^*)^2} \right) \left(\frac{\alpha Q_{i1}^*}{N_{i1}^* (1+Q_{i1}^*)^2} \right) \\
&\qquad - \frac{M_i}{1+Q_{i0}^*} \left(\frac{\alpha Q_{i1}^* (1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2N_{i1}^{*2} (1+Q_{i1}^*)^3} \right) \\
&= -\frac{(M_i-1)\alpha}{4\rho^2 P_{i0}^{*2} (1+Q_{i1}^*)} - \frac{\alpha^2 M_i Q_{i1}^*}{2\rho^2 P_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^* (1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4\rho^2 P_{i1}^{*2} (1+Q_{i1}^*)^3} \\
&= \frac{1}{\rho} \frac{\partial^2 \pi}{\partial P_{i0}^2}.
\end{aligned} \quad \text{(B.16)}$$

The last equality follows under symmetry and the fact that $N^*_{i0} \to \rho P^*_{i0}$, $N^*_{i1} \to \rho P^*_{i1}$ as $\beta \to 0$. Similarly,

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{j0} N_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial N_{j0} \partial N_{i0}} - \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{j1}}{\partial N_{j0}} + M_i \frac{\partial Pr_{i0}}{\partial N_{j0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} \\
&= \left(\frac{M_i}{1 + Q^*_{i1}} - \frac{1}{1 + Q^*_{j1}} \right) \frac{\alpha^2 Q^*_{i0}(1 - Q^*_{i0})}{\rho^2 P^*_{i0} P^*_{j0} (1 + Q_{i0})^3} + \frac{\alpha^2 Q^*_{i0} Q^*_{j1}}{\rho^2 P^*_{i0} P^*_{j1} (1 + Q^*_{i0})^2 (1 + Q^*_{j1})^2} \\
&\quad + \frac{\alpha^2 Q^*_{i0} Q^*_{i1} M_i}{\rho^2 P^*_{j0} P^*_{i1} (1 + Q^*_{i0})^2 (1 + Q^*_{i1})^2} \\
&= \frac{\alpha^2 M_i Q^*_{i1}}{\rho^2 P^*_{i1} P^*_{j0} (1 + Q_{i0})^2 (1 + Q^*_{i1})^2} + \frac{\alpha^2 Q^*_{i0} Q^*_{j1}}{\rho^2 P^*_{i0} P^*_{j1} (1 + Q^*_{i0})^2 (1 + Q^*_{j1})^2} \\
&= \frac{(M_i + 1)\alpha^2 Q^*_{i1}}{4\rho^2 P^*_{i1} P^*_{i0} (1 + Q_{i1})^2} \\
&= \frac{1}{\rho} \frac{\partial^2 \pi}{\partial P_{j0} N_{i0}}.
\end{aligned} \tag{B.17}$$

Therefore, from equations (B.15), (B.16) and (B.17), as $\beta \to 0$,

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} &= -\frac{\alpha(M_i - 1)}{4\rho^2 P^{*2}_{i0} (1 + Q^*_{i1})^2} - \left(\frac{(M_i - 1)\alpha^2 Q^*_{i1}}{4\rho^2 P^*_{i0} P^*_{i1} (1 + Q^*_{i1})^2} + \frac{\alpha M_i Q^*_{i1} (1 + 2\alpha - (2\alpha - 1) Q^*_{i1})}{4\rho^2 P^{*2}_{i1} (1 + Q^*_{i1})^3} \right) \\
&= -\frac{\alpha(M_i - 1)}{4\rho^2 P^{*2}_{i0} (1 + Q^*_{i1})^2} + \frac{1}{\rho} \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) < 0.
\end{aligned} \tag{B.18}$$

Further, one can show that

$$\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} = \frac{1}{\rho^2} \left(\frac{\partial^2 \pi}{\partial P_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P^2_{i0}} \right) < 0. \tag{B.19}$$

Step 3. We show that the denominator of the expression in equation (B.9),

$$\left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) \left(\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) - \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} \right) \left(\frac{\partial^2 \pi}{\partial P_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P^2_{i0}} \right),$$

is negative. From equations (B.12), (B.13) and (B.19), this reduces to showing that

$$\left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right)^2 - \rho^2 \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} \right)^2 < 0.$$

This holds, as equations (B.15) and (B.18) imply that

$$0 > \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) > \rho \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} \right).$$

Step 4. We show that the numerator of the expression in equation (B.9),

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}^2}\right)-\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right),$$

is positive in equilibrium. From equations (B.12), (B.13) and (B.19), the numerator reduces to

$$\rho^2\underbrace{\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}}_{<0}\underbrace{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)}_{<0}-\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\underbrace{\left(\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}\right)}_{<0}.$$

Now,

$$\begin{aligned}\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}&=\frac{\partial Pr_{i0}}{\partial P_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\beta}-\frac{\partial Pr_{j1}}{\partial\beta}\right)+M_iPr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\beta\partial P_{i0}}\\&=-\frac{\alpha^2(M_i-1)(N_{i0}^*-P_{i0}^*)Q_{i1}^*}{4P_{i0}(1+Q_{i1}^*)^2}-\frac{\alpha M_iQ_{i1}^*(\alpha\rho(N_{i0}^*-P_{i0}^*)(1-Q_{i1}^*)-N_{i1}^*(1+Q_{i1}^*))}{2N_{i1}^*(1+Q_{i1}^*)^3}\\&=\rho\left(-\frac{\alpha^2(M_i-1)(N_{i0}^*-P_{i0}^*)Q_{i1}^*}{4N_{i0}(1+Q_{i1}^*)^2}-\frac{\alpha^2M_iQ_{i1}^*(N_{i0}^*-P_{i0}^*)(1-Q_{i1}^*)}{2N_{i1}^*(1+Q_{i1}^*)^3}-\frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^*)^2}\right)\\&\qquad+\frac{\alpha Q_{i1}^*(M_i+\rho)}{2(1+Q_{i1}^*)^2}\\&=\rho\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}+\frac{\alpha Q_{i1}^*(M_i+\rho)}{2(1+Q_{i1}^*)^2}.\end{aligned}\tag{B.20}$$

To see that the numerator is always positive, first note that if $\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}>0$, then the numerator is positive. If, instead $\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}<0$, then from equations (B.15), (B.18) and (B.20),

$$0>\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}>\rho\frac{\partial^2\pi}{\partial\beta\partial N_{i0}},\quad 0>\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)>\rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)$$

implying that the numerator in equation (B.9) is positive. Since the denominator in equation (B.9)is negative (step 3), it follows that N_{i0}^* (and hence n_{i0}^*) is decreasing in β.

To determine the impact of increasing β on p_{i0}^*, we rewrite equation (B.8) below with the signs of some of the expressions contained in it.

$$\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}+\underbrace{\left(\frac{\partial^2\pi}{\partial N_{i0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}\right)}_{<0}\underbrace{\frac{dN_{i0}}{d\beta}}_{>0}+\underbrace{\left(\frac{\partial^2\pi}{\partial P_{i0}^2}+\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}\right)}_{<0}\frac{dP_{i0}}{d\beta}=0.$$

Then, it follows that $\frac{dP_{i0}}{d\beta}>0$ when $\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}>0$. Below, we show that $\frac{\partial^2\pi^i}{\partial\beta\partial P_{i0}}>0$ as long the challengers' budgets are not too large.

$$\begin{aligned}\frac{\partial^2\pi}{\partial\beta\partial P_{i0}} &= \frac{\partial Pr_{i0}}{\partial P_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\beta} - \frac{\partial Pr_{j1}}{\partial\beta}\right) + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\beta\partial P_{i0}} \\ &= -\frac{\alpha^2(M_i-1)(N_{i0}^*-P_{i0}^*)Q_{i1}^*}{4P_{i0}(1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^*(\alpha\rho(N_{i0}^*-P_{i0}^*)(1-Q_{i1}^*)-N_{i1}^*(1+Q_{i1}^*))}{2N_{i1}^*(1+Q_{i1}^*)^3} > 0 \\ &\iff -\frac{\alpha(M_i-1)(\rho-1)}{2} - \frac{\alpha(M_i-1)(\rho-1)(1-Q_{i1}^*)}{2Q_{i1}^*} + M_i > 0 \\ &\iff Q_{i1}^* > \frac{\alpha(M_i-1)(\rho-1)}{2M_i},\end{aligned}$$

i.e., if the challengers' budgets are not too high. The equivalences follow from $N_{i0}^* \to \rho P_{i0}^*$ as $\beta \to 0$, rearranging terms and substituting $\frac{P_{i0}^*}{P_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_iQ_{i1}^*}$ from equation (B.5). Also, following the same steps as above for the case where challengers only campaign negatively in the general election yields the result.

