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HIGH-THROUGHPUT SENSING OF BULK FLUIDS AND FREELY
FLOWING PARTICLES WITH OPTO-MECHANO-FLUIDICS

BY

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DISSERTATION

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ABSTRACT

High-Q optical resonators have revolutionized label-free sensing of nanoparticles, down to the resolution of individual viruses and single molecules. However, these laboratory demonstrations commonly encounter practical constraints that remain unresolved. Specifically, optical resonator detection techniques rely on random diffusion of particles to the sensing region, occasionally also necessitating adsorption onto the resonator. This implies severe reduction in detection speed and only a small number of analyte particles can be detected or quantified in a sample, at speeds no faster than a few particles per second. Furthermore, the mechanical properties of sample fluids or particles do not couple directly to optical fields.

Yet, many disease states correlate with mechanical properties such as compressibility and viscoelasticity (e.g. in anemia and cancers). These mechanical properties are usually measured with microelectromechanical sensors (MEMS). In particular, mass, compressibility, and viscoelasticity of analyte particles can be measured by binding them on a vibrating structure (Nat Commun 7:13452, 2016; Appl Phys Lett 108, 11, 2016). These methods, however, suffer from the severe reduction in detection speed as well. Fast non-contact measurement is made possible with a hollow type of MEMS device, which weighs bio-analytes in real-time as they flow through the internal channel of the device (Nat Commun 6:7070, 2015). However, the fastest achieved detection speed is still less than 10 events/s. Generating meaningful data on practical samples, for instance a liquid suspension containing a few viral nanoparticles amongst millions of particles of debris, requires ultra-high throughput measurements with nearly 100% detection efficiency.

To address this fundamental measurement challenge, we have developed a novel class of solid-liquid hybrid optomechanical resonators that perform phonon-mediated optical detection of fluids and nanoparticles at extremely high speed. The microcapillary-type resonators support ultrahigh-Q optical

modes that are coupled to co-localized mechanical (phonon) modes, while fluid analytes are flowed internally without influencing the optics. Thus we also call this resonator opto-mechano-fluidic resonator (OMFR). In this thesis, we first provide the detailed methodology to fabricate these OMFRs, perform optomechanical testing, and measure optomechanical vibrations.

Fundamentally, OMFR interfaces with bulk fluids acoustically through the shell-fluid hybrid mechanical modes. A numerical setup for an eigenfrequency analysis of the OMFR system in the acoustic regime is developed. This model unifies the solid shell domain and the inner fluid domain by applying interactive boundary conditions on the shell-fluid interface. As a result, we can simulate the hybrid fluid-shell vibrational modes and investigate the acoustic sensitivity of the OMFRs to fluid density and speed of sound. With the simulation results, we are able to identify of the hybrid fluid-shell modes from experiment and extract fluid properties. In addition, based on the simulation results, a perturbation model is built to predict the OMFR’s acoustic sensitivity to micro- and nano-particles. Experimental determination of fluid viscosity is also demonstrated, through optomechanical measurement of the vibrational noise spectrum of the resonator mechanical modes. A linear relationship between the spectral linewidth and root-viscosity is predicted and experimentally verified in the low viscosity regime.

The bioanalyte particles perturbs the acoustic field in the fluid domain by density and compressibility contrast with the ambient fluid. Phonons permeate the entire cross-section of the resonator, casting a near-perfect net for measuring particles flowing along the fluid streamlines. All particles in the sample must transit and perturb these phonon modes, in turn perturbing the optical readout due to the strong optomechanical coupling. In our first experimental report, we show that detection rates exceeding 10,000 particles-per-second are reachable with noise floor (size) better than 660 nm, without any binding, labeling, or reliance on random diffusion. This optomechanical method also uniquely quantifies mechanical parameters of single nanoparticles such as density, viscoelasticity, and compressibility, which are not accessible through traditional optical measurements. Later, an electro-optomechanical driving and lock-in technique is adopted and real-time operation of the OMFR is enabled. Particle detection with transit time scale as short as 490 μ s is experimentally demonstrated. Furthermore, the initial results on high-throughput measurement of particle compressibility using OMFR

are shown at last.

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CHAPTER 1

INTRODUCTION

1.1 Overview

¹ Optofluidic devices can incorporate optical sensing techniques into microfluidic systems for analysis of fluid or gaseous analytes. Recent technological advances in optofluidics [5] have enabled the use of refractive index measurement [6–8], fluorescence imaging [9], and surface enhanced Raman spectroscopy [10]. A variety of optofluidic structures have been explored, including ring resonators [11–14], liquid core capillaries [6], microtoroids [15], bubbles [16–18], droplets [19,20], photonic crystals [21,22], liquid core waveguides [23], surface plasmon resonance devices [24]. These structures enable sensitive detection of DNA, proteins, viruses, cells, and bacteria down to the level of single particles and even single molecules [15,21]. In addition, optical nanoparticle sizing and sorting [7,25] as well as flow control [26,27] have also been achieved. Resonator optical approaches are most widely used in optofluidics because of the high sensitivity resulting from their high Q factor and low mode volume. Unfortunately, the mechanical properties of sample fluids or particles do not couple directly to optical fields, so no mechanical properties can be derived.

As it is known that many disease states correlate with mechanical properties such as compressibility and viscoelasticity (e.g. in anemia and cancers), the motivation to pursue the high speed sensing of mechanical properties of single particles is quite pressing. With this goal in mind, microelectromechanical (MEMS) resonant sensors have been developed [28] that can weigh single particles. A very successful device in this category is the suspended microchannel resonator (SMR) [29–32] that weighs single cells in real-time as

¹Portion of this chapter is reprinted with permission from the author’s previously published material [1–4], and the author’s MS thesis (UIUC, August 2012).

they flow through a hollow cantilever, since the additional mass associated with a sample fluid or particle changes the resonant frequency. While SMRs are capable of weighing particles with attogram mass resolution, existing techniques are still relatively slow (>150 -ms transit) [32]. On the other hand, high-speed optical detection methods, such as flow cytometry, are routinely used for analysis of large populations of cells in buffers through measurements of their optical index, fluorescence, and scattering properties [33]. No mechanical sensor to date can approach the throughput of optical flow cytometers, which makes such measurements on large fluid volume, or particle populations impractical. Furthermore, no mechanical sensors till now have demonstrated the high-throughput sensing capability of measuring other important mechanical properties, such as Young’s modulus and compressibility, of single cells, either.

We propose that the solution lies in optomechanics. Optomechanical microresonators enable strong coupling between their photon modes and phonon modes through photothermal effects [34–36], radiation pressure (RP) force (see Section 1.5) [37–43], optical gradient force [44–47], and electrostriction mechanisms [48–54]. Therefore like the mechanical resonators, optomechanical microresonators can provide a mechanical interface with analytes. In addition, it also preserves the highly sensitive optical actuation and detection method. Furthermore, it is not burdened by parasitic feedthroughs that is common in other electric detection method, making the access to GHz mechanical modes easier. These capabilities have been harnessed for many fundamental experiments including optomechanical cooling [38–40, 52], induced transparency [55, 56], and dark modes [57]. Efforts have also been made towards sensing applications such as accelerometers [58, 59], mass sensors [60, 61], and force sensors [62, 63]. However, prior to my work, it was considered very difficult to perform optomechanics experiments with fluids. This is because direct liquid immersion of the optomechanical devices results in greatly increased radiative acoustic loss because of the higher impedance of liquids compared against air, and viscous loss as well. Recent demonstrations [64–66] have managed to reduce the mechanical loss due to direct liquid immersion by using small, thin resonators with mechanical modes that have motion mostly in the plane perpendicular to the thickness direction. However, their mechanical quality factors are still pretty small, usually much less than 100. The low mechanical Q limits the optomechanical application in

biosensing.

The introduction of the opto-mechano-fluidic resonators (OMFRs) [16, 54, 67, 68] offered a convenient solution to this challenge. These resonators have a hollow micro-bottle structure (Section 1.3), which by design is equipped for microfluidic experiments (Fig. 1.1). Both the optical whispering-gallery resonances [69] as well as mechanical resonant modes [70] can be co-localized on the resonator to provide the strong phonon-photon coupling needed for optomechanical experiments. Confinement of the liquid inside [29, 71] the capillary resonator, as opposed to outside it, prevents acoustic energy from leaking out. These mechanical excitations were shown to penetrate into the fluid within the device [54, 67], forming a shared solid-liquid resonant mode, thus enabling an opto-mechanical interface to the fluidic environment within.

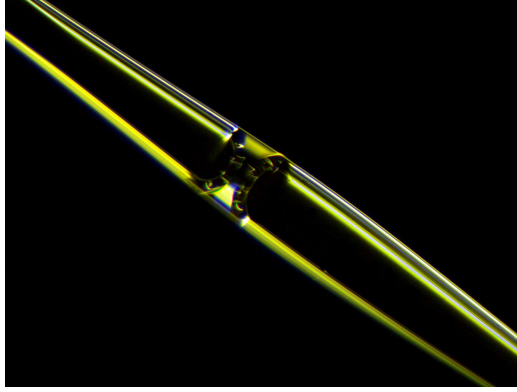


Figure 1.1: Image capturing the refraction of light through a 50 micron diameter opto-mechanical microfluidic resonator, with a water “plug” in the internal passage.

There are multiple advantages of using OMFRs in biosensing applications. Unlike in the optofluidic sensors, the optical field does not directly interact with the liquids, which means that the optomechanical biosensing doesn’t apply any constraints on the optical properties of the fluids or particles, and the optical Q is preserved. The high mechanical quality factor is ideal for high resolution frequency tracking [60]. In addition, since multiple families of mechanical resonant modes exist on OMFRs, including standing wave such as breathing modes, wine-glass modes (see [54] and Chapter 3), and traveling waves such as whispering-gallery acoustic modes [54], the possibilities for sensing are very expansive. These vibrational frequencies extend from a few MHz to 11 GHz regime, presenting an opportunity to study dynamics over

broad timescales and spacial scales.

1.2 Thesis organization

In this thesis we first describe, in Chapter 1 and 2 the basic sensing principle and fabrication for this novel optomechanical system. Then we introduce in Chapter 3 the numerical simulation methodology that I have developed for obtaining the coupled solid-liquid resonant modes, and for studying the eigenfrequency sensitivity to fluid properties (density and speed of sound). After that, an experimental investigation of the OMFR as a viscometer is shown in Chapter 4. In Chapter 5, we demonstrate the high-throughput sensing of freely flowing particles using an OMFR and introduce a perturbation model for the particle sensing mechanism. In Chapter 6, some initial results are shown for the measurement of particle compressibility using an OMFR. Finally, an electro-opto-mechanical technique that can be used to boost sensing speed is shown in Chapter 7.

1.3 Whispering gallery microresonators

Whispering gallery mode (WGM) optical resonators are a well-known type of miniaturized optical resonator [72]. The term “whispering gallery wave” was originally used by Lord Rayleigh to describe the acoustic phenomenon of the whispering gallery, which is a circular walkway, under the dome of St. Paul’s cathedral in London. A whisper uttered on one end of the dome can be heard across the dome, more than 30 meters away from the source, because the sound wave circulating around the curving wall attenuates much less than that in the free space [72]. Similarly, whispering gallery optical waves can also be supported by dielectric materials using total internal reflection with similar structures. Fig. 1.2 shows a simplistic ray-optics picture of light traveling inside an idealized whispering gallery – a 2D circular cavity.

The optical resonances in a whispering gallery optical resonator occur when the light comes back to the same spatial point, after integer number of round-trips, such that constructive interference occurs. These optical resonances are dependent on the radius of the structure, a , and the refractive index of the

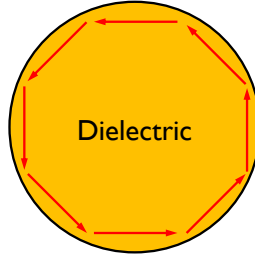


Figure 1.2: Ray of light propagation in a circular dielectric. Light is confined inside the whispering-gallery mode by total internal reflection.

dielectric, n . Since light circulates inside and close to the surface of the resonator, the traveling distance can be approximated as $2\pi a$ per round. If the traveling distance is exactly equal to multiple integer wavelengths in the dielectric, constructive interference occurs such that the resonance condition is,

$$2\pi a = l\lambda/n. \quad (1.1)$$

Here, l is an integer indicating the number of wavelengths per round trip, and λ is the wavelength in vacuum. If this condition is not met, the destructive interference occurs, as shown in Fig. 1.3, and the resonator is termed as being off resonance.

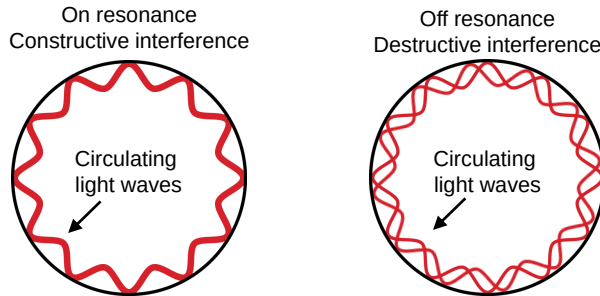


Figure 1.3: Constructive interference happens when the the resonator is exactly on resonance. Destructive interference happens when the resonator is off resonance.

Optical Q factor is a representation of the optical energy storage time, and

is defined as

$$Q = 2\pi \frac{\text{stored energy}}{\text{energy loss per cycle}} \quad (1.2)$$

Since the total internal reflection losses are negligible, when the material absorption is minimized, whispering gallery resonator can reach extraordinarily high quality factor. Recent demonstrations have shown a optical Q larger than 10^{11} in CaF_2 whispering gallery resonators [73]. The mode volume of whispering gallery modes can be much less than the volume of the resonator [74]. Thus very high energy density, which is proportional to the ratio between optical Q and mode volume, can be achieved in WGM.

Micro-bottle resonators like the OMFR are one type of whispering gallery resonator, in which both the whispering gallery mode and the “bouncing ball” mode are combined (Fig. 1.4). The curvature of the microbottle in the axial direction of the resonator limits the longitudinal light propagation. The mode shape of the resonator can be obtained by solving the Helmholtz equations for the electric field and magnetic field [75] and now can also be conveniently obtained through FEM simulation using commercial software like COMSOL. The mode shapes for the first several orders of optical WGMs in OMFRs are illustrated in Fig. 1.5.

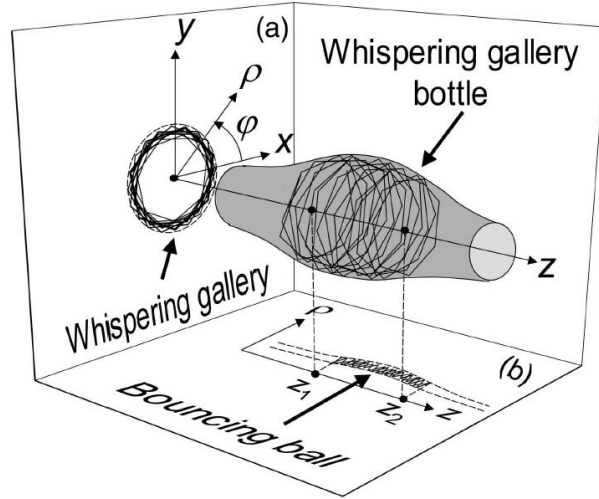


Figure 1.4: (Adapted from [76]) Optical modes in a microbottle resonator combine whispering gallery modes and “bouncing ball” modes.

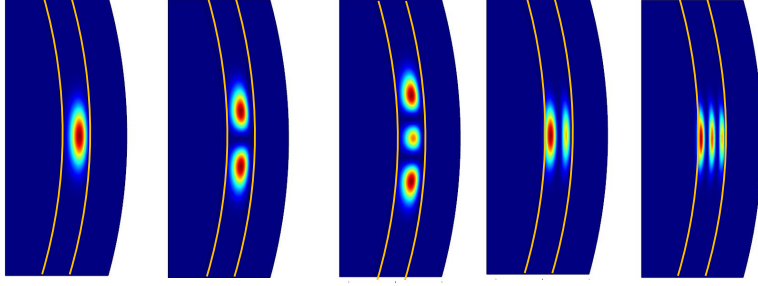


Figure 1.5: FEM simulation of the first several orders of optical WGMs in the shell of an OMFR (cross section view).

1.4 Motion sensing using light

To understand the optomechanical coupling, we first analyze the effect of the mechanical motion of the resonator on the light field. Consider a simple Fabry-Perot cavity as shown in Fig. 1.6 (a), which consists of a pair of highly reflective mirrors with free space in between, separated by a distance L . Constructive interference occurs for optical waves inside the cavity when incoming light frequency is equal to the resonance frequency

$$\omega_r = m\pi c/L. \quad (1.3)$$

Here, m is an integer, c is speed of light. The optical intensity (I) is maximized in this condition and the light is termed on resonance. When the resonance condition is not met ($\omega \neq \omega_r$), the intensity inside the cavity decreases. The relationship between the intensity inside the Fabry-Perot resonator and frequency of light (intensity transfer function) has a familiar Lorentzian shape (Fig. 1.6 (b)), characterized by the equation,

$$I(\omega) = \frac{1}{\pi} \frac{\delta\omega/2}{(\omega - \omega_r)^2 + (\delta\omega/2)^2}. \quad (1.4)$$

Here, ω is the pump laser frequency, ω_r is the resonant frequency, and $\delta\omega$ is the linewidth of the resonance (Fig. 1.6(b)). Next we define the laser detuning, Δ , as:

$$\Delta = \omega_r - \omega. \quad (1.5)$$

A signal with a smaller frequency than the resonance frequency is called red detuned ($\omega < \omega_r$); a signal with a larger frequency than the resonance frequency is called blue detuned ($\omega > \omega_r$).

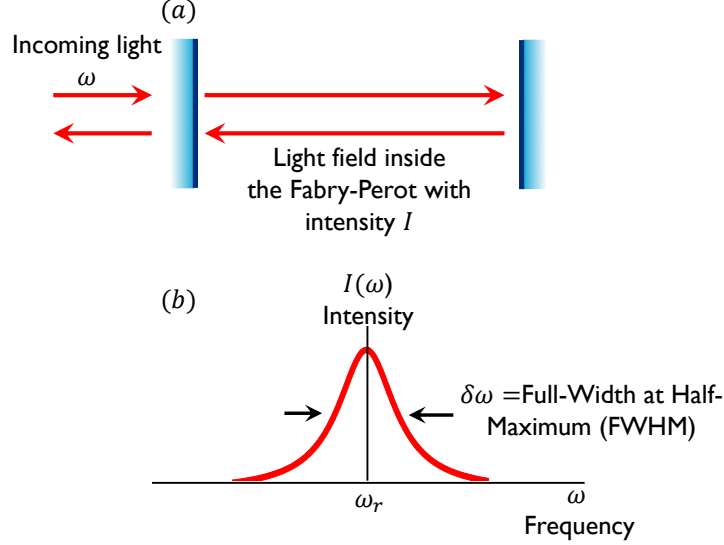


Figure 1.6: (a) Fabry-Perot cavity with length L between two mirrors. Light inside the Fabry-Perot cavity has intensity I . (b) Light intensity inside resonator as a function of incoming light wavelength.

We suppose now the mirror on the right hand side is movable (Fig. 1.7(a)). By moving the mirror by a small $x(t)$ to the right, the resonance frequency of the Fabry-Perot resonator becomes

$$\omega_r = m\pi c / (L + x(t)) \approx \omega_r - \frac{\omega_r}{L} x(t). \quad (1.6)$$

Here we define an optomechanical coupling coefficient as

$$G \equiv \frac{\partial \omega_r}{\partial x} = -\frac{\omega_r}{L}, \quad (1.7)$$

which has a unit of rad/m-s. The effect of the moving boundary to the light field inside the resonator with amplitude $\alpha(t)$ (intracavity field) can then be studied using a rate equation [77]

$$\dot{\alpha}(t) = [i(\Delta + Gx(t)) - \kappa/2]\alpha(t) + \frac{\kappa}{2}\alpha_{max}(t) \quad (1.8)$$

where, α is normalized such that $|\alpha|^2$ is the number of photons inside the

cavity, and κ is the total optical energy decay rate with both the external coupling loss (κ_e) and the intrinsic loss of the cavity (κ_0) accounted, and $\alpha(t) = \alpha_{max}(t) = \alpha_{max}e^{-i\omega t}$ when on resonance, which is determined by laser amplitude. The underlying assumptions here are that the pump wave is monochromatic at frequency ω , i.e., $b_{in}(t) = b_{in}e^{-i\omega t}$, its amplitude is constant, and the pump wave frequency is near resonance, i.e., $\omega \approx \omega_r$. Now we further assume the right mirror is driven at single mechanical frequency Ω such that $x(t) = x_a \sin(\Omega t)$, the solution of the intracavity optical field can be approximated as

$$\alpha(t) \approx \alpha_0(t) + \alpha_1(t) \quad (1.9)$$

$$\alpha_0(t) = \frac{\alpha_{max}(t)}{-i\Delta + \kappa/2} \quad (1.10)$$

$$\alpha_1(t) = \frac{Gx_a}{2} \frac{\alpha_{max}}{-i\Delta + \kappa/2} \left(\frac{e^{-i(\omega+\Omega)t}}{-i(\Delta + \Omega) + \kappa/2} - \frac{e^{-i(\omega-\Omega)t}}{-i(\Delta - \Omega) + \kappa/2} \right), \quad (1.11)$$

where $\alpha_0(t)$ is the steady state solution and $\alpha_1(t)$ consists of the modulation sidebands. Thus, the moving boundary acts as an optical modulator – during cavity expansion, circulating light experiences a red Doppler shift; during cavity shrinking, circulating light experiences a blue Doppler shift. Modulation of the device geometry at mechanical frequency Ω thus generates both upper (anti-Stokes) and lower (Stokes) optical sidebands of the pump light at $\omega \pm \Omega$ (Fig. 1.7(b)). The modulation of the intracavity field also results in a modulation of the intracavity intensity and thus of the output light intensity, since they both are proportional to $|\alpha(t)|^2$. Therefore we can monitor the mechanical motion of the system by measuring the output laser intensity using a variety of measurement techniques [78].

1.5 Optomechanical coupling through radiation pressure

In this section, we provide a brief overview of the optomechanical coupling through radiation pressure for the reader. The main source of this section is the Cavity Optomechanics book by Clerk et al [77]. In the analysis above, we have neglected the action of light on the mechanical system. While this is very small, every time a photon bounces off one of the end mirrors, it indeed

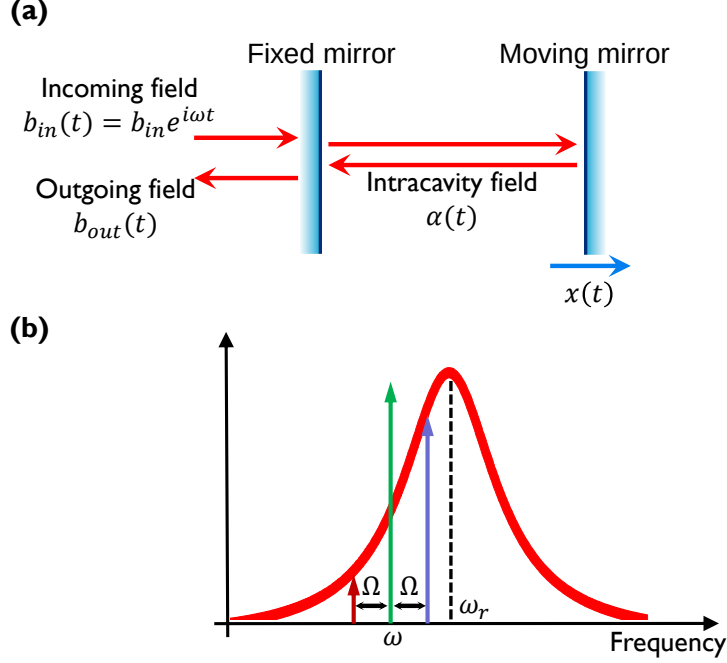


Figure 1.7: (a) Fabry-Perot cavity with a moving boundary. (b) Oscillating end mirror results in modulation of the laser field.

does transfer its linear momentum to the mirror. This force generated is called radiation pressure force and can be expressed as [77]

$$F_{rad} = \hbar G |\alpha|^2 \quad (1.12)$$

We can then analyze the mechanical response of the system to external force F_{ext} , by solving the equation of motion for the position of mechanical oscillator boundary (the right mirror for Fabry-Perot), $x(t)$, and rate equation of the intracavity light field, $\alpha(t)$ together, as shown in [77]:

$$\ddot{x}(t) = -\Omega^2 x(t) - \Gamma \dot{x}(t) + (F_{rad}(t) + F_{ext}(t))/m_{eff} \quad (1.13)$$

$$\dot{\alpha}(t) = [i(\Delta + Gx(t)) - \frac{\kappa}{2}]\alpha(t) + \frac{\kappa}{2}\alpha_{max}(t) \quad (1.14)$$

where for the mechanical resonator, Ω is the resonance frequency, Γ is the damping rate, and m_{eff} effective mass.

To solve the coupled equations, we linearize both $x(t)$ and $\alpha(t)$ around the new equilibrium point, $\bar{x}$ and $\bar{\alpha}$, after the laser is coupled into the cavity: $x(t) = \bar{x} + \delta x(t)$ and $\alpha(t) = \bar{\alpha} + \delta \alpha(t)$. When the system is at the new

equilibrium, all the time derivatives ($\dot{\alpha}(t), \dot{x}(t), \ddot{x}(t)$) vanish, and the system is stable such that $x(t) = \bar{x}$ and $\alpha(t) = \bar{\alpha}$. We can thus solve for the new equilibrium point

$$\bar{x} = -\frac{\hbar G |\bar{\alpha}|^2}{m_{eff} \Omega^2} \quad (1.15)$$

$$\bar{\alpha} = -\frac{\alpha_{max}}{i(\Delta + G\bar{x}) - \frac{\kappa}{2}} \quad (1.16)$$

The equilibrium solution for these two equations can be solved self-consistently [78].

We can then rewrite the coupled equations as:

$$\delta \ddot{x}(t) = -\Omega^2 \delta x - \Gamma \delta \dot{x} + \frac{\hbar G}{m_{eff}} [\bar{\alpha}^* \delta \alpha + \bar{\alpha} \delta \alpha^*] + F_{ext}(t)/m_{eff} \quad (1.17)$$

$$\delta \dot{\alpha}(t) = [i\bar{\Delta} - \kappa/2] \delta \alpha + iG\bar{\alpha} \delta x \quad (1.18)$$

where $\bar{\Delta} = \Delta + G\bar{x}$ is the effective detuning around the new equilibrium point of the system. Now the coupled equations are most conveniently solved in the frequency space. The details of the solving procedures can be found in [77]. Here we simply give the result of the mechanical response to external force:

$$\delta x[\omega] = \frac{F_{ext}[\omega]}{m_{eff}(\Omega^2 - \omega^2 - i\omega\Gamma) + \Sigma_{xx}(\omega)} \equiv \chi_{xx}(\omega) F_{ext}[\omega] \quad (1.19)$$

In this we have lumped the effect of the optomechanical interaction into the quantity Σ . The real part of Σ modifies the spring constant of the system (optical spring effect), causing the mechanical frequency to shift by:

$$\delta(\Omega^2) = \frac{1}{m_{eff}} \text{Re}\Sigma(\Omega) = \frac{\hbar G^2 |\bar{\alpha}|^2}{m_{eff}} \left[\frac{\Omega + \bar{\Delta}}{(\Omega + \bar{\Delta})^2 + (\kappa/2)^2} - \frac{\Omega - \bar{\Delta}}{(\Omega - \bar{\Delta})^2 + (\kappa/2)^2} \right] \quad (1.20)$$

The imaginary part of Σ adds additional damping to the system (optical damping effect):

$$\delta\Gamma = -\frac{1}{m_{eff}\Omega} \text{Im}\Sigma(\Omega) = \frac{\hbar G^2 |\bar{\alpha}|^2 \kappa}{2m_{eff}\Omega} \left[\frac{1}{(\Omega + \bar{\Delta})^2 + (\kappa/2)^2} - \frac{1}{(\Omega - \bar{\Delta})^2 + (\kappa/2)^2} \right] \quad (1.21)$$

When the input optical power is small, the optical spring and damping effects to the resonator behavior are negligible. However, the light can still be used as sensitive probe for the mechanical motion. With only the thermal noise as the external source, the transfer function of the system can be rewritten as:

$$\delta x[\omega] = \frac{F_{th}[\omega]}{m_{eff}(\Omega^2 - \omega^2 - i\omega\Gamma)} \equiv \chi_{xx}(\omega)F_{th}[\omega] \quad (1.22)$$

We can therefore write the power spectral density of the Brownian fluctuation $x(t)$ as [79]:

$$S_{xx}[\omega] = |\chi_{xx}(\omega)|^2 S_{FF}^{th}[\omega] \quad (1.23)$$

where S_{FF}^{th} is the power spectral density of the thermal noise force. In the high-temperature limit, S_{FF} is not frequency dependent and given by [79]

$$S_{FF} \approx 2m_{eff}\Gamma k_B T \quad (1.24)$$

where k_B is the Boltzmann constant and the T is the mode temperature. Furthermore, for high-Q resonator near the mechanical resonance, since $\omega \approx \Omega$, using $\Omega^2 - \omega^2 \approx 2(\Omega - \omega)\Omega$, we can approximate the transfer function as a Lorentzian function (Fig. 1.8):

$$\chi_{xx}(\omega) = (m_{eff}\Omega[2(\Omega - \omega) - i\Gamma])^{-1} \quad (1.25)$$

$$\text{and } |\chi_{xx}(\omega)|^2 = (m_{eff}^2\Omega^2[4(\Omega - \omega)^2 + \Gamma^2])^{-1} \quad (1.26)$$

On the other hand, when the detuning is positive, the optical damping can be negative and cancels the mechanical damping. When the input optical power is equal to the threshold power, the total effective damping is zero and parametric oscillation is initiated. The first study about optomechanical induced damping and frequency shift was done for the Fabry-Perot cavity of the Laser Interferometer Gravitational Wave Observatory (LIGO) [80]. Centrifugal radiation pressure induced optomechanical oscillations (OMOs) in toroidal WGRs were first described in [37, 81, 82]. The high quality factor of the optical WGM allows the amplification of radiation pressure inside the resonator, enhancing the mutual coupling between the optical and mechanical modes significantly and lowering the threshold power needed to introduce os-

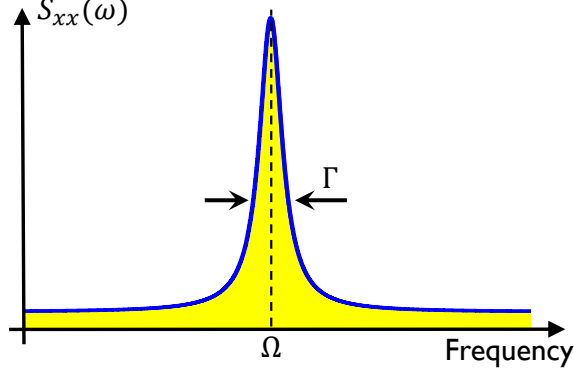


Figure 1.8: The spectrum for the thermal mechanical fluctuation of a damped harmonic oscillator.

cillation. Parametric optomechanical oscillation in OMFRs has been studied as well [70] and was applied for pressure sensing [3].

1.6 Light coupling to whispering gallery resonator

In this work, we make use of the evanescent waves to couple light into/out of a WGR from a waveguide efficiently. We can understand the generation of the evanescent wave using Snell's law for refraction. As shown in Fig. 1.9, we assume that when TE-mode light is transmitted from a high index (n_i) material to a low index (n_t) material, the refracted angle can be calculated as:

$$\sin(\theta_t) = \frac{n_i}{n_t} \sin(\theta_i) \quad (1.27)$$

$$\cos(\theta_t) = \sqrt{1 - \sin^2(\theta_t)} \quad (1.28)$$

The propagation vector of the transmitted wave can be written as:

$$\vec{k}_t = k_t \sin(\theta_t) \hat{x} + k_t \cos(\theta_t) \hat{z} \quad (1.29)$$

The critical incident angle, after which total reflection takes place, can be found using $1 = \frac{n_i}{n_t} \sin(\theta_{i,c})$, and thus $\theta_{i,c} = \sin^{-1}(\frac{n_t}{n_i})$. When the incident angle is larger than the critical angle, $\cos(\theta_t)$ becomes imaginary $\cos(\theta_t) =$

$i\sqrt{\sin^2(\theta_t) - 1}$ and we have

$$\vec{k}_t = k_t \frac{n_i}{n_t} \sin(\theta_i) \hat{x} + i k_t \sqrt{\sin^2(\theta_t) - 1} \hat{z} \quad (1.30)$$

Using the relation $k = \omega n/c$ (ω is light frequency, c is speed of sound in vacuum), the electric field of the transmitted wave can be expressed as

$$\vec{E}_t = \hat{y} E_{t0} e^{-\kappa z} e^{i(kx - \omega t)} \quad (1.31)$$

where E_{t0} is the field amplitude, decay parameter $\kappa = \frac{\omega}{c} \sqrt{n_i^2 \sin^2(\theta_i) - n_t^2}$, and propagation vector $k = \frac{\omega n_i}{c} \sin(\theta_i)$. Therefore, when the incident angle is larger than the critical angle, the transmitted wave propagates only along the interface. In the z direction, the wave amplitude decays with distance exponentially. The decay parameter κ is on the order of ω/c . For 200 THz light, the wave amplitude is reduced to $1/e$ of its original amplitude within $1 \mu\text{m}$. As shown in Fig. 1.10, since the evanescent field decays very rapidly, when two optical waveguides are far away from each other, there is no propagating wave generated in the bottom waveguide. However, when these two optical waveguides are close enough, light from one waveguide can be coupled to the other waveguide, due to frustrated total internal reflection. Many distinct evanescent coupling methods have been developed, for example, prism coupling [83], polished optical fiber coupling [84], and tapered fiber coupling [85]. In this thesis, single-mode tapered fiber coupling is used to couple light into OMFRs as shown in Fig. 1.11. The fabrication of tapered optical fiber and taper-coupling to WGM are discussed in detail in Chapter 2.

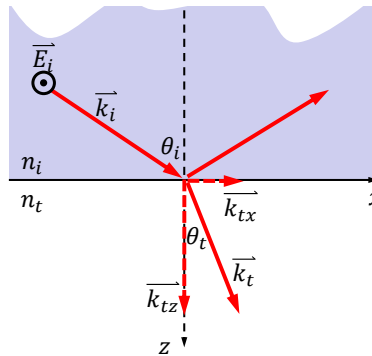


Figure 1.9: A TE mode light is transmitting from a high index material to a low index material.

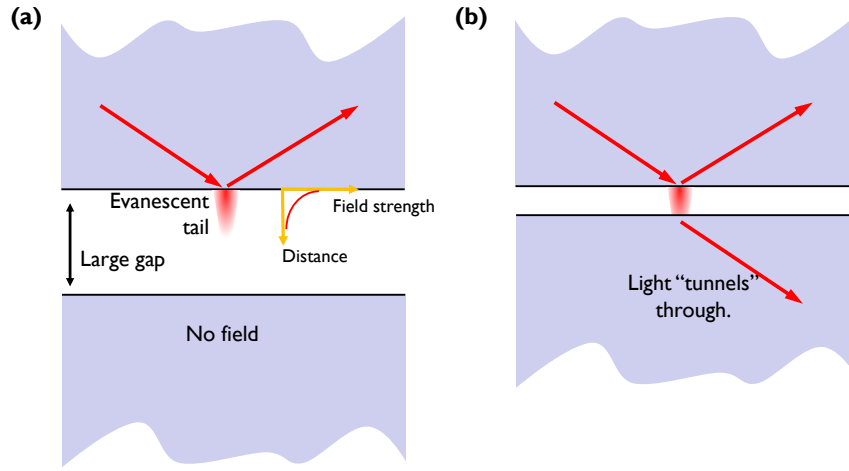


Figure 1.10: (a) Light cannot be coupled into the waveguide on the bottom since the evanescent field dies out when the gap between two waveguides is large. (b) The evanescent field does not decay much before it reach the waveguide on the bottom, resulting in a propagative mode in the bottom waveguide.

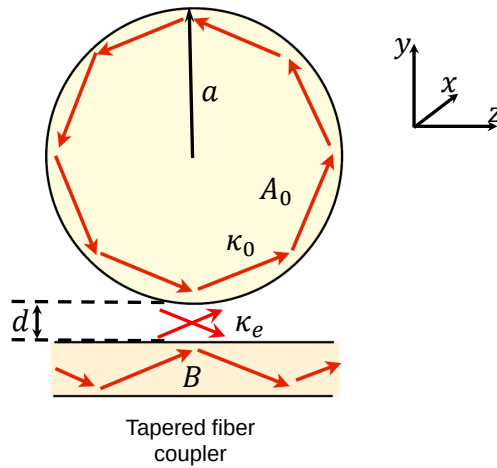


Figure 1.11: Light is evanescently coupled into the WGM resonator through a tapered fiber.

The formal way to study this coupling is to use the coupled-mode theory [86]. As shown in Fig. 1.11, the WGR has a radius of a and the distance between the waveguide and the WGR is d . With the detailed derivation available through [87], the photon energy loss rate from coupling, κ_e , is proved to decay with distance d :

$$\kappa_e \propto \exp\left(-\frac{\omega\sqrt{n^2-1}}{c}d\right), \quad (1.32)$$

where n is the refractive index of the waveguide material. Furthermore, the transmission due to the coupling has a Lorentzian shape with respect to laser detuning [79]:

$$T(\Delta) = \frac{(\kappa_e - \kappa_0)^2 + 4\Delta^2}{(\kappa_e + \kappa_0)^2 + 4\Delta^2}, \quad (1.33)$$

where κ_0 is the intrinsic loss rate of the resonator. Therefore, clearly, when on resonance, as the taper moves toward the resonator from far away, the transmission decreases first due to the increase of the coupling loss rate; critical coupling happens when the coupling loss rate is equal to the intrinsic loss rate, and the transmission is zero, meaning that all the light gets lost inside the resonator; when the tapered fiber moves further toward the resonator, the transmission increases again. The total loss rate can be expressed as:

$$\kappa = \kappa_e + \kappa_0 \quad (1.34)$$

The loaded quality factor of the system is defined as

$$Q = \omega_r / (\kappa_e + \kappa_0) \quad (1.35)$$

Clearly, the loaded quality factor decreases monotonically as the coupling loss increases. With the tapered fiber coupling, we can experimentally measure the optical resonance frequency and optical Q factor of the OMFR, by sweeping the wavelength of the laser and measuring the transmission of the incoming wave, and making use of the following expression:

$$Q \approx \frac{\omega_r}{\delta\omega}. \quad (1.36)$$

A typical measurement result is shown in Fig. 2.17. For 1550 nm light we

use, the typical optical Q for OMFR is about 10^7 .

1.7 Mechanical mode shape of the OMFR shell

The OMFR shell can take various eigenmodes, the mode shape of which can be easily simulated using Finite Element Method with geometrical and material properties of the OMFR. Some examples are shown in Fig. 1.12 [54]. When the OMFR is filled with fluid, however, the vibration of the OMFR shell entrains the fluid inside, form the fluid-shell hybrid mode, which we will discuss in detail in Chapter 3.

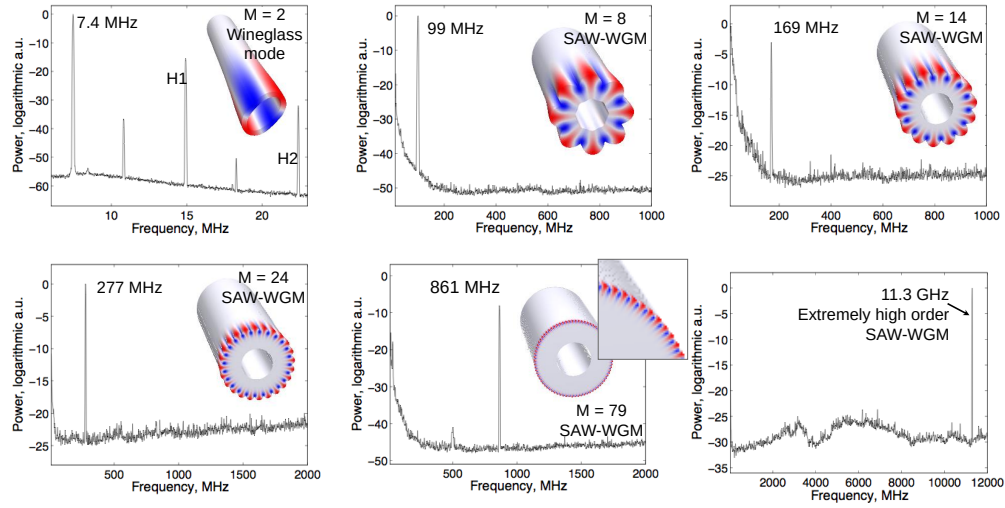


Figure 1.12: (Figure from [54]) FEM simulation and the spectrum of various mechanical modes of the OMFR shell.