Finally, we show that the increase in positive campaigning exceeds the decrease in negative campaigning so that total campaigning expenditures in the primary election increase. To see this, note that from equations (B.9) and (B.10) and the assertion in step 3, it follows that

$$\begin{aligned}\frac{dP_{i0}}{d\beta} > \left|\frac{dN_{i0}}{d\beta}\right| \iff & \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) \\ & < \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\end{aligned}$$

or,

$$\begin{aligned}&\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left[\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\right] \\ &< \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left[\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\right].\end{aligned} \tag{B.21}$$

In what follows, we show that this always holds at the equilibrium.To see this, first note that from equation (B.11) as $\rho \to 1$,

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} \to -\frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^{*2})}. \tag{B.22}$$

Therefore, from equations (B.20) and (B.22), as $\rho \to 1$,

$$\frac{\partial^2\pi}{\partial\beta\partial P_{i0}} \to \frac{\alpha M_i Q_{i1}^*}{2(1+Q_{i1}^{*2})} > \left|\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\right| \to \frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^{*2})}. \tag{B.23}$$

However, from equations (B.12), (B.13) and (B.18),

$$
\begin{aligned}
&\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\\
&\qquad=\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\\
&\qquad=\rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)+\frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2}-\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)\\
&\qquad\to\frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2}>0\quad(\text{when } \rho\to 1).
\end{aligned}
\tag{B.24}
$$

Similarly, from equations (B.14), (B.15), (B.17) and (B.18),

$$
\begin{aligned}
&\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}^2}\right)-\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\\
&=\rho^2\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)-\left[\rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{i0}^2}\right)+\frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2}\right]\\
&\qquad\to-\frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2}<0\quad(\text{when } \rho\to 1).
\end{aligned}
\tag{B.25}
$$

Thus, from equations (B.23), (B.24) and (B.25),

$$
\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}>\left|\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\right|\quad\text{and}
$$

$$
\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)=\left|\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}^2}\right)-\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\right|
$$

so that the inequality in (B.21) holds. ■

Proof of Proposition 7. With symmetry, $Q_{i0}^*=1, Q_{i1}^*=Q_{j1}^*$. Substituting these expressions and $\beta=1$ into equation (B.3) yields:

$$
\frac{M_iQ_{i1}^*}{(1+Q_{i1}^*)^2}\left(\frac{N_{i0}^*}{N_{i1}^*}-\frac{P_{i0}^*}{P_{i1}^*}+1\right)+\frac{Q_{i1}^*}{(1+Q_{i1}^*)^2}=0.
$$

Canceling terms on both sides, we get

$$
M_i\left(\frac{N_{i0}^*}{N_{i1}^*}-\frac{P_{i0}^*}{P_{i1}^*}+1\right)+1=0.
$$

Next, we re-arrange this equality to obtain

$$\left(\frac{N_{i0}^*}{N_{i1}^*} - \frac{P_{i0}^*}{P_{i1}^*}\right) = -\frac{M_i + 1}{M_i} = -1 - \frac{1}{M_i}.$$

Since $M_i > 1$, it follows that

$$\left(\frac{N_{i0}^*}{N_{i1}^*} - \frac{P_{i0}^*}{P_{i1}^*}\right) \in (-2, -1),$$

and hence

$$\left(\frac{N_{i0}^*}{N_{i1}^*} - \frac{P_{i0}^*}{P_{i1}^*}\right) < -1. \tag{B.26}$$

Now, recall from Proposition 4 that $p_{i1}^* = \frac{B_{i1}}{2} - \frac{\rho-1}{2\rho}$ and $n_{i1}^* = \frac{B_{i1}}{2} + \frac{\rho-1}{2\rho}$. Therefore,

$$P_{i1}^* = 1 + p_{i1}^* = \frac{\rho+1}{2\rho} + \frac{B_{i1}}{2} \quad \text{and} \quad N_{i1}^* = 1 + \rho n_{i1}^* = \frac{\rho+1}{2} + \frac{\rho B_{i1}}{2}, \quad \text{i.e.} \quad N_{i1}^* = \rho P_{i1}^*.$$

Substituting $N_{i1}^* = \rho P_{i1}^*$, the inequality in (B.26) simplifies to:

$$\left(\frac{N_{i0}^*}{\rho} - P_{i0}^*\right) < -P_{i1}^*. \tag{B.27}$$

Further, since $P_{i0}^* = 1+p_{i0}^*$ and $N_{i0}^* = 1+\rho n_{i0}^*$, it follows that $B_{i1} = \bar{B}_i-(p_{i0}^*+n_{i0}^*) = \bar{B}_i-(P_{i0}^*-1)-\frac{N_{i0}^*-1}{\rho}$, so that

$$P_{i1}^* = \frac{\rho+1}{2\rho} + \frac{B_{i1}}{2} = \frac{1}{\rho}\left(1 + \rho + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Hence, the inequality in (B.27) may be rewritten as

$$\left(\frac{N_{i0}^*}{\rho} - P_{i0}^*\right) < -\frac{1}{\rho}\left(1 + \rho + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Multiplying both sides by ρ and rearranging the right-hand side slightly yields:

$$N_{i0}^* - \rho P_{i0}^* < 1 - \rho - \left(2 + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right). \tag{B.28}$$

Now, recall from Proposition 4 that challengers devote resources to both negative and positive campaigning in the general election when $B_{i1} = p_{i1}^* + n_{i1}^* = \bar{B}_i - (p_{i0}^* + n_{i0}^*) \geq \frac{\rho-1}{\rho}$, i.e.

$$\bar{B}_i - (P_{i0}^* - 1) - \left(\frac{N_{i0}^* - 1}{\rho}\right) \geq \frac{\rho - 1}{\rho}. \tag{B.29}$$

Rearranging terms, we rewrite (B.29) as

$$1+\frac{\rho\bar{B}_i}{2}-\frac{\rho P_{i0}+N_{i0}}{2}\geq 0. \tag{B.30}$$

Hence, from (B.28) and (B.30),

$$N_{i0}^*-\rho P_{i0}^*<1-\rho-\underbrace{\left(2+\frac{\rho\bar{B}_i}{2}-\frac{\rho P_{i0}+N_{i0}}{2}\right)}_{>0\text{ from (B.30)}}<1-\rho,\quad\text{i.e.,}\quad N_{i0}^*-\rho P_{i0}^*<1-\rho.$$

Substituting $N_{i0}^*=1+\rho n_{i0}^*$ and $P_{i0}^*=1+p_{i0}^*$ to this inequality yields

$$1+\rho n_{i0}^*-(\rho(1+p_{i0}^*)<1-\rho\Leftrightarrow\rho(n_{i0}^*-p_{i0}^*)<0.$$

Continuity of n_{i0}^*,p_{i0}^* in β then implies that the result holds for all β sufficiently small. ∎

Proof of Lemma 1. At an interior optimum, $N_{i0}^*,P_{i0}^*,N_{j0}^*,P_{j0}^*$ satisfy

$$\underbrace{\frac{\partial^2\pi}{\partial\theta\partial N_{i0}}}_{\text{direct effect of a change in }\theta}+\overbrace{\frac{\partial^2\pi}{\partial {N_{i0}}^2}\frac{dN_{i0}}{d\theta}+\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in }i\text{'s actions}}+\underbrace{\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}\frac{dP_{j0}}{d\theta}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in }j\text{'s actions}}=0 \tag{B.31}$$

for $\theta\in\bar{X}_I,\bar{B}_I,\alpha$.