1.8 Experimental setup for optomechanical measurement

A basic experimental setup used for the optomechanical measurement in this thesis is shown in Fig. 1.13. Other hardware needed for different experiments is described in detail in the following Chapters. Tapered fiber couplers are used to couple 1550 nm light from the tunable laser into the WGM of OMFRs. As we have described above, the vibrating resonator can be treated as the optical modulator: changing optical path length modulates input laser,

resulting in a pair of optical sidebands. Photodetectors are used to convert the temporal interference between pump and scattered light to electronic RF signals as shown in Fig. 1.14. The temporal beating is analyzed by an electrical spectrum analyzer and the power spectrum of the mechanical vibration is obtained. In the low laser-power regime which we use the most for this thesis, for each mechanical mode, the electronic spectrum has a Lorentzian shape with the center frequency at the mechanical vibration frequency (Fig. 1.8). Due to the existence of different mechanical modes described in the last section, the total measured spectrum is the summation of the spectra of all the individual spectrum [78] since

$$S_{xx}^{tot} \approx \sum_n |\chi_{xx,n}|^2 S_{FF}^{th,n} + \text{measurement noise} \quad (1.37)$$

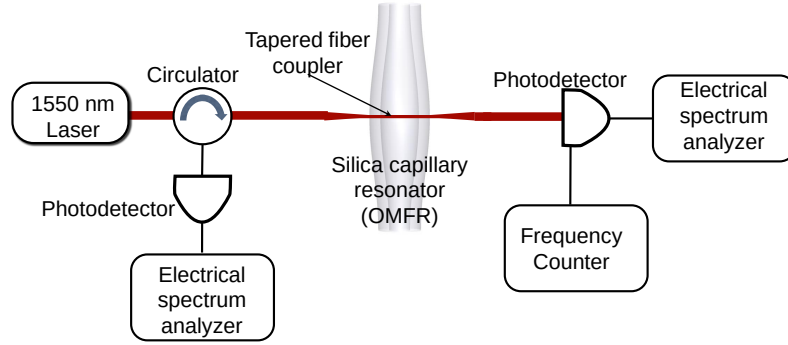


Figure 1.13: 1550 nm laser light is evanescently coupled into the WGM through a tapered fiber coupler. Photodetectors are used to convert the optical transmission to RF signal. Electrical spectrum analyzer and frequency counter can be used both in the forward and backward direction to analyze the signal.

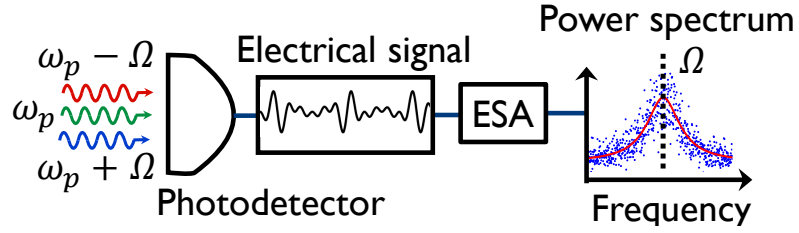


Figure 1.14: The optical pump and the scattered sidebands from a radiation pressure induced vibration are made to interfere on high speed photodetectors, thus generating beat notes at the mechanical vibration frequency. The beat note is then analyzed by the electrical spectrum analyzer (ESA) and power spectrum (Lorentzian shape, adapted from [3]) of the mechanical vibration is generated. The center frequency of the Lorentzian shape is at the mechanical vibration frequency.

CHAPTER 2

FABRICATION AND TESTING OF OPTO-MECHANO-FLUIDIC RESONATORS (OMFRS)

¹ Cavity optomechanical interactions have been demonstrated in many different optical systems, such as fibers [88], microspheres [49, 89], toroids [37, 90] and crystalline resonators [50, 91]. While these previous optomechanics demonstrations have focused on solid state sensors, recent demonstrations of opto-mechano-fluidic resonators (OMFRs) have enabled applications with liquids [54, 67] and gases [92]. These applications are accomplished by using the hollow capillary structure of the OMFR, which is very different from previous optical systems. In this chapter, we will discuss in detail how to manufacture OMFRs, how to mount the OMFRs for testing, how to build tapered fiber and couple light to WGMs of the OMFRs, and how to measure optomechanical vibrations.

2.1 Fabrication of OMFR

The geometry of the hollow microbottle resonator (MBR) [92–95] makes it ideal for sensing fluids or gases that flow through it, and it has been widely employed for opto-fluidic sensing applications. Often, these experiments are performed by means of detecting optical resonant frequency shift because of the effective refractive index change caused either by fluid/particle loading [93, 94], or by the geometry change due to internal pressure change [92, 95]. Fabrication of hollow MBRs has been achieved in various ways with a capillary preform as the starting stock. The curvature needed in the axial direction for stopping the axial light propagation can be created by softening the preform first and then applying a compression force axially [96]. This curvature can also be created by increasing the internal pressure while the

¹Portion of this chapter is reprinted with permission from Han, K., Kim, K. H., et al., (2014). “Fabrication and testing of microfluidic optomechanical oscillators,” *Journal of Visualized Experiments*, (87), e51497.

capillary is heated, which is especially useful for fabricating thin-walled resonators for enhancing the sensitivity to refractive index change [8]. However, neither the “soften-and-compress”, nor the “soften-and-blow” method can reduce the overall size of the capillary, which is critical for particle mass sensing since the mass sensitivity is inversely proportional to the total mass of the resonator. In this Chapter, we describe a “soften-and-pull” method to fabricate thin capillaries that we need for our application.

All the materials and instruments mentioned in this chapter are listed in Fig. 2.1. The OMFR is fabricated by heating a glass capillary preform with approximately 10 watts of CO₂ laser radiation at 10.6 microns wavelength, and drawing out the heated capillary linearly using motorized translation stages (Fig. 2.2). The silica glass material is a very good absorber to 10.6 micron laser light, making the laser heating possible. Meanwhile, silica glass is a low loss waveguide for the 1.5 micron laser light, providing the high Q needed for a strong photon-phonon coupling.

Preparation of capillary manufacturing setup

Fig. 2.3 shows the arrangement of the linear translation stages, the lasers, and the location of the capillary preform before the pulling process.

The two CO₂ lasers (for heating) and the two linear stages are simultaneously controlled by LabVIEW (National Instruments) program. The control software is written by the author. The two linear stages perform the drawing process for the laser-heated capillary. Before pulling, a brief preheating to the capillary preform must be applied to burn off the polymer jacket, and to increase the hot zone temperature such that the capillary is soft enough for pulling. During pulling, one of the linear stages must be fast (e.g. 5 mm/s) for the linear drawing process. More material to the heating zone is fed with the second, slower linear stage (e.g. 0.5 mm/s) since the capillary preform material gets depleted during the pulling process (Fig. 2.2(a)). Modulation of the laser power can be used to control the capillary radius as a function of length during the drawing process to form the ‘bottle’ resonators (Fig. 2.2(b)). The spacing between adjacent bottle resonators can be changed by modifying the pulling speed and modulation frequency. Control of pulling and feeding rate provides easy control over the capillary diameter. The sam-

Name of Material/ Equipment	Company	Catalog Number
Tunable IR laser	Newfocus	TLB-6328
Photodetectors	Newfocus	1811-FC (Low speed 125MHz) / 1611-FC-AC (High speed 1GHz)
Optical fiber	Corning	SMF28
Silica capillary	PolyMicro	TSP700850
10.6 um wavelength CO ₂ laser	Synrad	48-1KWM and 48-2KWM
UV-curing optical adhesive	Thorlabs	NOA81
Tubing	Tygon	EW-06418-01
Syringes	B-D	YO-07940-12
Needles	Weller	KDS201P
Electrical spectrum analyzer	Agilent Technologies	N9010A (EXA Signal Analyzer)
	Tektronix	6114A (RSA, Real-time spectrum analyzer)
Optical spectrum analyzer	Advantest	Q8384
Oscilloscope	Tektronix	DPO 4104B-L
Gold mirrors	II-VI Infrared	836627
Linear stage (slow)	DryLin	H1W1150
Linear stage (fast)	PBC Linear	MTB055D-0902-14F12
Fabry Perot optical spectrum analyser	Thorlabs	SA 200-14A (FSR: 1.5 GHz)

Figure 2.1: Materials' list for the fabrication of OMFR.

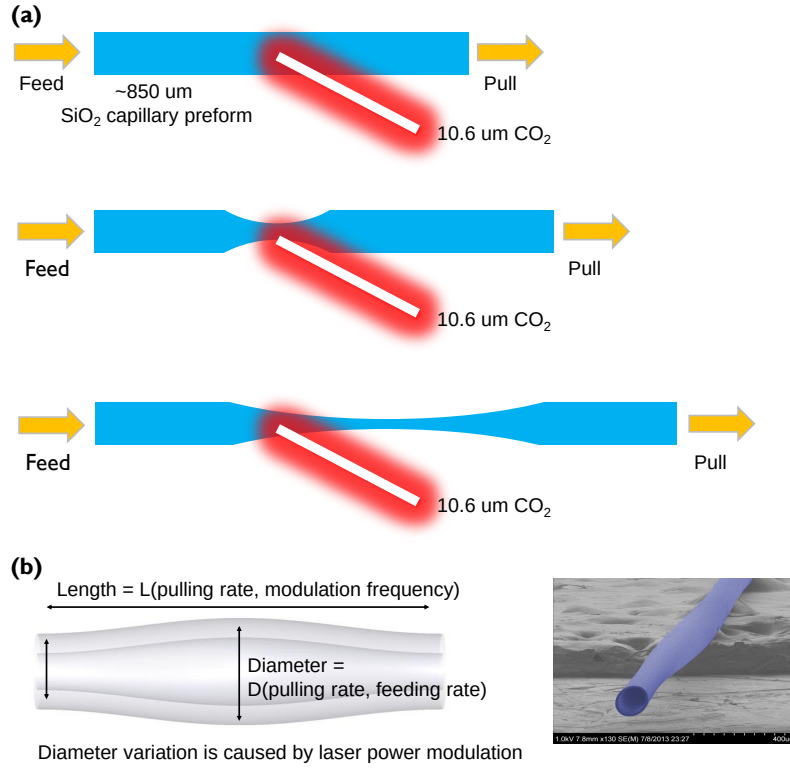


Figure 2.2: (a)Capillary drawing process: The OMFR is fabricated by heating a glass capillary preform CO₂ laser radiation and drawing out the heated capillary linearly using motorized translation stages. During pulling, one of the linear stages must be fast for the linear drawing process and the other linear stage is slow for feeding more material to the heating zone since the capillary preform material gets depleted during the pulling process. (b) Schematic and SEM of the ‘bottle’ resonators.

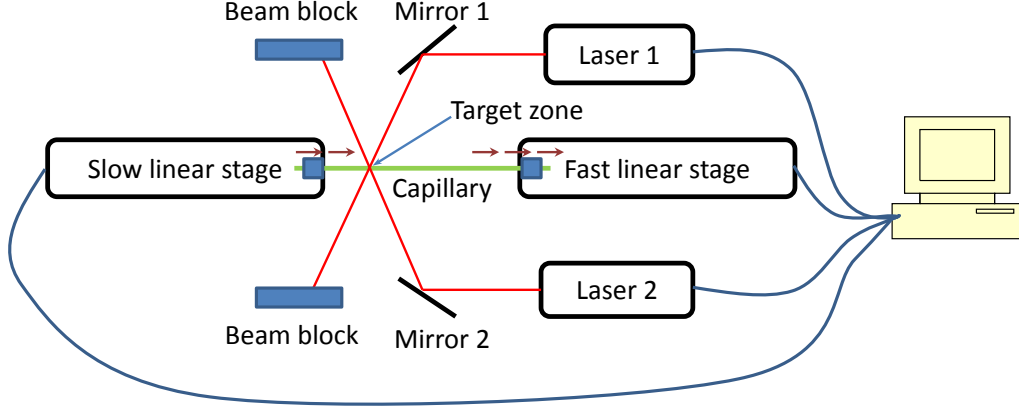


Figure 2.3: Schematic of the capillary pulling setup. The microfluidic optomechanical resonators are drawn from a larger capillary preform attached to two linear stages while the glass is heated by the CO₂ laser. Both laser beams are carefully aligned to the same spot of the capillary. Moving direction and relative speeds of the linear stages are indicated by the arrows.

ple holders are aligned on the linear stages along both vertical and horizontal axes. Both CO₂ laser beams are carefully aligned such that they target the same spot in space (between the sample holders). A piece of card paper or heat sensitive paper is useful for this process. Eye protection is used for laser safety. Beam blocks, fume exhaust, and fire protection are used suitably.

Measurement of the geometry of the OMFR

The measurement of the dimensions of the OMFR is performed under a microscope with the help of a camera control software. A sample image is shown in Fig. 2.4 (a). The software ruler can be used to measure the length (in pixels) between two points. Such length can be converted to real length (in μm) with the calibration of the microscope system. The conversion relation is

$$\text{length } (\mu\text{m}) = \text{pixels} \times Z, \quad (2.1)$$

where Z is a “metric per pixel” conversion factor for the entire system. By doing similar measurement, the dimensions (Fig. 2.4 (b)) of the OMFR can be determined, namely, length L , outer diameter at the widest point D_1 ,

inner diameter at the widest point $d1$, outer diameter at the thinnest point $D2$, and inner diameter at the thinnest point $d2$.

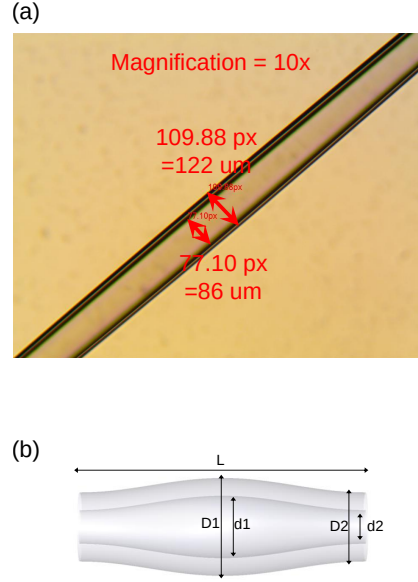


Figure 2.4: (a) Measurement of the outer and inner diameter of an OMFR from the side view. (b) Dimensions of an OMFR.)

A more accurate way to measure inner diameters is to cut the capillary and look at the cross section of it under microscope to avoid the influence of refraction when measuring from the side view. However, the accuracy of this measurement depends on the observation angle. To create the cross-section view of the capillary, a long segment of a pulled capillary is first taped down on a glass slide, with the capillary longer than the tape on one end (Fig. 2.5(a)). The tip of the capillary is then cut out with the tape such that we can handle the tip easily (Fig. 2.5(b)). Using a thick stack of glass slides as a holder (Fig. 2.5(c)), the capillary cross section can be viewed under the microscope (Fig. 2.6). The measurement results depend on where the cutting happens, which is not easy to control. A combination of measurements from both the side view and the cross section view should be performed for a more accurate measurement of the capillary diameter profile.

During our pulling process, we ensure that the minimum pulling power is used to prevent collapse (as described later in section 2.1.3). Therefore we can assume that the aspect ratio, which is the ratio between the inner diameter and outer diameter, remains consistent with the initial value [97] of the capillary preform. This is a weak assumption, but is fair to make for the

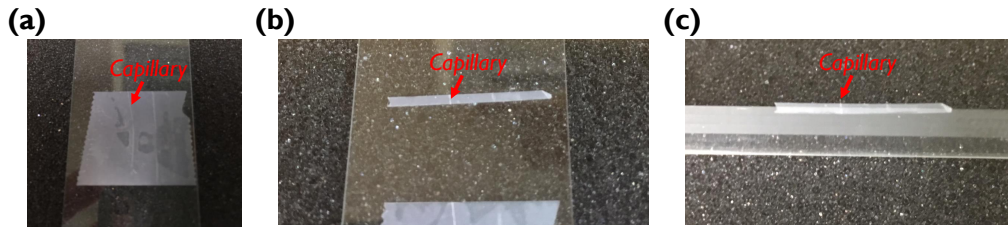


Figure 2.5: (a) A long capillary segment is first taped down on a glass slide, with the capillary end sticking out of the tape. (b) A small portion of the tape with the capillary is cut out from the end of the tape. (c) The shorter capillary segment is taped down on the side wall of a stack of glass slides. Then the capillary cross-section can be observed under microscope with the glass slide stack as a holder.

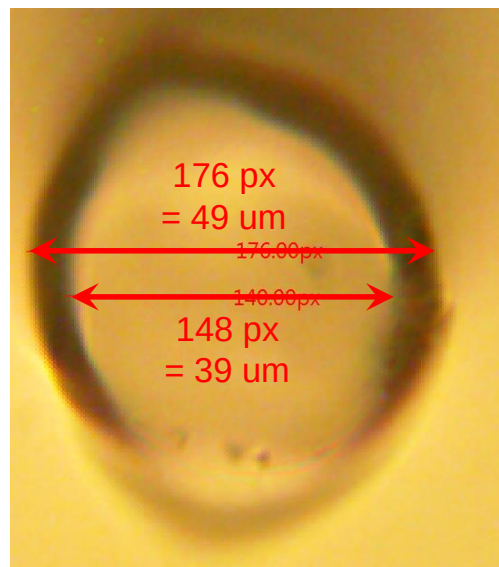


Figure 2.6: (a) Measurement of the outer and inner diameter of an OMFR from the cross section view.

purpose of this work. Thus using only measurement of outer diameter, and assuming that the initial aspect ratio is known, we can know all the important dimensions of OMFRs without destructive cross-section measurement.

Controllable parameters and their effects

As described in [98], the capillary pulling process can be approximately described by a set of Navier-Stokes and diffusion-convection equations. The final shape of the capillary depends on many parameters, such as, the pulling speed, the feeding speed, the viscosity and surface tension of the melt glass, gravity, and the process temperature at the hot zone. Among these parameters, we can control the pulling and feeding speed directly, and the process temperature and viscosity indirectly through controlling the laser power.

Capillary collapsing means that the capillary hole closes during the fabrication process due to surface tension force. It is very important to consider since it reduces the capillary aspect ratio and constricts flow. For OMFR sensing applications, we want the capillary wall to be as thin as possible to get a more effective fluid-shell coupling and thus higher mass sensitivity, as shown in Chapter 3. The main parameter that determines the capillary collapsing process is the ratio between glass surface tension and glass viscosity [98]. Surface tension is the major cause for collapse and viscosity is the major force opposing this collapse. Since the surface tension of silica is weakly temperature dependent while the viscosity of silica is highly inversely temperature dependent, when the pulling process temperature is higher, the capillary collapses more quickly. On the other hand, when the temperature is relatively low such that we can neglect the surface tension due to high glass viscosity, the capillary hole does not collapse and therefore the aspect ratio of the pulled capillary is consistent with the preform. The collapse happens slower with a higher feeding speed, due to the fact that the higher thermal mass associated with the higher feeding speed can lower the process temperature, i.e., the higher the feeding speed is, the shorter time the capillary stays in the hot zone. Similarly, the collapse happens faster with a lower pulling speed since the process temperature is higher. Thus, in practice, for every pulling and feeding speed combination, we usually first find out the lowest possible heating power needed for pulling, and then use this power for the

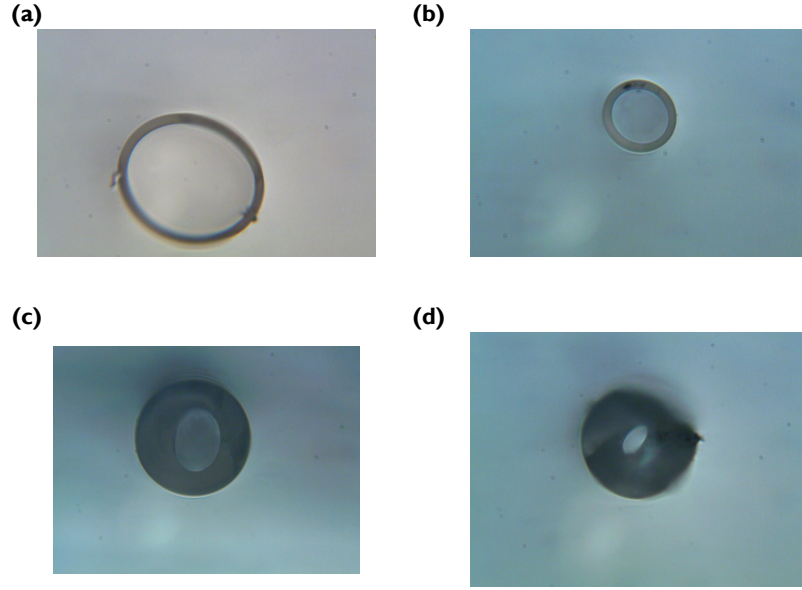


Figure 2.7: Pulled capillary cross section with the same pulling parameters except pulling speed decreasing from (a) to (d). The result shows a clear increase of wall thickness, i.e., collapsing, due to the decrease of the pulling rate.

pulling process to minimize the collapse. Examples of collapsing of different degrees are shown in Fig. 2.7. The effect of process temperature, which can be controlled by total heating power from the laser, is shown in Fig. 2.8.

The size of the capillary can be controlled by both the feeding speed and pulling speed. Fig. 2.9 shows that the feeding speed has a prominent effect on the overall size of the capillary. This can be understood from simple mass conservation – at the same pulling speed, the size of the pulled capillary has to increase to compensate the increase of incoming mass flow rate. Therefore, in reality, as the “coarse control”, we usually fix the feeding speed first according to the objective OMFR size range. We use the pulling rate as the “fine control” of the capillary size. As shown in Fig. 2.10, there is a strong trend of the capillary outer diameter decreasing when the pulling speed starts to increase from zero. We can make use of this fact to get the specific diameter that we want. This trend, however, flattens out when the pulling speed is high. In practice, we find that it is really hard to get a capillary size that is smaller than $30\text{ }\mu\text{m}$ with the TSP700850 capillary preform that we use for the OMFR, without breaking the capillary during the pulling process.

We need to preheat the capillary to burn the jacket off the capillary preform

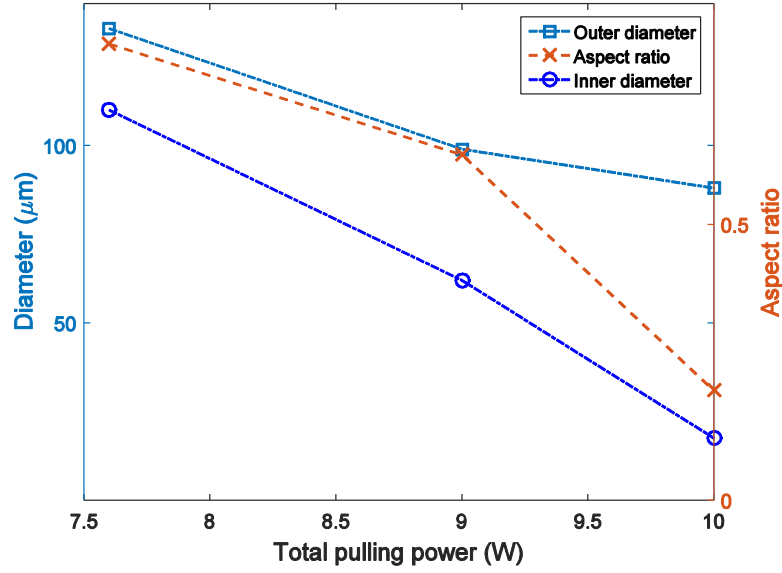


Figure 2.8: The aspect ratio decreases when the process temperature increases. The feeding rate is 0.5 mm/s, the pulling rate is 15 mm/s, and the preheating power is 7.6 W, and pulling power is 7.6 W without any power modulation during the pulling process.

and soften the material for pulling. Too much preheating is not good since it can cause collapsing. For our current setup and capillary preform, a 3s preheating with 7.6 W total laser power is a good combination for preheating without too much collapsing. We also need to apply power modulation to form the bottle shape of the OMFR. Usually, this is done by modulating laser power periodically. Modulation frequency can be varied from a few Hz to a few tens of Hz, depending on the application.

Furthermore, as shown in Fig. 2.7, there is some ellipticity with the pulled capillary due to gravity and the fact that we are only heating up the capillary from two sides. This ellipticity reduces the optical Q factor of the OMFR. One way to solve this problem is to rotate the capillary while heating and pulling [99], which is considerably harder to implement.

Reasonable parameters are selected for the drawing process depending on the applications. For example, the following parameters can be used to reliably produce a good capillary size - 10 mm/s pulling speed, 0.5 mm/s feed-in speed, 3 s preheating time, 4.5 W preheating powers for both lasers, and 5 W heating powers for both lasers. An example is shown in Fig. 2.11(c).

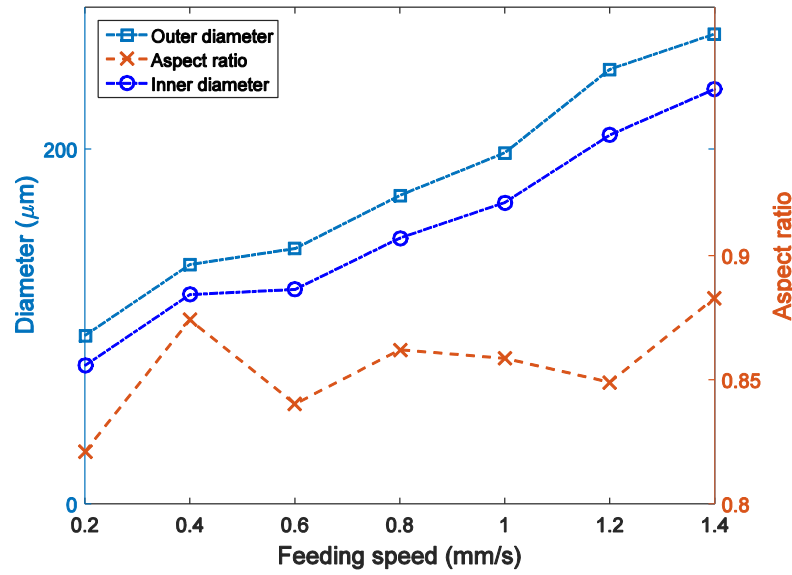


Figure 2.9: A small change in feeding speed can greatly increase the size of the diameter. The aspect ratio is only slightly increased by changing the feeding speed, probably because that the process temperature is relatively low here such that there is almost no collapsing happening. The pulling rate is 15 mm/s, preheating power is 7.6 W, and the pulling power is 7.6 W, without any power modulation during the pulling process.

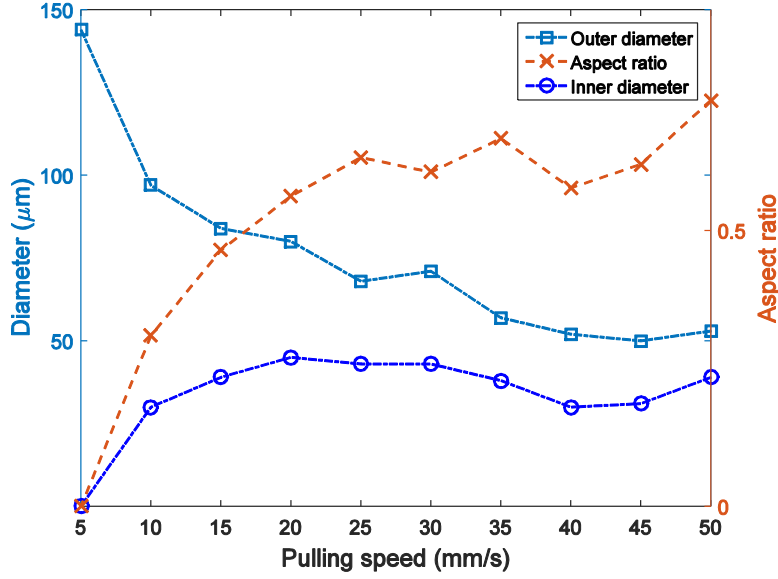


Figure 2.10: With the pulling speed increasing from zero, the outer diameter of the capillary drops quickly at first and then the curve becomes relatively flat. The aspect ratio increases as the pulling speed increases from 0. The feeding rate is 0.5 mm/s, preheating power is 7.6 W, and the pulling power is 10 W, without any power modulation during the pulling process.

The modulation parameters selected are: 3 Hz frequency, 6 W and 3 W laser powers for both lasers, and 50% duty cycle. The following parameters are for fabricating the smaller size capillary with roughly 50 μm in diameter at the largest diameter region as shown in Chapter 5: 50 mm/s pulling speed, 0.3 mm/s feed-in speed, 3 s preheating time, 4.5 W preheating powers for both lasers. The modulation parameters used are: 3 Hz frequency, 5 W and 4.5 W heating power for both lasers, and 50% duty cycle during pulling.

Fabrication of microfluidic optomechanical resonators

A sufficiently long segment (about 2 - 4 centimeters) of fused silica capillary is first cut such that it can reach the two holders attached to the linear translation stages. The capillary sample is mounted on the sample holders such that the laser target zone is roughly in the middle of the capillary. The capillary is pulled using the parameters as stated above. First the capillary is preheated for a few seconds (Fig. 2.11(a)), and then it is pulled with or without laser modulation (parameters in 2.1) as needed. The capillary draw-

ing is shown in detail in Fig. 2.12. The drawn capillary (Fig. 2.11(b)) is removed from the sample holder. The sample is handled with powder-free gloves at the two thick ends only, in order not to contaminate the clean resonator surface. The pulled capillary is glued onto a laser cut acrylic holder using UC curable optical adhesive (Fig. 2.11(c)). We note that the holder design is entirely depending on the specific application and thus very flexible. The pulling parameters are varied to fabricate capillaries with different diameters. Typically outer diameter varies from $30\text{ }\mu\text{m}$ to $200\text{ }\mu\text{m}$ depending on pulling conditions.

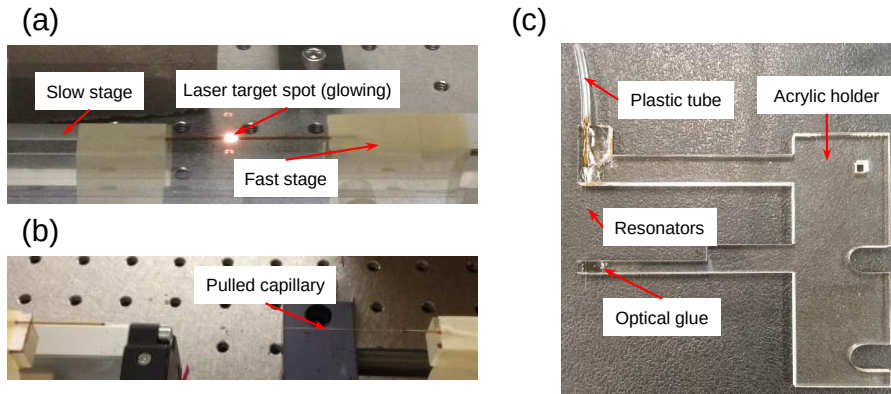


Figure 2.11: Optomechanical bottle resonator fabrication. (a) The capillary preform is pulled at a constant speed while being heated by means of CO_2 laser radiation. Note the glowing region is the laser target spot (where the beams heat the silica). When the required length and diameter are reached, (b) stop the linear stage motion and turn the lasers off. The pulled capillary is thin, clear, and very flexible. (c) Employ a laser cut acrylic holder to mount the microcapillary resonator device. The optomechanical bottle resonator is now ready to be taken to the experimental setup and connected to tubing that will provide analytes.

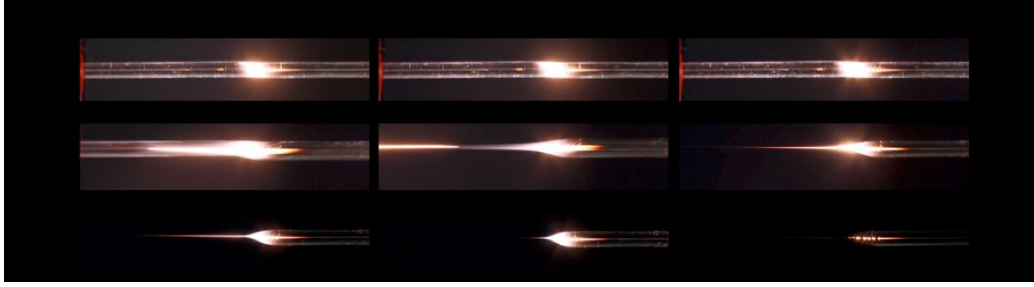


Figure 2.12: The capillary drawing process (sequence: from top left to bottom right). Photos' courtesy: Kaihao Zhang.

Clean OMFR fabrication protocols for particle sensing

At the beginning of the fabrication process, we cut the capillary preform segment from the capillary preform spool, we leave pieces of glass debris in the inner side of the preform (Fig. 2.13(a)). These pieces of glass debris will not cause any problems if we work with fluids (Chapter 4), or with gases [92]. However, as shown in Chapter 5, since the OMFR inner channel diameter is less than $50\ \mu m$, when working with particles (usually a few μm in diameter), a small piece of glass debris or dust can cause particles to clog around it, ruining the experiment (Fig. 2.13(b)). Therefore, we have to be very careful in fabricating and operating the OMFRs. In this part, I will briefly introduce the protocols and precautions that need to be followed when fabricating and using OMFRs for particle sensing.

After the capillary preform segment is initially cut from the spool, it should be carefully inserted into a slightly larger plastic tube (Fig. 2.14). The inner surface of the OMFR is then flushed several times with distilled water by connecting the plastic tube with a syringe. The OMFR is then blew with air to make sure there is no water left inside. The plastic tube segment is left on one end of the preform. This is done intentionally since frequent plugging and unplugging the capillary preform from the plastic tubing can also cause the generation of glass debris. Starting from this step, we need to operate the OMFR really carefully to prevent more glass debris from being generated. Next, the procedures in 2.1 are followed to fabricate the OMFR with the plastic tube segment. To accommodate the tube, a groove is made on one of the linear translation sample holders (Fig. 2.14). After pulling, the pulled capillary should be carefully glued on the sample holder with the tube segment. Then the particle suspension with proper concentration is

mixed and filtered such that there is no large dust or particle cluster existing in the suspension. Finally, extensional tube can be assembled as needed and the particle suspension can be infused for experiment. We note that all the instruments in contact with the particle suspension should be washed first with distilled water and ethanol, and air dried before using for the experiment. Cleanroom compatible wipes should be used to prevent dust particles (Kimwipes should not be used).

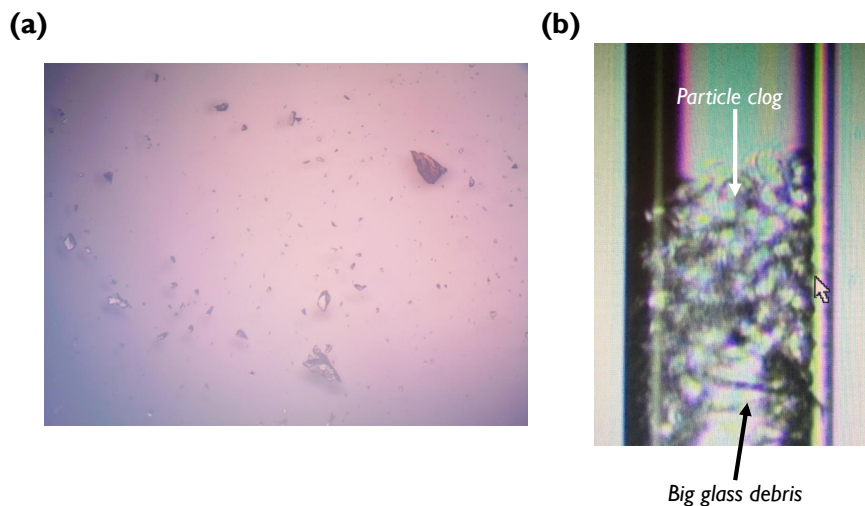


Figure 2.13: (a) Pieces of silica glass debris that is washed out from the capillary preform segment before pulling. (b) Particles are clogged due to a big glass debris in the OMFR.

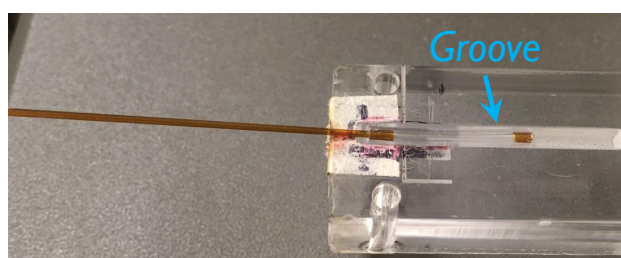


Figure 2.14: (a) A groove is cut on the capillary holder of the pulling stage to accommodate the tubing segment pre-installed.

2.2 Experimental setup for optomechanical testing

Fabrication of tapered optical fiber

A single mode telecom-band optical fiber is prepared of desired length (e.g. a few feet). Fiber segment should be long enough to be both mounted in the tapering area and connected to the setup (Fig. 2.16). The tapering method explained here is similar to what is suggested and demonstrated in [85]. The prepared fiber segment is connected to the rest of the experimental setup using any convenient fiber-splicing method. The spliced fiber segment is mounted onto two linear pullers that face each other. The fiber jacket is then stripped off in the center of the mounted fiber fragment to expose cladding area. This is where taper is fabricated. The stripped area is cleaned with methanol. The tunable laser is turned on to see real-time transmission on an oscilloscope. Attenuators are used so that photodetectors are not damaged. A narrow nozzle hydrogen gas burner is placed immediately underneath theunjacketed portion of the fiber. All recommended safety procedures must be followed when working with pressurized flammable gases such as hydrogen. Other clean burning sources of flame or ceramic heaters could also be used. Before lighting up the gas, the flow rate is checked so that flame will not be too large (a 1-2 cm tall flame is adequate). Note that the flame is mostly invisible but may be seen as a faint bluish-orange glow in a dark room. The hydrogen flow rate should be set to a point where lighted flame will adequately soften the glass fiber. As soon as flame is on, the fiber pulling is started using motorized stages (Fig. 2.15). Appropriate pulling speed depends on flow rate of hydrogen gas and vicinity of the flame. Transmission through the fiber will begin to show temporal oscillation behavior as pulling continues. This indicates multimode operation. When oscillatory behavior stops and shows an unchanging signal over time, the pulling is stopped and the flame is turned off immediately. This is when single-mode taper is obtained [101]. The transmission through the taper is checked then. If transmission is too low, the procedure is repeated from 2.2, with modified gas flow rate, flame size, and flame location. On occasion, low transmission could be due to bad alignment at step 2.2, or due to contamination of the exposed cladding. If resultant transmission through the taper is satisfactory, the taper is cooled down by waiting a few minutes. The taper is inspected

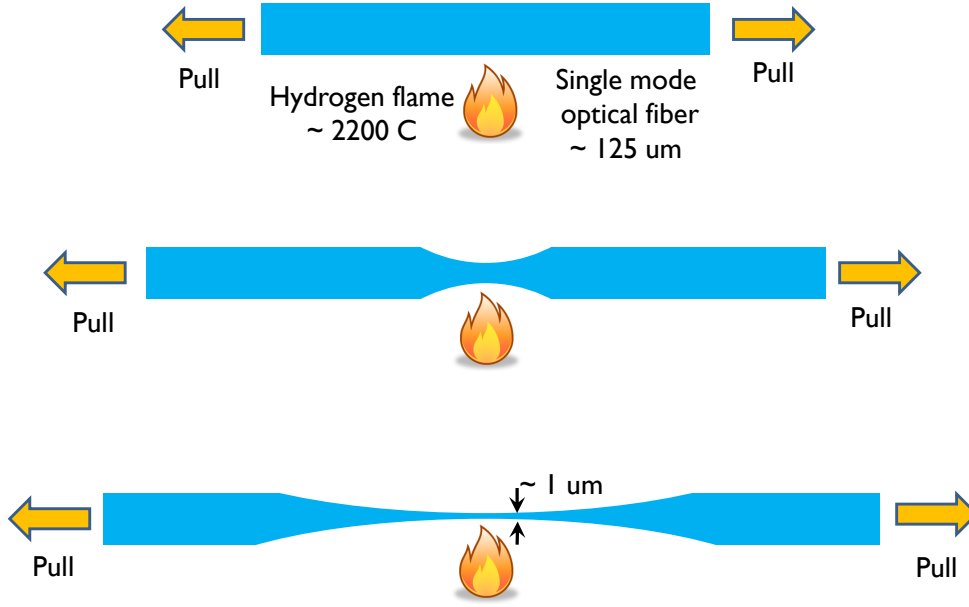


Figure 2.15: Taper is pulled until the diameter is about $1\ \mu\text{m}$, when the single-mode taper is obtained [100,101].

under a microscope. For 1550 nm operational wavelength, typical diameter of the single-mode taper is in the order of $1 - 2\ \mu\text{m}$. After fabrication, the taper is glued down on another laser-cut acrylic holder and then the holder is mounted on a nano-positioner such that the taper position can be controlled with high precision (Fig. 2.16 (b)). The OMFR device holder is then mounted on a XYZ near the taper (Fig. 2.16 (a)). The typical arrangement for the experiment setup is shown in Fig. 2.16 (c).

Taper-coupling to WGM and searching for electronic signals indicating vibration.

The experiment is set up in the configuration shown in Fig. 1.13. Mechanical vibrations can be generated through both SBS and RP by the same experimental configuration. In order to clearly detect back-scattered signals as in the case of backward-SBS [49, 102], a circulator is used between taper and tunable laser (Fig. 1.13). The tunable IR laser is turned on and stabilized. A function generator is used to sweep the frequency of the input IR laser. The resonator holder is mounted on a nanopositioning stage (Fig. 2.16 (b)). The resonator is carefully brought close to the tapered fiber in

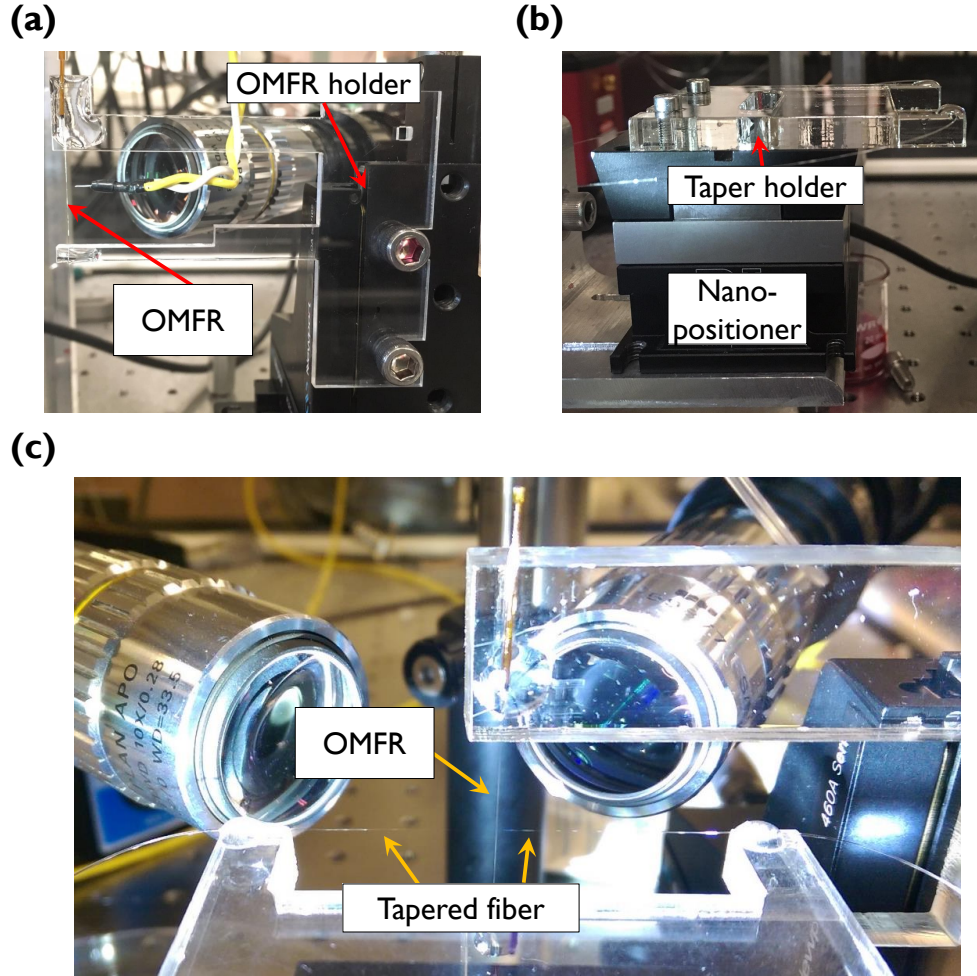


Figure 2.16: (a) The OMFR is holder is mounted on a XYZ. (b) The tapered fiber is first glued on a customized acrylic holder and then the holder is mound on top of a nano-positioner for high precision position control. (c) The OMFR is mounted near the tapered fiber so that light can be evanescently coupled into the resonators.

order to obtain evanescent coupling. As the laser frequency is swept, optical resonances will appear as dips in transmission in the oscilloscope, as in Fig. 2.17. The photodetector output is connected to an electrical spectrum

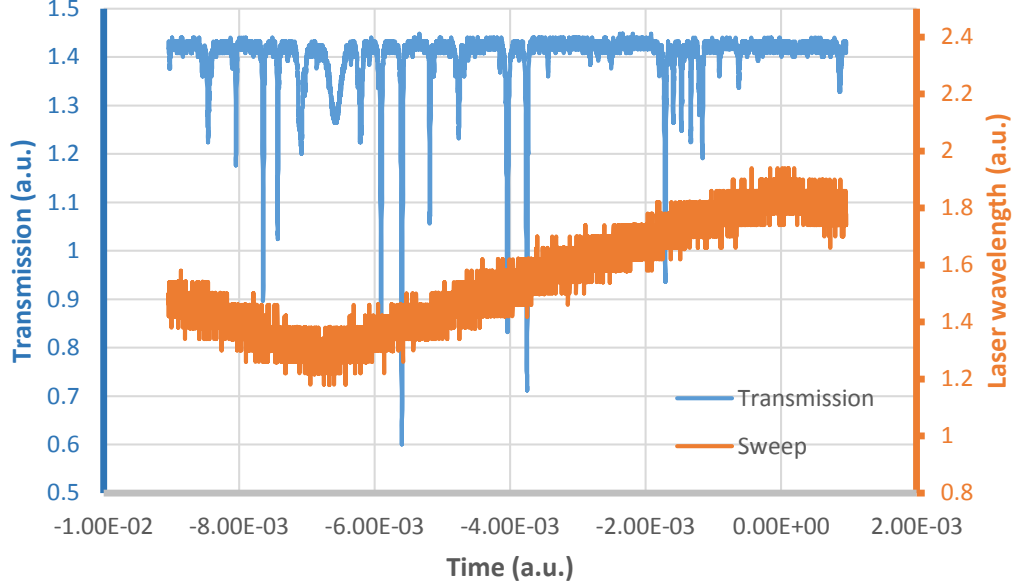


Figure 2.17: The transmission (blue line) changes as the laser wavelength sweeps (orange line).

analyzer (ESA), where the temporal interference (i.e. beat note) between the input laser light and the scattered light can be observed if mechanical oscillation takes place. This temporal interference occurs at the mechanical oscillation frequency. The peak hold function on the spectrum analyzer is often useful in the initial search for mechanical vibrations. Higher input power is used while performing the initial search for mechanical vibration, especially when liquids are present inside the device. Typically, input power in the order of $100 \mu\text{W}$ to the device is sufficient to excite mechanical vibration. If mechanical oscillation is observed, the laser frequency scan is turned off and the laser wavelength is controlled in CW mode to attempt to lock to the relevant optical mode. Here, both oscilloscope and spectrum analyzer are useful in tandem. Periodic signals appear on the oscilloscope when a mechanical mode is present, as seen in Fig. 2.18 and [51, 103].

2.3 Measuring optomechanical vibrations

As described in 2.2, mechanical oscillations will be observed when the taper and device are correctly coupled, the device optical and mechanical modes have sufficient Q-factors, and sufficient input optical power is provided. If oscillations in the range of 10MHz - 1GHz are not observed, polarization is changed to investigate different resonances, the input power from tunable laser is increased in order to overcome the minimum threshold for oscillation. When increasing the input power, we make sure not to saturate or damage the photodetectors. Also, as described in [89], coupling distance is a key factor for exciting different RP modes. If mechanical modes are still not observed, optical quality factor can be measured. For microfluidic optomechanical resonators, results show that optical quality factor of 10^6 is sufficient to excite parametric oscillations [67]. Usually, RP modes will manifest as electronic oscillations on the spectrum analyzer accompanied by their harmonics, as seen in Fig. 2.18. A scanning Fabry-Perot interferometer or high-resolution optical spectrum analyzer is used to detect the optical side bands that are generated due to amplitude and phase modulation, which is in turn induced by the periodic cavity deformation. An example measurement may be seen in Fig. 3h of [37].

2.4 Discussion

We have fabricated and tested a new device that bridges between cavity optomechanics and microfluidics by employing high-Q optical resonances to excite (and interrogate) mechanical vibration. It is surprising that multiple excitation mechanisms are available in the very same device, which generate a variety of mechanical vibrational modes at rates spanning 2 MHz to 11300 MHz. Centrifugal radiation pressure supports both wineglass modes and breathing modes in the 2 MHz – 200 MHz span, forward stimulated Brillouin scattering allows mechanical whispering gallery modes in the 50 MHz – 1500 MHz range, and lastly, backward stimulated Brillouin scattering excites mechanical whispering gallery modes near 11,000 MHz.

The methods that are described in the current work enable the fabrication of these microfluidic resonators with ultra-high optical quality factors of

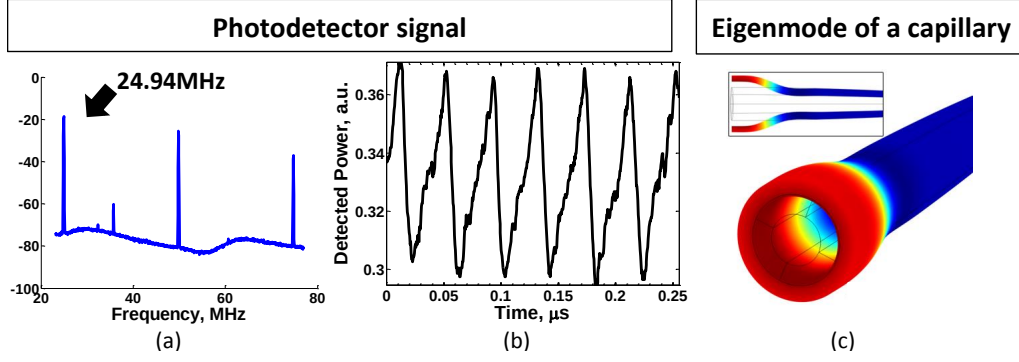


Figure 2.18: Representative results (Figure from [2]) (a) A breathing mechanical mode at 24.94 MHz in the microcapillary is excited by centrifugal radiation pressure by light circulating in an optical mode. Modulation of the input light by this mechanical vibration is observable on an electrical spectrum analyzer through beat-note generation on a photodetector placed in the forward-scattering direction. (b) An oscilloscope trace of the photodetector output signal (i.e. transmitted power) shows the periodic temporal interference of the input light and scattered light. (c) Finite element simulation for the corresponding breathing mode confirms that the observed optical modulation corresponds to an eigenmechanical frequency. Colors represent deformation and the simulation is sliced at the capillary mid-point for presentation.

about 10^8 . Simultaneously, since liquids are now confined within the device, acoustic losses are brought under control and the device is able to maintain a high mechanical quality factor as well.

There are a few practical challenges associated with this fabrication method. For instance, the capillary material must be a good absorber for the 10.6 micron CO₂ laser radiation so that it can heat up sufficiently for the pulling process to take place. In this regard, the materials that have been tested for capillary fabrication are silica and quartz. Furthermore, the circular symmetry of the capillary is dictated by the relative power balance between the two lasers that are employed during the pulling step, and by the location of the capillary in the laser target zone. Since the circular symmetry of the device is a key parameter for maintaining high optical and mechanical quality factor, misalignment of the capillary preform in the CO₂ laser target zone before pulling or during pulling can be a concern and care must be taken to keep this under control.

On the other hand, this fabrication method provides a lot of flexibility in the fabrication of silica-based optomechanical capillary resonators. By

modulating the CO₂ laser power, the capillary diameter can be varied quite easily to suit the application. On demand spacing between adjacent bottle resonators is possible thanks to the high degree of computer control. Finally, control of the rate of pulling and the rate of feed in of the capillary preform provides an easy knob for controlling the capillary diameter.

In conclusion, the silica-based microcapillary platform as described is a low-cost, high-performance optical and optomechanical system that can be applied to a variety of studies with non-solid phase materials, including superfluids, and bio-analytes such as living cells. These devices can additionally leverage the very large body of literature on surface acoustic wave sensing of gases and liquids. As a result, this is an enabling technology for optical sensing applications.

CHAPTER 3

NUMERICAL SIMULATION OF OMFRS

3.1 Introduction

Motivation

¹ When I first joined our group, Dr. Bahl’s research results had shown that the parametric excitation of mechanical oscillations in fluid-filled OMFRs is possible. High-Q mechanical resonances ranging from 2 MHz – 11 GHz can be excited in this manner [54, 67]. It had also been shown that the mechanical oscillation frequency is sensitive to the fluid present within the device, proving that light can be used to sense mechanical properties of the fluid. One may then imagine a system in which a single cell passes through the device, and its acoustic properties are sensed in a label-free manner. However, at that point, no predictive analytical or computational model existed that can elucidate how the optical physics, solid mechanics, and fluid mechanics are inter-related in this highly complex system.

Existing analytical models, at that time, for opto-mechanical coupling only described very simplified situations, and therefore, numerical FEA approaches were generally used to compute the optical and acoustic modes of these resonators [70, 104] separately. Efforts on computationally modeling the OMFR platform had shown that the mechanical modes take the form of “breathing modes”, “wineglass modes”, and “whispering-gallery modes” [54, 70, 105], while the optical modes take on fairly complex shapes. Nonetheless, it was reasonable to assume that properties of the fluid present inside the OMFR, such as density and viscosity, will have a nontrivial effect on the

¹Portion of this chapter is reprinted with permission from Zhu, K., Han, K., et. al., (2014), “Opto-acoustic sensing of fluids and bioparticles with optomechanofluidic resonators,” The European Physical Journal Special Topics, 223(10), 1937-1947.

dynamics of the overall coupled system, especially considering the experimental data (Figure 6 of [54]). No previous modeling efforts incorporated the liquid or its properties. No published models existed at that time that could capture the complexity of the optical-mechanical-fluidic interactions within OMFRs, and that could predict the non-monotonic behavior of the sensor [54]. The speed of sound balance formulation suggested in [9] was at best a guess, with two free parameters that were fitted to experimental data. Clearly, a more scientific approach was needed. Therefore, prior to my experimental exploration of the OMFR, the first theoretical task was to come up with a theoretical/numerical model of the opto-mechano-fluidic system, to understand how the solid shell interacts with the fluid inside the OMFR. More importantly if we were to use the OMFR as a sensor, what is the effect of fluid properties to the measurable resonant behavior, i.e., the mechanical resonance frequency and mechanical Q factor.

Single harmonic oscillator model

A solid microstructure system that interacts with viscous fluid can be modeled as a single harmonic oscillator with the equation of motion [106]:

$$m_0\ddot{x}(t) + d_0\dot{x}(t) + k_0x(t) = F_f(t) + F_{dr}(t), \quad (3.1)$$

where m_0 is the effective mass of the system in vacuum, d_0 is the damping coefficient of the system in vacuum, k_0 is the effective spring constant of the system in vacuum. Thus the undamped resonance frequency of the system in vacuum is $\Omega_{uo} = \sqrt{k_0/m_0}$, while the resonance frequency in vacuum is given by

$$\Omega_{r0} = \Omega_{uo} \sqrt{1 - 2\zeta_0^2}, \quad (3.2)$$

where, $\zeta_0 = d_0/2\sqrt{m_0k_0}$ is the damping ratio in vacuum, which is less than $1/\sqrt{2}$ for an oscillator, but typically negligible for a high Q system. $F_{dr}(t)$ is the driving force on the system, and $F_f(t)$ is the fluidic force acting on the resonator. $F_f(t)$, also known as “hydrodynamic loading”, results from the reaction force when the solid microstructure interacts with the fluid and the surrounding fluid is entrained by the microstructure. We can further

divide the hydrodynamic loading into viscous force and pressure force. The portion of the hydrodynamic loading, which is proportional to velocity of the microstructure, adds additional damping to the system. The portion of the hydrodynamic loading, which is proportional to the acceleration of the microstructure, adds additional mass to the system. The portion of the hydrodynamic loading, which is proportional to the displacement of the microstructure, adds additional spring constant to the system. By incorporating the hydrodynamic loading to the left hand side of Equation 3.1, and rewriting it in frequency domain, we get the amplitude transfer function

$$\frac{X(j\omega)}{F_{dr}(j\omega)} = \frac{1}{-\omega^2 m + j\omega d + k}. \quad (3.3)$$

$m = m_0 + \Delta m$ is the effective mass of the system in viscous fluid, where Δm is called induced mass; $d = d_0 + \Delta d$ is the effective damping coefficient of the system in viscous fluid, where Δd is the additional damping caused by fluidic force; $k = k_0 + \Delta k$ is the effective mass of the system in viscous fluid, where Δk is called added spring. Thus the undamped resonance frequency of the system in viscous fluid is $\Omega_u = \sqrt{k/m}$, while the resonance frequency in viscous fluid is given by

$$\Omega_r = \Omega_u \sqrt{1 - 2\zeta^2}, \quad (3.4)$$

where, $\zeta = d/2\sqrt{mk}$ is the damping ratio in viscous fluid. We can also obtain the mechanical quality factor, Q , by:

$$Q = \frac{\sqrt{mk}}{d} \quad (3.5)$$

Thus, to theoretically characterize the resonant behavior of the OMFR, we have to study the hydrodynamic loading, F_f , in detail. However, the general problem of a mechanical system interacting with an viscous fluid is extremely complex. Both the elasticity theory for the solid part and the Navier-Stokes equations for the fluid domain have to be solved together with the correct boundary conditions set for the fluid-solid interface. Especially, to solve the Navier-Stokes equations for fluid within the OMFR with diameter less than 200 μm , both the viscous effect and the compressibility of the fluid are important to get the right hydrodynamic loading. In this case, even if

we know the precise material and geometric properties, it is not possible to obtain an analytical solution for the coupled system.

Summary of most recent development of immersible optomechanical devices

After the OMFR was demonstrated, its sensitivity to fluid density, speed of sound [54], and viscosity has also been shown [3]. After our demonstrations, more optomechanical devices have appeared [64–66] (Figure 3.1). They are capable of working with fluids through direct liquid immersion. These devices are designed to be very small, with masses of picograms, to increase their mass sensitivity. The amplitude of the thermal-mechanical fluctuations of these devices is on the order of picometer or less [64], thus traditional optical or electrical detection methods are not easily applied. For example, Laser Doppler Vibrometers (LDVs) can be used to measure the amplitude/velocity response of the system to a known driving source such that both resonant frequency and Q factor of the system can be extracted easily as shown in [106]. However, LDVs can only attain nm amplitude resolution. Therefore, optomechanical transduction is used in these studies due to its high sensitivity.

Moreover, to reduce the hydrodynamical loading and preserve the mechanical Q factor, these researchers [64–66] have chosen to use small, thin resonators, such as wheel resonator [64], disk resonator [65], or nano-beam resonator [66]. Due to their small masses, these resonators' eigen-frequencies are high and hence their Stokes boundary layer thickness is small (< 100 nm). The mechanical modes used in these studies are the modes with mostly in-plane motion, such as breathing mode, in the plane perpendicular to the thickness direction to further reduce the fluid entrainment.

To analyze the resonant behavior of these devices, several assumptions are made in these studies. First, in order to analytically/numerically study the interaction between the microstructure and its ambient fluid, the mode shape of the solid microstructure is first analytically/numerically solved and it is assumed to be unchanged after fluid immersion. Furthermore, the interaction between microstructure and its ambient fluid is broke down into a viscous interaction and an acoustic interaction, and studied separately. In the viscous regime, the fluid is assumed to be incompressible without any bulk viscosity,

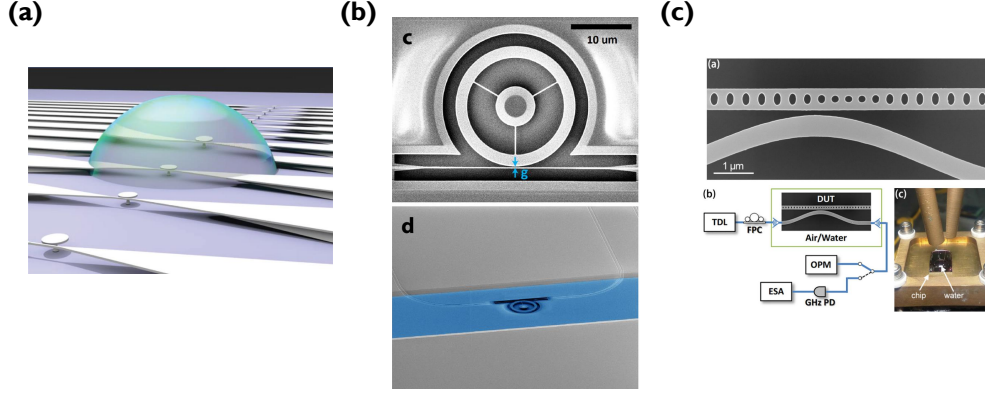


Figure 3.1: (a) (Reproduced from [65].) Nano-optomechanical disks. (b) (Reproduced from [64].) Nano-optomechanical wheel resonator. (c) (Reproduced from [66].) Nanobeam optomechanical crystal resonator.

therefore the microstructure dissipates energy via fluid friction associated with fluid shear viscosity. In the acoustic regime, the fluid is assumed to be compressible with both bulk and shear viscosity to be zero, therefore, the microstructure only radiates energy out into the ambient fluid by the means of sound waves.

When immersed in viscous fluid, if the microstructure is still relatively large (with size on the order of $10\ \mu\text{m}$ and mechanical frequency on the order of $100\ \text{MHz}$), and the viscous interaction dominates the acoustic interaction, the fluid is often studied only in the viscous regime. In the study by Fong et. al [64], the hydrodynamic loading is assumed to have a linear response to the small displacement of the microstructure, such that the linearized Navier-Stokes equation can be solved by adopting a ‘boundary integral method’. Both resonance frequency shift and the mechanical Q factor can be predicted with roughly 20% error.

When the immersed microstructure gets smaller (with size on the order of $1\ \mu\text{m}$ and mechanical frequency larger than $1\ \text{GHz}$), both the viscous and acoustic interaction are commensurately important and therefore have to be taken into account together. Gil-Santos et. al [65] proposed that since the exact solution of fluid field and the hydrodynamic loading force for a sphere vibrating in a viscoelastic liquid along one direction has been analytically solved, the vibrating microstructure can be decomposed in elementary vibrating spheres, and the total hydrodynamical loading can be obtained by summing up the forces on each elementary spheres. The result by applying

this method in the viscous regime shows that both mechanical frequency shift and damping rate are a function of the fluid density and viscosity. Also the damping rate is primarily proportional to $\sqrt{\rho_f \mu}$ (ρ_f is the density of fluid and μ is the dynamic viscosity of fluid), which agrees with the observation of viscosity measurement we made with the OMFR. Furthermore, their findings show that mechanical Q of their device decreases by a factor of 10 when the overall size shrinks by a factor of 100, while the device mass is reduced by a factor of 10^6 . Therefore, in the viscous regime, miniaturization of microstructure helps to increase the mechanical Q factor. Their analytical results agree well with their simulation results. In the acoustic regime, their analytical model reveals that both density and speed of sound of the fluid are important in determining the mechanical resonance frequency shift and Q factor. However, in the acoustic regime, their analytical results fail in quantitative fitting the simulation results. This is because the method used does not take the acoustic interferences of sound waves into account.

Although these results are very illuminating, we cannot simply apply these methods in analyzing the OMFR system. In these studies, the mechanical mode shape of microstructure is solved first analytically/numerically and is assumed to be unchanged by hydrodynamic loading. This assumption is acceptable when the resonance frequency shift of the system is very small (within 5%) due to fluid loading. However, due to the special bottle shape of the OMFR, both the solid wall and the liquid inside form an acoustic cavity together. The resonance frequency shift of the system due to fluid loading can be larger than 20%. Therefore, the elasticity theory for the solid part and the Navier-Stokes equations for the fluid domain have to be solved together to obtain the right mode shape, in both the solid and fluid domain. Since it is hard to get the analytical expression for a geometry as complex as the OMFR, next I show the numerical simulation setup I have developed for an eigenfrequency analysis of the OMFR system, in the acoustic regime. The hybrid fluid-shell mechanical modes are solved, with only the basic Pressure Acoustic module and Solid Mechanics module provided in the commercial Multiphysics Simulation software, COMSOL. This model unifies the solid shell domain and the inner fluid domain by applying interactive boundary conditions on the shell-fluid interface. As a result, I was able to investigate the acoustic sensitivity of the OMFRs to fluid properties (density and speed of sound), and identify the hybrid fluid-shell modes and fluid properties.

Also, as shown in Chapter 5, based on the simulation results, I am able to build a perturbation model to predict the OMFR's acoustic sensitivity to micro- and nano-particles.

3.2 Numerical simulation setup

Eigenfrequency analysis on the OMFR is performed through finite element analysis using COMSOL Multiphysics (version 4.3b) software. The OMFR consists of two physics domains, a solid shell made of fused-silica glass and the fluid contained inside. The shell has a bottle-like shape, which we can approximate using a radius of curvature, β . At the largest diameter region of the bottle, the capillary has an outer diameter of D , and an inner diameter of d . Therefore, for simplicity, a single aspect ratio parameter can be defined as $\alpha = d/D$, which describe how thick the the wall is compared with capillary diameter. In the real experiment, the tapered fiber is usually placed at the large diameter region of the OMFR. Thus, in this work, we will only simulate vibrational modes that are confined at the large region of the OMFR. The length of the OMFR is chosen to be long enough such that the effect of the boundary conditions at the two ends to the mechanical eigenfrequencies is minimized. The parameter used in the following simulation is summarized in Table 3.1. The boundary conditions used for the simulations are shown in Figure 3.2.

Table 3.1: Dimensions of the OMFR

D (μm)	β (μm)	α (μm)	L (μm)
100	5000	0.85	600

A governing equation for the eigenfrequency study is build respectively, for the solid-domain displacement field and the fluid-domain acoustic pressure field. For the solid domain, we use the Solid Mechanics interface of the Structural Mechanics Module in COMSOL, with the OMFR shell defined as linear elastic material. COMSOL solves the governing equation for eigenfrequency of the displacement as:

$$\rho_s \omega^2 \mathbf{u} - \nabla \cdot \boldsymbol{\sigma} = \mathbf{F}_V \quad (3.6)$$

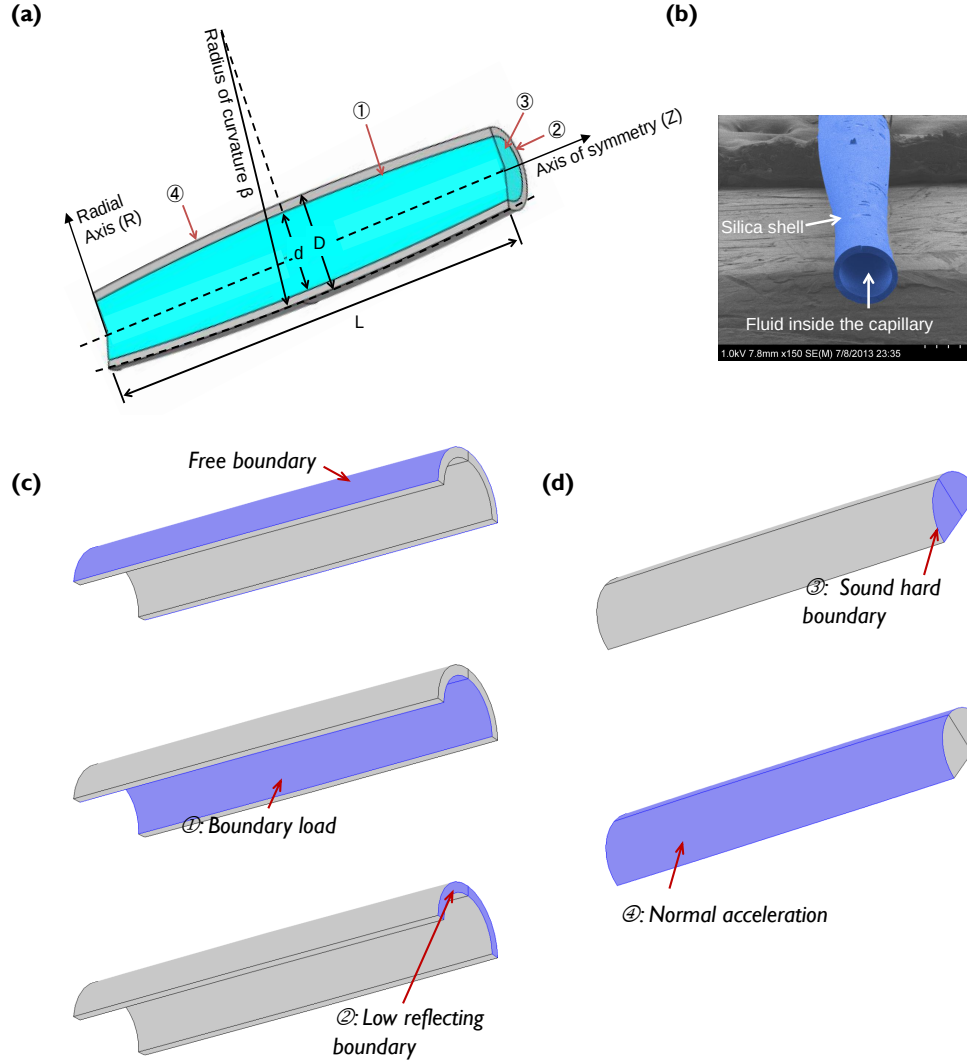


Figure 3.2: Geometry and boundary conditions for the OMFR.
 Geometry parameters: outer diameter of the shell, D ; inner diameter of the shell, d ; length, L ; aspect ratio, $\alpha = d/D$; radius of curvature, β ; Boundary conditions: ① Pressure loads to the shell and normal accelerations to fluid phase are applied at the fluid-solid interface, ② low-reflecting acoustic boundary condition at the ends of the fluid phase ③ sound hard boundary condition at the ends of the solid shell (normal acceleration vanishes) and ④ free boundary condition at outer surface of the shell. (b) Scanning electron micrograph of an OMFR. (c) Boundary conditions in the solid domain. (d) Boundary conditions in the fluid domain.

where ρ_s is the density of the solid material, ω is the eigenfrequency of a certain mode, $\mathbf{u}$ is the displacement tensor of the wall, $\boldsymbol{\sigma}$ is the stress tensor, and F_V is the external force (per unit volume) applied on the OMFR shell. The strain of the shell, $\boldsymbol{\epsilon}$, has the relationship with displacement field as $\boldsymbol{\epsilon} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$. To enable interaction between the two physics domains, a boundary load on the shell inner wall resulting from the pressure of fluid (p), which has the form of $-p\mathbf{n}$, is applied, where $\mathbf{n}$ is the outward-pointing unit normal vector of shell inner wall. Free boundary condition is applied on the outer surface of the shell since we neglect the interaction between air and the OMFR. Either fixed boundary condition or low-reflecting boundary condition can be applied to the two far ends of the OMFR, since the choice doesn't affect the mechanical modes confined at the large diameter region of the OMFR very much.

In the fluid domain, we use the Pressure Acoustic interface of the Acoustic module in COMSOL. COMSOL is set to solve the inhomogeneous form of Helmholtz equation given as,

$$\nabla \cdot \left(-\frac{\nabla p}{\rho_f} \right) - \frac{\omega^2 p}{\rho_f c^2} = Q \quad (3.7)$$

where ρ_f and c is the density and speed of sound of the fluid, respectively, and Q is the acoustic source term. In this case, this source term comes from normal accerleration of the OMFR inner surface, $\mathbf{a}_0$, such that, $\mathbf{n} \cdot \left(-\frac{\nabla p}{\rho_f} \right) = -\mathbf{n} \cdot \mathbf{a}_0$, where $\mathbf{n}$ is the outward-pointing unit normal vector of the outer surface of the fluid domain. When Q is positive, the shell is inputing energy into the fluid domain with an acceleration field $\mathbf{a}_0$. Sound hard boundary is applied to the two far ends of the fluid domain, where the normal acceleration is zero ($-\mathbf{n} \cdot \mathbf{a}_0 = 0$), or equivalently by linearized acoustic Euler equation, the normal pressure gradient is zero ($-\mathbf{n} \cdot \nabla p = 0$).

3.3 Hybrid fluid-shell vibrational modes

Due to the way we have set numerically, the resulting computationally evaluated modes are fluid-shell coupled vibrational modes with a single frequency. Some examples of the hybrid mode shapes are shown in Figure 3.3. Apparently, it is most conveniently to describe the system in cylindrical coordinate.

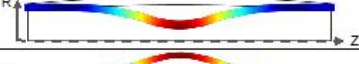
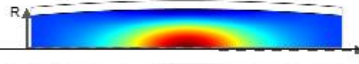
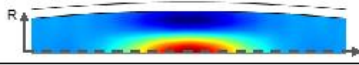
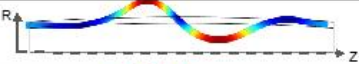
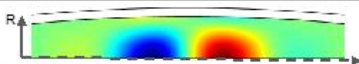
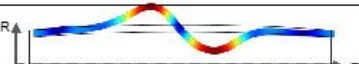
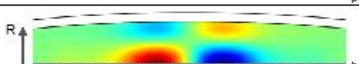
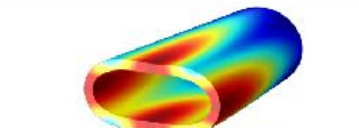
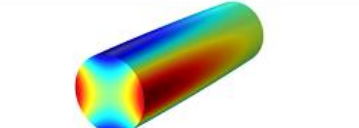
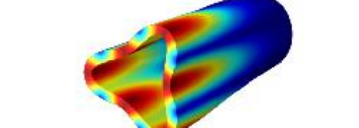
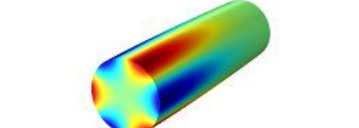
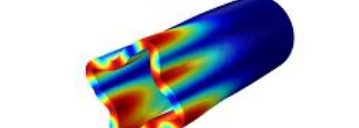
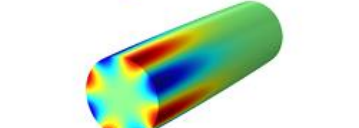
Mode Order	Mode Shape (cross-section view)		Mode Family
	Shell Displacement Field	Fluid Pressure Field	
(N,L,M)			
(1,1,0)			Breathing Modes
(2,1,0)			
(1,2,0)			
(2,2,0)			
(1,1,2)			Wineglass Modes
(1,1,3)			
(1,1,4)			

Figure 3.3: Mode shapes of the OMFR vibrational modes. $(N, L, M) = (\text{radial order}, \text{axial order}, \text{azimuthal order})$. Breathing modes are shown in the R-Z cross-section plane. Wineglass modes are cut at the $Z=0$ plane and the cross-section view is shown. Red color in the solid domain represents high displacement and blue color represents the low displacement. Red color in the fluid domain represents the positive acoustic pressure and the blue color shows the negative acoustic pressure. Black solid lines represent the undeformed boundaries of the capillary.

We have used 3 non-negative mode orders to describe the modeshape, namely, radial order (N), axial order (L), and azimuthal order (M). M describes the antinode number in the azimuthal direction. When $M = 0$, we call mechanical mode family breathing modes since the entire shell is expanding or contracting at the same time. When $M > 0$, we call the mechanical mode family wineglass mode. Similarly, N describes the antinode number in the radial direction. The only antinode in the radial direction when $N = 1$ occurs at the solid-fluid interface. Also, as we can see from mode (1,2,0) and (2,2,0), although their shell displacement fields are very similar, their acoustic pressure fields are entirely different. The mode shape plays an important role on the relationship between the eigenfrequency and the density of fluid.

3.4 Sensitivity of hybrid fluid-shell modes to fluid properties

Eigenfrequency dependence on fluid density: We now proceed to study the relationship of the eigenfrequency with the fluid density and speed of sound of these hybrid modes, respectively, through the parametric study in COMSOL. Figure 3.4 shows how fluid density can affect the eigenfrequencies of different vibrational modes differently. In this simulation here, the speed of sound of the fluid is fixed at 1500 m/s . With the fluid density sweeping from 400 kg/m^3 to 2200 kg/m^3 , we can clearly see that different mode families have different frequency trend and sensitivity to density. For example, $N = 1$ breathing modes show decreasing frequency trend against density (Figure 3.4a), while $N = 2$ breathing modes show increasing frequency trend against density (Figure 3.4b). Such opposite resonance frequency dependence on fluid density is a result of the different pressure mode shape from both families. In contrast, $N = 1$ breathing mode family and wineglass mode family both exhibit a decreasing frequency dependence on fluid density (Figure 3.4a,c). Their sensitivities to fluid density though are slightly different.

Eigenfrequency dependence on fluid sound speed: The frequency dependence on the speed of sound of the fluid, however, has a similar trend for different mode families. As we can see from figure 3.5, with the density of the fluid fixed at 1000 kg/m^3 , both breathing mode with $N = 1$ and $N = 2$,

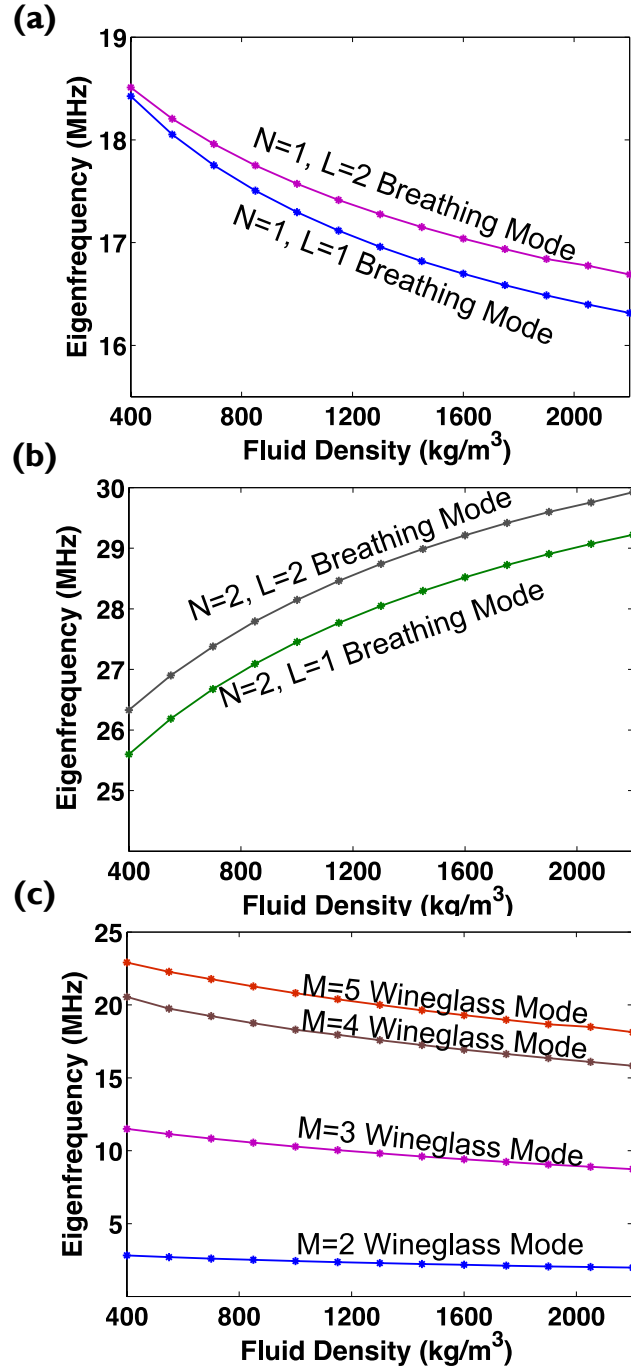


Figure 3.4: Eigenfrequency dependence on density of the fluid for (a) first radial order ($N=1$) breathing modes, (b) second radial order ($N=2$) breathing modes, and (c) wineglass modes with various azimuthal (M) orders. In this simulation, the speed of sound is set at 1500 m/s.

as well as the wineglass mode family are all monotonically increasing when the speed of sound is swept from 1000 to 1800 m/s . In order to understand why the eigenfrequency depends on the fluid density and speed of sound differently, we can simplify our system as a spring-mass system, where the resonance frequency is proportional to the effective stiffness of the system, but inversely proportional to the effective mass of the system. In the OMFR, the effective mass is proportional to the density of the fluid to the first order (if the mode shape remains the same when the density varies). Therefore, an increase of the density of fluid causes the eigenfrequency to decrease. The effective stiffness of the system is proportional to bulk modulus of the fluid, which can be expressed as $B = \rho_f c^2$. Evidently, the density of the fluid plays an opposite role here – an increase of the density, which causes the bulk modulus to increase, results in an increase of the eigenfrequency. These two opposing effects make the frequency tuning with density highly dependent on the vibrational mode shape, and can take on both increasing and decreasing trends. On the contrary, the frequency tuning with speed of sound only has a single effect, which is consistent with our experimental observation.