From equation (B.6) it follows that when $\beta=0$, under assumptions of symmetry, $\frac{dN_{i0}}{d\theta}=\rho\frac{dP_{i0}}{d\theta}=\rho\frac{dP_{j0}}{d\theta}=\frac{dN_{j0}}{d\theta}$, at the equilibrium. Therefore, equation (B.31) may be rewritten as

$$\frac{\partial^2\pi}{\partial\theta\partial N_{i0}}+\frac{dN_{i0}}{d\theta}\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)=0. \tag{B.32}$$

Then, if

$$\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}<0 \tag{B.33}$$

$\frac{\partial^2\pi}{\partial\theta\partial N_{i0}}$ and $\frac{dN_{i0}}{d\theta}$ have the same sign. Similarly, for changes in $\bar{B}_i,\bar{B}_j$, or $\bar{X}_i,\bar{X}_j$ or M_i,M_j, $\frac{dn_{i0}}{d\theta_i}+\frac{dn_{i0}}{d\theta_j}$, $\frac{\partial^2\pi}{\partial\theta_i\partial n_{i0}}+\frac{\partial^2\pi}{\partial\theta_j\partial n_{i0}}$ have the same sign. Thus, the direct effect is reinforced by, or if in the opposite direction, outweighs the indirect effect, and in what follows, we show that the inequality above indeed holds at equilibrium under symmetry. To see this, note that from equations (B.13), (B.14), (B.16) and

(B.17), the left-hand side of (B.33) reduces to

$$\begin{aligned} &\frac{-(M_i-1)\alpha}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)} - \frac{\alpha M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2\rho^2 P_{i1}^{*2}(1+Q_{i1}^*)^3} - \frac{(M_i-1)\alpha^2 Q_{i1}^*}{2\rho^2 P_{i0}^* P_{i1}^*(1+Q_{i1}^*)^2} \\ &= \frac{\alpha}{4\rho^2(1+Q_{i1}^*)}\left(-\frac{(M_i-1)}{P_{i0}^{*2}} - \frac{2M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{P_{i1}^{*2}(1+Q_{i1}^*)^2} - \frac{2(M_i-1)\alpha Q_{i1}^*}{P_{i0}^* P_{i1}^*(1+Q_{i1}^*)}\right). \end{aligned} \tag{B.34}$$

Multiplying both numerator and denominator by P_{i0}^{*2}, and substituting $\frac{P_{i0}^*}{P_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, from equation (B.5) under symmetry yields

$$\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[-1-\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i}\right]. \tag{B.35}$$

Therefore, (B.33) holds if

$$-1-\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i} < 0. \tag{B.36}$$

Simplifying the inequality in (B.36) yields

$$Q_{i1}^* > -\frac{(1+2\alpha)(M_i-1)}{3M_i-1},$$

which always holds as $M_i > 1$.

Similarly, one can show that when the resources in the general election are low enough that a challenger only campaigns negatively, the left-hand side of (B.33) reduces to

$$\frac{-(M_i-1)\alpha}{4N_{i0}^{*2}(1+Q_{i1}^*)} - \frac{\alpha M_i Q_{i1}^*(1+\alpha-(\alpha-1)Q_{i1}^*)}{N_{i1}^{*2}(1+Q_{i1}^*)^3} - \frac{(M_i-1)\alpha^2 Q_{i1}^*}{2N_{i0}^* N_{i1}^*(1+Q_{i1}^*)^2}.$$

As before, multiplying by N_{i0}^2, substituting the first-order conditions and rearranging terms reveals that (B.33) holds as long as

$$1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{M_i} + \frac{\alpha(M_i-1)}{M_i} > 0,$$

or,

$$Q_{i1}^* > -\frac{(1+\alpha)(M_i-1)}{2M_i-1},$$

which always holds in equilibrium. ■

Proof of Proposition 8. From Lemma 1, we only need to focus on the partial derivative capturing the direct effect of a change in a parameter. Thus, N_{i0}^* increases in the challengers' reputations $\bar{X}_i, \bar{X}_j$ if and only if $\frac{\partial^2 \pi}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{X}_j \partial N_{i0}} > 0$. We show that this holds in equilibrium:

$$\frac{\partial^2 \pi}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{X}_j \partial N_{i0}} = (M_i Pr_{i1} - Pr_{j1}) \left(\frac{\partial^2 Pr_{i0}}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 Pr_{i0}}{\partial \bar{X}_j \partial N_{i0}} \right) +$$
$$\frac{\partial Pr_{i0}}{\partial N_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \bar{X}_i} - \frac{\partial Pr_{j1}}{\partial \bar{X}_j} \right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{X}_i \partial N_{i0}} + M_i \frac{\partial Pr_{i1}}{\partial N_{i0}} \left(\frac{\partial Pr_{i0}}{\partial \bar{X}_i} + \frac{\partial Pr_{i0}}{\partial \bar{X}_j} \right).$$

Now, under symmetry, $\frac{\partial Pr_{i0}}{\partial \bar{X}_i} + \frac{\partial Pr_{i0}}{\partial \bar{X}_j} = \frac{Q_{i0}^*}{\bar{X}_i (1+Q_{i0}^*)^2} - \frac{Q_{i0}^*}{\bar{X}_j (1+Q_{i0}^*)^2} = 0$ and since $Q_{i0}^* = 1$, it follows that $\frac{\partial^2 Pr_{i0}}{\partial \bar{X}_i \partial N_{i0}} = -\frac{\alpha Q_{i0}^* (1-Q_{i0}^*)}{\bar{X}_i N_{i0}^* (1+Q_{i0}^*)^3} = 0$. Therefore,

$$\frac{\partial^2 \pi}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{X}_j \partial N_{i0}} = \frac{\partial Pr_{i0}}{\partial N_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \bar{X}_i} - \frac{\partial Pr_{j1}}{\partial \bar{X}_j} \right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{X}_i \partial N_{i0}}$$
$$= \frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^*)^2 \bar{X}_i} \left(\frac{M_i - 1}{2N_{i0}^*} + \frac{M_i (1-Q_{i1}^*)}{N_{i1}^* (1+Q_{i1}^*)} \right).$$

Multiplying both numerator and denominator by N_{i0}^* and substituting $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$ from the first-order conditions yields

$$\frac{\partial^2 \pi}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{X}_j \partial N_{i0}} = \frac{\alpha (M_i - 1)}{4N_{i0}^* (1+Q_{i1}^*)^2 \bar{X}_i} > 0. \tag{B.37}$$

Therefore, N_{i0}^*, P_{i0}^* and hence, total campaigning expenditures in the primary rise with the challenger's reputation.

The probability of winning the primary is always half in a symmetric equilibrium. Therefore, the probability of being elected to office only varies with a change in a candidate's chances against the incumbent in the general election. Since $Pr_{i1}^* = \frac{1}{1+Q_{i1}^*}$, it suffices to derive the impact on Q_{i1}^*. Since increasing $\bar{X}_i$ reduces campaigning in the general election, N_{i1}^*, P_{i1}^*, the net impact on the probability that candidate i wins the general election depends on the relative change in $\bar{X}_i$ and N_{i1}^*, P_{i1}^*. We now show that the direct effect (improvement in reputation) dominates the indirect effect (decrease in campaigning expenditures in the general election), implying that the probability of ousting the incumbent rises. To see this, recall that $Q_{i1}^* = \frac{\bar{X}_I (P_I^* N_I^*)^\alpha}{\bar{X}_i (P_{i1}^* N_{i1}^*)^\alpha} = \frac{\bar{X}_I (N_I^*)^{2\alpha}}{\bar{X}_i (N_{i1}^*)^{2\alpha}}$, so that

$$\left(\frac{dQ_{i1}}{d\bar{X}_i} + \frac{dQ_{i1}}{d\bar{X}_j} \right) < 0 \iff 1 + \frac{2\alpha \bar{X}_i}{N_{i1}} \left(\frac{dN_{i1}}{d\bar{X}_i} + \frac{dN_{i1}}{d\bar{X}_j} \right) > 0 \iff 1 - \frac{2\alpha \bar{X}_i}{N_{i1}} \left(\frac{dN_{i0}}{d\bar{X}_i} + \frac{dN_{i0}}{d\bar{X}_j} \right) > 0. \tag{B.38}$$

The last equivalence in (B.38) follows from

$$N_{i1}^{*} = 1+\rho n_{i1}^{*} = \frac{\rho+1}{2} + \frac{\rho B_{i1}}{2} = 1+\rho+\frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0}+N_{i0}}{2} = 1+\rho+\frac{\rho\bar{B}_i}{2} - N_{i0} \quad (\text{since } N_{i0}^{*}=\rho P_{i0}^{*}).$$

Now, from equation (B.32),

$$\frac{dN_{i0}}{d\bar{X}_i} + \frac{dN_{i0}}{d\bar{X}_j} = -\frac{\frac{\partial^2\pi^i}{\partial\bar{X}_i\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial\bar{X}_j\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}}.$$