Eigenfrequency dependence on both fluid density and sound speed:

To see the the resonance frequency dependence on the both density and speed of sound together, we can simultaneously vary both of them and plot the resulting resonance frequency in a 3D surface plot. Figure 3.6a, and b shows an $N=2, L=1$ breathing and an $M=5, L=1$ wineglass mode, respectively. We can therefore use the fact that two distinct surfaces will have at least one intersection contour, to uniquely determine fluid properties through measuring the resonant frequency of two modes. Let us imagine that we experimentally measure two distinct mechanical modes at 20.8 MHz and 27.5 MHz. Let us also assume that from the simulation, we know that the 20.8 MHz mode is an $N=0, M=5, L=1$ wineglass mode and the 27.5 MHz mode is an $N=2, M=0, L=1$ breathing mode. With the 3D surfaces from both modes, we can easily plot a constant-frequency contour plot for both modes like shown in figure 3.6c at their measured frequency. In other words, figure 3.6c shows the possible combinations between density and speed of sound such that the desired resonant frequency can be met for both modes. From the intersection of two curves, we can therefore uniquely determine the only density and speed of sound combination that simultaneously satisfy both modes. In this

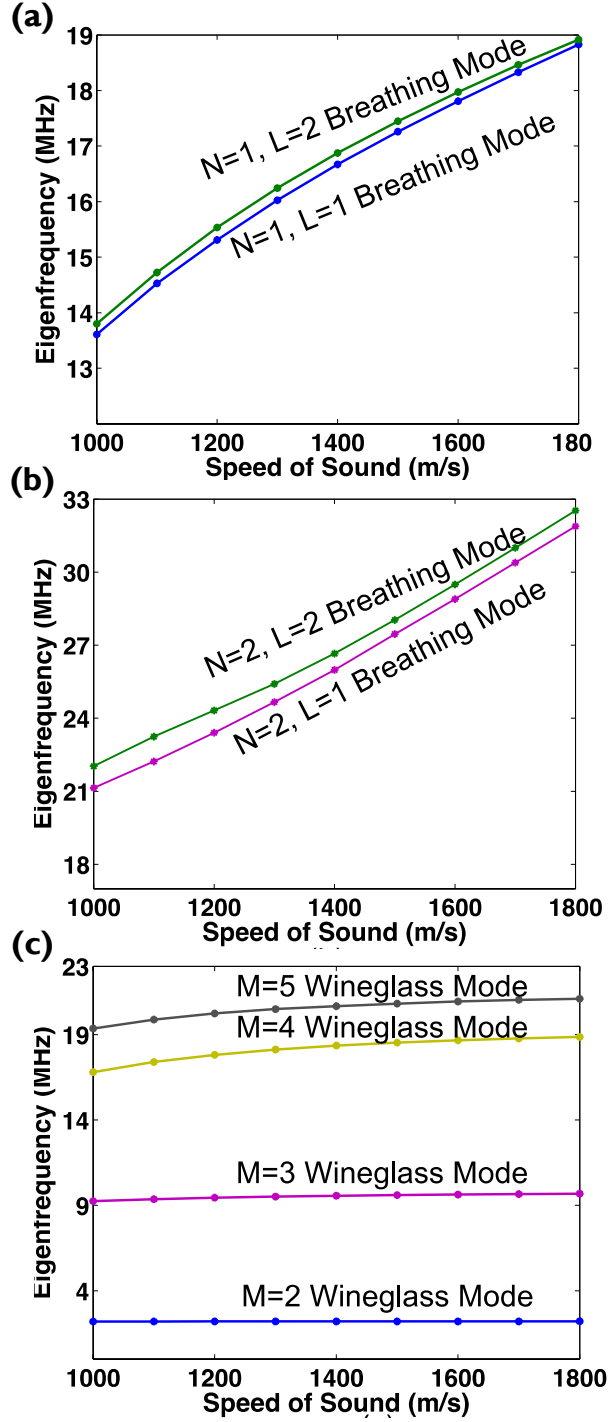


Figure 3.5: Eigenfrequency dependence on speed of sound of the fluid for (a) first radial order ($N=1$) breathing modes (b) second radial order ($N=2$) breathing modes (c) wineglass modes with various azimuthal (M) numbers. The density is set at 1000 kg/m^3 in this simulation.

case, the obtained density is 1000 kg/m^3 , and the speed of sound is 1500 m/s .

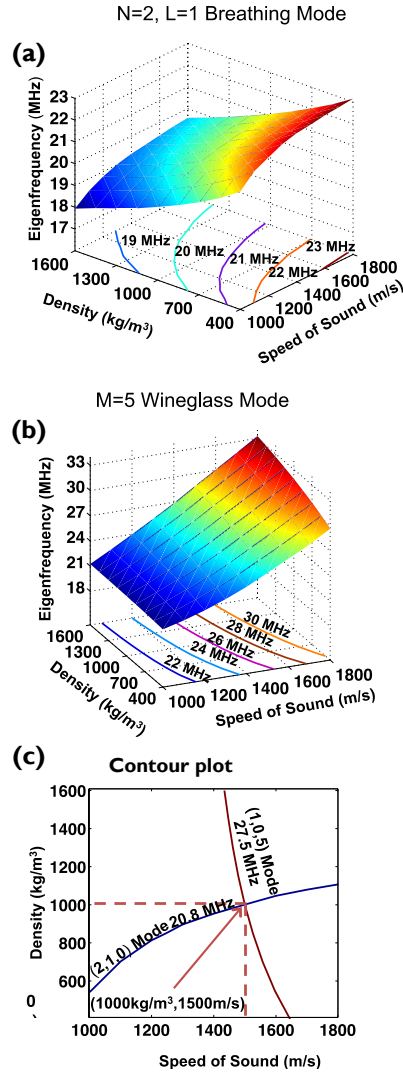


Figure 3.6: (a) 3D surface plot for frequency of the N=2, L=1 breathing mode as a function of fluid density and speed of sound. (b) 3D surface plot for frequency of the M=5 wineglass mode as a function of fluid density and speed of sound. (c) Constant frequency contour plot of both modes in (a) and (b). The intersection shows the only combination of density and sound speed that agrees with the proposed measured frequency.

3.5 Identification of hybrid fluid-shell vibrational modes and fluid properties

In order to use the aforementioned technique to measure the fluid properties like the density and speed of sound, one important assumption is that we actually know the correct identification of the vibrational modes measured. Unfortunately, this is hard to achieve for the OMFR system for two reasons. For the first, mode shapes in microdevices are hard to directly observe. For the second, it is hard to know the exact dimensions of the OMFR like the thickness of the capillary wall, and the radius of curvature of the capillary. Usually, one must rely on educated guess for the identification of the vibrational modes in microdevices. However, it is easy for us to monitor multiple mechanical modes with the same fluid at the same time, and measure the fluid properties using the above mentioned method for each of the multiple modes. We can therefore verify the measured fluid properties through comparing the results obtained for different mechanical modes.

For example, let us take a specific experimental result that we will look at in more details later in the next chapter [3]. The parameter of the OMFR is summarized below as well as the multiple resonant frequencies measured Table 3.2. The test fluid here is a NIST-calibrated standard viscosity oil with a known density of 0.884 g/ml. We would like to determine the speed of sound of this oil without knowing the exact corresponding vibrational modes for the measured mechanical frequencies. We first make an initial guess of the speed of sound to be 1300 m/s. For the 11.54 MHz mode, we can find computationally four candidate mechanical modes around this frequency (Figure 3.7a). Then we can calculate their eigenfrequency dependence on the speed of sound from 1100 m/s to 1600 m/s. It is therefore easy to narrow down our candidate list, as shown in Figure 3.7a, to two mechanical modes ($M=4, L=3$ mode and $M=4, L=2$ mode), since the 11.54 MHz horizontal line only intersects with the curves of these two modes. Thus we have obtained the possible speed of sound to be 1275 m/s and 1560 m/s. Similarly, we can find the possible speed of sound to be 1205 m/s, 1290 m/s, and 1455 m/s for the 17.53 MHz mode. The only possible speed of sound range, by comparing two candidate lists, is approximately from 1275 m/s to 1290 m/s. The uncertainty of the result is due to inaccuracies in the geometry of the OMFR and simplistic model that we use for simulation. We can also

conclude that the experimentally measured 11.54 MHz mode is an $M = 4$, $L = 3$ wineglass mode and the 17.53 MHz mode is an $M = 6$, $L = 1$ breathing mode. Furthermore, as another confirmation, we found that near the 13.91 MHz range, a $N = 2$, $L = 1$ breathing is the only possible computational solution. The corresponding speed of sound (Figure 3.7c) obtained for 13.91 MHz is 1240 m/s, which is close to 1275 m/s and thus acceptable.

Table 3.2: Dimensions of the OMFR and measured resonant frequencies

D (μm)	170
β (μm)	8000
α (μm)	0.823
L (μm)	600
Resonant frequency 1 (MHz)	11.54
Resonant frequency 2 (MHz)	13.91
Resonant frequency 3 (MHz)	17.53

3.6 Problem with analysis of sensitivity to particles in [1]

With this frequency simulation method, we have successfully extracted the density and speed of sound for some test fluids. We also did an estimation of the device sensitivity to single flowing particles in the fluid in the original paper [1], based on mass-loading effect of the particle due to the particle density difference with the ambient fluid. We have defined a spatial mass sensitivity as the fractional shift of the eigenfrequency per mass of fluid replaced through the equation

$$\frac{1}{f} \frac{df}{dm} = \frac{1}{2m_{eff}} \left(\frac{|\bar{P}|}{\max|P|} \right)^2 \quad (3.8)$$

where $\bar{P}$ is the local acoustic pressure at the particle center location and $\max|P|$ is the largest acoustic pressure inside the fluid. However, this analysis based on mass-loading theory, as it turned out later (Chapter 5), was incomplete, resulting in largely underestimating the sensitivity to micro-particles. As we will see next, the mass-loading effect is only a minor reason for the eigenfrequency to shift. The major effect, is the spring-hardening

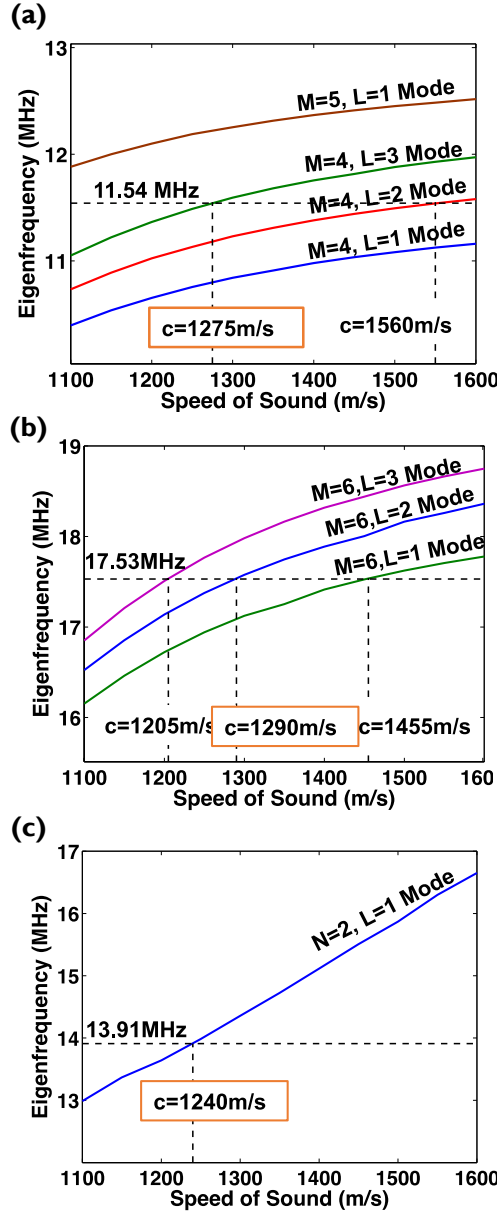


Figure 3.7: Computational identification of experimentally measured vibrational modes. We compute the eigenfrequencies of candidate modes around our experimental measurements of (a) 11.54 MHz, (b) 17.53 MHz, and (c) 13.91 MHz using a test fluid with known density of 884 kg/m^3 but unknown speed of sound. Here we conclude that the test fluid has speed of sound around 1275 m/s to 1290 m/s, and that the 11.54 MHz mode is an $M = 4$, $L = 3$ wineglass mode and the 17.53 MHz mode is an $M = 6$, $L = 2$ wineglass mode. The point of intersection in (c) from a $N = 2$, $L = 1$ breathing mode gives speed of sound of the fluid as 1240 m/s, which value is consistent with the one obtained from (a) and (b) within a small margin of error.

effect caused by the compressibility difference between the particle and the ambient fluid.

3.7 Useful techniques for running simulation

In this part, I want to briefly introduce some useful techniques when running simulation for the fluid-shell hybrid vibrational modes. In addition to the simulation setup that we have explained above, additional boundary conditions are needed for computing different mechanical modes, depending on their mode shape.

Techniques for getting wineglass modes: To obtain a wineglass mode more conveniently, since the mode shape is periodic in the azimuthal direction, we can only simulate a small wedge portion of the OMFR. For example, for an $M = 6$ wineglass mode, the periodicity angle in the azimuthal direction is $360^\circ/M = 60^\circ$. Therefore when setting up the geometry, we can first draw a 2D geometry with the shape of a half cross-section of the OMFR and revolve it with the symmetry axis of the OMFR (Figure 3.8a) with an angle of 60° . Next, with all the boundary conditions set as described above, we apply additional periodic boundary condition to both the fluid and solid domain (Figure 3.8b). Such a periodic condition on the two sides of the wedge can help enforce the value of the solution to be the same on the periodic boundaries. The result is shown in Figure 3.8c.

Techniques for getting breathing modes: For a breathing mode, the periodic boundary condition is still valid with an arbitrary periodicity angle. But since the shell of the OMFR is only expanding and contracting, we can apply a roller boundary condition on the two sides of the solid part (Figure 3.9a). The result is shown in Figure 3.8b.

Evaluation of local properties through virtual particles: Furthermore, as we will describe in later chapters for particle sensing, we often need to predict the frequency perturbations caused by particle transits at different locations. As we will see later in Chapter 5, this prediction can be made by evaluating the acoustic kinetic energy and potential energy of the unperturbed pressure field in the particle volume, which normally requires

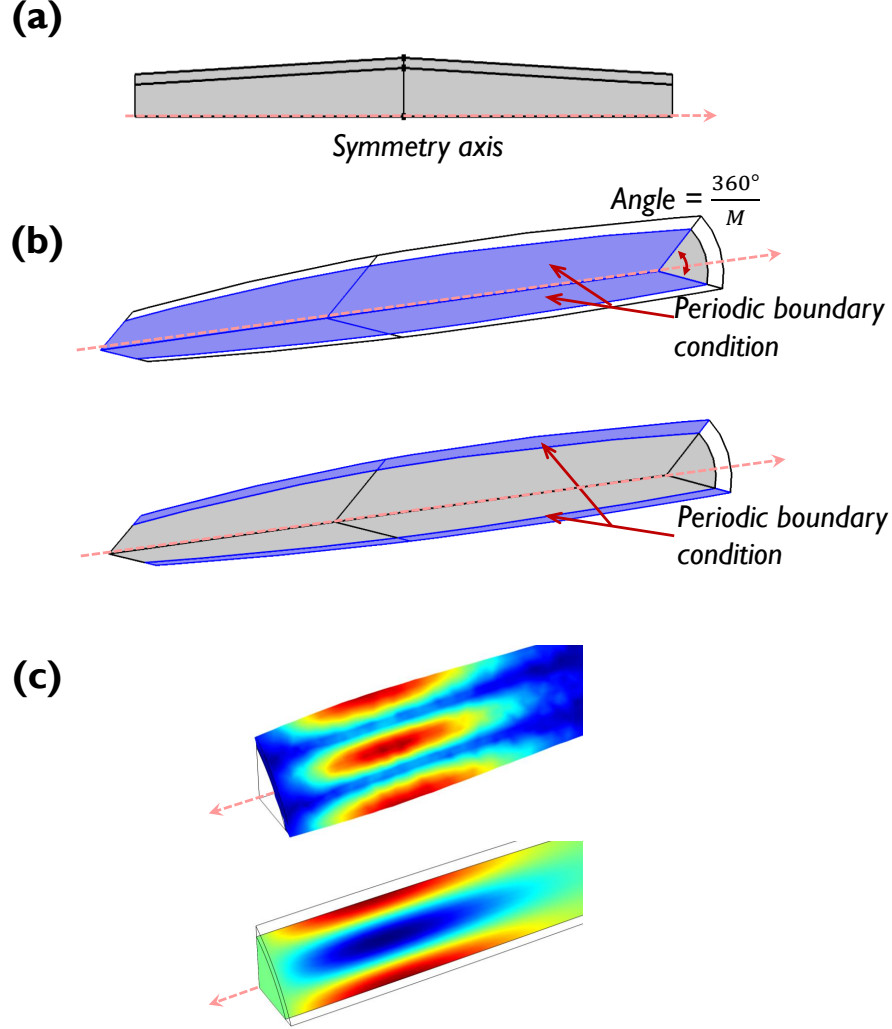


Figure 3.8: (a) A 2D geometry with the shape of half cross-section of the OMFR is first drawn and then revolved with respect to the revolving axis to get a wedge-like portion of the OMFR. (b) Periodic boundary condition is set to ensure the periodicity in the azimuthal direction for wineglass mode. The angle of the simulated wedge is set to be $360^\circ/M$, where M is the azimuthal order. In this case, $M = 6$. (c) Simulation result for $L = 2$, $M = 6$ wineglass mode. Only half of the wedge is shown here.

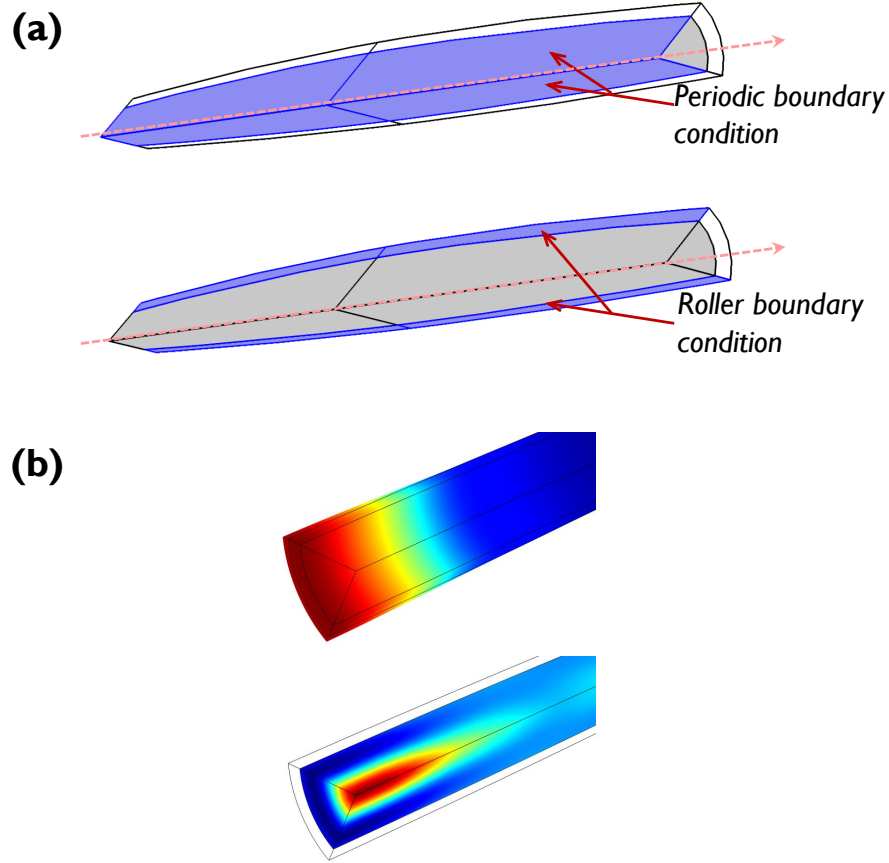


Figure 3.9: Periodic boundary condition is set on two sides of the fluid domain to ensure the periodicity in the azimuthal direction. Roller boundary condition is set on two sides of the solid domain to ensure the OMFR shell is only expanding and contracting in the radial direction. (b) Simulation result for $L = 1$, $M = 0$ breathing mode. Only half of the wedge is shown here.

exporting the simulation data and processing outside COMSOL. However, we can also do such integration in the simulation directly, by creating virtual particles at the desired radial location like shown in Figure 3.10. These virtual particles are simply entities that have the same simulation settings with the ambient fluid, the existence of which ideally should not affect our simulation results but provide us a easy ‘knob’ such that we can perform data manipulation easily within COMSOL.

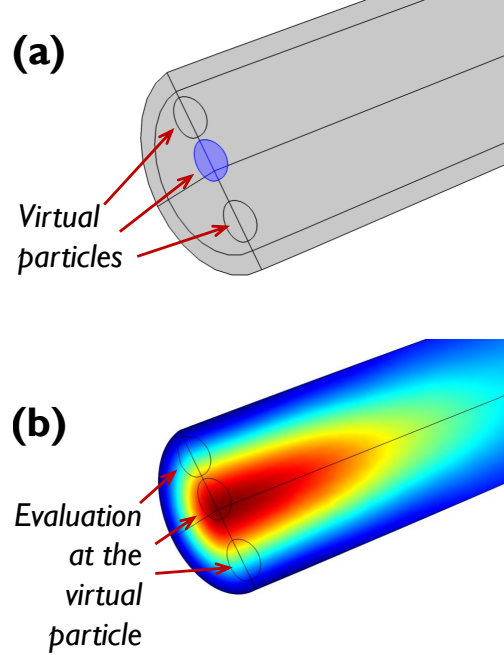


Figure 3.10: (a) Multiple virtual particles can be created in the fluid domain, by creating particle-like entities and keeping the simulation settings in these particles the same with the ambient fluid. (b) The existence of the virtual particles doesn’t affect the numerically simulated result. But we can use these entities to evaluate the properties such as acoustic kinetic energy and acoustic potential energy inside an entity (entities) by doing an volume integration of these properties over the desired entity (entities), without exporting the pressure field data.

Effect of mesh settings: The mesh setting of the simulation also plays an important role. Not only does the mesh setting determine the computational power the simulation requires, it also is important for us to successfully get the right mode. What I usually do is to set the sequence type as ‘User-controlled mesh’, and under the element size settings, use calibration for ‘Fluid dynamics’ with predefined ‘Extremely coarse’ mesh size. Also, the

overlap between a virtual particle entity and a OMFR symmetry line can create a small volume that requires much finer mesh than usual during the simulation (Figure 3.11a), and causes problematic simulation results. We can solve this issue by moving the particle entity slightly, such that this small volume disappears (Figure 3.11b), or becomes large enough (Figure 3.11c).

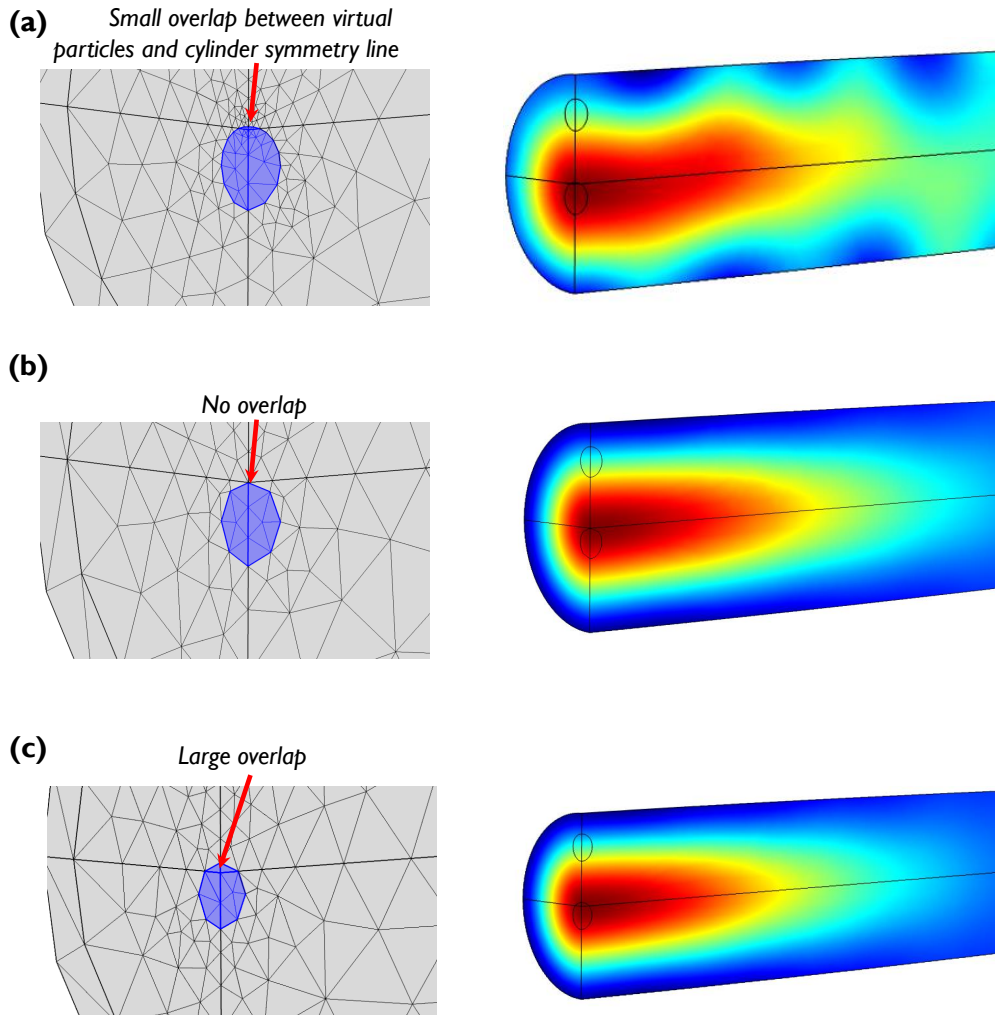


Figure 3.11: (a) Overlap between the virtual particle entity and the OMFR entity symmetry line causes problematic simulation result. Move the virtual particle entity slightly so that the overlap (b) disappears or (c) becomes large enough help solve the problem.

Scale of the eigenfrequency simulation is random: I would also like to note that the quantities associated with the mode shape of eigenfrequency study results using COMSOL, like the pressure field and the displacement

field, are at a random scale. This is because if you scale of the mode shape by any factor it is still a mode shape. Therefore, we cannot use the absolute simulated quantities like the pressure field or displacement field directly for our calculation when using eigenfrequency study. However, if we divide two quantities, which have the same scale, the scaling factor cancels. For example, term A and B introduced in Chapter 5 are defined as:

$$A = \frac{\int_{V_s} |p|^2 dV}{\int_{V_c} |p|^2 dV} \quad \text{and} \quad B = \frac{\int_{V_s} |\nabla p|^2 dV}{k_l^2 \int_{V_c} |p|^2 dV} . \quad (3.9)$$

It is thus more appropriate for us to use these two normalized terms in our perturbation calculation.

CHAPTER 4

MEASURING FLUID VISCOSITY WITH OMFRS

4.1 Introduction

¹ In previous chapters, we have shown both theoretically and experimentally the sensitivity of OMFR to fluid density and speed of sound. The natural question to ask next is that whether we can extract the fluid viscosity as well from the OMFR measurement. In this work, we demonstrate an experimental system for optomechanically measuring the dynamic viscosity, μ , of various test fluids using sample volumes in the nanoliter regime.

4.2 Setup and working principle

In this work, the device we use has a $\sim 170 \mu\text{m}$ diameter and $\sim 15 \mu\text{m}$ wall thickness. One end of the device is left open while the other end is connected to a syringe through which analytes can be infused (Fig. 4.1). The sensing volume contained within is about 20 nl, and can be reduced by changing the fabrication parameters.

Continuous-wave 1550 nm laser light is coupled into the optical whispering-gallery modes (Q-factor $\sim 10^7$) of the OMFR by evanescent coupling [85] through a tapered optical fiber (Fig. 4.1). The taper is not in contact with the device, which prevents additional damping effects. The radiation pressure of light is capable of actuating eigenmechanical oscillations through the optomechanical parametric instability [67] in this device (Fig. 4.2(a)). Mechanical modulation of the device geometry generates optical sidebands of the input light. Even when the parametric actuation threshold power is not reached, stochastic thermal fluctuations (Langevin noise force) provide a

¹Portion of this chapter is reprinted with permission from Han, K., Zhu, K., and Bahl, G. (2014). “Opto-mechano-fluidic viscometer,” *Applied Physics Letters*, 105(1), 14103.

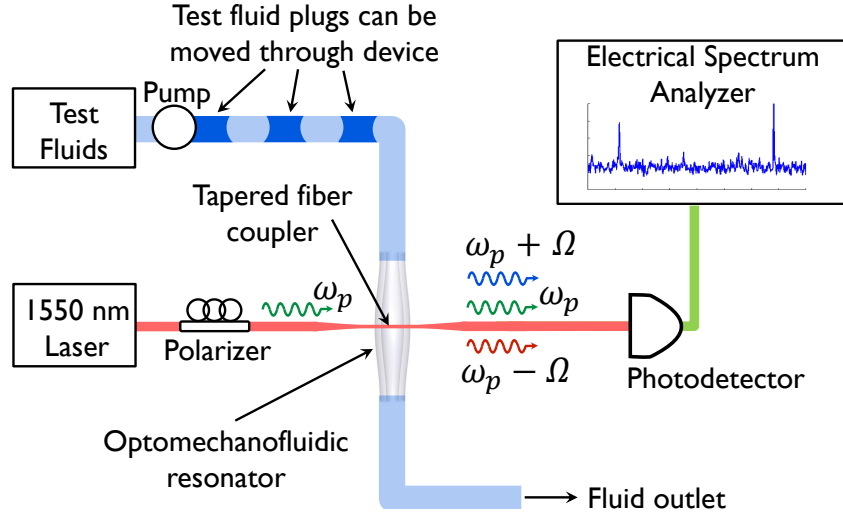


Figure 4.1: Experimental overview: The temporal interference of optical signals at the photodetector generates an electronically measurable signal, allowing the mechanical power spectrum to be measured using an electrical spectrum analyzer. Test fluids can be pumped in and out of the resonator, changing the effective mass and stiffness as well as the damping loss rate.

detectable amount of quiescent energy to the mechanical degrees of freedom. We can electronically measure the noise spectrum of the mechanical mode by observing the beating between input and scattered light on a photodetector (Fig. 4.1).

In this work, $\Omega \approx 11$ MHz, 13 MHz, and 17 MHz vibrational modes are selected (Fig. 4.2(a)). According to computational models, these modes are high-order wineglass modes and a breathing mode, where both fluid and shell are coupled into a hybrid eigenmode (Fig. 4.2(b)). For a continuously driven system, the mechanical damping losses in these hybrid shell-fluid modes can be obtained through optical measurement of the linewidth of the Lorentzian shaped mechanical noise spectrum as described in Fig. 4.3. Information on both the mechanical damping rate and the effective mass and stiffness of the hybrid system are embodied in the vibrational noise spectrum. By measuring the linewidth of the spectrum, Γ , we can quantify the mechanical damping rate of the system; by measuring the center frequency of the spectrum, Ω , we can quantify the oscillator effective mass and stiffness.

Optomechanical self oscillation [67, 107], however, narrows the vibrational linewidth due to amplification, which affects the ability to measure intrinsic

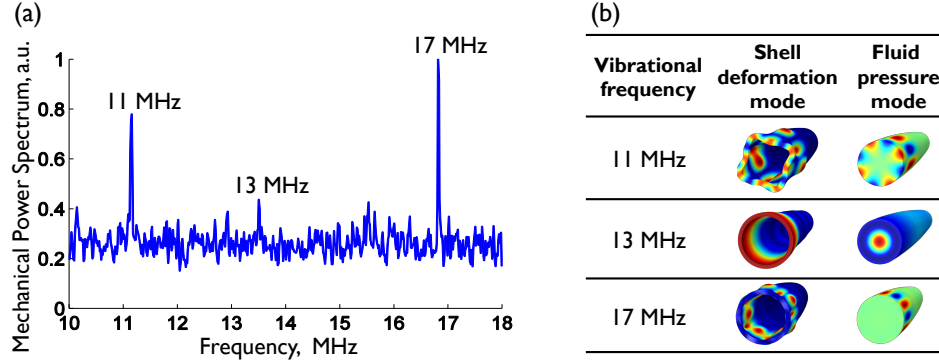


Figure 4.2: (a) Mechanical power spectrum using above-threshold, continuous-wave laser excitation power shows three mechanical modes. (b) Multiphysical simulations of solid OMFR shell and coupled pressure waves in fluid for the 11 MHz (high order wineglass mode), 13 MHz (breathing mode), and 17 MHz (high order wineglass mode) vibrational resonances.

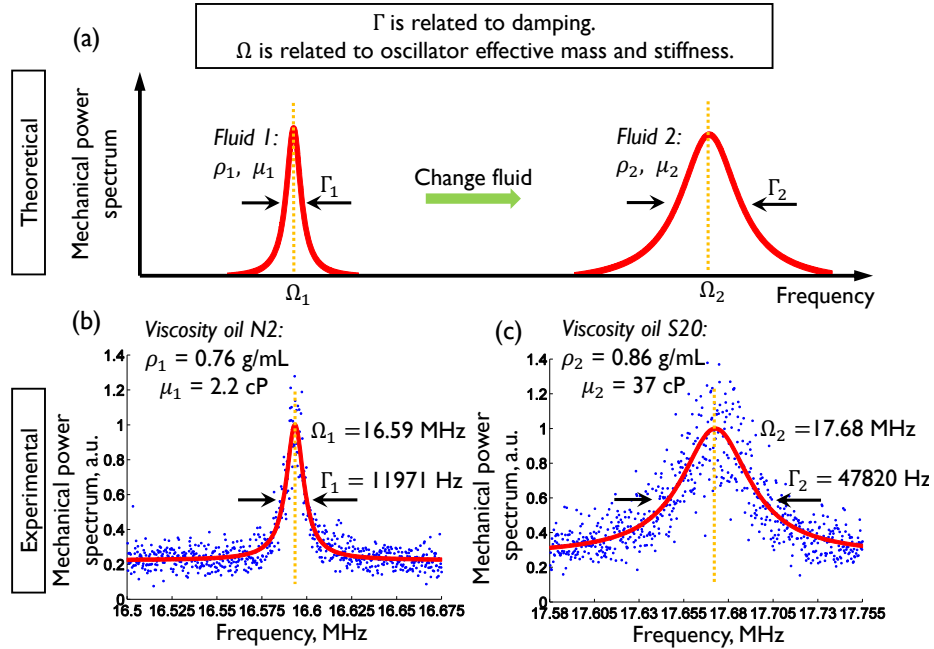


Figure 4.3: (a) Theoretically, the vibrational noise spectrum contains information about both the mechanical damping rate and the effective mass and stiffness of the hybrid system. The linewidth (-3 dB bandwidth) of the spectrum, Γ , is related to damping; and the vibrational frequency, Ω , is related to the oscillator effective mass and stiffness. Example measurement of the vibrational noise spectrum of ~ 17 MHz mode with (b) N2 viscosity oil, and with (c) S20 viscosity oil. In both cases, there are linewidth and center frequency changes.

oscillator damping. Here we make sure to employ very low subthreshold input optical power to avoid amplifying the mechanical motion to the self oscillation point. Specifically, the typical threshold input optical power for oscillation for our system is in the range of 10 mW when the OMFR is loaded with liquids, and we ensured that the input optical power was always less than 0.5 mW. In such a low-power situation, as described in [108], the influence of the coupled optical power on the linewidth can be neglected. This preserves the intrinsic damping and natural linewidth of the vibrational spectrum, allowing us to quantify intrinsic loss rates.

4.3 Results

To calibrate the optomechanical viscometers, we use seven viscosity standard oils (Cannon Instrument Company – Table 4.1). For each of the three vibrational modes in Fig. 4.2(a), we plot in Fig. 4.4(a) the density normalized experimentally measured mechanical mode linewidth $\Gamma/\sqrt{\rho}$, against the square root of the viscosity, $\sqrt{\mu}$.

Table 4.1: Properties of the calibration viscosity oils

Sample	Density (g/mL)	Viscosity (cP)
N2	0.762	2.2
N4	0.787	5.2
S6	0.878	10
N10	0.884	21
S20	0.863	37
N26	0.820	47
N35	0.868	75

The results in Fig. 4.4(a) can be understood by considering the nature of viscous damping in OMFR. The geometry of the resonator is a shell, which locally resembles the thin plate case discussed in [109]. Because of liquid entrainment within the resonator, viscous damping, associated with both shear and normal motion of the fluid relative to the resonator wall, occurs near the solid-fluid interface. For thin shells or plates, this damping is dominated by the shear motion of the fluid relative to the resonator wall. The attenuation rate due to viscous damping is proportional to $\sqrt{\rho\mu}$ at low values of viscosity [109]. At high viscosity, attenuation saturates due to the

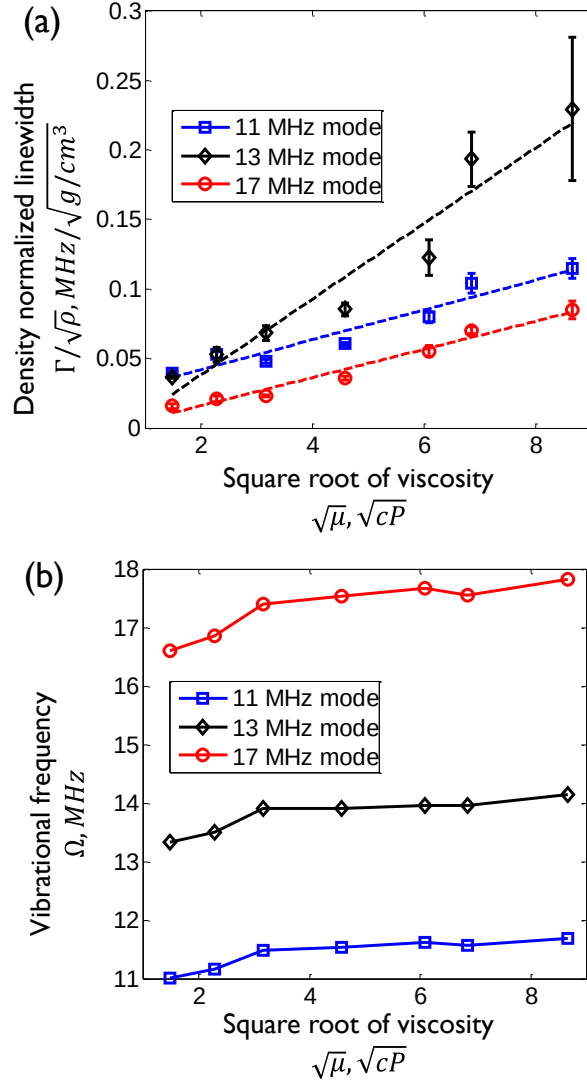


Figure 4.4: (a) Measured linewidth of selected mechanical modes operating with viscosity standard oils shows that the damping loss rate Γ increases linearly with square root of viscosity $\sqrt{\mu}$. Slope differences between different modes indicate different modes (Fig. 4.2(b)). Dashed lines are the linear fits. (b) Measured frequency of selected mechanical modes operating with viscosity standard oils. The three selected modes have similar frequency trend. This increasing frequency trend is likely caused by variation in density and the speed of sound of the test fluids [67].

viscoelastic nature of the fluid. Using Maxwell's model of a viscoelastic fluid, a critical viscosity separating low and high viscosity regimes can be defined by $\mu_c = \tau G_\infty$ at which $2\pi\Omega\tau = 1$, where Ω is the mechanical vibrational frequency, τ is the viscoelastic relaxation time in the liquid, and G_∞ is the high-frequency elastic rigidity modulus. Assuming a typical G_∞ value of 1 GPa [109], the μ_c for a 10 MHz device is 1.6×10^4 cP, indicating that our experiment is well within the linear regime. Since the intrinsic losses of the mechanical modes without liquids are very low ($Q_m \sim 10^3$ - 10^4 without fluids), the acoustic energy loss primarily arises from viscous damping associated with the fluid ($Q_m \sim 10^1$ - 10^3 with fluids). By electronically measuring the vibrational noise spectrum of the mechanical mode, the mechanical mode linewidth, Γ , is obtained (Fig. 4.3(b),(c)). Since Γ is proportional to loss rate, it is also proportional to $\sqrt{\rho\mu}$ at low values of viscosity. In order to isolate the effects of viscosity, we normalize the linewidth against $\sqrt{\rho}$ to obtain Fig. 4.4(a). We note that the linewidth slopes of the 11 MHz and 17 MHz modes are similar, but are lower than that of the 13 MHz mode, potentially indicating a difference in the mechanical mode families. This can be intuitively understood as a greater amount of mechanical energy of the 13 MHz breathing mode is stored in the fluid and is subject to viscous dissipation.

The conclusion that the loss rate due to viscous damping is mainly proportional to $\sqrt{\rho\mu}$ agrees with the expression given in [65] like we have summarized in Chapter 3. However, like we have discussed in Chapter 3, to more accurately describe the attenuation rate of the system, both viscous loss associated with the viscous interaction, and acoustic radiation loss have to be taken into account. Due to the complex geometry of the OMFR and the strong fluid-shell coupling, analytically exact solution is out of reach. However, the numerical methods used in [65] for estimating the viscous loss and radiation loss separately, in the future, can be adopted to provide more insights of the problem.

In addition, the measured relationship between the line center frequency Ω and $\sqrt{\mu}$ is plotted in Fig. 4.4(b). We see that the frequencies of all three modes are not constant but follow the same increasing trend. As we have discussed in Chapter 3, the mechanical frequency is affected by density, viscosity, and speed of sound altogether. Again, complete study of the hydrodynamic loading is required to fully understand of the mechanical frequency

shift of the OMFR.

4.4 Influence of the shell thickness

We now proceed to analyze how the sensitivity and dynamic range are influenced by the OMFR shell thickness. As discussed in [109], the attenuation rate is inversely proportional to the plate thickness due to the fact that a thicker plate can transmit more wave energy compared with the amount lost in viscous dissipation. Thus, in the OMFR case, a thinner shell should be used to increase the viscosity sensitivity. In contrast, a thicker shell should be used to expand the viscosity measurement dynamic range. However, as revealed by [70], thick shells can prevent the acoustic excitation from interacting with the liquid. So does increasing the mechanical frequency since it can decrease the radial depth of the acoustic mode. Therefore a suitable balance between operational frequency and shell thickness must be sought. Finally, we note that thin-shell resonators are also subject to potentially undesirable pressure effects [92] generated by pumped liquids (Chap. 3).

4.5 Comparison with other MEMS viscometers

Several types of fully-contained microfluidic oscillating microstructures for measuring fluid viscosity have been developed previously. They employ a variety of actuation methods, including electromagnetic [110], electrostatic [111], thermal [112], and piezoelectric [113] techniques. Our viscometer uses an optical interface, but the principle of viscosity measurement is still the damping of a vibrational mode. As a result, we expect similar limitations to dynamic range, noise performance, and sensitivity when compared against existing MEMS technologies.

Recent technological advances in optofluidics [5] have enabled biochemical sensors that employ many different optical techniques such as refractive index measurement [6–8], fluorescence [9], and surface enhanced Raman spectroscopy [10]. The recent introduction of optically-interfaced acoustics into optofluidic devices [54,105], inspired by lab-on-a-chip mechanical sensors [29], is providing researchers with a previously inaccessible mechanical degree of

freedom by which to perform such biochemical analyses. In this work we have invoked these opto-mechano-fluidic techniques to develop a microfluidic optomechanical sensor of liquid viscosity. Our result supplements established passive [114,115] and active [115,116] microrheological techniques for the optical measurement of fluid viscoelasticity, but does not contaminate the fluid with dispersed particles. Another advantage of our all-silica fiber-interfaced device is the operability in high temperature, remote, and electromagnetically noisy environments, such as engines, oil wells, and reaction chambers. Finally, the GHz-regime multifrequency capability of OMFRs [54] can enable high-frequency probes for mapping viscoelastic properties of fluids and boundary layers. In the long-term, we envision high-throughput viscoelastic measurements on flowing living cells, essentially enabling an optical mechanism for high-frequency acoustic flow cytometry.

CHAPTER 5

HIGH-THROUGHPUT SENSING OF FREELY FLOWING PARTICLES WITH OMFRS

5.1 Introduction

¹ As one type of biosensors, nanoparticle sensors for bio-analyte has become more and more important. Depending on whether is bioanalyte is labeled, we can also divide the biosensors into two groups. While labeled detection helps increase the sensitivity, the labeling processes require large effort and they also interfere with the function of a biomolecule. Therefore label-free detection is desired and it has been achieved in various ways. On the ensemble level, the bioanalytes are detected based on their concentration, like to detect the glucose concentration using glucose sensor. Single particle sensing, on the other hand, is very important for our understanding of biological processes, as well as for diagnostic and therapeutic applications. This, however, is harder to do since the sensitivity required for single particle sensing is very high. These days, with the development of the fabrication technique, highly sensitive electrical impedance sensors [117,118], optical cavities [119,120], and mechanical resonators [29,106,121] have emerged for single-particle sensing.

The sensitivity of resonant optical sensors to nanoparticles can be improved by either reducing mode volume [122,123] or by increasing the light-particle interaction time. The latter method often employs high-Q whispering gallery resonators (WGRs) [119,120,124], like those shown in Fig. 5.1a, with which even single viruses [125,126] and single molecules have been measured [127–129]. While extremely capable, such submerged-WGR methods [129,130] cannot provide 100% detection efficiency since they rely on random diffusion processes to bring particles to the sensor. Additionally, the randomized particle arrival location only enables statistical or binary mea-

¹Portion of this chapter is reprinted with permission from Han, K., Kim, J., and Bahl, G. (2016). “High-throughput sensing of freely flowing particles with optomechanofluidics,” *Optica*, 3(6), 585.

surements, and computation-intensive techniques [131] must be invoked for more information. Further, the need for particle adsorption and frequent rinsing are prohibitive for analysis of large numbers of particles. Optofluidic ring resonators (OFRRs) [68,132,133] improve on this significantly by confining the analyte within an internal microfluidic channel that ensures thorough analysis. However, analytes are still undetectable deep in the core of OFRRs since the resonant optical field only has short range of 1-2 μm in the proximity of the resonator shell. In contrast, commercial flow cytometers, which are non-resonant labeled optical sensors, deliver near-perfect capture efficiency on very large populations of bio-particles with impressive speeds of $>10,000$ particles/second [33], but cannot reach sensitivity comparable to WGRs.

An orthogonal sensing problem is the measurement of particle mechanical properties such as mass density, elastic moduli, and viscoelastic dissipation. The idea that a shift of a resonator's vibrational frequency can be used to infer mass of a particle, that is attached on top of the resonator, has been widely adopted in the cantilever-like (cantilevers, drums, and doubly clamped beams) microelectromechanical (MEMS) device [29,134,135]. Using this method, exquisite Yoctogram resolution by shooting mass on vibrating carbon nanobube in vacuum has been achieved [136]. But this method is inherently slow and its sensitivity drops when the device is immersed in fluids due to the mechanical damping loss increase. Recently, this technique has been extended to acquire other mechanical properties like stiffness and viscosity of the particle [134,137,138] due to the fact that a bioanalyte particle is not a perfect rigid body, and the resonant behaviors of the resonator are related to the stiffness and viscosity of the particle as well. We will discuss the application of these MEMS resonators in measuring the stiffness in detail in next chapter. Fast non-contact measurement is made possible with another MEMS device called suspended microchannel resonator (SMR) [29–32]. By confining liquid inside the suspended microchannel resonator (SMR) rather than submerging the resonator in the fluid, the aqueous environment needed for biological applications can be provided to cells and other bio-particles without performance degradation of the SMR from viscous damping. SMRs have shown impressive mass-measurement capabilities with very good temporal resolution using kHz-MHz vibrational modes. They have been employed for a wide range of biological applications, for example, monitoring cell growth [139] and studying single cell biophysical properties [140].

On the other hand, since the mass density and compressibility of particles do not couple directly to optical fields, it becomes necessary for optical techniques to invoke photon-phonon interactions that are sensitive to such properties. These optomechanical couplings [79, 141] also uniquely offer access to GHz-frequency vibrational modes that can help increase temporal resolution far beyond the capabilities offered by MEMS. Till date, all optomechanical particle and protein sensors [142, 143] have required adsorption of the analytes in order to perform detection (Fig. 5.1a) and no system has demonstrated a sensitivity to compressibility. Further, existing optomechanical systems are still handicapped by random diffusion and it has proven impractical to decouple the optical mode perturbation from the mechanical effect [143]. Thus, a fundamental scientific and technological gap persists on performing such mechanical measurements optically on micro/nano-particulate solutions with extremely high throughput and near-perfect capture efficiency.

This work presents a new approach to perform resonantly-enhanced optical sensing of *freely flowing particles* through the action of long-range phonons that extend between solid and fluid phases of the sensor and sample (Fig. 5.1b). We demonstrate this new principle by flowing analyte solutions confined within a simple microchannel optomechanical device [54, 105], thereby also eliminating reliance on random diffusion processes. Being similar to OFRRs, these opto-mechano-fluidic resonators (OMFRs) simultaneously confine light and sound in high-Q modes of their ‘bottle’ structure. The vibrational modes extend across both solid (shell) and fluid (core) phases [1, 3]. Sensing is thus mediated through photon coupling to these phonon modes that permeate the entire fluid volume (Fig. 5.1b). This potentially allows the OMFR sensor to cast a perfect net that captures measurements on every particle that is present in the sample. In stark contrast to all previous optical sensors, we show here that OMFRs exhibit the greatest sensitivity when the analytes are located *furthest* from the optical mode, deep in the core of the device. Light in the whispering-gallery resonances is coupled to the phonon modes through modulation of the optical path length (Fig. 5.1c). Although opto-mechanics enables amplification of these phonon modes through radiation pressure and Brillouin scattering [54, 105], here we only perform sub-threshold scattering measurements of the thermally occupied phonon mode (Fig. 5.1c). Previous experiments by this method have already con-

firmed that bulk fluid density, speed of sound, and viscosity [3, 54, 105] can be sensed. We explore the particle-sensing capabilities of the OMFR using bakers' yeast and two types of microbeads. Multimode sensing capability is demonstrated, which permits transit measurement of multiple particles with redundancy, and indicates future potential for inertial imaging [144]. The system also detects losses associated with individual particles, likely related to viscoelastic properties of the soft material and boundary loss at interface of particle and liquid. We estimate the fundamental throughput limit of the demonstrated sensor to exceed 10,000 events/sec, at noise-equivalent particle diameter that can approach 660 nm without any environmental controls or advanced instrumentation.

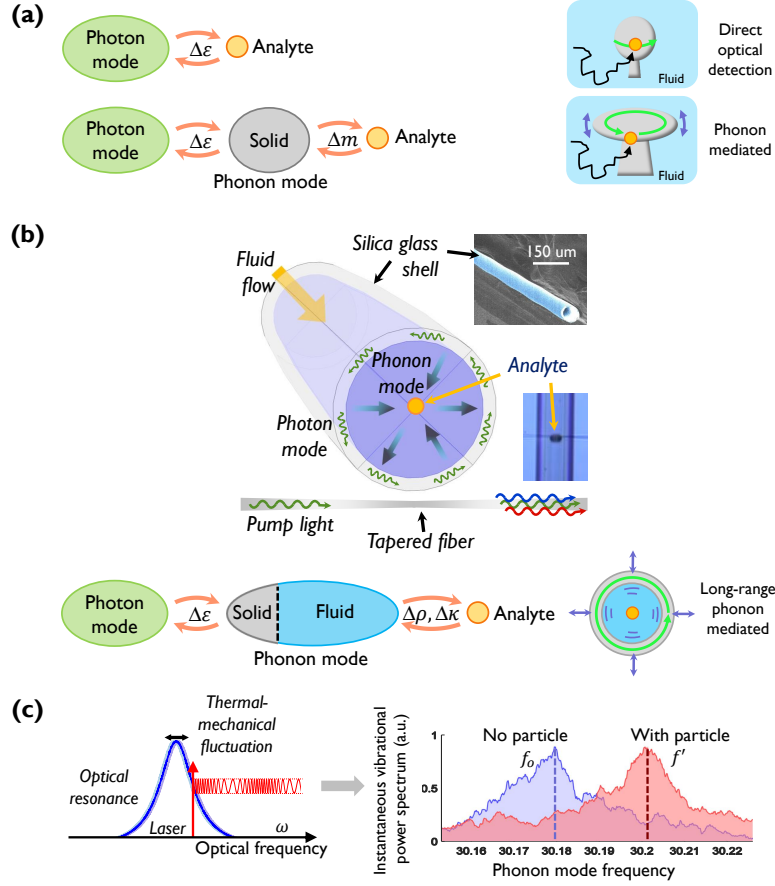


Figure 5.1: Opto-mechano-fluidic resonators (OMFRs) and principle of long-range phonon-mediated sensing. (a) In the past, particle detection through the ultra-high-Q whispering gallery resonator is done either by perturbing the photon mode directly (upper figure), or by mass-loading the optically induced phonon mode. Both methods requires the binding of particles on the sensor. (b) High-Q photon and phonon modes are simultaneously confined at large-diameter points of the OMFR. The phonons mediate a long-range interaction between light and the analyte particles flowing deep within the microfluidic channel. (c) Thermal-mechanical fluctuation of the OMFR phonon modes couple to the photonic resonance. The phonon mode spectrum is imprinted onto the scattered light, and is affected by the presence or absence of the particle. Particle properties can be inferred from frequency and linewidth fluctuations with high throughput.

5.2 Results

Our experiments are performed on silica OMFRs of diameter 40-60 μm , prepared through a linear drawing process (see Methods). The spectrum of the OMFR phonon modes is measured through opto-mechanical coupling with the light propagating in an adjacent tapered fiber (Fig. 5.2). The phonon spectrum is tracked in real-time and shows perturbations occurring during the particle transits (Fig. 5.3a). Two microscopes help verify each particle transit and triangulate the radial location within the OMFR (see Methods). For each spectrogram, curve fitting to a Lorentzian function is performed to obtain center-frequency traces as shown in Fig. 5.3 (see Methods). It is immediately observed that most transit events modify the phonon mode frequency to higher values. Less frequent events occur that shift the phonon frequency lower, though the magnitude of this shift is much smaller. These observations are consistently made using 6 (± 0.15) μm melamine resin particles (Fig. 5.3a), 11 (± 0.77) μm carboxyl magnetic polystyrene particles (Fig. 5.3b), and even household yeast of diameters ranging between 3-4 μm (Fig. 5.3c). The experiments shown here were performed using two OMFRs with diameter of 55 μm ($f_o = 30.18$ MHz) and 47 μm ($f_o = 40.75$ MHz).

Two novel insights are generated from the above experiments. First, the phonon frequency shifts observed generated by heavier-than-water particles do not follow the known mass-loading sensing mechanism used by mechanical [31] and opto-mechanical oscillators [142]. Second, the sensing mechanism is entirely due to an interaction with the phonon mode since the optical mode is not perturbed during a particle transit. This conclusion is reached since the signal strength remains unaffected during transits indicating that there is no dispersive or dissipative modification of the ultra-high-Q optical mode.

We employ a multiphysical finite-element model to compute the phonon eigenmodes of the hybrid resonator [1,3] (see Methods). The first-order mode shape crosssection for a 55 μm OMFR is visualized in Fig. 5.4a, indicating that the observed 30.18 MHz mode is a radial breathing mode. Here, the motion of the capillary wall creates a radially symmetric pressure field in fluid, resulting in storage of kinetic energy in both fluid and shell. A pressure maximum occurs at the center of the OMFR far away from the WGR optical mode of the shell. There is also a notable pressure node that appears near the shell inner wall. Next we will show an experimental verification of this

computed mode shape.

The two camera viewpoints captured during each particle transit of the OMFR sensing region enable triangulation of the particle radial position to within $5\ \mu\text{m}$ (see Methods). Fig. 5.4b plots the phonon mode frequency shift for both $6\ \mu\text{m}$ melamine resin particles and $11\ \mu\text{m}$ carboxyl magnetic polystyrene particles. It is immediately observed that this perturbation data follows the general shape of the standing pressure wave induced within the resonator with the maximum sensitivity at the center of the OMFR at the greatest distance from the optical mode, and a sensing null near the inner OMFR wall.

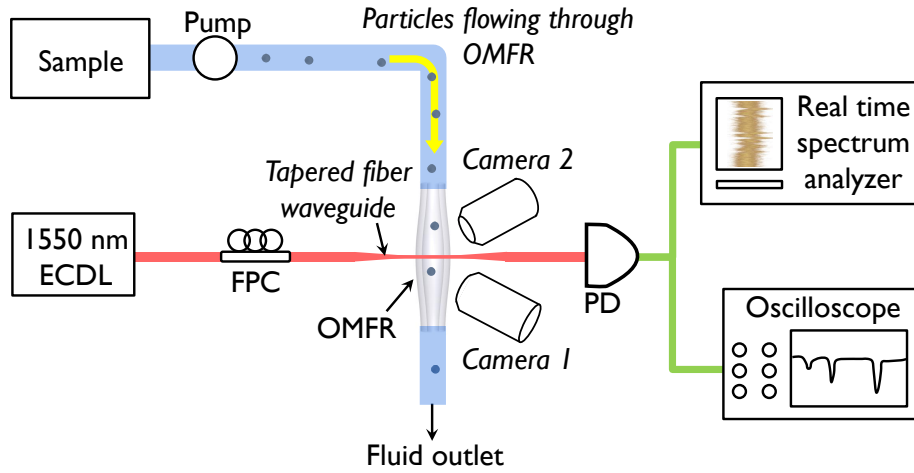


Figure 5.2: Experimental setup: Analytes are flowed through an OMFR using a syringe pump. A continuous wave fiber-coupled external cavity diode laser (ECDL) is used to probe the high-Q optical whispering gallery modes of the OMFR via tapered optical fiber. A photodetector performs heterodyne measurement of the forward scattered light, which is monitored using a real-time electronic spectrum analyzer (RSA). Perturbations of the vibrational phonon mode due to the particle thus can be tracked rapidly.

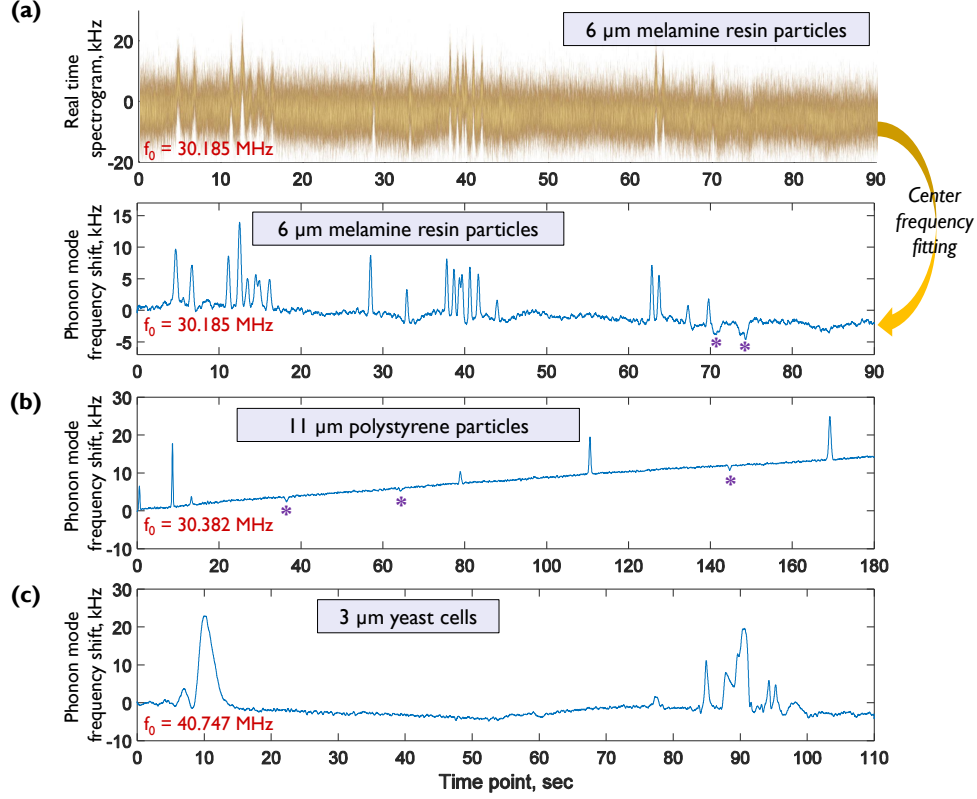


Figure 5.3: Optomechanical measurement of high speed particle transits. (a) Real time spectrogram (spectrum vs time) of the phonon mode is recorded during transits of 6 μm melamine resin particles through a 55 μm diameter OMFR. The color intensity corresponds to the noise power spectral density. The lower figure more clearly shows the center-frequency extracted from the spectrogram. The lower figure more clearly shows the center-frequency extracted from the spectrogram. (b) A similar dataset captured for 11 μm carboxyl magnetic polystyrene particles. Subfigures a and b are obtained with the same OMFR with phonon mode center-frequency $f_o \approx 30.18$ MHz. (c) Phonon mode center-frequency trace captured for yeast cells (3 to 4 μm diameter) transiting through a 47 μm OMFR. The phonon frequency is $f_o \approx 40.7$ MHz. The * icon points out the rarer downward frequency shifts.

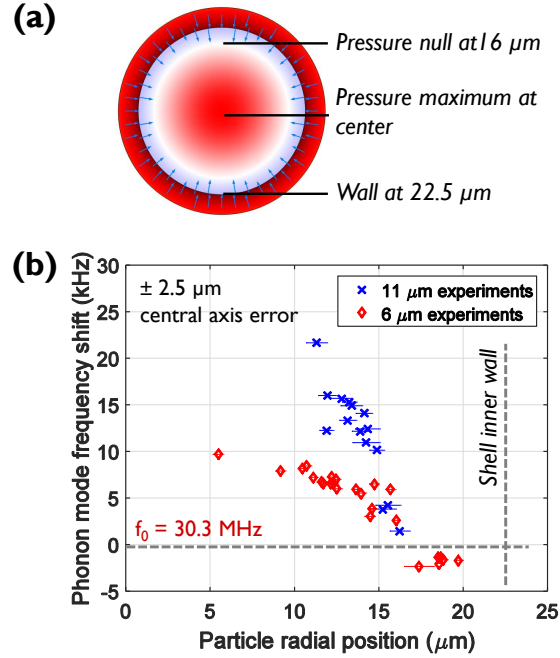


Figure 5.4: Characterizing frequency perturbation as a function of particle location. (a) Multiphysical finite-element model shows a phonon mode with eigenfrequency of 30 MHz. This radial breathing mode of the shell results in a standing pressure wave pattern with a pressure maximum at the center of the OMFR, and a pressure node at roughly 16 μm radial distance from the center. (b) The relationship between phonon mode frequency shift and particle radial position for both 6 μm and 11 μm particles matches the shape of the standing pressure wave within the resonator. The particle position of each particle is subject to both the fitting error as shown by the error bar here and a roughly $\pm 2.5 \mu\text{m}$ error in determining central axis of the OMFR, independently (see Methods). We note that the frequency shift is also subject to the size uncertainty of the particles – 2.5 % for the 6 μm particles and 7 % for the 11 μm particles.

5.3 Model for estimating the frequency perturbation

The interaction between an acoustic resonant cavity and a particle present inside the cavity manifests as a resonance frequency shift and acoustic potential acting on the particle [145]. Such mutual interaction has been used to achieve acoustic levitation for containerless studies and for material processing [146]. Recent studies reveal that such interactions not only depend on the properties of the particle and the fluid (size, density, compressibility) [147], but also on the phonon mode shape and location of the particle [145, 148]. Here we build a perturbation model to predict the frequency modifications by particle transits. As we will see from the deviation below, for this perturbation model, the critical assumption is that the introduction of the particle brings only small perturbation such that the perturbed pressure field in fluid, p' , is approximately equal to the unperturbed pressure field in fluid, i.e., $p' \approx p$.

For an inviscid fluid contained in an acoustic resonant chamber having a rigid wall, the angular resonant phonon frequency, Ω , can be found by solving the Helmholtz equation for the pressure field, p , of the fluid [149]:

$$\left(\frac{\Omega}{c_l}\right)^2 = \frac{\int_{V_c} |\nabla p|^2 dV}{\int_{V_c} |p|^2 dV}, \quad (5.1)$$

where subscript l stands for the fluid, the c_l is the speed of sound of the fluid, and the integrations are performed over the entire cavity fluid volume (V_c). This equation can be rewritten in the form:

$$\frac{\frac{1}{2} \int_{V_c} \frac{|\nabla p|^2}{\rho_l \Omega^2} dV}{\frac{1}{2} \int_{V_c} \frac{|p|^2}{\rho_l c_l^2} dV} = \frac{K_{max}}{V_{max}} = 1, \quad (5.2)$$

where ρ_l is the density of fluid. The numerator is equal to the maximum kinetic energy in the pressure field (K_{max}), and the denominator is equal to the maximum potential energy in the pressure field (V_{max}). Both K_{max} and V_{max} are equal to the total stored energy in the system therefore the ratio is always equal to one.

In the OMFR system, however, the resonator shell does participate in the phonon mode. Thus we will have to take interaction between the shell and the fluid into account while solving the Helmholtz equation, which manifests itself by the form of a surface integral over the fluid-shell boundary. As

we will show below, the integral remains unchanged if the perturbation of the particle is so small such that this integral term is essentially constant. Inspired by previous work [147], we find that for the two slightly different cavities, i.e., the sample-free cavity (case 1) and the sample-loaded cavity (case 2), we can rewrite the Helmholtz equation as

$$-\kappa_1\Omega_1^2p_1 = \vec{\nabla} \cdot \left(\frac{1}{\rho_1}\vec{\nabla}p_1\right), \quad (5.3)$$

$$-\kappa_2\Omega_2^2p_2 = \vec{\nabla} \cdot \left(\frac{1}{\rho_2}\vec{\nabla}p_2\right). \quad (5.4)$$

Here, ρ is the density of the material and κ represents the compressibility of the material. In the OMFR system, the introduction of the sample particle perturbs the resonance properties of the system, changing the pressure distribution within the OMFR from p_1 to p_2 . In case 1, $\rho_1 = \rho_l$ and $\kappa_1 = \kappa_l = 1/\rho_l c_l^2$ everywhere within the cavity. In case 2, density and compressibility are modified to $\rho_2 = \rho_s$ and $\kappa_2 = \kappa_s$ only at the sample location. Here subscript s indicates sample particle. ρ_s is the density of the particle. κ_s is the compressibility of the particle, given by $1/\rho_s(c_p^2 - \frac{4}{3}c_s^2)$, where c_p and c_s are the P-wave and S-wave (acoustic) velocity in the particle material, respectively. After multiplying Eqn. 5.3 by p_2 and Eqn. 5.4 by p_1 and integrating both equations over the entire cavity volume within the OMFR shell (V_c), we get

$$-\int_{V_c} \kappa_1\Omega_1^2p_1p_2dV = \int_{V_c} \vec{\nabla} \cdot \left(\frac{1}{\rho_1}\vec{\nabla}p_1\right)p_2dV, \quad (5.5)$$

$$-\int_{V_c} \kappa_2\Omega_2^2p_1p_2dV = \int_{V_c} \vec{\nabla} \cdot \left(\frac{1}{\rho_2}\vec{\nabla}p_2\right)p_1dV. \quad (5.6)$$

Applying divergence theorem to the right hand side of Eqn. 5.5 and Eqn. 5.6, we get

$$\begin{aligned} & -\int_{V_c} \kappa_1\Omega_1^2p_1p_2dV = \\ & \int_{S_c} \frac{p_2}{\rho_1}(\vec{\nabla}p_1 \cdot \vec{n})dS - \int_{V_c} \frac{1}{\rho_1}\vec{\nabla}p_1 \cdot \vec{\nabla}p_2dV, \end{aligned} \quad (5.7)$$

$$\begin{aligned}
& - \int_{V_c} \kappa_2 \Omega_2^2 p_1 p_2 dV = \\
& \int_{S_c} \frac{p_1}{\rho_2} (\vec{\nabla} p_2 \cdot \vec{n}) dS - \int_{V_c} \frac{1}{\rho_2} \vec{\nabla} p_1 \cdot \vec{\nabla} p_2 dV,
\end{aligned} \tag{5.8}$$

where S_c represents the surface of the cavity volume within the OMFR shell, and $\vec{n}$ is the outward-pointing unit normal vector seen from inside the fluid domain. Now if we further assume the perturbation due to sample loading is very limited such that $p_1 \approx p_2 \approx p$, the surface integral terms in Eqn. 5.7 and Eqn. 5.8 are both equal to $\int_{S_c} p/\rho_l (\vec{\nabla} p \cdot \vec{n}) dS$. Subtracting Eqn. 5.7 from Eqn. 5.8, we find

$$\begin{aligned}
& \int_{V_c} \kappa_1 \Omega_1^2 |p|^2 dV - \int_{V_c} \kappa_2 \Omega_2^2 |p|^2 dV = \\
& \int_{V_c} \frac{1}{\rho_1} |\vec{\nabla} p|^2 dV - \int_{V_c} \frac{1}{\rho_2} |\vec{\nabla} p|^2 dV.
\end{aligned} \tag{5.9}$$

Since the surface integrals cancel, we can see that the effect of the shell can be neglected if the pressure field change is small. Recall that $\kappa_1 = \kappa_2 = \kappa_l$ and $\rho_1 = \rho_2 = \rho_l$ in the fluid fraction. We thus can rewrite Eqn. 5.9 with a little rearrangement as

$$\begin{aligned}
& \kappa_l (\Omega_1^2 - \Omega_2^2) \int_{V_c} |p|^2 dV + (\kappa_l - \kappa_s) \Omega_2^2 \int_{V_s} |p|^2 dV = \\
& \left(\frac{1}{\rho_l} - \frac{1}{\rho_s} \right) \int_{V_s} |\vec{\nabla} p|^2 dV,
\end{aligned} \tag{5.10}$$

where V_s represents the sample volume. Finally, by defining $\Omega_2 = \Omega' = 2\pi f'$ and $\Omega_1 = \Omega_o = 2\pi f_o$, we obtain the perturbed resonance frequency

$$f' = f_o \sqrt{\frac{1 - \frac{\rho_s - \rho_l}{\rho_s} B}{1 + \frac{\kappa_s - \kappa_l}{\kappa_l} A}}, \tag{5.11}$$

$$\text{where } A = \frac{\int_{V_s} |p|^2 dV}{\int_{V_c} |p|^2 dV} \quad \text{and} \quad B = \frac{\int_{V_s} |\vec{\nabla} p|^2 dV}{k_l^2 \int_{V_c} |p|^2 dV},$$

where $k_l = 2\pi f_o/c_l$ is the unperturbed wavenumber associated with phonons in the fluid fraction of the OMFR. It is easy to see that A and B are propor-

tional to acoustic potential and kinetic energy associated with the particle volume, respectively.

For the 11 μm polystyrene particle, $\rho_s = 1.5 \text{ g/cm}^3$, $c_p = 2350 \text{ m/s}$, and $c_s = 1120 \text{ m/s}$ [150]. We can thus calculate $(\rho_s - \rho_l)/\rho_s = 0.0476$ and $(\kappa_s - \kappa_l)/\kappa_l = -0.457$. This allows us to estimate the frequency perturbation for a given vibrational mode through Eqn. 5.11, by computing values of A and B . We note that the evaluation of A and B can only be done numerically. As discussed in Chapter 3, we can create virtual particles at different positions and integrate the quantity of interest over the virtual particle volume. Example plots for both A and B , along with the frequency perturbation for the 11 μm polystyrene particles are presented in Figure 5.5a. The perturbed frequency clearly depends on the position of the particle, since both A and B are functions of position related to the pressure mode shape p . The frequency perturbations for both particles are plotted in Figure 5.5b, along with the experimental data as a comparison.

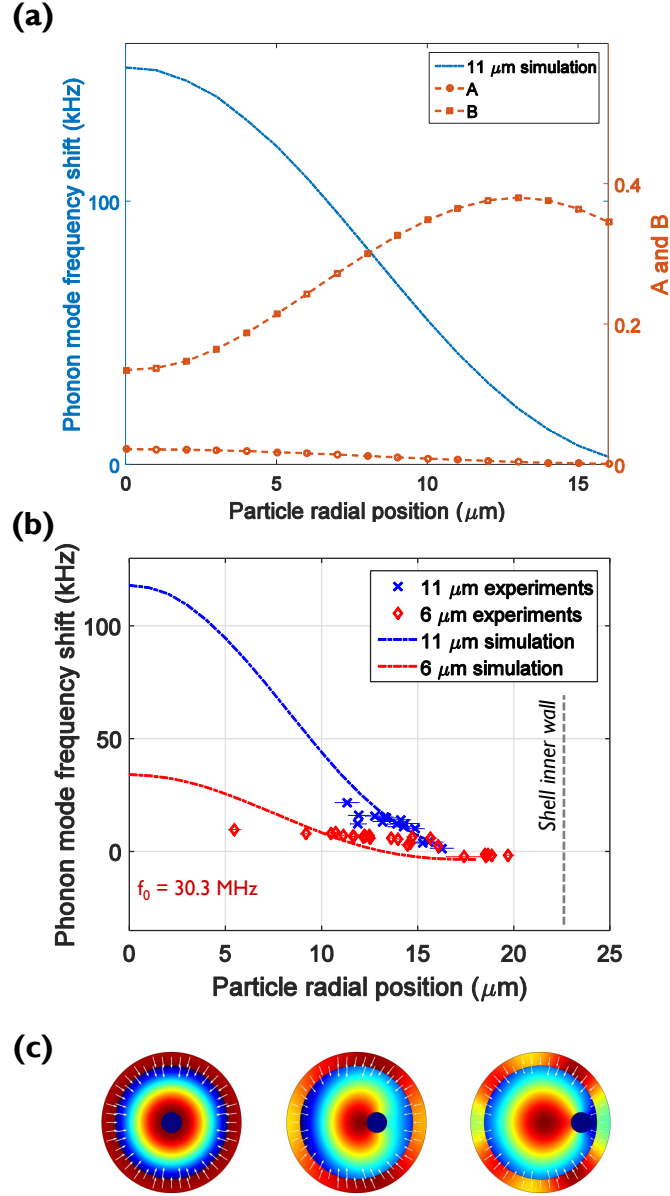


Figure 5.5: (a) The phonon frequency shift is affected by the position-dependent A and B parameters for the phonon mode (Eqn. 5.11) for the 11 μm particles. (b) The frequency predictions for both 6 μm and 11 μm particles are compared with the experimental results. (c) The phonon mode can be modified significantly by sound-hard spherical particles (dark blue circle) differently at various radial positions.

5.4 Discussion

Effect of particle radial location: Eqn. 5.11 shows that the perturbed frequency is affected by the density contrast $(\rho_s - \rho_l)/\rho_s$ and the compressibility contrast $(\kappa_s - \kappa_l)/\kappa_l$ of the particle in relation to the ambient carrier fluid. For heavier-than-water particles, the density contrast is positive whereas the compressibility contrast is generally negative. Particles at the center of the OMFR where the pressure is maximized create the largest perturbation in A , while B remains relatively small. This results in a frequency *increase* when a particle is present. On the other hand, when the particle is placed near a pressure node (a fluid velocity maximum), a decrease in the phonon frequency is expected.

Measuring density and compressibility: Using Eqn. 5.11, the density or the compressibility of the sample particle can be measured if the vibrational mode shape and other properties of the particle material are known. This can be especially useful for determining the material compressibility of particles without contact since density can generally be acquired through simple weighing. For example, for a 11 μm polystyrene particle at 14 μm radial offset from the OMFR center, we measured the phonon frequency shift to be 14.09 kHz. Given the density of the polystyrene, we can calculate the compressibility contrast to be -0.471. Thus, the polystyrene particles have estimated compressibility of $2.41 \times 10^{-10} \text{ Pa}^{-1}$.

Measuring small particles: Another interesting observation of the above analysis is that both the compressibility κ_s and the density ρ_s of the particle can be determined independently if the particle is much smaller compared with the acoustic wavelength. For example, A vanishes when the particle is at the pressure field maximum, resulting in the frequency perturbation being only sensitive to the density contrast. On the other hand, B vanishes when the particle is at the pressure node allowing measurement of the compressibility contrast. Ultimately, the sensitivity to particle location can be eliminated by implementing a sheath flow that brings particles through the sensor region via the same streamline. Such techniques are already implemented in flow cytometers.

Error sources: Residual errors in our current prediction model come from

multiple sources. First, we note that since neither the precise geometry of the OMFR, nor the accurate material properties of the OMFR are known, the calculation of A and B are only based on an estimated FEM model. Second, we have assumed that the pressure field remains unchanged upon addition of the particle. This assumption is not accurate, especially when the particle has an acoustic scattering effect that cannot be ignored. This can occur when the compressibility contrast is large, or when the particle size is similar to or larger than the acoustic wavelength. As we can see from Figure 5.5c, when the particle is relatively large and sound-hard, the acoustic scattering from the particle can modify the phonon modeshape severely. A better calibration of the system could be performed using particles with known material properties in the future.

High-throughput limit: Fig. 5.6a presents our fastest measured single particle transit over a 20 ms timescale. This measurement was limited by the available flow rate, and suggests a present maximum throughput of 25 particles/second. However, the instrumentation limit of the present experimental setup is far higher at 1000 particles/second, set by the sampling rate of the real-time spectrum analyzer (easily noted in the large number of individual samples in Fig. 5.6a). Fundamentally, the time constant τ associated with the center-frequency shift of energy stored, in response to mode perturbation, is related to the modal quality factor. This response timescale can be inferred by direct measurement of the dissipation rate from the phonon mode spectrum. For the presented 55 μm device the phonon mode linewidth $\Gamma = 1/\pi\tau$ is roughly 10 kHz. This implies a maximum sensing rate limit $1/\tau$ for this device is about 30,000 particles/second. This number could be reached in the future if sample flow control and faster signal processing hardware can be implemented. We also note that modes with higher dissipation rate are available and can greatly increase the sensing rate limit since the device resonance frequency typically exceeds 30 MHz.

Smallest measurable particle: To further understand the sensitivity limit, we characterized the phonon mode frequency stability of the sensor operating in air, without environmental protection or feedback stabilization. The Allan deviation data (shown in Fig. 5.18) indicates a noise-equivalent particle diameter (NEPD) of around 840 nm over an averaging time of 2 seconds.