From equations (B.35) and (B.37), the right-hand side may be written as

$$\begin{aligned} &\frac{\frac{\alpha(M_i-1)}{4N_{i0}^{*}\bar{X}_i(1+Q_{i1}^{*})^2}}{\frac{\alpha(M_i-1)}{4\rho^2P_{i0}^{*2}(1+Q_{i1}^{*})}\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^{*}}{Q_{i1}^{*}}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\ &= \frac{\frac{N_{i0}^{*}}{\bar{X}_i(1+Q_{i1}^{*})}}{\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^{*}}{Q_{i1}^{*}}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]}, \text{ since } N_{i0}^{*}=\rho P_{i0}^{*}. \end{aligned}$$

One can show that the last inequality in (B.38) holds by rearranging terms and substituting for $\frac{N_{i0}^{*}}{N_{i1}^{*}} = \frac{(M_i-1)(1+Q_{i1}^{*})}{2M_iQ_{i1}^{*}}$. Note that these results extend to the case where a challenger only campaigns negatively in the general election:

$$\frac{dQ_{i1}}{d\bar{X}_i} + \frac{dQ_{i1}}{d\bar{X}_j} = \left(\frac{d}{d\bar{X}_i}+\frac{d}{d\bar{X}_j}\right)\left(\frac{\bar{X}_I N_I 2\alpha}{\bar{X}_i(\rho N_{i1})^{\alpha}}\right) < 0 \iff \frac{dN_{i1}}{d\bar{X}_i} + \frac{dN_{i1}}{d\bar{X}_j} = -2\left(\frac{dN_{i0}}{d\bar{X}_i}+\frac{dN_{i0}}{d\bar{X}_j}\right) > -\frac{N_{i1}}{\alpha\bar{X}_i},$$

since

$$N_{i1}^{*} = 1+\rho B_{i1} = 2+\rho+\rho\bar{B}_i-\rho P_{i0}^{*}-N_{i0}^{*} = 2+\rho+\rho\bar{B}_i-2N_{i0}^{*}.$$

Now,

$$\begin{aligned} \left(\frac{dN_{i0}}{d\bar{X}_i}+\frac{dN_{i0}}{d\bar{X}_j}\right) &= \frac{\frac{\alpha(M_i-1)}{4N_{i0}^{*}(1+Q_{i1}^{*})^2\bar{X}_i}}{\frac{\alpha(M_i-1)}{4\rho^2P_{i0}^{*2}(1+Q_{i1}^{*})}\left[1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^{*}}{Q_{i1}^{*}}\right)\frac{M_i-1}{M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\ &= \frac{\frac{N_{i0}^{*}}{\bar{X}_I(1+Q_{i1}^{*})}}{\left[1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^{*}}{Q_{i1}^{*}}\right)\frac{M_i-1}{M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \end{aligned}$$

Rearranging terms and substituting for $\frac{N_{i0}^{*}}{N_{i1}^{*}}$ from the first order conditions yields the result.

Similarly, consider the impact of increasing the challengers' budgets. Under symmetry,

$$\begin{aligned}\frac{\partial^2 \pi}{\partial \bar{B}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{B}_j \partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \bar{B}_i} - \frac{\partial Pr_{j1}}{\partial \bar{B}_j} \right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{B}_i \partial N_{i0}} \\ &= \frac{\alpha Q_{i1}^*}{4P_{i1}^*(1+Q_{i1}^*)^2} \left[\frac{\alpha(M_i - 1)}{N_{i0}^*} + \frac{M_i}{N_{i1}^*(1+Q_{i1}^*)} (1 + 2\alpha - (2\alpha - 1)Q_{i1}^*) \right] \\ &= \frac{\alpha(M_i - 1)(1 + 2\alpha + Q_{i1}^*)}{8N_{i0}^* P_{i1}^* (1+Q_{i1}^*)^2} > 0\end{aligned} \tag{B.39}$$

at equilibrium.

Again, since $P_{i0}^* = \frac{N_{i0}^*}{\rho}$, increases in the challengers' budgets raise P_{i0}^*. Thus, N_{i0}^*, P_{i0}^* and total primary campaigning expenditure rise with the challengers' budgets and reputations.

We next argue that increasing the challengers' budgets also increases P_{i1}^*, N_{i1}^* (and hence the probability of winning the general election), i.e., the increase in primary campaigning expenditures does not entirely exhaust the increase in the challengers' budgets. This follows if and only if $\frac{dN_{i0}}{d\bar{B}_i} + \frac{dN_{i0}}{d\bar{B}_j} < \frac{\rho}{2}$ as $N_{i1}^* = 1+\rho+\frac{\rho \bar{B}_i}{2} - N_{i0}$. As before, one can show that given equations (B.32), (B.35) and (B.39), this always holds in equilibrium. This result also carries over to when a challenger only campaigns negatively in the general election.

Lastly, consider the effect of an increase in M_i, which captures the relative payoffs for candidate i from being elected to office.

$$\begin{aligned}\frac{\partial^2 \pi^i}{\partial M_i \partial N_{i0}} &= Pr_{i0} \frac{\partial Pr_{i1}}{\partial N_{i0}} + Pr_{i1} \frac{\partial Pr_{i0}}{\partial N_{i0}} \\ &= \frac{\alpha}{(1+Q_{i0}^*)(1+Q_{i1}^*)} \left(\frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)} - \frac{Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)} \right) \\ &= \frac{\alpha Q_{i0}^*}{M_i N_{i0}^* (1+Q_{i0}^*)^2 (1+Q_{j1}^*)} > 0 \quad \text{(from equation (4))}.\end{aligned}$$

Thus, N_{i0}^* is increasing in M_i. Since $M_i = \frac{U_i}{V_i} > 1$, it follows that an increasing U_i increases negative campaigning in the primary election, while increasing V_i, leads to a decrease in N_{i0}^*. It also follows that equal changes in U_i, V_i lower M_i and hence N_{i0}^*. Since, $P_{i0}^* = \frac{N_{i0}^*}{\rho}$, the same results apply to P_{i0}^*, too. Again, via the budget constraint, P_{i1}^*, N_{i1}^* are decreasing in M_i. Consequently, the chances of winning the general election decrease in M_i. As above, this also holds when challengers only campaign negatively in the general election.

Note that changes in n_{i0}^*, p_{i0}^* mirror changes in N_{i0}^*, P_{i0}^*. Finally, the continuity of the derivatives in β, implies that there exists a $\bar{\beta}$ such that the results hold for all $\beta \in [0, \bar{\beta})$. ■

Proof of Proposition 9. From the continuity of the derivatives in $\bar{X}_i, \bar{B}_i, M_i$, it suffices to prove the result when challengers are symmetric. Further, from Lemma 1, we only need to focus on the partial derivative capturing the direct effect of a change in a parameter on N_{i0}^*. Consider the impact of increasing α.