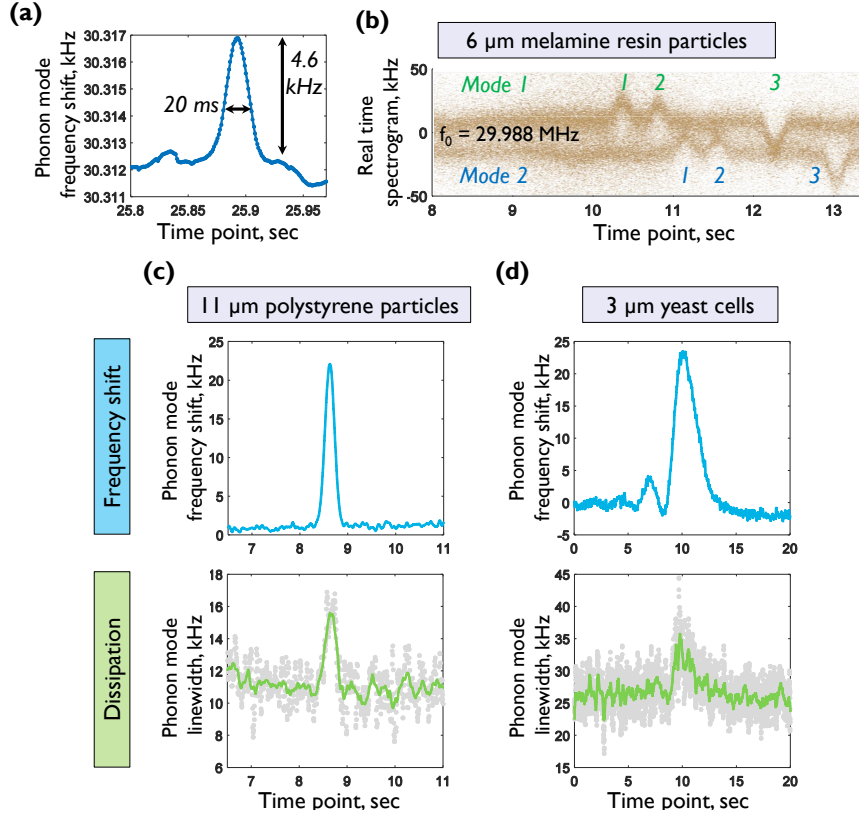


Figure 5.6: High throughput, multi-mode sensing, and dissipation measurement: (a) A fast flow measurement demonstrates a 20 ms timescale single particle transit. (b) A spectrogram example showing simultaneous sensing of two phonon modes. The transits of three consecutive 6 μm particles (with the first two in middle of the OMFR and the last one near the capillary side wall) result in similar frequency shift patterns for both modes. The time delay indicates differing spatial location of the phonon modes. (c)(d) Figures show the correlated fluctuations of phonon mode center-frequency and dissipation rate, both derived from the real-time spectrogram data. The data presented have the same time axes with Fig. 5.4b,c, depending on case.

We also note that bringing the device into oscillation mode as shown in [105] could lower the NEPD to 660 nm with an averaging time of only 0.2 seconds. Significant improvements are possible through feedback stabilization and environmental control. Smaller diameter OMFRs will indeed have higher sensitivity and can reach substantially smaller NEPD limit since the oscillation frequency scales as the inverse of particle diameter and the density and compressibility perturbations are more appreciable for a smaller device.

Multimode sensing: The wide spatial extent of the optical mode in bottle-shaped OMFRs also enables the simultaneous measurement of spatially distinct phonon modes. Fig. 5.6b shows an example measurement of two frequency-adjacent modes that react to transiting particles with a time lag. Correlating the two spectra provides information on flow speed and spatial separation, with the additional potential to make two independent measurements of the same particle. Additionally, this also enables multimode sensing [32] and inertial imaging [144] capabilities with OMFR devices.

Dissipation associated with particles: The spectrum of scattered photons also carries information on the phonon dissipation associated with the particle. This can be gleaned from linewidth observations [3] of the real-time optomechanical spectrum, where higher linewidth indicates greater phonon mode dissipation rate. Fig. 5.6c,d present direct evidence of increased phonon dissipation during transits previously shown in Fig. 5.3. The 30% increase of phonon dissipation rate measured during the transit of a 11 μm particle (Fig. 5.6c) is more than an order-of-magnitude greater than would be estimated from modification of the stored energy in the resonator by the particle. This implies that the increased dissipation is likely related to viscoelastic properties of the particle material, and boundary loss at interface of particle and liquid.

5.5 Methods

Device fabrication and setup

Opto-mechano-fluidic resonators (OMFRs) are fabricated [3] from fused-silica capillaries (Polymicro Technologies TSP-700850) by linear drawing under localized heating from 10.6 μm CO_2 lasers. The diameter of an OMFR can be varied along its length by modulating the laser power during the linear drawing process. This enables the fabrication of localized microbottle resonators with diameters spanning 40-60 μm in the widest region, where the optical and mechanical energy can be simultaneously confined [54]. One end of the centimeter-length device is connected to a fluid reservoir and syringe

pump for particle infusion, while the other end is left open as the outlet. The OMFR is vertically oriented such that even when the pumping is stopped, particles with a density larger than that of the fluid media continue to flow under the influence of gravity. In this manner, spurious effects due to applied internal pressure [92] can be minimized.

Optomechanical measurement of phonon modes

A continuous wave fiber-coupled external cavity diode laser (ECDL) at 1550 nm is used to provide the pump light to the OMFR resonant system. This light is coupled into high-Q optical whispering gallery modes of the OMFR via a tapered optical fiber. The thermal-mechanical fluctuations of the device modify the optical path length of the WGR modes at frequencies around 30–50 MHz, depending on device size. The resulting modulation of light in the fiber waveguide generates optical sidebands to the pump. The spectrum of the phonon mode can then be resolved via temporal interference of the pump and these sidebands on a photodetector, and can be measured by a real-time electronic spectrum analyzer (RSA) – Tektronix model RSA6120A. By tracking this vibrational power spectrum in real time using the RSA, perturbations of the vibrational phonon mode frequency and linewidth due to the particle can be observed. The obtained spectra are then fitted to a Lorentzian lineshape to find the spectral parameters during particle transits.

Center frequency tracking and frequency shift determination

Center-frequency tracking is performed through Lorentzian fitting of the instantaneous vibrational power spectrum (Figure 5.7). To do this, the spectrogram data in .mat format should be exported from the real-time spectrum analyzer first. Then the phonon mode center frequency trace can be tracked using Matlab code ‘Plotspectrogram2.m’ as attached in Appendix B. Such post-processing allows us to see details of the frequency perturbation during particle transits. Figure 5.8 shows the center-frequency trace matched with particle longitudinal position at various times. The phonon mode linewidth can also be extracted from the Lorentzian fit, allowing us to probe the additional dissipation induced by particles.

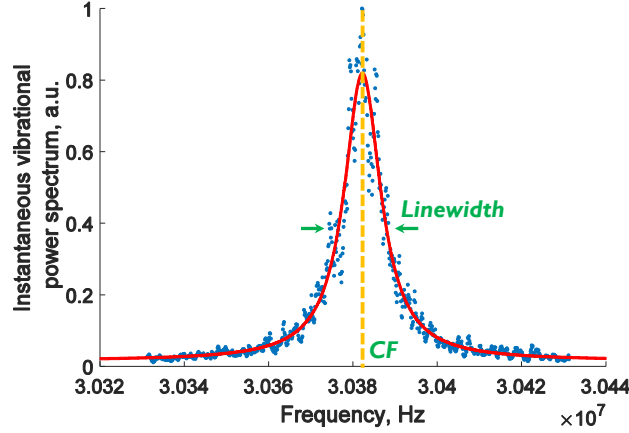


Figure 5.7: Finding the center frequency and linewidth of the spectrum using Lorentzian curve fitting – Instantaneous vibrational power spectra (blue dots) are fitted using a Lorentzian function (red curve). The center frequency (CF) of the oscillation is the instantaneous phonon mode frequency, while the -3 dB bandwidth is the phonon mode linewidth.

After finding out the phonon mode center frequency trace, center frequency shift caused by particle transit can then be determined by Gaussian fitting the peak (dip) of the center frequency trace (Figure 5.9d). This can be done manually. However, it is not a trivial task when there are more than a few particle transits happening. To automatically find all the frequency perturbations caused by particle transits, the first step is to find peaks (dips) using ‘findpeaks’ command built in with Matlab (Figure 5.9c). Since the data is usually very noisy, I filter the signal first and then find the peaks. Savitzky-Golay filtering is used to remove noise in the signal (Figure 5.9b). Depending on the nature of the frequency trace, peaks of interest can be accurately found by thresholding the peak minimum height and prominence through specifying a ‘MinPeakHeight’ and ‘MinPeakProminence’ when using ‘findpeaks’ command. Similarly, all the negative frequency shifts (dips) can be found by simply inverting the frequency trace and repeat the above process. After finding the peaks, Gaussian fitting can be done to each of them and both the frequency shift caused by particle transit and its uncertainty can be saved for later use. The Matlab code ‘FindFrequencyShift.m’ used are attached in Appendix C.

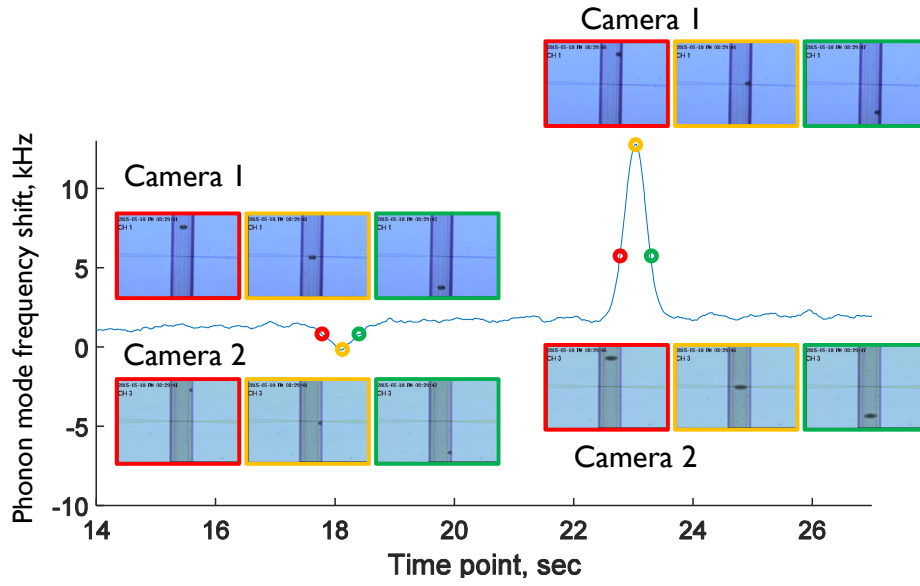


Figure 5.8: Phonon mode frequency shift measured while particles are flowing through the OMFR – Particles flowing through the OMFR are observed simultaneously by two cameras. We show here the images from both cameras matched in time domain with the phonon mode frequency shift. The observed frequency shift depends on both the longitudinal and radial position of the particle. The frequency shifts at the apexes are collected for analyzing the effect of particle radial location.

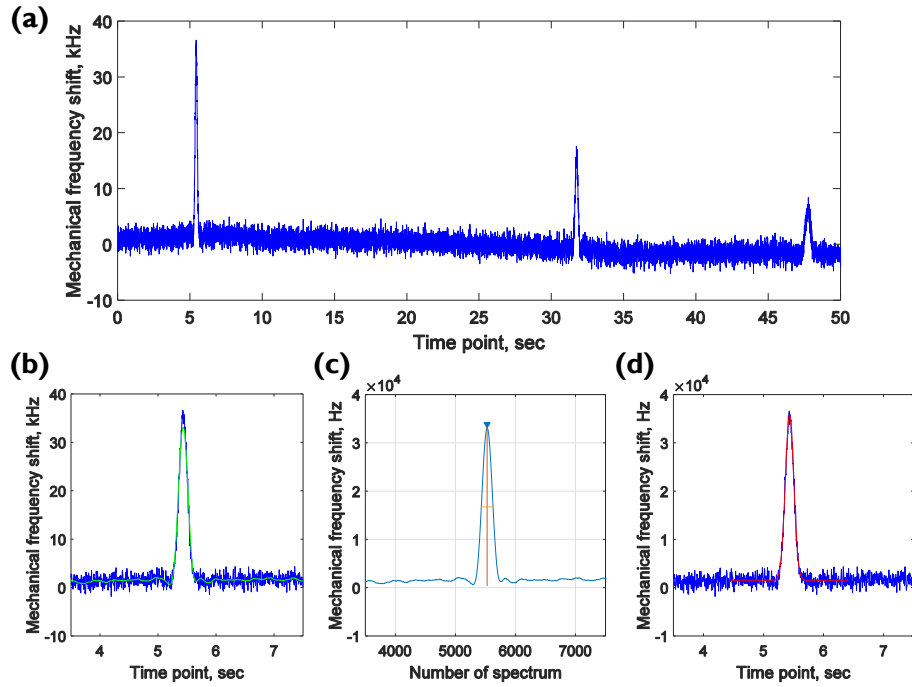


Figure 5.9: Frequency shift determination – (a) Example of center frequency trace found. (b) Savitzky-Golay filtering removes the noise in the frequency trace. (c) Command ‘findpeaks’ is applied to the filtered trace, which gives the index of the peak position. (d) Gaussian fit is done to a segment of data near the peak, from which the frequency shift can be determined.

Particle apparent position finding in 2D projection images

Two digital microscopes are used to simultaneously monitor the apparent position of each particle during transits from distinct points of view at the same height. For each view the 2D symmetry line for the OMFR image, which is projection of the 3D center line of the OMFR, is first determined (Figure 5.10a, middle yellow line), as the reference for the particle position finding. To do this, I first cropped a small area near the left edge of the OMFR projection image (Figure 5.10a). The darkest spot in each pixel row of this cropped area are found and indicated by a blue line (Figure 5.10a). A linear fit for this blue line is then done as the left edge line. Similarly, the right edge line can be found and the slope and intercept of the symmetry line (yellow) is then determined by averaging two edge lines' slopes and intercepts. The Matlab code for finding the symmetry lines are attached in Appendix D.

After finding out the 2D symmetry line for each view, the particle transit images from both cameras associated with the same frequency perturbation are then exported. A two dimensional Gaussian fit to the greyscale intensity plot of the each particle image is performed, and the apex of the Gaussian fit provides the apparent position of the particle center (Figure 5.10b,c). The distance between the particle and the symmetry line for each particle image, denoted as d_{1i} (camera channel 1) and d_{2i} (camera channel 2), can then be evaluated in the unit of pixels (Figure 5.10d). Since during the data analysis, there will usually be tens of particles that need processing, I have developed the Matlab code for batch image processing, as attached in Appendix E.

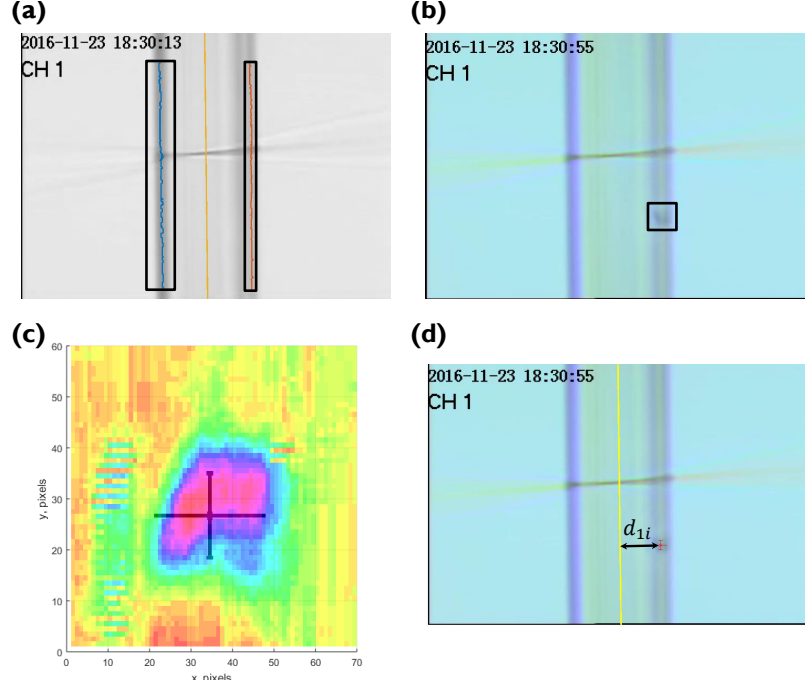


Figure 5.10: Image processing method – (a) To determine the 2D OMFR image symmetry line, two edge lines (blue, orange) are first determined with a linear fit edge search. The slope and intercept of the symmetry line (yellow) is then determined by averaging the two edge fits. Since the OMFR image is affected by illumination intensity, angle, and focal plane position, we estimate uncertainty for the edge determination of about $2.5 \mu\text{m}$. Therefore the uncertainty for each 2D symmetry line determination is about $2.5 \mu\text{m}$ as well, and the 3D OMFR central axis is located in the circle of roughly $2.5 \mu\text{m}$ radius. The particle position determination is therefore subject to $\pm 2.5 \mu\text{m}$ error due to the central axis offset. (b) A particle in transiting from camera 1 is shown here as an example. To determine the particle position in this 2D image, we first use this image to subtract a background image without particle, to get a better contrast between the particle and its surroundings. A small area shown by the black outline is then cropped out. (c) The cropped color images from (b) are converted to a greyscale intensity plot. This false color image represents darkest regions in purple and lightest regions in yellow. A 2D Gaussian fit is applied, with the peak representing the particle image center. The full width at half maximum (FWHM) of the fit in both x and y direction are indicated by the black bars. For each particle, the larger x-axis FWHM of the particle image fit from two cameras is used as the approximate uncertainty of the particle radial location in Figure ?? . (d) Finally, for two particle images associated with the same particle transiting from two cameras, the distance between the particle center (Gaussian fit peak) and the symmetry line can be easily calculated in the unit of pixels.

In situ measurement of capillary diameter

In this part, we introduce one useful technique to measure diameter of the capillary in situ. As described, the OMFR has a bottle shape. Therefore, it is the best to measure the OMFR diameter in situ such that we know diameter of the OMFR where the optical mode resides for our experiment. During the experiment, like we have discussed in Figure 5.2, we are recording live video of the experiment. A live video image of the OMFR is shown in Figure 5.11. As we can see, the OMFR is vertically oriented, very close to the tapered fiber. Also, as shown in Figure 5.2, we mount the tapered fiber horizontally on a piezo-driven nano-positioner, from which we can displace the taper with sub-micron precision. Thus we can measure real vertical distance of the microscope image, Y , by moving the taper from the top edge of the image to the bottom edge of the image using the nano-positioner (Figure 5.11). Since we know there are n pixels in the y -direction (vertical direction) of the image, a “metric per pixel” conversion factor in y -direction, Z_y , is defined as:

$$Z_y = \frac{Y}{n}. \quad (5.12)$$

From now on, we call this factor “Z-factor” in short. We are given that the pixel size of camera is $y \text{ } \mu\text{m}$ in height and $x \text{ } \mu\text{m}$ in width. we can then calculate the Z-factor in x -direction as:

$$Z_x = Z_y * \frac{x}{y}. \quad (5.13)$$

Next, with image processing software like Matlab or ImageJ, we measure the pixel distance from one edge of the OMFR to the other edge, D_i , by drawing a line segment that is perpendicular to edges of the OMFR. By assuming that the OMFR is placed vertically such that we can consider this line segment as horizontal, we can convert the pixel distance to find the diameter of the OMFR, D , as

$$D = Z_x * D_i. \quad (5.14)$$

For example, for the image shown in Figure 5.11, we have measured Y to be $148 \text{ } \mu\text{m}$ using the nano-positioner, and there are in total 480 pixels in the y direction of the image. For our Sentech STC620AII camera, the pixel

dimension is $9.8 \mu\text{m}$ in height and $8.4 \mu\text{m}$ in width. Therefore, the Z-factor in x direction is $Z_x = \frac{148}{480} * \frac{8.4}{9.8} = 0.26 \mu\text{m}/\text{pixel}$. Since we have measured that there are 186 pixels from the left edge of the OMFR to the right edge of it, the diameter of the OMFR is $D = 186 * 0.26 = 50.2 \mu\text{m}$.

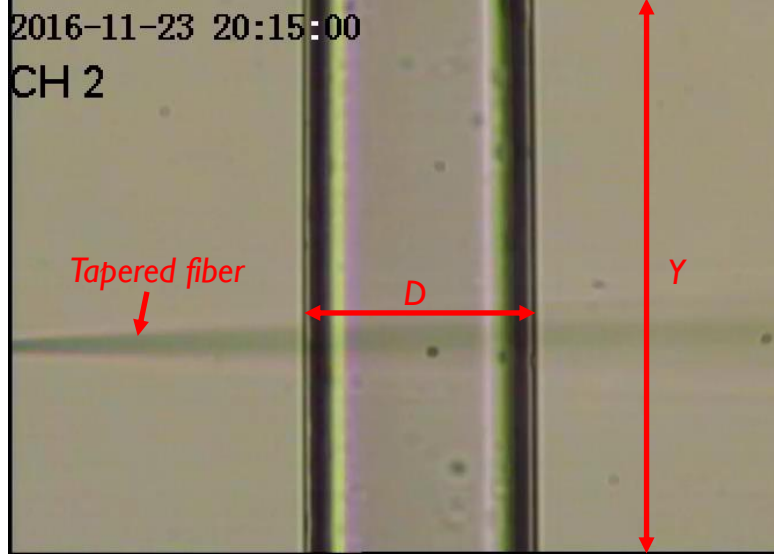


Figure 5.11: In situ capillary diameter measurement – As described in Figure 5.2, the tapered fiber is horizontally fixed on a piezo-driven nano-positioner, and the capillary is vertically oriented, in contact of the tapered fiber. Using the nano-positioner, we can move the taper from the top edge of the image to the bottom edge of the image, such that we know the real range of the image in the vertical direction, denoted by Y (μm). The pixel size of camera is $y \mu\text{m}$ in height and $x \mu\text{m}$ in width. Since we know there are n pixels in the y -direction of the image, we can calculate the Z-factor in x direction by $Z_x = \frac{Y}{n} * \frac{x}{y}$. Next, with the capillary well focused in the image, we can measure the diameter of the OMFR in pixels, denoted by D_i (pixels). The real diameter of the capillary is therefore given by D (μm) = $Z_x * D_i$.

As a confirmation, in our system, we have an 10x Mitutoyo type long working distance infinity corrected objective, since we are using the maximum magnification possible from 12x Narvitar UltraZoom system, the total system magnification is 33.31 (Figure A.1). Then the Z-factor in x direction is $Z_x = 8.4/33.31 = 0.25 \mu\text{m}/\text{pixel}$.

A more direct (but not in situ) calibration method of the Z-factor can be done with a stage micrometer (Thorlabs R1L3S2P). Figure 5.12 shows the images of the vertically oriented stage micrometer and the horizontally

oriented micrometer. Since we know exactly the distance between two major ticks on the stage micrometer, we can calibrate the Z-factor in the x direction as $0.27 \mu\text{m}/\text{pixel}$, which agrees with the above analysis.

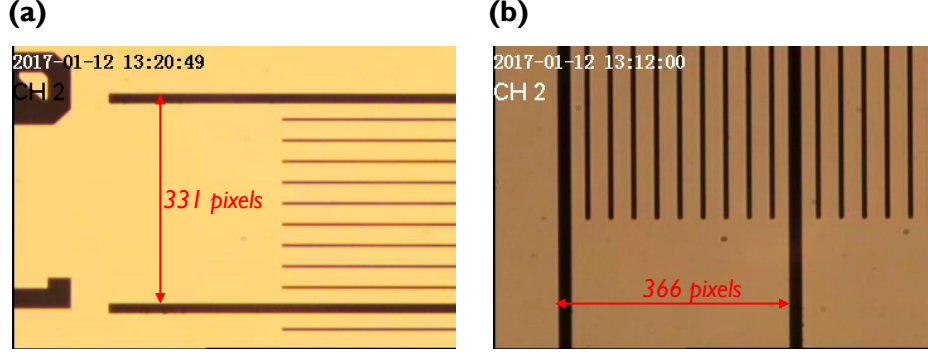


Figure 5.12: (a) The pixel distance between two major tick marks of the vertically oriented micrometer is 331 pixels. Since the distance between two major ticks is $100 \mu\text{m}$, the Z-factor in y direction is $100/331 = 0.30 \mu\text{m}/\text{pixel}$. (b) The pixel distance between two major tick marks of the horizontally oriented micrometer is 366 pixels. Similarly, the Z-factor in x direction is $100/366 = 0.27 \mu\text{m}/\text{pixel}$.

Particle radial location finding

Under far-field assumption, the actual location of the particle can easily be triangulated from the projections on each camera (Figure 5.13). In this part, we introduce the method for triangulation in details.

We first set up the coordinate for the two cameras' point of view, respectively, in the cross-section plane of the OMFR (Figure 5.14). Then coordinates are normalized using the radius of the OMFR, $R = D/2$ (Figure 5.14). The y-axis for each coordinate is set to be parallel to the light line that enters each camera (outside the OMFR) (Figure 5.13). The two z-axes coincide with each other and with the center line of the OMFR, whose projections are the two 2D symmetry lines shown in Figure 5.10. Also we have already obtained the pixel distance between the particle projection image and the symmetry line for each view as d_{1i} and d_{2i} . Again by assuming the OMFR

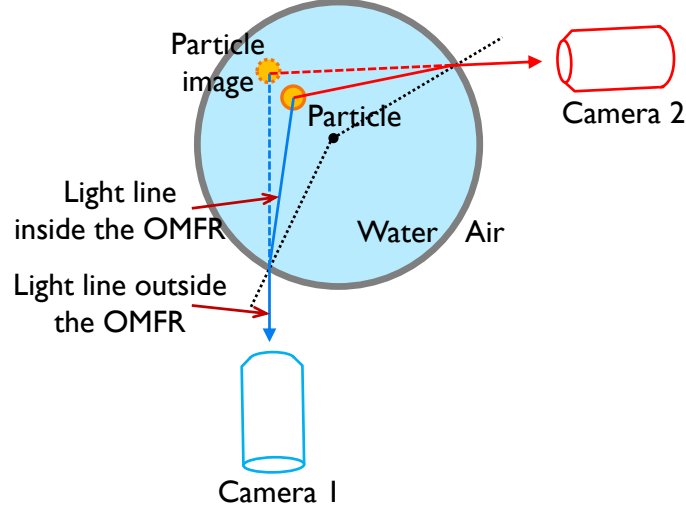


Figure 5.13: Determining true particle position – The particle position relative to the OMFR central axis can be determined by triangulating the apparent particle locations from the two cameras, while accounting for refractive index.

is vertically oriented, we can convert this pixel distance to real distance by

$$d_1 = d_{1i} Z_x \quad (5.15)$$

$$d_2 = d_{2i} Z_x \quad (5.16)$$

We can then find the light line exit point on the circumference of the OMFR for each view to be,

$$(x_1, y_1) = \left(\frac{d_1}{R}, -\sqrt{1 - \frac{d_1^2}{R^2}} \right) \quad (5.17)$$

$$(x_2, y_2) = \left(\frac{d_2}{R}, -\sqrt{1 - \frac{d_2^2}{R^2}} \right). \quad (5.18)$$

Finally, the angle between two coordinates, ϕ , is the angle between two cameras.

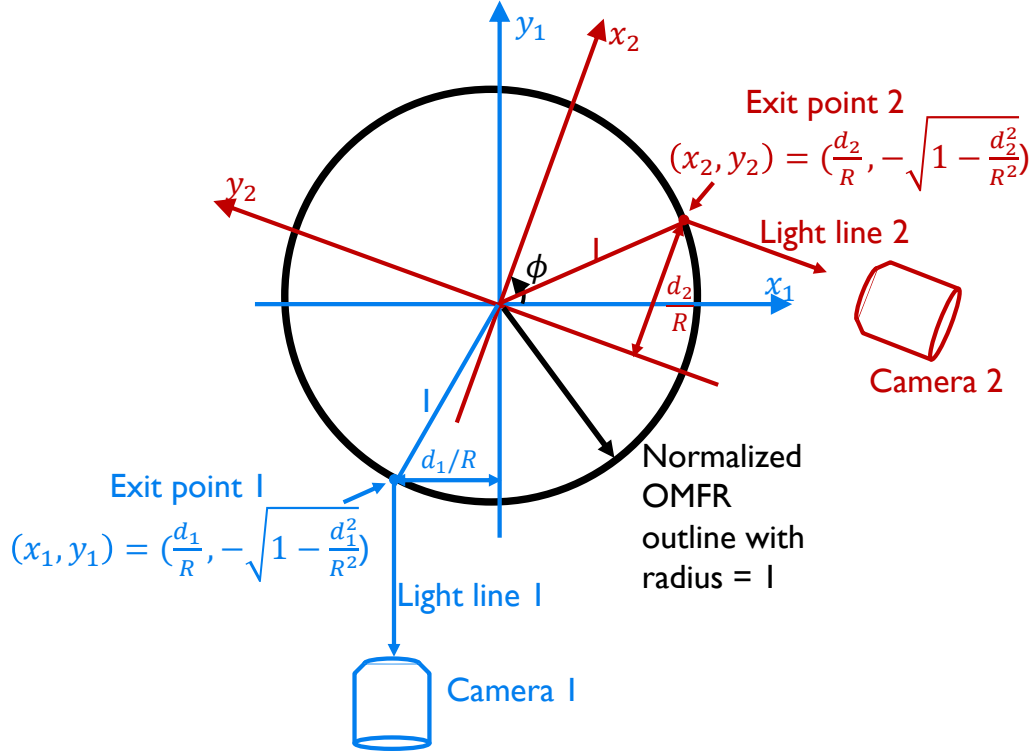


Figure 5.14: We first set up coordinate for the two cameras' point of view, respectively, in the cross-section plane of the OMFR. The black circle here is the normalized outline of the OMFR such that its radius is 1. The y-axis for each coordinate is parallel to the light line that enters each camera. The origins of two coordinates coincide with each other such that the z-axis is the OMFR center line. Since from above analysis, we have obtained for each point of view that the distance between the particle projection image and the symmetry line is $d_1(d_2)$, we can easily determine position of the light line exit point $(x_1, y_1)((x_2, y_2))$ on the normalized outline of the OMFR, in each coordinate. The angle between two coordinates is ϕ .

Single-refraction approximation: At the interface between the OMFR and air, refraction happens and it happens at the interface between OMFR shell and water. But since the shell is relatively thin and the refractive index of water ($n = 1.33$) is similar to that of silica glass ($n = 1.45$), a single effective refractive index, $n_{eff} = 1.35$, is used here to simplify the math, and the refraction is assumed to happen once at the outer circumference of the OMFR (Figure 5.15). Next, with the help of Snell's law, we show the calculation of the light line function inside of the OMFR in coordinate 1. The process is exactly the same for the other coordinate.

First, we define that when the particle image is on the left of the symmetry line (Figure 5.14), $d_1 < 0$. The refracted angle, θ_1 , can be found by trigonometry as (Figure 5.15),

$$\theta_1 = \sin^{-1}(x_1/1) = \sin^{-1}(x_1). \quad (5.19)$$

Then the incident angle, γ_1 , can be related to the the refracted angle, θ_1 , by Snell's law,

$$\gamma_1 = \sin^{-1}\left(\frac{n_{air}\sin(\theta_1)}{n_{eff}}\right). \quad (5.20)$$

Therefore, when $x_1 > 0$, $\theta_1 > 0$ and $\gamma_1 > 0$ (Figure 5.15b); when $x_1 < 0$, $\theta_1 < 0$ and $\gamma_1 < 0$ (Figure 5.15a). The angle between the normal line and x-axis can be found as:

$$0 < \alpha_1 = \tan^{-1}\left(\frac{|y_1|}{-x_1}\right) < \frac{\pi}{2}, \quad \text{when } x_1 < 0, \quad (5.21)$$

$$\frac{\pi}{2} < \alpha_1 = \tan^{-1}\left(\frac{|y_1|}{-x_1}\right) + \pi < \pi, \quad \text{when } x_1 > 0. \quad (5.22)$$

This two equation can be conveniently realized together using the 'atan2' Matlab command as, for example, 'atan2(abs(y1), -x1)'. Thus as shown in Figure 5.15a, when $x_1 < 0$,

$$\beta_1 = |\gamma_1| + \alpha_1, \quad \text{thus } \beta_1 = -\gamma_1 + \alpha_1. \quad (5.23)$$

Similarly as shown in Figure 5.15b, when $x_1 > 0$,

$$\alpha_1 = \gamma_1 + \beta_1, \quad \text{thus } \beta_1 = -\gamma_1 + \alpha_1. \quad (5.24)$$

Thus we have successfully reduced both cases to one equation, which makes our calculation much easier.

The light line function inside the OMFR in coordinate 1 for camera 1 (Figure 5.15) is then

$$y = \tan(\beta_1)x + y_1 - \tan(\beta_1)x_1. \quad (5.25)$$

Similarly, the light line function inside the OMFR in coordinate 2 for camera 2 is then

$$y = \tan(\beta_2)x + y_2 - \tan(\beta_2)x_2. \quad (5.26)$$

In order to find the intersection between both light lines, we need to convert the light line function for camera 1 from coordinate 1 to coordinate 2. To do this, we need to first convert the exit point. The transformation of exit point 1 from coordinate 1 to coordinate 2 is given by

$$\begin{bmatrix} x'_1 \\ y'_1 \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad (5.27)$$

The angle between the light line 1 and the x-axis of coordinate 2 is simply (Figure 5.16)

$$\beta'_1 = \beta_1 - \phi. \quad (5.28)$$

Then expression of light line 1 inside the OMFR in coordinate 2 is

$$y = \tan(\beta'_1)x + y'_1 - \tan(\beta'_1)x'_1. \quad (5.29)$$

Therefore, using Equation 5.26 and 5.29 the intersection point coordinate

between two lines can be found as

$$x_{intersection} = \frac{y'_1 - \tan(\beta'_1)x'_1 - y_2 + \tan(\beta_2)x_2}{\tan(\beta_2) - \tan(\beta'_1)} \quad (5.30)$$

$$y_{intersection} = \tan(\beta_2)x_{intersection} + y_2 - \tan(\beta_2)x_2 \quad (5.31)$$

Eventually, the distance, L , between the particle and the centerline of the OMFR is (Figure 5.16)

$$L = R\sqrt{x_{intersection}^2 + y_{intersection}^2} \quad (5.32)$$

This entire part's Matlab code is listed in 'RadialLocationFinding.m' as attached in Appendix F.

We note that the determination of the particle radial position is subject to uncertainty in determining the 3D OMFR central axis, in determining the apparent position of the particle center, and as well as in finding the particle radial position (Figure 5.10). In this work, there is roughly 5 μm uncertainty in knowing the central axis (see Fig. 5.10). The error bar in Fig. 5.4 shows the full width at half maximum (FWHM) of the Gaussian fit of the particle greyscale image in x direction (see Fig. 5.10b).

Double-refraction method: With a similar procedure like we did above, we can include the second refraction at the interface between water and the glass shell into account if we know the shell thickness, which provides a more accurate way to calculate particle positions. The comparison between the results with single-refraction approximation and double-refraction method is shown in Figure 5.17 for some 6 μm polystyrene particles with a 55 μm device as we will see in Chap. 6. As we can see, the single-refraction approximation works very well for $n_{eff} = 1.35$. The single-refraction method is more convenient since the shell thickness is hard to know without destructive measurement.

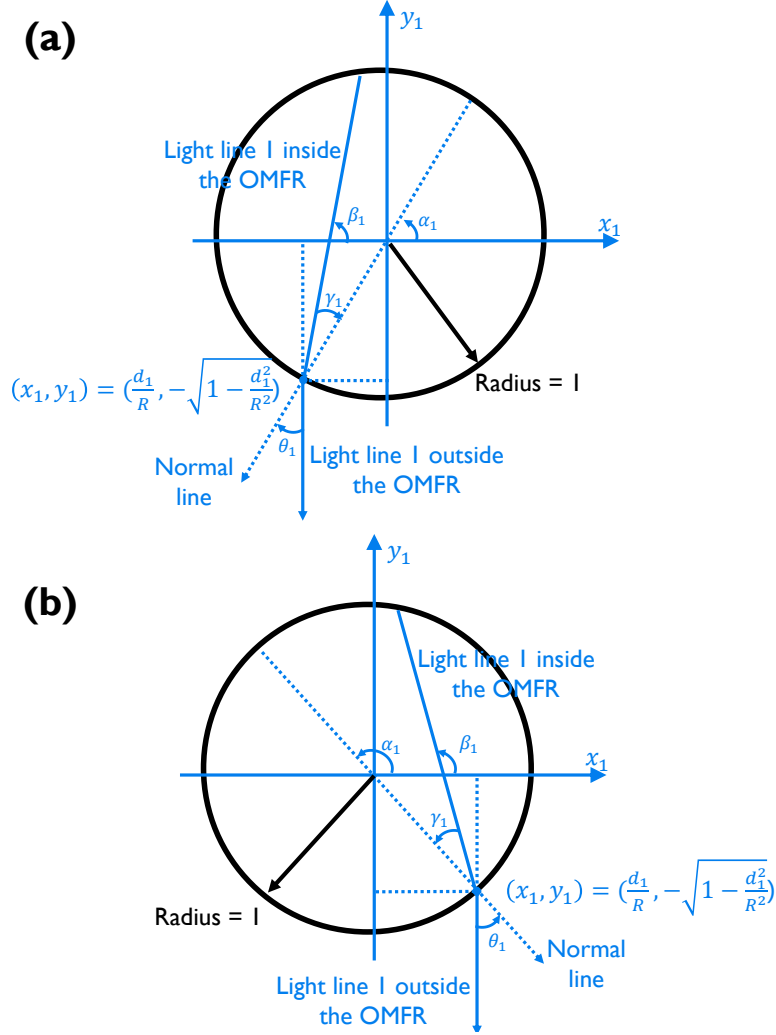


Figure 5.15: The light line function inside the OMFR can be determined by the light exit point and the slope of the light line inside the OMFR. (a) When $x_1 < 0$, the refracted angle, θ_1 , and the incident angle, γ_1 , are defined to be negative, and related by the Snell's law. The angle between the light line inside the OMFR and x -axis is $\beta_1 = |\gamma_1| + \alpha_1 = -\gamma_1 + \alpha_1$. (b) When $x_1 > 0$, the refracted angle, θ_1 , and the incident angle, γ_1 , are defined to be positive. The angle between the light line inside the OMFR and x -axis is $\beta_1 = -\gamma_1 + \alpha_1$.

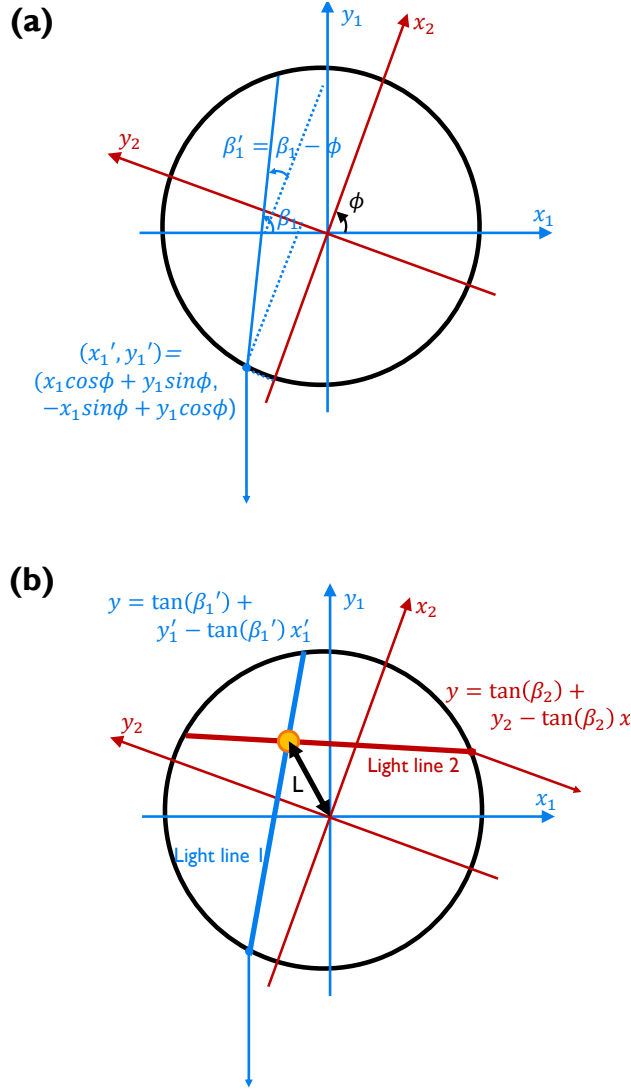


Figure 5.16: (a) The light line function inside the OMFR for camera 1 needs to be expressed in coordinate 2. The coordinate for exit point 1 in coordinate 2 can be found using simple trigonometry. The angle between the light line with x_2 axis can be found by subtracting the rotating angle between two coordinates from the angle between light line for camera 1 and x_1 axis. (b) With the light line functions both expressed in coordinate 2, we can find the coordinate of their intersection, and thus the distance between particle and the symmetry line of the OMFR.

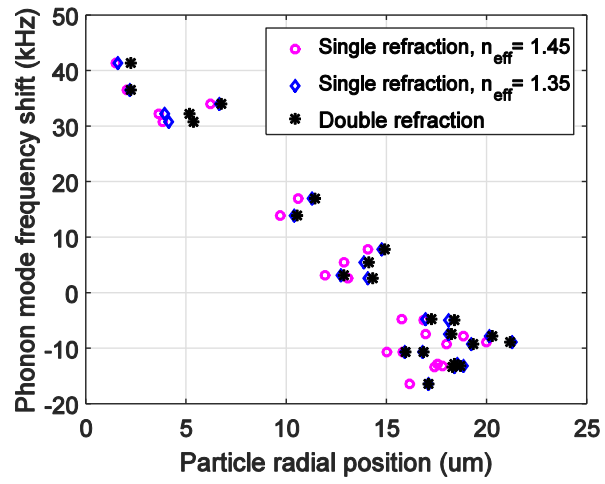


Figure 5.17: Comparison between the particle positions found with the single effective refractive index of 1.35 and 1.45 and with the double-refraction method.