$$\begin{aligned}
\frac{\partial^2 \pi^i}{\partial\alpha\partial N_{i0}} &= (M_i Pr_{i1} - Pr_{j1})\frac{\partial^2 Pr_{i0}}{\partial\alpha\partial N_{i0}} + \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\alpha} - \frac{\partial Pr_{j1}}{\partial\alpha}\right) \\
&\qquad + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\alpha\partial N_{i0}} + M_i\frac{\partial Pr_{i0}}{\partial\alpha}\frac{\partial Pr_{i1}}{\partial N_{i0}} \\
&= (M_i Pr_{i1} - Pr_{j1})\left[\frac{Q_{i0}^*\left(1 - \ln\left(\frac{\bar{X}_j}{\bar{X}_i Q_{i0}^*}\right) + \left(1 + \ln\left(\frac{\bar{X}_j}{\bar{X}_i Q_{i0}^*}\right)\right)Q_{i0}^*\right)}{N_{i0}^*(1+Q_{i0})^3}\right] \\
&\quad + \frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2}\left(\frac{M_i Q_{i1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)}{(1+Q_{i1}^*)^2} - \frac{Q_{j1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_j Q_{j1}^*}\right)}{(1+Q_{j1}^*)^2}\right) - \frac{M_i Q_{i0}^* Q_{i1}^*\ln\left(\frac{\bar{X}_j}{\bar{X}_i Q_{i0}^*}\right)}{N_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2} \\
&\quad + \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)^3(1+Q_{i0}^*)}\left[-1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) - \left(1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)\right)Q_{i1}^*\right] \\
&= \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}\left(1 - \ln\left(\frac{\bar{X}_j}{\bar{X}_i Q_{i0}^*}\right) + Q_{i0}^*\right) \\
&\quad + \frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2}\left(\frac{M_i Q_{i1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)}{(1+Q_{i1}^*)^2} - \frac{Q_{j1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_j Q_{j1}^*}\right)}{(1+Q_{j1}^*)^2}\right) \\
&\quad + \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)^3(1+Q_{i0}^*)}\left[-1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) - \left(1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)\right)Q_{i1}^*\right] \\
&= \frac{M_i Q_{i1}^*\ln\left(\frac{\bar{X}_i Q_{i0}^*}{\bar{X}_j}\right)}{N_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2} + \frac{\left(M_i - \frac{(1-Q_{i1}^{*2})}{1+Q_{j1}}\right)Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) \\
&\qquad + \frac{Q_{i0}^* Q_{j1}^*}{N_{i0}^*(1+Q_{i0}^*)^2(1+Q_{j1}^*)^2}\ln\left(\frac{\bar{X}_j Q_{j1}^*}{\bar{X}_I}\right) \\
&= \frac{M_i - (1-Q_{i1}^*)}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) - \frac{Q_{i1}^*}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) \text{ (when } Q_{i0}^* = 1, Q_{i1}^* = Q_{j1}^*) \\
&= \frac{M_i - 1}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) \gtreqless 0,
\end{aligned} \tag{B.40}$$

according to whether $\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) = \left(\frac{N_{i1}^*}{N_I}\right)^{2\alpha} \gtreqless 1$, i.e., according to whether $\bar{B}_i \gtreqless \bar{B}_i^*$. The impact of raising the sensitivity of reputations to campaigning on the probability of winning the general election similarly depends on the challengers' budgets. To see this, note that

$$\frac{dQ_{i1}}{d\alpha} \gtreqless 0 \Longleftrightarrow -\frac{dN_{i0}}{d\alpha} = \frac{dN_{i1}}{d\alpha} \lesseqgtr \frac{N_{i1}^*}{\alpha}\ln\left(\frac{N_I^*}{N_{i1}^*}\right).$$

From equations (B.32), (B.35) and (B.40),

$$
\begin{aligned}
-\frac{dN_{i0}}{d\alpha} &= \frac{\frac{2\alpha(M_i-1)}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{N_I^*}{N_{i1}^*}\right)}{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\
&= \frac{\frac{2N_{i0}^*}{(1+Q_{i1}^*)}\ln\left(\frac{N_I^*}{N_{i1}^*}\right)}{1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}}.
\end{aligned}
$$

As before, by rearranging terms and substituting for $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, one can show that if $\ln\left(\frac{N_I^*}{N_{i1}^*}\right) \gtreqless 0$, i.e. if $\left(\frac{N_I^*}{N_{i1}^*}\right) \gtreqless 1$, so that N_{i0}^* is decreasing, invariant or increasing in α respectively, then $\frac{dQ_{i1}}{d\alpha} \gtreqless 0$; that is, the chances of winning the general election are decreasing, invariant or increasing in α, respectively. Thus, when $\bar{B}_i < \bar{B}_i^*$, i.e., when the challengers' budgets are relatively low, their campaigning expenditures in the general election are relatively low, so that increasing α causes them to reduce negative primary campaigning N_{i0}^* and to increase N_{i1}^*, but this is insufficient to offset the incumbent's increased advantage from his greater budget due to the heightened sensitivity of reputations to campaigning—so that the challenger is less likely to win the general election. The reverse holds when challengers' budgets are high. When $\bar{B}_i = \bar{B}_i^*$, an increase in α has no impact on campaign levels or the probability of a win for the challengers. One can show that these results carry over to when the challenger's campaign in the general election is exclusively negative.

Next, consider the impact of improving the incumbent's reputation. Note that N_{i0}^* decreases in $\bar{X}_I$ if and only if $\frac{\partial^2\pi}{\partial\bar{X}_I\partial N_{i0}} < 0$, which holds in equilibrium:

$$
\begin{aligned}
\frac{\partial^2\pi^i}{\partial\bar{X}_I\partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial}{\partial\bar{X}_I}(M_i Pr_{i1} - Pr_{j1}) + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{X}_I\partial N_{i0}} \\
&= \frac{\alpha Q_{i0}^*}{N_{i0}(1+Q_{i0}^*)^2}\left(\frac{Q_{j1}^*(1+Q_{i1}^*)^2 - Q_{i1}^*(1+Q_{j1}^*)^2 M_i}{\bar{X}_I(1+Q_{i1}^*)^2(1+Q_{j1}^*)^2}\right) - \frac{\alpha Q_{i1}^*(1-Q_{i1}^*)M_i}{\bar{X}_I N_{i1}^*(1+Q_{i0}^*)(1+Q_{i1}^*)^3} \\
&= \frac{\alpha Q_{i0}^*}{\bar{X}_I(1+Q_{i0}^*)^2 N_{i0}^*}\left(\frac{Q_{j1}^*}{(1+Q_{j1}^*)^2} - \frac{M_i}{(1+Q_{11}^*)^2} + \frac{1-Q_{i1}^*}{(1+Q_{i1}^*)(1+Q_{j1}^*)}\right) \\
&= \frac{\alpha Q_{i0}^*}{\bar{X}_I(1+Q_{i0}^*)^2 N_{i0}^*}\left(\frac{-(M_i-1)(2Q_{j1}^*+1) + 2Q_{i1}^*Q_{j1}^* - M_i Q_{j1}^{*2} - Q_{i1}^{*2}}{(1+Q_{i1}^*)^2(1+Q_{j1}^*)^2}\right) < 0.
\end{aligned}
\tag{B.41}
$$

Similarly, it is easy to show that negative campaigning in the primary falls with the incumbent's budget, $\bar{B}_I$:

$$\begin{aligned}
\frac{\partial^2\pi^i}{\partial\bar{B}_I\partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\bar{B}_I} - \frac{\partial Pr_{j1}}{\partial\bar{B}_I}\right) + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{B}_I\partial N_{i0}} \\
&= \frac{\alpha Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2}\left(\frac{-\alpha M_i Q_{i1}^*}{P_I^*(1+Q_{i1}^*)^2} + \frac{\alpha Q_{j1}^*}{P_I^*(1+Q_{j1}^*)^2}\right) - \frac{\alpha^2 M_i Q_{i1}^*(1-Q_{i1}^*)}{P_{i1}^*(1+Q_{i0}^*)N_I^*(1+Q_{i1}^*)^3} \\
&= \frac{\alpha^2}{\rho P_I^*(1+Q_{i0}^*)}\left[\frac{Q_{i0}^*}{P_{i0}^*(1+Q_{i0}^*)}\left(\frac{-M_i Q_{i1}^*}{(1+Q_{i1}^*)^2} + \frac{Q_{j1}^*}{(1+Q_{j1}^*)^2}\right) - \frac{M_i Q_{i1}^*(1-Q_{i1}^*)}{P_{i1}^*(1+Q_{i1}^*)^3}\right] \\
&= \frac{\alpha^2 Q_{i0}^*}{\rho P_I^* P_{i0}^*(1+Q_{i0}^*)^2}\left(\frac{-M_i Q_{i1}^*}{(1+Q_{i1}^*)^2} + \frac{Q_{j1}^*}{(1+Q_{j1}^*)^2} - \frac{(M_i Pr_{i1}^* - Pr_{j1}^*)(1-Q_{i1}^*)}{(1+Q_{i1}^*)}\right) \\
&= -\frac{\alpha^2 Q_{i0}^*\big((M_i-1)(1+Q_{j1}^*)^2 + (Q_{i1}^* - Q_{j1}^*)^2\big)}{\rho P_I^* P_{i0}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2(1+Q_{j1}^*)^2} < 0.
\end{aligned} \tag{B.42}$$

The fourth equality follows from equation (B.5). Since $P_{i0}^* = \frac{N_{i0}^*}{\rho}$, it follows that$P_{i0}^*$ decreases in $\bar{X}_I, \bar{B}_I$ too. Then, total primary campaigning expenditures fall with the incumbent's budget and reputation, too. Further, since P_{i1}^*, N_{i1}^* decrease in N_{i0}^*, P_{i0}^*, they rise with $\bar{X}_I, \bar{B}_I$, implying that challengers now spend more on campaigning in the general election than before.