Phonon mode simulations:

Our vibrational phonon mode simulations are performed using Comsol Multiphysics using a customized solid-fluid interaction model as described in Chapter 3 [1]. Coupling between the solid and fluid components is engaged by local pressure in the fluid that exerts force on the shell, and by the normal acceleration of the shell that launches pressure waves in the fluid. The phonon eigenmodes describing the displacement field in the solid domain and the pressure wave field in the fluid domain can thus be determined. This allows calculation of the kinetic acoustic energy and potential acoustic energy stored in fluid domain.

5.6 Sensing limit

In order to estimate the sensing limit of a 55 μm OMFR used in this work, the Allan deviation of the center frequency is measured over 180 seconds (Figure 5.18). No feedback control or stabilization is applied. The device is operated in lab air with ambient environmental fluctuations causing the measured frequency fluctuations.

For small particles, the frequency perturbation is generally proportional to the volume V_s of the particle (Eqn. 5.11). For the 6 μm particles used in this work, our model predicts a fairly high frequency perturbation of about 40 kHz when the particle is at the OMFR center. On the other hand, our experiments indicate a trend towards 15 kHz - 40 kHz frequency shift when the particle is at the OMFR center (Figure 5.4). We thus use a conservative value of 20 kHz for this estimation. We can now convert the Allan deviation noise measurement to a Noise-Equivalent Particle Diameter (NEPD) as shown in Figure 5.18.

We note also that the data in the present work is gathered when the phonon mode optomechanical oscillation is still in the sub-threshold regime (i.e. ‘cold cavity’), with very low optical pump power. Temperature variations dominate the frequency fluctuations here, resulting in the long averaging time of 2.03 seconds before the minimum NEPD of 840 nm is achieved. An OMFR with similar frequency was reported previously in the above-threshold optomechanical oscillation regime [105], with a much lower Allan deviation

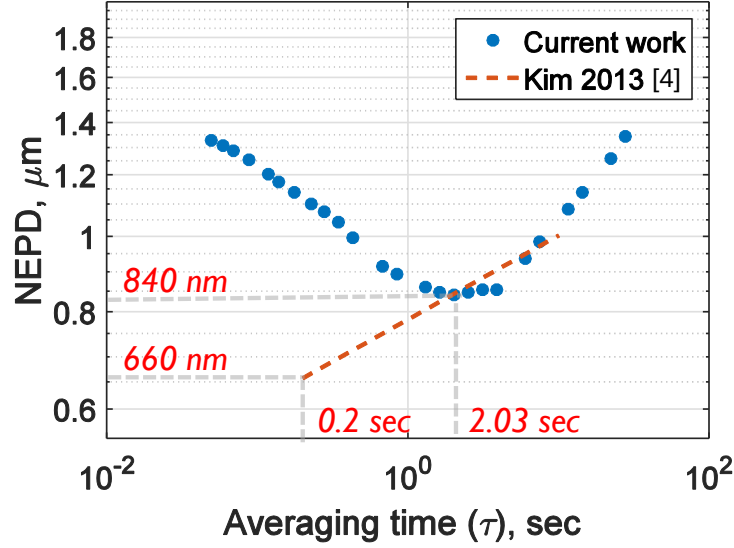


Figure 5.18: Sensitivity estimations of the OMFR system – The Noise-Equivalent Particle Diameter (NEPD) defines the minimum sensing noise floor of an OMFR. The data presented here are for a 55 μm diameter OMFR (30.3 MHz center frequency) in the form of an Allan deviation plot with x-axis representing averaging time. We estimate a minimum sensing level of 840 nm with averaging time of 2 seconds. A similarly sized 31 MHz OMFR was demonstrated previously in [105] operating in the above-threshold optomechanical oscillation regime. Using their Allan deviation data (see Fig. 3b in that work) we estimate an improved minimum NEPD of 660 nm at 0.2 seconds averaging time (red dashed line) for that mode of operation.

floor. We can thus estimate that a minimum NEPD of 660 nm could be achieved when operated in this above threshold regime over only 0.2 seconds averaging time. Feedback stabilization of the oscillator, and environmental controls on temperature and pressure can improve these noise floors significantly.

5.7 Conclusion

Broadly speaking, this work presents a new phonon-mediated optical technique for performing high-throughput mechanical measurements on free-flowing particles in fluid. The technique enables resonant measurements without reliance on random diffusion, and spatially decouples the optical mode from the sensing volume. Specific to our demonstration of this concept,

the ‘breathing’ phonon modes we show have uniquely maximized sensitivity when the particles are at the center of the OMFR sensor. Indeed, many more vibrational modes spanning 10 MHz - 12 GHz regimes can be accessed in a single OMFR structure [54] which will have different spatial sensitivity functions. A combination of modes can thus be used to ensure perfect capture and inertial imaging of every particle that flows through the device. At the same time, the less-sensitive ‘Brillouin modes’ of the OMFR [54] may be employed as 11 GHz frequency references that can help cancel out common mode fluctuations and even enable feedback stabilization towards substantial improvement of the detection limit.

CHAPTER 6

MEASURING PARTICLE COMPRESSIBILITY WITH OMFRS

6.1 Introduction

In this Chapter, I demonstrate the preliminary results of the compressibility measurements using the OMFR.

Mechanical properties of cells such as density, Young's modulus, and compressibility (Figure 6.1) are important in determining how a cell interacts with its environment. These mechanical properties are known to correlate with many diseases such as anemia, malaria, and cancer [151]. For example, malignant cancer cells are found to be more compressible than benign cells [151], which is one of the reason why cancer cells can squeeze through small capillaries and spread easily [152]. The mechanical properties of cells are also very different at different stages of cell differentiation. Understanding the difference of mechanical properties between cells can help us with cell separation and can be used for developing next generation disease diagnostics.

To test the mechanical properties of cells, forces such as tensile forces, shear forces, twisting forces, and bending forces have to be applied, and the mechanical responses (e.g. deformation) of the cell can then be measured to determine the properties of interest. This force application can be done statically by direct contact using techniques like atomic force microscopy (AFM) [153,154], optical tweezers [155], and glass micropipettes [156]. These methods, however, may cause damage to the cells due to the direct contact, and are inherently slow due to the nature of the static measurement. With the development of micro-fabrication techniques, highly sensitive mechanical resonators have appeared [134,137,138]. In these studies, a cell is loaded on top of or inside a mechanical resonator. The resonant behaviors of the resonator are related to the density, stiffness and viscosity of the cell and

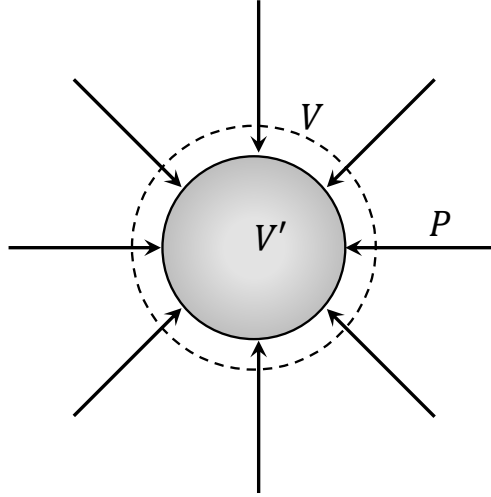


Figure 6.1: Compressibility of a particle measures the easiness to deform the particle under a pressure applied evenly.

therefore these properties of the cell can be measured. However, these methods are still slow since the measurement requires the binding of the cell. On the other hand, as we discussed in Chapter 5, acoustic radiation force can interact with cells while cells are flowing through an acoustic cavity without any physical contact being made with the cells themselves, therefore expediting the measurement speed. Hartono et. al. demonstrated [157] compressibility measurement of cells using the fact that cells can be moved by acoustic radiation force, and the trajectories of cells are related to their size, density, and compressibility. However, in their measurement, the development of the trajectory usually takes more than 5 seconds, which fundamentally limits the throughput of such measurement. Particle sensing using OMFRs, on the other hand, demonstrates a fundamental limit of more than 10,000 measurements/s, providing a high-speed tool for analyzing mechanical properties of cells.

As we showed in Chap. 5, both density and compressibility of the particle affect resonant behavior of the OMFR. Unlike the cell density measurement, for which several methods can be applied [28], cell compressibility measurement is seldomly done previously and thus the goal of this study. Chap. 5 shows that parameters like the size, position, density, and compressibility of the particle, and the vibrational modeshape, are all coupled in the resonant frequency measurement using OMFRs. In order to successfully measure the

compressibility of particles, we need to decouple these effects. Using perturbation model (Eqn. 5.11), the density or the compressibility of the sample particle can be measured if the vibrational mode shape and other properties of the particle material are known. However, as shown by Fig. 5.5, there is indeed some residual error in our current prediction models, which originates from multiple sources. First, we note that since neither the precise geometry of the OMFR, nor the exact material properties of the OMFR are known, the calculation of A and B is only based on an estimated FEM model. Second, we have assumed that the pressure field remains unchanged upon addition of the particle. This assumption is acceptable when the particle is much smaller than the acoustic wavelength. However, when this condition is not met, severe deviation between the data and the prediction can happen. To be able to more accurately predict the frequency perturbation, we need to incorporate effects like scattering and acoustic radiation pressure force into our calculation. We can do this by using an energy method as I will show in the next section.

6.2 Energy method

Energy method with fixed particles

For any resonator, the average kinetic energy per oscillation cycle has to be equal to the average potential energy per oscillation cycle (Rayleigh-Ritz method [158]). For the OMFR, when there is no particle in the resonator, this relation can be expressed as:

$$\langle W_{wp} \rangle + \langle W_{lp} \rangle = \langle W_{wk} \rangle + \langle W_{lk} \rangle, \quad (6.1)$$

where W_{wp} (W_{wk}) is the elastic strain (kinetic) energy in the shell, W_{lp} (W_{lk}) is the acoustic potential (kinetic) energy in the fluid region, and the $\langle \cdot \rangle$ denotes the time average over a period T : $\frac{1}{T} \int_t^{t+T} \cdot dt$. We can expand this expression as follows:

$$\langle W_{wp1} \rangle + \left\langle \int_{V_l} \frac{1}{2} \kappa_l P_1^2 dV \right\rangle = \left\langle \int_{V_w} \frac{1}{2} \rho_w \Omega_1^2 |\vec{U}_1|^2 dV \right\rangle + \left\langle \int_{V_l} \frac{1}{2} \frac{|\nabla P_1|^2}{\rho_l \Omega_1^2} dV \right\rangle, \quad (6.2)$$

where subscript “1” denotes the unperturbed situation, κ_l is the compressibility of the fluid, ρ_l is the density of the fluid, and ρ_w is the OMFR wall density. V_l is the volume of the fluid cavity and V_w the volume of the solid shell. By assuming the oscillation to be time harmonic, i.e., the elastic displacement of the shell $\vec{U}(r, t) = \vec{u}(r) \cos(\Omega t)$ and the pressure field $P(r, t) = p(r) \cos(\Omega t)$, we can further simplify this expression as:

$$\langle W_{wp1} \rangle + \frac{\kappa_l}{4} \int_{V_l} p_1^2 dV = \frac{1}{4} \rho_w \Omega_1^2 \int_{V_w} |\vec{u}_1|^2 dV + \frac{1}{4} \frac{1}{\rho_l \Omega_1^2} \int_{V_l} |\nabla p_1|^2 dV. \quad (6.3)$$

When there is a particle inside the resonator, which is assumed to be fixed at some location in the equator plane of the OMFR, we can rewrite this energy balance as:

$$\begin{aligned} \langle W_{wp2} \rangle + \frac{\kappa_l}{4} \int_{V_l-V_s} p_2^2 dV + \frac{\kappa_s}{4} \int_{V_s} p_2^2 dV = \\ \frac{1}{4} \rho_w \Omega_2^2 \int_{V_w} |\vec{u}_2|^2 dV + \frac{1}{4} \frac{1}{\rho_l \Omega_2^2} \int_{V_l-V_s} |\nabla p_2|^2 dV + \frac{1}{4} \frac{1}{\rho_s \Omega_2^2} \int_{V_s} |\nabla p_2|^2 dV, \end{aligned} \quad (6.4)$$

where we have treated the particle as fluid but having density and compressibility of the solid material. The subscript “s” denotes the particle such that κ_s represents the particle compressibility and V_s represents the particle volume. We further assume that the elastic displacement field of the shell does not change, i.e., $\vec{u}_1 = \vec{u}_2 = \vec{u}$, and therefore $\langle W_{wp1} \rangle = \langle W_{wp2} \rangle$. We can then subtract Eqn. 6.4 from Eqn. 6.3 and obtain:

$$\begin{aligned} \frac{\kappa_l}{4} (T_1 - T_2) + \frac{1}{4} (\kappa_l t_1 - \kappa_s t_2) - \frac{1}{4 \rho_l \Omega_1^2} (G_1 - G_2) - \frac{1}{4} \left(\frac{g_1}{\rho_l \Omega_1^2} - \frac{g_2}{\rho_s \Omega_2^2} \right) = \\ \frac{F}{4} (\Omega_1^2 - \Omega_2^2) + \frac{G_2}{4 \rho_l} \left(\frac{1}{\Omega_1^2} - \frac{1}{\Omega_2^2} \right), \end{aligned} \quad (6.5)$$

where we have used the following notations: $T = \int_{V_l-V_s} p^2 dV$, $t = \int_{V_s} p^2 dV$, $G = \int_{V_l-V_s} |\nabla p|^2 dV$, $g = \int_{V_s} |\nabla p|^2 dV$, and $F = \rho_w \int_{V_w} |\vec{u}|^2 dV$. Since $\Omega_1 \approx \Omega_2$, we can make the following approximations: $\frac{g_1}{\rho_l \Omega_1^2} - \frac{g_2}{\rho_s \Omega_2^2} \approx \frac{1}{\Omega_1^2} \left(\frac{g_1}{\rho_l} - \frac{g_2}{\rho_s} \right)$, and $\Omega_1^2 - \Omega_2^2 \approx -2\Omega_1 \Delta\Omega$, where $\Delta\Omega = \Omega_2 - \Omega_1$. Therefore we can obtain the

frequency perturbation as:

$$\frac{\Delta\Omega}{\Omega_1} = \frac{\frac{\kappa_l}{4}(T_1 - T_2) + \frac{1}{4}(\kappa_l t_1 - \kappa_s t_2) - \frac{1}{4\rho_l\Omega_1^2}(G_1 - G_2) - \frac{1}{4\Omega_1^2}(\frac{g_1}{\rho_l} - \frac{g_2}{\rho_s})}{-\frac{\Omega_1^2 F}{2} + \frac{G_2}{2\rho_l\Omega_1^2}}. \quad (6.6)$$

Generally, the terms $T_1 - T_2$ and $G_1 - G_2$ are not zero due to the scattering effect. Theoretically, by simulating both the perturbed and unperturbed situations, we can use Eqn. 6.6 to calculate the frequency perturbation. However, due to the random scaling of the eigenfrequency simulation in COMSOL, we have to perform a renormalization between two simulations. One possible choice is to normalize the elastic kinetic energy in both cases to be the same as assumed. However, this assumption is not entirely accurate since the kinetic energy of the shell should also be varied by the scattering wave, although the change may be very small. Furthermore, all of the difference terms in the numerator of Eqn. 6.6 are so small that they are on the same order of the computational error, which makes the numerical calculation using Eqn. 6.6 impractical. However, if we can theoretically calculate the scattering field, we should be able to get a much better prediction.

If we neglect the scattering effects and assume that $p_1 = p_2 = p$, then $T_1 = T_2$, $G_1 = G_2$, $t_1 = t_2 = t$, and $g_1 = g_2 = g$. Therefore we can further simplify Eqn. 6.6 to:

$$\frac{\Delta\Omega}{\Omega_1} = \frac{\frac{t}{4}(\kappa_l - \kappa_s) - \frac{g}{4\Omega_1^2}(\frac{1}{\rho_l} - \frac{1}{\rho_s})}{-\frac{\Omega_1^2 F}{2} + \frac{G}{2\rho_l\Omega_1^2}}. \quad (6.7)$$

Finally, by defining $\Omega_2 = \Omega' = 2\pi f'$ and $\Omega_1 = \Omega_o = 2\pi f_o$, and treating $G = \int_{V_l-V_s} |\nabla p|^2 dV \approx \int_{V_l} |\nabla p|^2 dV$ for the denominator, we obtain the perturbed resonance frequency

$$\frac{\Delta f}{f_o} = -\frac{\kappa_s - \kappa_l}{2\kappa_l} C - \frac{\rho_s - \rho_l}{2\rho_s} D, \quad (6.8)$$

$$\text{where } C = \frac{\langle W_{sp} \rangle}{\langle W_{lk} \rangle - \langle W_{wk} \rangle}, \quad D = \frac{\langle W_{sk} \rangle}{\langle W_{lk} \rangle - \langle W_{wk} \rangle}.$$

W_{sp} and W_{sk} is the acoustic potential and kinetic energy in the particle volume, respectively, evaluated with the fluid material properties. Meanwhile,

by Taylor expanding Eqn. 5.11, we can find the prediction of the frequency perturbation as:

$$\frac{\Delta f}{f_o} = -\frac{\kappa_s - \kappa_l}{2\kappa_l} A - \frac{\rho_s - \rho_l}{2\rho_s} B, \quad (6.9)$$

$$\text{where } A = \frac{\langle W_{sp} \rangle}{\langle W_{lp} \rangle}, \quad B = \frac{\langle W_{sk} \rangle}{\langle W_{lp} \rangle}.$$

As we can see, the frequency perturbation prediction equations derived from the energy balance (Eqn. 6.8) and the Helmholtz equation (Eqn. 6.9) have very similar form. Basically, the frequency perturbation is due to the modification of system potential energy through compressibility contrast and the modification of system kinetic energy through density contrast. The energy term of the denominator is the only difference given by the two methods. The influence of this difference makes in the frequency perturbation predictions is negligible as shown in Figure 6.2.

Energy method with moving particles

When a particle is in a standing wave pressure field, on the first order, the particle experiences a harmonic acoustic radiation force [159]. This force arises because the gradients in the pressure field exert a greater force on one side of the particle than the other. The direction of the force reverses every half cycle and as a result, the particle vibrates spatially. To illustrate the effect of this particle vibration, we assume that a particle is vibrating in the radial direction only, in the equator plane of the OMFR, with a equilibrium radial location r_0 . This vibration has a time harmonic amplitude of $S(r, t) = s_0(r) \cos(\Omega_2 t)$ and therefore the particle position is $r_s = r_0 + s_0(r_0) \cos(\Omega_2 t) \approx r_0 + s_0(r_0) \cos(\Omega_1 t)$. Here, we have assumed the vibration of the particle is in phase with the pressure field for the sake of showing the effect of particle moving. However, in reality, we need to solve the dynamic equation for this particle vibration to find out its vibrating amplitude and phase with respect to the pressure field. Thus the mean kinetic energy associated with the particle vibration is $\langle \frac{1}{2} m v^2 \rangle = \frac{1}{4} m s_0^2 \Omega_2^2 \approx \frac{1}{4} m s_0^2 \Omega_1^2$, where m is mass of the particle. Again we assume the pressure field and shell displacement field are

unchanged upon particle addition. Since the particle is moving in this case, i.e., V_s is a function of time, we need to rewrite the energy balance (Eqn. 6.1) when there is no particle as

$$\langle W_{wp} \rangle + \frac{1}{T} \int_0^T dt \int_{V_l} \frac{\kappa_l P^2}{2} dV = \frac{\Omega_1^2 F}{4} + \frac{1}{T} \int_0^T dt \int_{V_l} \frac{|\nabla P|^2}{2\rho_l \Omega_1^2} dV, \quad (6.10)$$

and with particle as

$$\begin{aligned} \langle W_{wp} \rangle + \frac{1}{T} \int_0^T dt \int_{V_l - V_s(t)} \frac{\kappa_l P^2}{2} dV + \frac{1}{T} \int_0^T dt \int_{V_s(t)} \frac{\kappa_s P^2}{2} dV = \\ \frac{\Omega_2^2 F}{4} + \frac{1}{T} \int_0^T dt \int_{V_l - V_s(t)} \frac{|\nabla P|^2}{2\rho_l \Omega_2^2} dV \\ + \frac{1}{T} \int_0^T dt \int_{V_s(t)} \frac{|\nabla P|^2}{2\rho_s \Omega_2^2} dV + \frac{1}{4} m s_0^2 \Omega_1^2. \end{aligned} \quad (6.11)$$

We subtract Eqn. 6.11 from Eqn. 6.10 and obtain

$$\begin{aligned} \frac{\kappa_l - \kappa_s}{2} \frac{1}{T} \int_0^T dt \int_{V_s(t)} P^2 dV = \\ - \frac{\Omega_1 \Delta \Omega F}{2} + \frac{\Delta \Omega}{\Omega_1^3} \frac{1}{T} \int_0^T dt \int_{V_l - V_s(t)} \frac{|\nabla P|^2}{2\rho_l} dV \\ + \frac{1}{2\Omega_1^2} \left(\frac{1}{\rho_l} - \frac{1}{\rho_s} \right) \frac{1}{T} \int_0^T dt \int_{V_s(t)} |\nabla P|^2 dV - \frac{1}{4} m s_0^2 \Omega_1^2. \end{aligned} \quad (6.12)$$

By using $\Omega_1 \approx \Omega_2$ and $\int_{V_l - V_s(t)} |\nabla p|^2 dV \approx \int_{V_l} |\nabla p|^2 dV$ again, we get

$$\begin{aligned} \frac{\kappa_l - \kappa_s}{2} \frac{1}{T} \int_0^T \cos^2(\Omega_1 t) dt \int_{V_s(t)} p^2 dV \approx \\ \frac{2\Delta \Omega}{\Omega_1} (\langle W_{lk} \rangle - \langle W_{wk} \rangle) - \frac{1}{4} m s_0^2 \Omega_1^2 \\ + \frac{1}{2\Omega_1^2} \left(\frac{1}{\rho_l} - \frac{1}{\rho_s} \right) \frac{1}{T} \int_0^T \cos^2(\Omega_1 t) dt \int_{V_s(t)} |\nabla p|^2 dV. \end{aligned} \quad (6.13)$$

We further divide both sides by $(\langle W_{lk} \rangle - \langle W_{wk} \rangle)$ and get

$$\begin{aligned} \frac{\kappa_l - \kappa_s}{2\kappa_l} \frac{4}{T} \int_0^T \cos^2(\Omega_1 t) C(r_s) dt = \\ \frac{2\Delta \Omega}{\Omega_1} - \frac{\frac{1}{4} m s_0^2 \Omega_1^2}{\langle W_{lk} \rangle - \langle W_{wk} \rangle} + 4 \frac{\rho_s - \rho_l}{2\rho_s} \frac{1}{T} \int_0^T \cos^2(\Omega_1 t) D(r_s) dt, \end{aligned} \quad (6.14)$$

where $C(r_s)$ and $D(r_s)$ mean that both C and D are evaluated at particle radial location $r_s \approx r_0 + s_0(r_0) \cos(\Omega_1 t)$. Both $C(r)$ and $D(r)$ can be easily simulated and approximated as polynomials with respect to r . If we assume a third order polynomial for $C(r)$ as

$$C(r) = a_1 r^3 + a_2 r^2 + a_3 r + a_4, \quad (6.15)$$

then $C(r_s)$ can be expanded as

$$\begin{aligned} C(r_s) = & a_1[r_0^3 + 3r_0^2 s_0 \cos(\Omega_1 t) + 3r_0 s_0^2 \cos^2(\Omega_1 t) + s_0^3 \cos^3(\Omega_1 t)] \\ & + a_2[r_0^2 + 2r_0 s_0 \cos(\Omega_1 t) + s_0^2 \cos^2(\Omega_1 t)] \\ & + a_3[r_0 + s_0 \cos(\Omega_1 t)] \\ & + a_4. \end{aligned} \quad (6.16)$$

Therefore,

$$\frac{1}{T} \int_0^T \cos^2(\Omega_1 t) C(r_s) dt = \frac{1}{2} C(r_0) + \frac{3}{8} (3a_1 r_0 + a_2) s_0^2 \quad (6.17)$$

Similarly, with the same assumption

$$D(r) = b_1 r^3 + b_2 r^2 + b_3 r + b_4, \quad (6.18)$$

we can express

$$\frac{1}{T} \int_0^T \cos^2(\Omega_1 t) D(r_s) dt = \frac{1}{2} D(r_0) + \frac{3}{8} (3b_1 r_0 + b_2) s_0^2 \quad (6.19)$$

Finally, we have

$$\begin{aligned} \frac{\Delta f}{f_o} = & -\frac{\kappa_s - \kappa_l}{2\kappa_l} C(r_0) - \frac{\rho_s - \rho_l}{2\rho_s} D(r_0) \\ & - \frac{\kappa_s - \kappa_l}{2\kappa_l} \frac{3}{4} (3a_1 r_0 + a_2) s_0^2(r_0) \\ & - \frac{\rho_s - \rho_l}{2\rho_s} \frac{3}{4} (3b_1 r_0 + b_2) s_0^2(r_0) \\ & + \frac{\frac{\pi^2}{2} m f_o^2}{\langle W_{lk} \rangle - \langle W_{wk} \rangle} s_0^2(r_0). \end{aligned} \quad (6.20)$$

With this expression, if we know the particle vibration amplitude $s_0(r)$ due to the acoustic radiation pressure force, we will be able to calculate the frequency shift caused by this particle vibration.

6.3 Experiment

This experiment is performed exactly the same way as we show in Chapter 5 with an OMFR with roughly 55 μm outer diameter ($f_o = 34.6$ MHz). Two tests are made using 6 μm polystyrene particles and 6 μm silica particles separately. The properties of the materials ¹ used are summarized in Table 6.1, where c_P and c_S are the P-wave and S-wave (acoustic) velocity in the particle material, respectively. The frequency perturbations caused by these particles are shown in Figure 6.2 with respect to particle radial locations.

Table 6.1: Material properties for silica and polystyrene

Material	Density (kg/m^3)	c_P (m/s)	c_S (m/s)	κ (Pa^{-1})
Water	1000	1482	/	4.55×10^{-10}
Silica	2200	5968	3764	2.72×10^{-11}
Polystyrene	1060	2350	1120	2.45×10^{-10}

To compare the results from both Helmholtz method and energy method, we first performed an simulation like we have discussed in Chapter 3. The parameters used for the simulation are summarized in Table 6.2, where β is the radius of curvature of the bottle shape and α is the aspect ratio (see Chap. 3). The simulation results are shown in Figure 6.3(a). A third-order-

Table 6.2: Dimensions of the OMFR

Diameter (μm)	β (μm)	α (μm)	Length (μm)
55	15000	0.75	495

polynomial curve fit is performed for A , B , C , and D , with respect to radial position of the particle, respectively. The relationships are given as:

$$\frac{A(r)}{0.0035} = 4.49 \times 10^{-4}r^3 - 1.06 \times 10^{-2}r^2 - 6.90 \times 10^{-3}r + 0.99 \quad (6.21)$$

¹From <http://www.rfcafe.com/references/general/velocity-sound-media.htm>

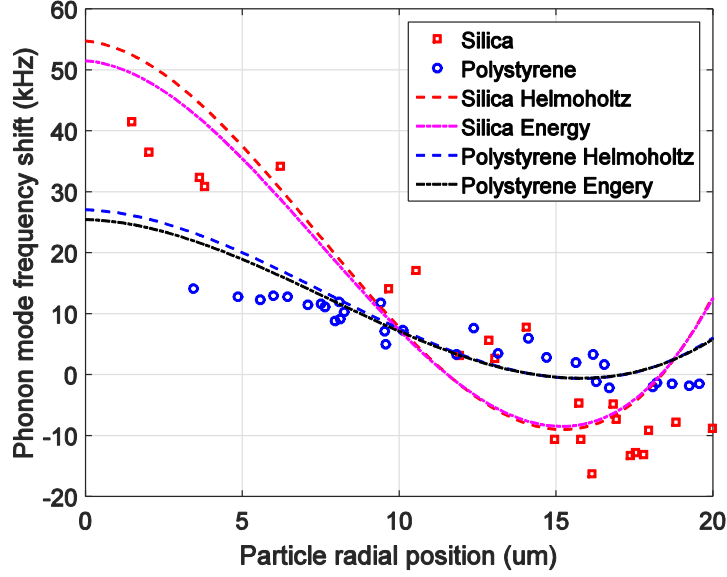


Figure 6.2: Phonon resonance frequency shifts using 6 μm polystyrene and silica particles (experiments) are shown along with the frequency perturbation predictions.

$$\frac{B(r)}{0.0012} = -8.11 \times 10^{-4}r^3 + 1.54 \times 10^{-2}r^2 + 1.08 \times 10^{-2}r + 0.041 \quad (6.22)$$

$$\frac{C(r)}{0.0033} = 4.56 \times 10^{-4}r^3 - 1.08 \times 10^{-2}r^2 - 5.27 \times 10^{-3}r + 0.98 \quad (6.23)$$

$$\frac{D(r)}{0.0011} = -8.15 \times 10^{-4}r^3 + 1.55 \times 10^{-2}r^2 + 9.93 \times 10^{-2}r + 0.0415 \quad (6.24)$$

where unit of r is μm . We note that the aspect ratio the OMFR plays an important role for the simulation results as we can see from Figure 6.3. We see that as the wall becomes thinner, A , B , C , and D become larger, resulting in an increase of the sensitivity to particle. For this study, $\alpha = 0.75$ is used.

The simulated terms A , B , C , and D are then used to find out the frequency perturbation predictions using Eqn. 6.8 and 6.9, and the frequency perturbation predictions are also plotted in Figure 6.2. The test results show a similar trend as what we have seen in Chapter 5. Again, while the predictions from the model can be used to qualitatively describe the experimental results, there is appreciable deviation between the predictions and the mea-

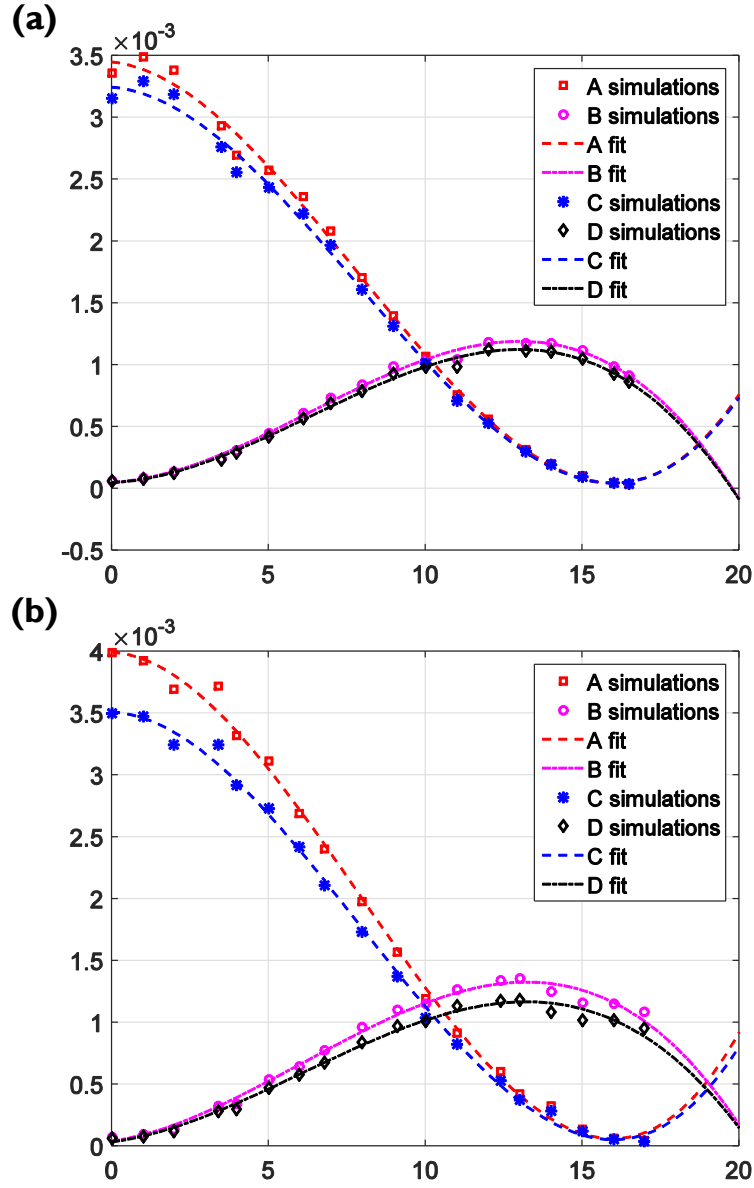


Figure 6.3: A , B , C , and D results obtained from simulation, and their curve fittings for an OMFR having (a) $\alpha = 0.75$, and (b) $\alpha = 0.82$.

surements – the prediction overestimates frequency shifts when the particles are close to the center of the OMFR, and underestimates frequency shifts when particles are away from the center. As discussed above, this deviation happens for many reasons. One of the reasons is that the particle is actually vibrating along with the pressure field, resulting in three additional terms in Eqn. 6.20 compared with Eqn. 6.8 for fixed particles. We note that we need to know the real amplitude of the pressure wave to calculate $s_0(r)$, while the energy terms are obtained through simulation with a random scale. Therefore to correctly evaluate the fifth term on the right hand side of Eqn. 6.8, we need to renormalize the eigenfrequency simulation result (e.g. the pressure field) with respect to its real amplitude. Here we only provide an example calculation to see the effect of the third and fourth term in Eqn. 6.20. When the particle is at the center of the OMFR, since it experiences the same pressure from all direction all the time, the particle vibration amplitude should be zero. Similarly, the particle vibration amplitude should also be zero when the particle is in between the pressure node and the OMFR inner wall (i.e., $r \approx 19 \mu m$) where the pressure forces are balanced from all direction. On the other hand, when the particle is in between the pressure node and antinode, the existence of pressure gradient can cause the imbalance of the pressure force on two sides of the particle, resulting in an particle vibration. Here, we simply assume that

$$s_0(r) = s_{amp} \times \sin(\pi/19 \times r), \quad (6.25)$$

where s_0 , s_{amp} , and r are all with the unit of μm . By writing this, we basically prescribe that the particle vibration amplitude increases from 0 at $r = 0 \mu m$ to s_{amp} at $r = 9.5 \mu m$, and the vibration amplitude decreases to 0 at $r = 19 \mu m$. The results with $s_{amp} = 0, 0.1, 0.2, 0.3$ and $0.4 \mu m$ are shown in Figure 6.4. As we can see from the result, this moving particle hypothesis seems to explain some of the mismatch between data and predictions.

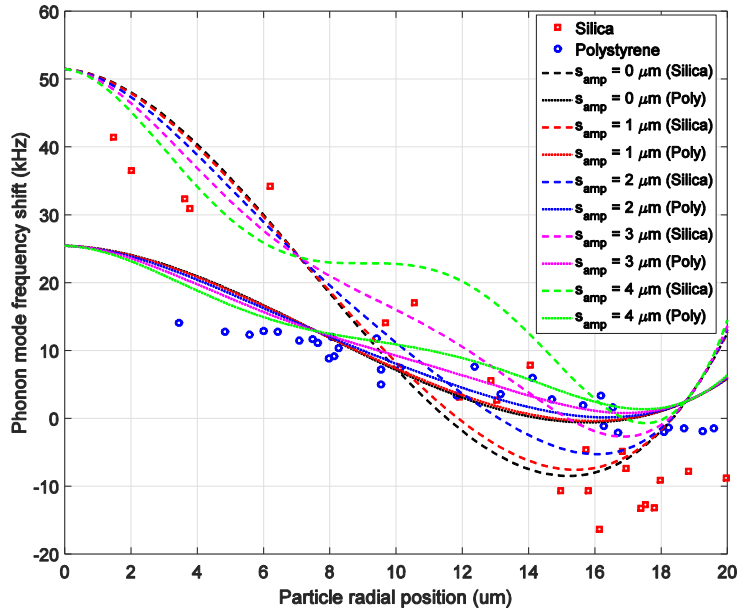


Figure 6.4: Frequency perturbation predictions (dashed lines) using Eqn. 6.20 with $s_{amp} = 0, 0.1, 0.2, 0.3$ and $0.4 \mu m$ are plotted in comparison with the experimental results (dots) for silica and polystyrene materials, respectively. Particle vibrational kinetic energy is neglected.

6.4 Empirical method for determining particle compressibility

We have observed that the predictions using Eqn. 6.9 with A and B (or Eqn. 6.8 with C and D) directly from simulation mismatch experimental frequency perturbations. Furthermore, this mismatch has similar feature for both silica and polystyrene particles. Without precise knowledge of mechanisms that cause this mismatch, here we show that by calibrating the term A and B first with one type of sample particles, we can extract the particle compressibility with very high accuracy. We thus propose to add a single empirical scaling parameter N into our Equation 6.9 such that:

$$\Delta f = N f_0 \left(-\frac{\rho_s - \rho_l}{2\rho_s} B(r) - \frac{\kappa_s - \kappa_l}{2\kappa_l} A(r) \right) \quad (6.26)$$

In this study, we used the silica particle as the sample particles to obtain N . To do this, we perform a least-square curve fitting using the silica test data with N as an unknown and all the other parameters as known (Figure 6.5a). Here, we use the material properties from Table 6.1 and the third-order polynomials for A and B given in Equation 6.21 and 6.22. $N = 0.768 \pm 0.171$ is thus obtained. We then use the extracted N as known and apply the least-square curve fitting for Equation 6.26 again to a new set of polystyrene data with the compressibility of polystyrene as the unknown (Figure 6.5b). The compressibility of polystyrene is thus obtained as $(2.53 \pm 0.26) \times 10^{-10} \text{ Pa}^{-1}$. The error compared with data provided in Table 6.1 is 3%. As a comparison, if we do a curve fitting for Equation 6.26 again with $N = 1$, the compressibility extracted is $(2.97 \pm 0.20) \times 10^{-10} \text{ Pa}^{-1}$, which is 22% different compared with data provided in Table 6.1. We thus conclude that our single-parameter empirical method has improved the compressibility prediction accuracy significantly.