Thus, improving the incumbent's reputation induces challengers to campaign more extensively in the general election. However, the probability that a challenger wins the general election still falls with $\bar{X}_I$, i.e., the direct effect dominates the indirect strategic effect on challengers campaigning in the primary. The incumbent's campaigning levels depend on his budget alone, so

$$\frac{dQ_{i1}}{d\bar{X}_I} = \frac{d}{d\bar{X}_I}\left(\frac{\bar{X}_I N_I^{2\alpha}}{\bar{X}_i N_{i1}^{*2\alpha}}\right) > 0 \iff \frac{dN_{i1}}{d\bar{X}_I} = -\frac{dN_{i0}}{d\bar{X}_I} < \frac{N_{i1}}{2\alpha\bar{X}_I} \iff \frac{\frac{dN_{i1}}{N_{i1}}}{\frac{d\bar{X}_I}{\bar{X}_I}} < \frac{1}{2\alpha}. \tag{B.43}$$

Intuitively, increasing both $\bar{X}_I$ and N_{i1}^* have opposing effects on Q_{i1}^*, so that the relative strength of the two effects decides the net impact on the probability that the challenger wins the general election. Now, from equation (B.41), substituting the symmetry conditions, $Q_{i0}^* = 1, Q_{i1}^* = Q_{j1}^*$, yields

$$\frac{dN_{i0}}{d\bar{X}_I} = \frac{\alpha(M_i-1)}{4\rho P_{i0}^*\bar{X}_I(1+Q_{i1}^*)^2} \tag{B.44}$$

Therefore, from equations (B.32), (B.35) and (B.44),

$$
\begin{aligned}
-\frac{dN_{i0}}{d\bar{X}_I} &= \frac{\frac{\partial^2 \pi^i}{\partial \bar{X}_I \partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}}} \\
&= \frac{\frac{\alpha(M_i-1)}{4\rho P^*_{i0}\bar{X}_I(1+Q^*_{i1})^2}}{\frac{\alpha(M_i-1)}{4\rho^2 P^{*2}_{i0}(1+Q^*_{i1})}\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q^*_{i1}}{Q^*_{i1}}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\
&= \frac{\frac{N^*_{i0}}{\bar{X}_I(1+Q^*_{i1})}}{\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q^*_{i1}}{Q^*_{i1}}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]}.
\end{aligned}
$$

One can show that the inequality in (B.43) holds by rearranging terms and substituting $\frac{N^*_{i0}}{N^*_{i1}} = \frac{(M_i-1)(1+Q^*_{i1})}{2M_iQ^*_{i1}}$.

Similarly, one can show that increasing the incumbent's budget reduces a challenger's chances of winning the general election. This holds if -$\frac{dN_{i0}}{dB_I} < \frac{\rho N_{i1}}{2N_{I1}}$ and one can show that this holds the equilibrium, given equations (B.32), (B.35) and (B.42). These results extend to when a challenger only campaigns negatively in the general election.

Finally, note that for a given ρ, an increase (decrease) in N^*_{it}, P^*_{it}, $t \in \{0,1\}$, implies an increase (decrease) in n^*_{it}, p^*_{it}. Again, by the continuity of the derivatives in β and other parameters, there exists a $\bar{\beta}$ such that these results hold for all $\beta \in [0, \bar{\beta})$ in the neighborhood of symmetry. ■

Proof of Proposition 11.

At the equilibrium $(N^*_{i0}, P^*_{i0}, N^*_{j0}, P^*_{j0})$,

$$
\underbrace{\frac{\partial^2 \pi^i}{\partial \theta \partial N_{i0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2}\frac{dN_{i0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\frac{dP_{j0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0
$$

and

$$
\underbrace{\frac{\partial^2 \pi^j}{\partial \theta \partial N_{j0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^j}{\partial N_{i0}\partial N_{j0}}\frac{dN_{i0}}{d\theta} + \frac{\partial^2 \pi^j}{\partial P_{i0}\partial N_{j0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}\frac{dP_{j0}}{d\theta} + \frac{\partial^2 \pi^j}{\partial N^2_{j0}}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0.
$$

Since $\rho P^*_{i0} = N^*_{i0}$ and $\rho P^*_{j0} = N^*_{j0}$ at an interior optimum when $\beta = 0$ (from equation (B.6)), the equations reduce to,

$$
\frac{\partial^2 \pi^i}{\partial \theta \partial N_{i0}} + \left(\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}}\right)\frac{dN_{i0}}{d\theta} + \left(\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\right)\frac{dN_{j0}}{d\theta} = 0
$$

and

$$\frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}+\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\frac{dN_{i0}}{d\theta}+\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)\frac{dN_{j0}}{d\theta}=0.$$

Next, solve these equations for

$$\frac{dN_{i0}}{d\theta}=\frac{\frac{\partial^2\pi^i}{\partial\theta\partial N_{i0}}\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)-\frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)}{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi^i}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)} \tag{B.45}$$

and similarly for $\frac{dN_{j0}}{d\theta}$. It follows from the second-order conditions and equation (B.16) (which holds in the neighborhood of symmetry from the continuity of the derivatives in $\bar{X}_i, \bar{B}_i, M_i$, etc.) that

$$\frac{\partial^2\pi^i}{\partial N_{i0}^2}\frac{\partial^2\pi^i}{\partial P_{i0}^2}-\left(\frac{\partial^2\pi^i}{\partial N_{i0}\partial P_{i0}}\right)^2\to\left(\rho\frac{\partial^2\pi^i}{\partial N_{i0}^2}\right)^2-\left(\frac{\partial^2\pi^i}{\partial N_{i0}\partial P_{i0}}\right)^2>0\Rightarrow\frac{\partial^2\pi^i}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}<0. \tag{B.46}$$

Also, recall from equation (B.17) that

$$\frac{\partial^2\pi^i}{\partial N_{j0}N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}N_{i0}}>0. \tag{B.47}$$

Further, given the continuity of derivatives in the parameters, it follows from Lemma 1 that in the neighborhood of symmetry,

$$\frac{\partial^2\pi}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}+\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}<0. \tag{B.48}$$

Then, from equations (B.47) and (B.48) it follows that

$$0<\frac{\partial^2\pi^i}{\partial N_{j0}N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}N_{i0}}<\left|\frac{\partial^2\pi^i}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right|$$

Analogous inequalities hold for challenger j, implying that the denominator of the expression on the right-hand side in equation (B.45) is negative. Below, we reproduce equation (B.45) along with the signs of expressions contained in it:

$$\frac{dN_{i0}}{d\theta} = \frac{\frac{\partial^2\pi^i}{\partial\theta\partial N_{i0}}\overbrace{\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}^{<0} - \frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}\overbrace{\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)}^{>0}}{\underbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi^i}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}_{<0}}. \tag{B.49}$$

Similarly,

$$\frac{dN_{j0}}{d\theta} = \frac{\frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}\overbrace{\left(\frac{\partial^2\pi^i}{\partial N_{i0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)}^{<0} - \frac{\partial^2\pi^i}{\partial\theta\partial N_{i0}}\overbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)}^{>0}}{\underbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)-\left(\frac{\partial^2\pi^i}{\partial {N_{i0}}^2}+\frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2}+\frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}_{<0}}. \tag{B.50}$$

Now, consider the impact of improving *only* challenger i's reputation. We first show that as long as a challenger's payoff from winning office significantly exceeds his payoff if his primary rival wins, so that $M_i > 3$, then $\frac{\partial^2\pi^j}{\partial\bar{X}_i\partial N_{j0}} > 0$. To show this, we first note that,

$$\pi^j = -(M_jPr_{j1} - Pr_{i1})Pr_{i0} + M_jPr_{j1}$$

so that,

$$\begin{aligned}
\frac{\partial^2\pi^j}{\partial\bar{X}_i\partial N_{j0}} &= -\frac{\partial^2 Pr_{i0}}{\partial\bar{X}_i\partial N_{j0}}(M_jPr_{j1}-Pr_{i1}) - M_j\frac{\partial Pr_{j1}}{\partial N_{j0}}\frac{\partial Pr_{i0}}{\partial\bar{X}_i} + \frac{\partial Pr_{i0}}{\partial N_{j0}}\frac{\partial Pr_{i1}}{\partial\bar{X}_i}\\
&= (M_jPr_{j1}-Pr_{i1})\frac{\alpha Q_{i0}^*(1-Q_{i0}^*)}{\bar{X}_iN_{j0}^*(1+Q_{i0}^*)^3} - \frac{\alpha Q_{i0}^*Q_{i1}^*}{\bar{X}_iN_{j0}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}\\
&\qquad + \frac{\alpha M_jQ_{i0}^*Q_{j1}^*}{\bar{X}_iN_{j1}^*(1+Q_{i0}^*)^2(1+Q_{j1}^*)^2}\\
&\to \frac{\alpha Q_{i1}^*(M_iN_{i0}^*-N_{i1}^*)}{4\bar{X}_iN_{i0}^*N_{i1}^*(1+Q_{i1}^*)^2} \quad (\text{as } Q_{i0}^*=1, Q_{i1}^*=Q_{j1}^*)\\
&= \frac{\alpha(M_i-1+(M_i-3)Q_{i1}^*)}{8\bar{X}_iN_{i0}^*(1+Q_{i1}^*)^2} > 0,
\end{aligned} \tag{B.51}$$

when $M_i \geq 3$. Recall that N_{i0}^* increases in M_i. Therefore, N_{i1}^* decreases in M_i, so that $M_iN_{i0}^* - N_{i1}^* > 0$ when M_i is high. In fact, the last equality in (B.51), which is obtained by dividing both numerator and denominator by N_{i1}^* and substituting $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_iQ_{i1}^*}$, indicates that $M_i > 3$ suffices, and in what follows, we assume that this holds. Similarly,

$$\begin{aligned}\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} &= \frac{\partial^2 Pr_{i0}}{\partial \bar{X}_i \partial N_{i0}}(M_i Pr_{i1} - Pr_{j1}) + M_i \frac{\partial Pr_{i1}}{\partial N_{i0}}\frac{\partial Pr_{i0}}{\partial \bar{X}_i} + M_i \frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial Pr_{i1}}{\partial \bar{X}_i} + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial \bar{X}_i \partial N_{i0}} \\ &= (M_i Pr_{i1} - Pr_{j1})\frac{\alpha Q^*_{i0}(1-Q^*_{i0})}{\bar{X}_i N^*_{i0}(1+Q^*_{i0})^3} + \frac{\alpha M_i Q^*_{i0} Q^*_{i1}}{\bar{X}_i N^*_{i0}(1+Q^*_{i0})^2(1+Q^*_{i1})^2} + \frac{\alpha M_i Q^*_{i1}(1-Q^*_{i1})}{\bar{X}_i N^*_{i1}(1+Q^*_{i0})(1+Q^*_{i1})^3} \\ &\qquad - \frac{\alpha M_i Q^*_{i1} Q^*_{i0}}{\bar{X}_i N^*_{i1}(1+Q^*_{i0})^2(1+Q^*_{i1})^2} \\ &\to \frac{\alpha M_i Q^*_{i1}}{4\bar{X}_i N^*_{i0}(1+Q^*_{i1})^2} + \frac{\alpha M_i Q^*_{i1}(1-Q^*_{i1})}{2\bar{X}_i N^*_{i1}(1+Q^*_{i1})^3} - \frac{\alpha M_i Q^*_{i1}}{4\bar{X}_i N^*_{i1}(1+Q^*_{i1})^2} = \frac{\alpha(M_i-3)(\frac{M_i-1}{M_i-3} - Q^*_{i1})}{8\bar{X}_i N^*_{i0}(1+Q^*_{i1})^2}.\end{aligned} \tag{B.52}$$

Now, note that $\frac{M_i-1}{M_i-3}$ decreases in M_i, while Q^*_{i1} increases in M_i (when M_i increases, N^*_{i0}, P^*_{i0} increase and N^*_{i1}, P^*_{i1} decrease implying Q^*_{i1} increases). Therefore, when M_i is high, $\frac{M_i-1}{M_i-3} - Q^*_{i1}$ is low. If it is positive, i.e. if $Q^*_{i1} < \frac{M_i-1}{M_i-3}$, then so is $\frac{\partial^2\pi^i}{\partial \bar{X}_i \partial N_{i0}}$, and from equations (B.49) and (B.50), $\frac{dN_{i0}}{d\bar{X}_i} > 0$, $\frac{dN_{j0}}{d\bar{X}_i} > 0$. If it is negative, i.e., if $Q^*_{i1} > \frac{M_i-1}{M_i-3}$, so that $\frac{\partial^2\pi^i}{\partial \bar{X}_i \partial N_{i0}} < 0$, then $\frac{dN_{j0}}{d\bar{X}_i} > 0$, since from equations (B.51) and (B.52),

$$\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} > \left| \frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} \right|$$

while, as $\beta \to 0$,

$$\left(\frac{\partial^2 \pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{i0}\partial N_{j0}} \right) = \left(\frac{\partial^2 \pi^i}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}} \right) < \left| \frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}} \right|$$

from Lemma 1. Hence, the numerator of expression (B.50) is still negative. Thus, challenger j always raises his campaigning levels in response to an improvement in the reputation of the other challenger. However, the analysis of the direction of change in N^*_{i0} is not so straightforward anymore. From equation (B.49),

$$\frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \iff \frac{\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}}{\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}} \gtrless \frac{\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}}}{\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}}.$$

Adding one to both sides and substituting

$$\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}} = \frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}},$$

which holds as $\beta \to 0$, yields

$$\frac{\frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}} \gtrless \frac{\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} + \frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}} \tag{B.53}$$

From Lemma 1, as $\beta \to 0$, the expression on the left-hand side of (B.53) goes to

$$\frac{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}\right]}{\frac{(M_i+1)\alpha^2 Q_{i1}^*}{2\rho^2 P_{i1}^* P_{i0}^*(1+Q_{i1})^2}}.$$

Rearranging terms and substituting $\frac{P_{i0}^*}{P_{i1}^*} \to \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, this reduces to

$$\frac{M_i}{\alpha(M_i+1)}\left[-1 - \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i}\right]. \tag{B.54}$$

From equations (B.51) and (B.52), after canceling terms, the right-hand side of (B.53) is $\frac{\frac{2(M_i-1)}{M_i-3}}{\frac{M_i-1}{M_i-3}-Q_{i1}^*}$. Thus, when $Q_{i1}^* > \frac{M_i-1}{M_i-3}$,

$$\frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \iff \frac{M_i}{\alpha(M_i+1)}\left[-1 - \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i}\right] \gtrless \frac{\frac{2(M_i-1)}{M_i-3}}{\frac{M_i-1}{M_i-3} - Q_{i1}^*}$$

or,

$$\begin{aligned} \frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \iff \frac{(1+2\alpha)(M_i-1) + Q_{i1}^*(3M_i-1)}{2\alpha Q_{i1}^*(M_i+1)} + \frac{2(M_i-1)}{M_i-1-(M_i-3)Q_{i1}^*} \lessgtr 0 \\ \iff \Big((1+2\alpha)(M_i-1) + Q_{i1}^*(3M_i-1)\Big)\Big(M_i-1-(M_i-3)Q_{i1}^*\Big) + 4(M_i^2-1)\alpha Q_{i1}^* \gtrless 0. \end{aligned} \tag{B.55}$$

It is easy to see that there exists a $\bar{Q}$, such that for $Q_{i1}^* \in (\frac{M_i-1}{M_i-3}, \bar{Q})$, the above expression is positive implying that negative primary campaigning by i increases in $\bar{X}_i$, and from the analysis above, it is increasing for $Q_{i1}^* < \frac{M_i-1}{M_i-3}$ as well. Thus, if the incumbent is weak so that he is likely to be ousted by a challenger (either due to a poor reputation or a low budget), i.e., $Q_{i1}^* \in (0, \bar{Q})$, then the challenger's primary campaigning levels increase as his reputation improves, while the opposite holds if the incumbent is strong.

Next, consider the impact of increasing challenger i's budget. We first show that it results in increased negative campaigning by both challengers in the primary when challengers have nearly identical endowments, preferences and reputations. To see this, first note that when $\beta = 0$,

$$\frac{\partial^2 \pi^j}{\partial \bar{B}_i \partial N_{j0}} = \frac{\partial Pr_{i0}}{\partial N_{j0}}\frac{\partial Pr_{i1}}{\partial \bar{B}_i} = \frac{-\alpha Q_{i0}^*}{N_{j0}^*(1+Q_{i0}^*)^2}\frac{\alpha Q_{i1}^*}{P_{i1}^*(1+Q_{i1}^*)^2} \to \frac{-\alpha^2 Q_{i1}^*}{4N_{i0}^* P_{i1}^*(1+Q_{i1}^*)^2} < 0, \tag{B.56}$$

and

$$\begin{aligned}
\frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}} &= M_i\frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial Pr_{i1}}{\partial\bar{B}_i} + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{B}_i\partial N_{i0}} \\
&= \frac{\alpha^2 M_i Q_{i0}^* Q_{i1}^*}{N_{i0}^* P_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2} + \frac{\alpha M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2N_{i1}^* P_{i1}^*(1+Q_{i0}^*)(1+Q_{i1}^*)^3} \\
&= \frac{\alpha^2\rho M_i Q_{i1}^*}{4N_{i0}^* N_{i1}^*(1+Q_{i1}^*)^2} + \frac{\alpha\rho M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4N_{i1}^{*2}(1+Q_{i1}^*)^3} \\
&= \frac{\alpha\rho(M_i-1)}{8N_{i0}^{*2}(1+Q_{i1}^*)}\left(\alpha + \frac{(M_i-1)(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2M_iQ_{i1}^*}\right) \\
&= \frac{\alpha\rho(M_i-1)((M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*)}{16N_{i0}^{*2}M_iQ_{i1}^*(1+Q_{i1}^*)} > 0.
\end{aligned} \tag{B.57}$$

Then, it follows from equation (B.49) that as $\beta \to 0$,

$$\begin{aligned}
\frac{dN_{i0}}{d\bar{B}_i} > 0 &\Longleftrightarrow \frac{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial\bar{B}_i\partial N_{j0}}}{\frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}}} \\
&\Longleftrightarrow \frac{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial\bar{B}_i\partial N_{j0}} + \frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}}}.
\end{aligned}$$

From equations (B.54), (B.56) and (B.57), and substituting for $\frac{N_{i0}^*}{N_{i1}^*} \to \frac{(M_i-1)(1+Q_{i1}^*)}{2M_iQ_{i1}^*}$, this may be rewritten as

$$\begin{aligned}
&\frac{M_i}{\alpha(M_i+1)}\left[-1-\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i}\right] < \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*} \\
&\text{or, } -\frac{(1+2\alpha)(M_i-1)+Q_{i1}^*(3M_i-1)}{2\alpha Q_{i1}^*(M_i+1)} - \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*} < 0,
\end{aligned}$$

which always holds as $M_i > 1$. Similarly,

$$\begin{aligned}
&\frac{dN_{j0}}{d\bar{B}_i} > 0 \Longleftrightarrow \frac{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial\bar{B}_i\partial N_{j0}} + \frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial\bar{B}_i\partial N_{i0}}} \\
&\text{or, } \quad \frac{1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}}{1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}+2\alpha} < \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*} \\
&\qquad\qquad\Longleftrightarrow M_i(M_i-1)(1+2\alpha)+M_iQ_{i1}^*(M_i-3) > 0
\end{aligned}$$

which always holds. Thus, negative primary campaigning by challengers initially increases with challenger i's budget in the neighborhood of symmetry.

Finally, consider the impact of an increase in the relative payoff, M_i, of challenger i. Note that $\frac{\partial^2\pi^j}{\partial M_i\partial N_{j0}} = 0$, while $\frac{\partial^2\pi^i}{\partial M_i\partial N_{i0}} > 0$, as shown earlier. Then, from equation (B.49) and (B.50) it follows that

$\frac{\partial N_{i0}}{\partial M_i} > 0, \frac{\partial N_{j0}}{\partial M_i} > 0$. Since $\rho P_{i0}^* = N_{i0}^*, \rho P_{j0}^* = N_{j0}^*$, positive campaigning levels increase as well. Thus, the challengers' campaigning levels and expenditure in the primary increase. Note that the probability of a win for either challenger in the general election declines.

By the continuity of derivatives in β, there exists a $\tilde{\beta}$ such that for all $\beta \leq \tilde{\beta}$ the result holds. Lastly, it is easy to show that these results carry over to the case where the challenger campaigns only negatively in the general election. ■

References

1. Alba, J., Lynch, J., Weitz, B., Janiszewski, C., Lutz, R., Sawyer, A. and Wood, S. (1997), "Interactive Home Shopping: Consumer, Retailer and Manufacturer Incentives to Participate in Electronic Marketplaces", *Journal of Marketing*, 61, 38-53.

2. Bernhardt, D. and Ghosh, M. (2014), "Quality provision and Pricing in the face of Online Competition", Working Paper.

3. Brown, J. R. and Goolsbee, A. (2002), "Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry", *Journal of Political Economy*, 110, 481-507.

4. Brueckner, J. K. and K. Lee (2013), "Negative Campaigning in a Probabilistic Voting Model", *CESifo Working Paper*, No. 4233.

5. Chakrabarti, S. (2007), "A note on negative electoral advertising: Denigrating character vs. portraying extremism", *Scottish Journal of Political Economy*, Vol. 54, pp. 136-149.

6. Danaher, P. J., Wilson, I. W. and Davis, R. A. (2003), "A Comparison of Online and Offline Consumer Brand Loyalty", *Marketing Science*, 22, 461-476.

7. Ellison, G. (2005), "A model of add-on pricing", *The Quarterly Journal of Economics*, 120(2), 585-637.

8. Goldman, M. B., Leland, H. E., and Sibley, D. S. (1984), "Optimal nonuniform prices", *The Review of Economic Studies*, 51(2), 305-319.

9. Harrington, J. E. and G.D. Hess (1996), "A Spatial Theory of Positive and Negative Campaigning", *Games and Economic Behavior*, Vol. 17, No. 2, pp. 209-229.

10. Lal, R. and Sarvary, M. (1999), "When and How is the Internet Likely to Decrease Price Competition?", *Marketing Science*, 18, 485-503.

11. Loginova, O. (2009), "Real and Virtual Competition", *Journal of Industrial Economics*, 57(2), 319-342.

12. Maskin, E., and Riley, J. (1984), "Monopoly with incomplete information", *The RAND Journal of Economics*, 15(2), 171-196.

13. Mussa, M. and Rosen, S. (1978), "Monopoly and Product Quality", *Journal of Economic Theory*, 18, 301-317.

14. Peterson, D. A. M. and P. A. Djupe (2005), "When primary campaigns go negative: The determinants of campaign negativity", *Political Research Quarterly*, Vol. 58, No. 1, pp. 45-54.

15. Peterson, R. A., Balasubramanian, S. and Bronnenberg, B. J. (1997), "Exploring the Implications of the Internet for Consumer Marketing", *Journal of the Academy of the Marketing Science*, 25, 329-346.

16. Polborn, M. and D. T. Yi (2006), "Informative Positive and Negative Campaigning", *Quarterly Journal of Political Science*, Vol. 1(4), pp. 351-371.

17. Ratchford, B. T., Talukdar, D. and Lee, M. (2001), "A Model of Consumer Choice of the Internet as an Information Source"', *International Journal of Electronic Commerce*, 5, 7-21.

18. Rochet, J. C., and Stole, L. (1997), "Competitive nonlinear pricing", *Working Paper*.

19. Rochet, J. C. and Stole, L. (2002), "Nonlinear Pricing with Random Participation", *Review of Economic Studies*, 69(1), 277-311.

20. Sengupta, A. and Wiggins, S. (2006), "Airline Pricing, Price Dispersion and Ticket Characteristics On and Off the Internet", *NET Institute Working Paper*, No. 06-07.

21. Skaperdas, S. and B. Grofman (1995), "Modeling Negative Campaigning", *The American Political Science Review*, Vol. 89, No. 1, pp. 49-61.

22. Soubeyran, R. (2009), "Contest with attack and defense: does negative campaigning increase or decrease voter turnout?", *Social Choice and Welfare*, Vol. 32, No. 3, pp 337-353.

23. Spence, A. M. (1980), "Multi-product quantity-dependent prices and profitability constraints", *The Review of Economic Studies*, 47(5), 821-841.