We note that the above method cannot provide accurate sensitivity prediction when the sensitivity of the particle is much smaller than the ambient fluid. As we can see from Eqn. 6.26, since $\frac{\kappa_s - \kappa_l}{2\kappa_l} = \frac{\kappa_s}{2\kappa_l} - \frac{1}{2}$, when $\kappa_s \ll \kappa_l$, a small change in the compressibility contrast due to fitting requires a large relative change in κ_s , resulting in a poor estimation of κ_s . For example, we apply the least-square curve fitting for Equation 6.26 again to the silica data with the compressibility of silica as the unknown and use $N = 0.768$ found

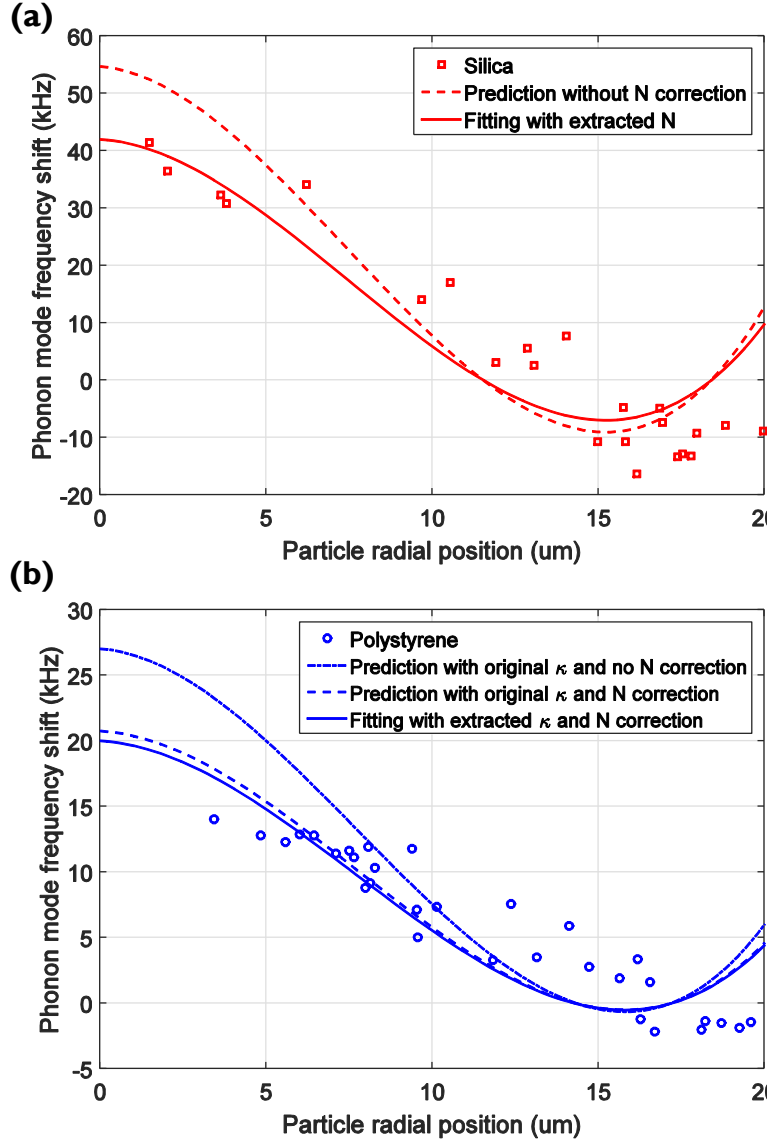


Figure 6.5: (a) Least-square curve fitting for Equation 6.26 is applied to the silica data with N as the unknown. The material properties are from Table 6.1, and the functions for A and B are given in 6.21 and 6.22. We obtain $N = 0.768$. The prediction with $N = 1$ is given as a comparison. (b) The obtained N is used for the least-square curve fitting to the polystyrene data with the compressibility of polystyrene as the unknown. The rest material properties are from Table 6.1, and the functions for A and B are given in 6.21 and 6.22. We obtain the compressibility of polystyrene as $2.53 \times 10^{-10} \text{ Pa}^{-1}$, 3% different from the data in Table 6.1. The predictions with compressibility from Table 6.1, with and without scaling factor N , are given as comparisons.

for silica particles in the first step. The compressibility extracted is $(1.75 \pm 9.00) \times 10^{-11} \text{ Pa}^{-1}$. The huge error bar indicates that we simply cannot trust this extracted value.

6.5 Future work

As we can see from the last section, the successful extraction of the compressibility of the test particles requires knowledge of both terms A and B with respect to r for a certain particle size. This knowledge can be obtained either through simulation or through direct calibration using sample particles. However, this knowledge of the spatial distribution of the pressure field, or terms A and B , is redundant. Theoretical, if we can accurately place each particle such that every particle transits through the same radial location inside the OMFR, we can decouple the effects from the particle radial location. Therefore, only two different calibration particles are needed, corresponding to two unknowns, A and B . We propose that this particle positioning can be achieved by implementing a sheath flow (Figure 6.6) that brings particles through the sensor region via the same streamline. Such techniques are already implemented in flow cytometers. With A , B , and density of the sample particle available, we can then measure the compressibility of any sample particle using Eqn. 5.11.

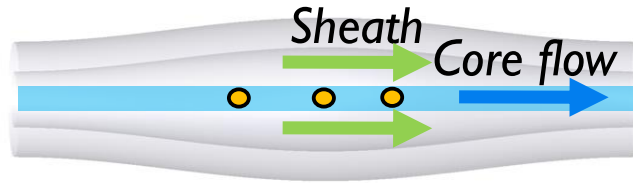


Figure 6.6: Illustration of using sheath flow to confine particles in the core flow region.

Another interesting observation of the above analysis is that both the compressibility κ_s and the density ρ_s of the particle can be determined independently if the particle is much smaller compared against the acoustic wavelength. For example, B vanishes when the particle is at the pressure field maximum, resulting in the frequency perturbation being only sensitive

to the compressibility contrast. On the other hand, A vanishes when the particle is at the pressure node allowing measurement of the density contrast. Therefore, to measure the compressibility of any micro-particle with known size, that is small enough compared with the acoustic wavelength, we only need one type of calibration particles for finding out the A term, given that we are flowing the particles through the pressure field maximum.

However, for any natural bioanalyte like cells, the size variability has to be taken into account for the compressibility measurement. With both the size and the compressibility as unknowns, we need at least two separate measurements. Luckily, as we have shown in Chapter 5, the OMFR indeed exhibits multimode sensing capability. Figure 6.7 shows that a particle transit is simultaneously measured by four different mechanical modes. These mechanical modes are mostly likely breathing modes of different axial orders. With two of these vibrational modes, the effect of size and compressibility can be decoupled, and the value of each can be obtained using the inverse problem algorithm similar to that in [134].

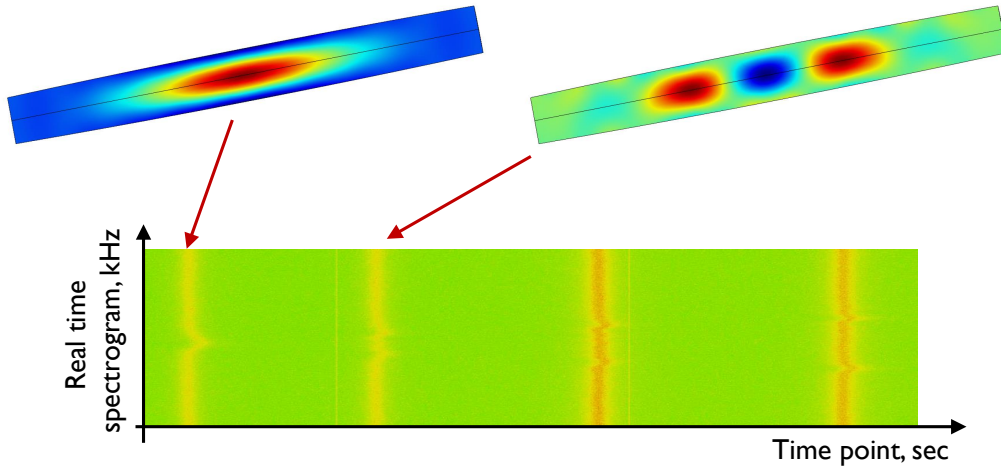


Figure 6.7: A particle transit is simultaneously recorded by four different mechanical modes. The frequency perturbation pattern for each mode (from left to right) shows the signature of breathing mode of axial order $L = 1, 3, 5$, and 7 , respectively.

CHAPTER 7

OTHER APPLICATIONS WITH OMFRS

7.1 Fast particle sensing using OMFR

Introduction

¹ In the previous two Chapters, we have introduced the a novel class of solid-liquid hybrid optomechanical resonators that perform phonon-mediated optical detection of nanoparticles at extremely high speed and at long range without adsorption. The microcapillary-type resonators support ultrahigh-Q optical modes that are coupled to co-localized mechanical (phonon) modes, while fluid analytes are flowed internally without influencing the optics. Phonons permeate the entire cross-section of the resonator, casting a near-perfect net for measuring particles flowing along the fluid streamlines. All particles in the sample must transit and perturb these phonon modes, in turn perturbing the optical readout due to the strong optomechanical coupling.

We have also shown that the detecting rates are ultimately limited by phonon lifetime, and are potentially exceeding 10,000 particles-per-second. However, since the measurement is done by measuring the thermal-mechanical fluctuation spectrum, the vibrational mode signal is close to the noise floor. Therefore the center-frequency tracking of the vibrational modes relies on curve fitting of the spectrum, which can only be done after experiment in the post-processing. Such post-processing requirement significantly slows down the experiment. This measurement technique thus needs to be improved if we want to implement real-time measurement of particles. In this section, an electro-opto-mechanical driving and lock-in technique, for which the author

¹Portion of Section 7.1 is reprinted with permission from Suh, J., Han, K., Peterson, C. W., and Bahl, G. (2017). “Invited Article: Real-time sensing of flowing nanoparticles with electro-opto-mechanics,” *APL Photonics*, 2(1), 10801.

has participated in the development, is briefly introduced.

Measurement principle and setup

As discussed in Chapter 5, to track the center frequency of the phonon modes, the phonon spectra with a wide frequency range around the phonon mode center frequency are collected and curve fitting to each of them is done to extract the phonon mode center frequency at each time instance. Both the frequency sweep and the curve fitting take time and slow down the measurement. The reason why we have to do the curve fitting to extract the frequency, as opposed to just taking the maximum of the spectrum, is that the phonon mode signal strength is very weak since we are monitoring the thermal-mechanical fluctuations of the vibrational modes. We can improve the signal-to-noise ratio by exciting the radiation-pressure induced oscillation like we did in [92]. To excite the radiation-pressure induced oscillation, the optical power circulating inside the resonator has to be larger than the threshold power. However, this is not easy to achieve due to the high mechanical energy loss (viscous loss and radiation loss) associated with the fluids [3]. Instead, since silica is an insulator, we make use of electrostatic actuation method to drive the resonator actively. As shown in Figure 7.1, a large single-tone RF stimulus is sent to a pair of wire electrodes near the OMFR, applying electrostatic force to the OMFR at frequency ω_{RF} near the mechanical resonance frequency. Through optomechanical coupling, the electrostatically excited oscillation can be seen from the spectrum (Figure 7.1) as a sharp peak on top of thermal-mechanical fluctuation spectrum of the phonon mode. Compared with the thermal-mechanical fluctuations, the signal-to-noise ratio of the electrostatically excited oscillation is enhanced by more than 50 dB. The RF frequency is chosen at which the RF amplitude response has the largest slope to maximize the sensitivity. As we have discussed in Chapter 5, a photodetector performs heterodyne measurement of the forward scattered light, which is monitored using both a real-time electronic spectrum analyzer, and a lock-in amplifier (Figure 7.2). Part of the RF stimulus is sent directly to the reference port of the lock-in amplifier such that the transfer function from the RF signal input to the optical modulation on the photodetector at frequency ω_{RF} can be monitored in real-time

through an oscilloscope (Figure 7.2). During the particle transits, the frequency perturbations of the phonon modes translate into both amplitude and phase perturbations of the transfer function. In this work, we monitor only the amplitude of the transfer function. Changes in amplitude are converted back to the center frequency shift of the phonon mode through a calibration done before the experiment.

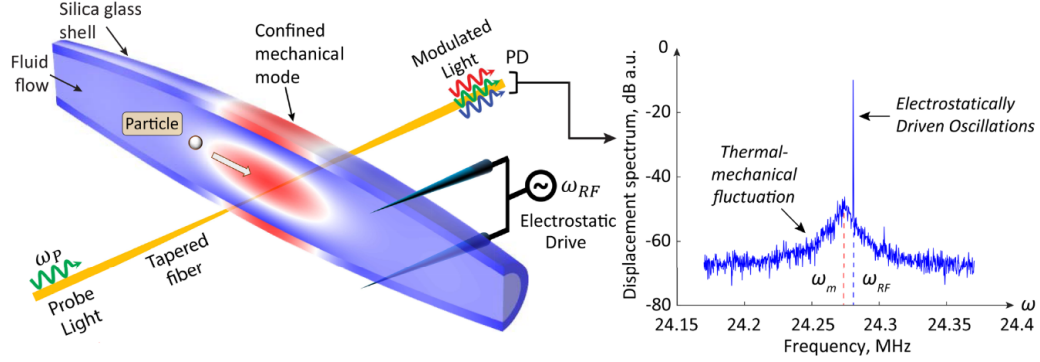


Figure 7.1: (Reproduced from [160] with permission) **Working principle of the electro-static driven OMFR.** The difference between the working principle of the electro-static driven OMFR with the normal OMFR like we have seen in Chapter 5, is that the resonator is actively driven by a single-tone electrostatic force at ω_{RF} by two 100 μm wire electrodes. The resulting spectrum is shown on the right. On top of the signal from the thermal-mechanical fluctuation with center frequency ω_m , a sharp signal from the electrostatically driven oscillation at ω_{RF} is shown with a much higher signal-to-noise ratio. Frequency perturbations of the phonon mode due to particle transits exhibit as amplitude perturbations to signal of the electrostatically driven oscillation.

Results

The OMFR used for this test has a 70 μm outer diameter and a 50 μm inner diameter. The measurement is performed using a 24.26 MHz mechanical mode with monodisperse 3.62 μm silica particles mixed in water. As shown in Figure 7.3(a), the frequency perturbation can be clearly seen above the noise floor, without any post-processing. Two measured frequency perturbations are zoomed in to show the particle transit time (Figure 7.3(b)) with flow rate at 50 $\mu\text{l}/\text{min}$. The shortest transit time achieved is about 490 μs , which is

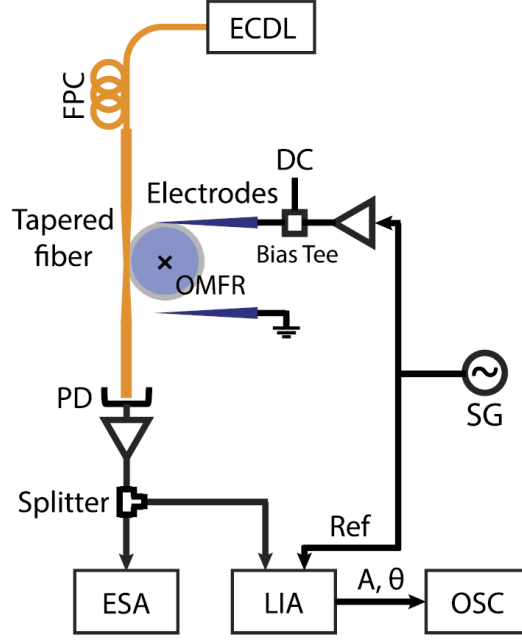


Figure 7.2: (Reproduced from [160] with permission) **Experimental setup.** An RF stimulus at ω_{RF} generated from the signal generator (SG) is split and sent to one of the electrodes and the reference port of the lock-in amplifier (LIA). A DC signal is added to the RF stimulus to the OMFR to enhance the electrostatic drive force. The electrical signal from the photodetector (PD) is split and sent to the electrical spectrum analyzer (ESA) and the input port of LIA. The output from the LIA are monitored in real-time on a oscilloscope. Both amplifiers are used to increase the signal-to-noise ratio.

corresponding to more than 1000 particles/s, 40x faster than what is shown in Chapter 5. We note that this measurement speed is only limited by the available syringe pump flow rate.

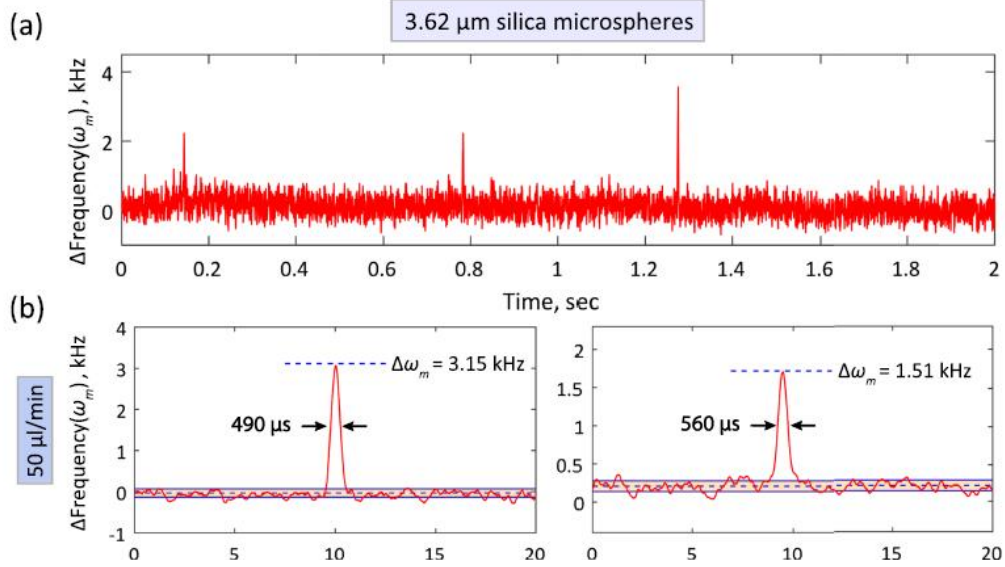


Figure 7.3: (Reproduced from [160] with permission) **Experimental results.** (a) Phonon frequency trace is recorded during the transit of 3.62 μm silica particles through an OMFR with 70 μm outer diameter and 50 μm inner diameter. (b) Two quick transits are measured at flowrate 50 μl/min.

The system exhibits similar sensitivity and ultimate throughput limit like we have shown in Chapter 5. Since the sensitivity to particle relies on the radial location of the particles, we cannot make any measurement of the particle properties, like density and compressibility, unless we know the particle position. We have used cameras to triangulate the particle radial position in Chapter 5. However, for fast transits with a time scale of ms or less, the particle transits are too fast to record even for high-speed camera. Therefore, in the future, it is important for us to implement hydrodynamic focusing (sheath flow) to help confine the particle to a known radial location. It is also possible, since the OMFR is made of transparent silica glass, to tag the flowing particle with fluorescent dyes, in the same way as traditional flow cytometry. When the laser light is focused onto the sample stream, the scattered light due to the particle transits can be measured to provide the particle radial location. Furthermore, the simultaneous extraction of optical and mechanical responses for single bioparticles may provide us previously unavailable information.

7.2 Fabrication of highly stretchable nanowire

² Besides for fabricating OMFR, the author has been involved in adapting the laser drawing setup for fabricating ultra-long Pd micro- and nanowires [161]. The OMFR has been designed to be hollow for microfluidic experiments. Here in this section, we show that this hollow structure can be exploited for this newly developed laser draw-cast (LDC) process.

The nanowire fabrication process is the same with the OMFR fabrication process (Fig. 7.4(a)) except that before pulling, a Pd metal wire of 25.4 μm diameter is inserted inside of a thick-walled fused silica capillary preform (ID: 50 μm , OD: 800 μm). Pd material is chosen here mainly because it has a melting temperature (1828 K) similar to the glass transition temperature of silica (1943 K) such that during pulling, the Pd wire can be melted and remain molten without much evaporation. As we can see from Fig. 7.4(b), in the preheating stage, the molten Pd wire balls up and seals the inside channel of the OMFR because of surface tension. After that, when the pulling starts, the diameter of the capillary is reduced and the molten Pd is squeezed and driven by the shear force at the inner wall of the capillary. This shear force resulting from surface tension between the molten metal and the capillary is critical since it helps to stretch the molten metal into the thinner part of the capillary. Similar to the OMFR fabrication process, the preheating power and time is just enough to soften the silica glass. The shape of the capillary maintains to confine the flow of the Pd melt. In other words, as the silica capillary is being drawn, it becomes a dynamic mold in which the molten Pd can be cast. The final cast metal therefore exists within the drawn capillary meniscus region (Fig. 7.4(c)) from where the necking starts to where the silica dimension stops changing. Therefore the drawn metal also has the same conical shape as the silica meniscus. For a typical drawn Pd wire shown in Fig. 7.5(a), the diameter of the drawn metal wire varies from 1203 to 180 nm over 3.75 mm length along the wire. As we can see from the microscopic view of the metal wire (Fig. 7.5(b)), the surface is very smooth as well.

Unlike the OMFR fabrication process where thin-walled preform is used to

²Portion of Section 7.2 is reprinted with permission from Zhang, K., Han, K., Shi, S., Bahl, G., and Tawfik, S. (2016). “Highly Stretchable Conductors Made by Laser Draw-Casting of Ultralong Metal Nanowires,” *Advanced Electronic Materials*, 2(6), 1600003.

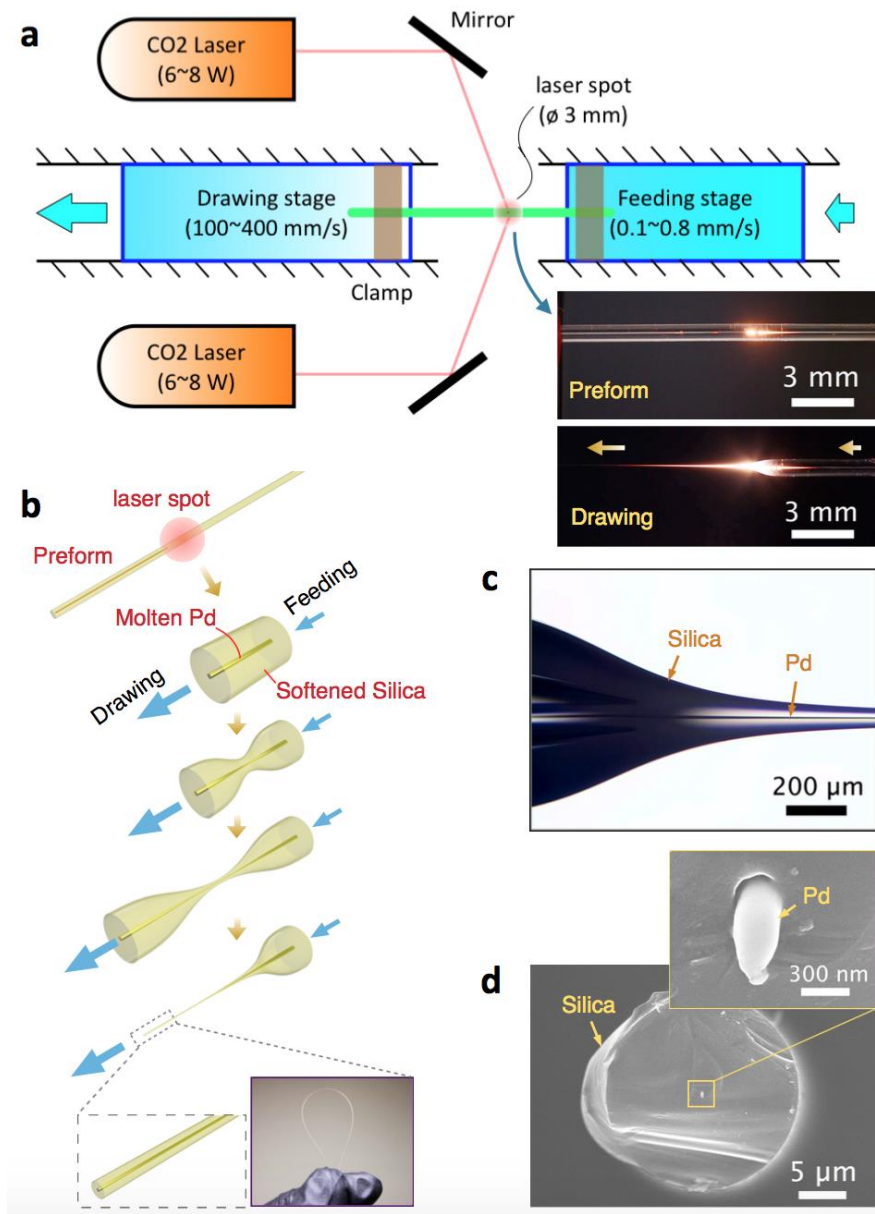


Figure 7.4: (Reproduced from [161] with permission) (a) The metal nanowire fabrication setup is the same with the OMFR fabrication setup, except that the pulling speed is much higher for the nanowire fabrication. (b) The metal fabrication process is almost the same with the OMFR fabrication except that a Pd metal wire is inserted to thick-walled silica capillary preform before pulling. During the pulling process, the molten Pd is stretched and reformed by the shear stress at the silica-Pd interface. (c) Image of the capillary meniscus with the Pd wire inside. (d) SEM image of the cross section of drawn capillary shows that the Pd wire is reformed and filled inner channel of the OMFR.

get thin-walled device, a really thick-walled preform is used here so that the molten Pd material is able to fill up the inner channel. We cannot simply use smaller thin-walled capillary preform, e.g. with ID: 50 μm , OD: 300 μm , since absorption area to laser may become too small to get the process temperature high enough.

The viscosity of the softened silica is 10^8 higher than that of the molten Pd at the process temperature, the shear force caused by the Pd to the capillary is negligible. Therefore, the metal material can be considered as a thermal mass during pulling. When designing the geometry of the pulled nanowire, we just need to control the reduction of the capillary inner diameter using the same controllable parameters such as pulling speed, feeding speed, and process temperature the same way as we did previously in Chap. 2 for the OMFR diameter control. For example, the pulling speed is set at the extreme speed that the current motor can provide, 400 mm/s, to reduce the size of nanowire. Fig. 7.5(b) shows the relationship between average drawn metal wire thickness and the feeding speed. In agreement with the analysis we did above, as the feeding speed increases, the average diameter of the drawn wire increases.

Due to the special fabrication process, the nanowire exhibit single-crystalline nature across the cross section of the wires in a bamboo-shaped grain structure. This single-crystalline nature allows the nanowire to have 53% local strain without breaking, which is one order of magnitude better than that of the short Si nanowires grown by vapo-liquid-solid process [162]. Such good endurance to local strain enable the LDC fabricated nanowire as stretchable material. One way to realize such stretchable conductor is to deposit the nanowire on a pre-strained substrate. After deposition, The nanowire starts to buckle when the pre-strained substrate is relaxed. From 100% to 700% strain, the conductivity of the nanowire is only changed by 0.4%, which indicates a great potential of the LDC fabricated nanowires as a highly stretchable conductor material.

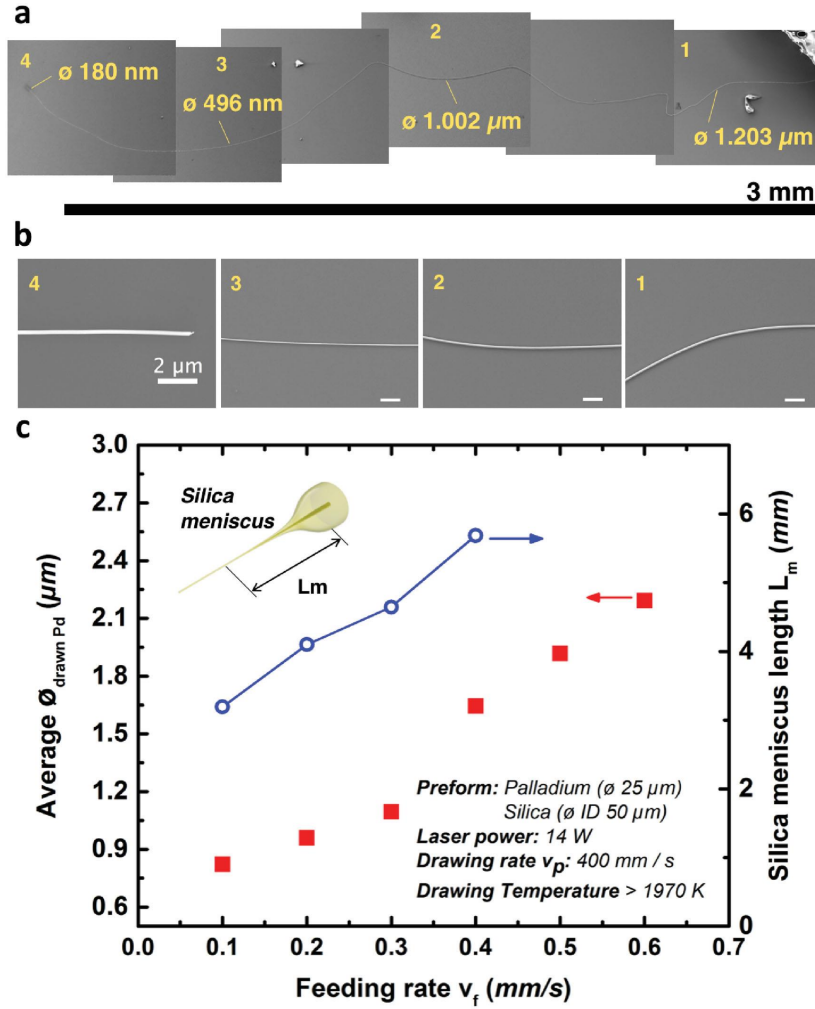


Figure 7.5: (Reproduced from [161] with permission) (a) The conical morphology is transferred from the capillary meniscus. (b) SEM images of drawn Pd wire at different positions. (c) The relationship between average diameter of the drawn metal wire and the feeding speed.

APPENDIX A

FIELD OF VIEW MATRIX FOR THE NAVITAR 12X ULTRAZOOM SYSTEM

12X UltraZoom Field of View Matrix

12X UltraZoom Field of View Matrix for 1-50502, 1-50503 and 1-50504 (mm)

Objective Lens (Mitutoyo) Long W.D.	W.D. (mm)	Camera Formats/ Parameters	1X Adapter Low - High	1.33X Adapter Low - High	2X Adapter Low - High	3.3X Adapter Low - High
4X 0.14 NA* 1-55075	20	Mag.	(1) 2.60X - 13.35X	(2) 2.61X - 17.75X	2.19X - 26.66X	3.61X - 44.04X
		1/4" Sensor	1.38 - 0.30	1.53 - 0.23	1.83 - 0.15	1.11 - 0.09
		1/3" Sensor	2.07 - 0.45	2.30 - 0.34	2.74 - 0.22	1.66 - 0.14
		1/2" Sensor	2.76 - 0.60	3.06 - 0.45	3.65 - 0.30	2.21 - 0.18
		2/3" Sensor	3.79 - 0.82	4.21 - 0.62	5.02 - 0.41	3.04 - 0.25
5X 0.14 NA* 1-60226	34	Mag.	(1) 3.57X - 16.66X	(2) 3.26X - 22.16X	2.77X - 33.31X	4.52X - 55.05X
		1/4" Sensor	1.12 - 0.24	1.22 - 0.18	1.44 - 0.12	0.89 - 0.07
		1/3" Sensor	1.68 - 0.36	1.84 - 0.27	2.17 - 0.18	1.33 - 0.11
		1/2" Sensor	2.24 - 0.48	2.45 - 0.36	2.89 - 0.24	1.77 - 0.15
		2/3" Sensor	—	2.45 - 0.49	3.97 - 0.33	2.44 - 0.20
10X 0.28 NA* 1-60227	33	Mag.	(1) 7.14X - 33.31X	(2) 6.50X - 44.30X	5.54X - 66.63X	9.03X - 110.10X
		1/4" Sensor	0.56 - 0.12	0.61 - 0.09	0.72 - 0.06	0.44 - 0.04
		1/3" Sensor	0.84 - 0.18	0.92 - 0.13	1.08 - 0.09	0.66 - 0.05
		1/2" Sensor	1.12 - 0.24	1.23 - 0.18	1.44 - 0.12	0.89 - 0.07
		2/3" Sensor	—	1.23 - 0.25	1.99 - 0.17	1.22 - 0.10
20X 0.42 NA* 1-60228	20	Mag.	(1) 14.28X - 64.63X	(2) 13.10X - 85.96X	11.08X - 133.25X	18.07X - 220.21X
		1/4" Sensor	0.28 - 0.06	0.30 - 0.04	0.36 - 0.03	0.22 - 0.02
		1/3" Sensor	0.42 - 0.09	0.46 - 0.07	0.54 - 0.04	0.33 - 0.03
		1/2" Sensor	0.56 - 0.12	0.61 - 0.09	0.72 - 0.06	0.44 - 0.04
		2/3" Sensor	—	0.61 - 0.13	0.99 - 0.08	0.61 - 0.05
50X 0.55 NA* 1-60229	13	Mag.	(1) 35.66X - 166.57X	(2) 40.00X - 221.54X	27.50X - 333.13X	45.17X - 550.52X
		1/4" Sensor	0.11 - 0.02	0.10 - 0.02	0.14 - 0.01	0.09 - 0.007
		1/3" Sensor	0.17 - 0.04	0.15 - 0.05	0.22 - 0.02	0.13 - 0.010
		1/2" Sensor	0.22 - 0.05	0.20 - 0.04	(2) 0.17 - 0.03	0.18 - 0.010
		2/3" Sensor	—	0.20 - 0.05	0.40 - 0.03	0.24 - 0.020

NOTE: This system is not recommended for use with a 2/3" Sensor.

(1) Zoom setting at 1.5X.

(2) Zoom setting at 1.0X.

*NA at high mag. NA varies with zoom setting.

Figure A.1: The field of view matrix for the Navitar 12x UltraZoom lens (1-50502) system. For the 10X Infinity corrected objective (Edmund optics 59-877) and 1X adapter tube (Navitar 1-6218) used in our experiment, the magnification provided by the system is 33.31X when the maximum magnifying power is used from the Navitar 12x UltraZoom lens as highlighted (<https://navitar.com/products/imaging-optics/high-magnification-imaging/12x-zoom/>)

APPENDIX B

MATLAB CODE FOR PHONON MODE CENTER FREQUENCY TRACKING

Main code: 'Plotspectrogram2.m'

```
1 % Program reads the RSA spectrogram .mat file and plots a spectrogram and
2 % the CF trace from Lorentzian fit.
3 % Then do a Lorentzian fit for trace to find out the center frequency. If
4 % the data trace is too noisy, smooth with a reasonable span.
5
6 clear all;
7 close all;
8 display(['Start time:' datestr(clock, 0)])
9 load('1123S8.mat');
10 display('=====');
11
12
13 %% First determine how many spectra are there in the file
14
15 % — OPTION 1: user manually enters this by looking at file —
16 % NumSpectra = 24120; % 0 to 24119
17
18 % — OPTION 2: automated slow code
19 for i = 1:100000 % increase the number if larger file
20     if ~exist(['S', num2str(i)], 'var')
21         break
22     end
23 end
24 NumSpectra = i;
25 display(['Number of spectrum variables is ' num2str(NumSpectra)]);
26 display(['Duration of the data is ' num2str(NumSpectra)]);
27
28
29 % — calculate —
30 MaxTime = NumSpectra * TDelta;
31 TimeArray = [0:NumSpectra-1] * TDelta; % this is for the entire data set
32 FrequencyValues = linspace(SpectraCenter-SpectraSpan/2, SpectraCenter+
    SpectraSpan/2, 801)';
33 display(['Data set duration = ' num2str(TimeArray(end)) ' seconds']);
34
35
36
37 %% Input the start and stop times
38 % — OPTION 1: user inputs the start and stop times for the plot
```

```

39     StartTime = 0;    % seconds
40     StopTime = 55;   % seconds
41
42 % — OPTION 2: prompt the user
43 %     display('=====');
44 %     StartTime = input('Enter starting time in seconds : ');
45 %     StopTime = input('Enter ending time in seconds : ');
46
47
48 %% continue
49
50 % — variable descriptions inside the RSA file —
51 % TDelta = step between adjacent spectra
52 % SpectraCenter = CF
53 % SpectraSpan = span
54
55
56 % — calculate the starting and stopping index of the dataset
57 % based on times provided by the user
58 StartDistance = abs(TimeArray - StartTime);
59 StartPos = find(StartDistance == min(StartDistance));
60 StopDistance = abs(TimeArray - StopTime);
61 StopPos = find(StopDistance == min(StopDistance));
62
63 VariableSuffixes = [StartPos:StopPos];    % these are the S1, S2. .. indices
64 TimeValues = TimeArray(StartPos:StopPos)';
65
66 %% — now construct a 2D array —
67 DataSet = ones(801, length(VariableSuffixes)); % this is the 2D set
68 CenterFreq=ones(length(VariableSuffixes),2); % this is the 2D column of
        center frequency and uncertainty set
69 LineWidth=ones(length(VariableSuffixes),2);
70 NoiseFloor=ones(length(VariableSuffixes),2);
71 Amplitude=ones(length(VariableSuffixes),2);
72 Scaling=ones(length(VariableSuffixes),2);
73 counter = 0;
74 Span=20; % Smoothing span
75 FitPoints=101:750; %extract only the center part of the spectrum for better
        fitting
76 for thisSuffix = VariableSuffixes
77     counter = counter+1 % to display the current trace
78     % construct the variable name
79     thisVariableName = ['S',num2str(thisSuffix)]; % S1, S2 etc...
80     % evaluate the variable
81     this_data = eval(thisVariableName);
82     % dump it in the 2D array
83     DataSet(:,counter) = this_data;
84     % evaluate the center frequency and uncertainty by calling LorFit
        function
85     % Fit the first curve using frequency at the maximum power (or something
        that will give a good fit)
86     if counter==1;
87     [CenterFreq(counter,1), CenterFreq(counter,2), LineWidth(counter,1),
        LineWidth(counter,2),...

```

```

88         NoiseFloor(counter,1),NoiseFloor(counter,2),Amplitude(counter,1),
            Amplitude(counter,2), Scaling(counter)]...
89         =LorFit2(FrequencyValues, smooth(this_data,Span), FrequencyValues(
            this_data == max(this_data)));
90 %     % Pass the previous Fo as the initial guess for the next fitting
91     else
92         [CenterFreq(counter,1), CenterFreq(counter,2), LineWidth(counter,1),
            LineWidth(counter,2),...
93         NoiseFloor(counter,1),NoiseFloor(counter,2),Amplitude(counter,1),
            Amplitude(counter,2), Scaling(counter)]...
94         =LorFit2(FrequencyValues(FitPoints), smooth(this_data(FitPoints),
            Span), CenterFreq(counter-1,1));
95     end
96
97 end
98
99 filename=['FrequencyTrace_SmoothingSpan',num2str(Span),'_',num2str(StartTime
    ),'_s to ',num2str(StopTime),'s.mat'];
100 save(filename,'TimeValues','CenterFreq','LineWidth','NoiseFloor','Amplitude'
    ,'Scaling')
101 display('done loading');
102
103
104 %% Plot the spectrogram
105 figure(1)
106 df=(FrequencyValues-SpectraCenter)/1e3;
107 surf(TimeValues,df, DataSet, 'EdgeColor','none');
108 % note the kilohertz conversion
109 % also note that we only measure shift from center
110 set(gca,'fontsize',16);
111 ylabel('Frequency shift, kHz','fontsize',16);
112 xlabel('Time point, sec','fontsize',16);
113 title(['Center frequency = ', num2str(SpectraCenter/1e6) ' MHz']);
114 axis([StartTime, StopTime, min(df), max(df), min(min(DataSet)), max(max(
    DataSet))+10]);
115 % make the axis a tight fit, but add 10 dB to amplitude for better
    visualization
116 view(2); % sets top view
117
118
119 %% Plot the center frequency/Linewidth versus time
120 figure(2)
121 plot(TimeValues,((CenterFreq(:,1)-SpectraCenter)/1e3))
122 xlabel('Time point, sec','fontsize',16);
123 ylabel('Frequency, kHz','fontsize',16);
124 title(['Center frequency = ', num2str(SpectraCenter/1e6) ' MHz']);
125
126 figure(3)
127 plot(TimeValues,LineWidth(:,1)/1e3)
128 xlabel('Time point, sec','fontsize',16);
129 ylabel('LineWidth, kHz','fontsize',16);
130
131 %%
132 display(['Stop time:' datestr(clock,0)])

```

Function for Lorentzian fitting: 'Lorfit2.m'

```

1 function [PeakFreq,UncertainFreq,LineWidth,UncertainLW,NoiseFloor,
    UncertainNF,Amplitude,UncertainA, scaling] = LorFit2(xdata, ydata,FoLast
    )
2 %%in this version, we will pass the fitting frequency (Fo) from previous
3 %fitting to the next fitting as the initial guess.
4 % input frequency as xdata, power as y data, both x and y should be a
    column of data
5
6 %%
7 lineardata = double(10.^(ydata/10)*1e6); % linear power data in W
8 scaling = max(lineardata); %scaling
9 lineardata = lineardata/scaling; % normalize the spectrum for curve fitting
10
11
12 %% Lorentzian fitting function
13
14 opts = fitoptions('Method', 'NonlinearLeastSquares');
15 % vars : A, Fo, Gamma (alphabetical order) , tox
16 opts.Lower = [ 0, min(xdata), 10, 0];
17 opts.Upper = [ 1, max(xdata), 1e8, 1];
18
19 % set up the initial fit
20 opts.StartPoint = [0.5, FoLast, (xdata(end)-xdata(1))/10, 0.1]; %
    sometimes there might me more than one maximum
21 ftype = fitype('A*(Gamma/2)^2 ./ ((f-Fo)^2 + (Gamma/2)^2) + Z', ...
22 'options', opts, ...
23 'independent', 'f', ...
24 'dependent', 'power');
25 fitresult = fit(xdata, lineardata, ftype);
26
27 % set up the second fit for refining the fitting
28 opts.StartPoint = [fitresult.A, fitresult.Fo, fitresult.Gamma, 0.1];
29 ftype = fitype('A*(Gamma/2)^2 ./ ((f-Fo)^2 + (Gamma/2)^2) + Z', ...
30 'options', opts, ...
31 'independent', 'f', ...
32 'dependent', 'power');
33 fitresult = fit(xdata, lineardata, ftype);
34
35 %% return the center frequency, centerfrequency uncertainty, linewidth,
    linewidth uncertainty
36 ci=confint(fitresult, 0.95); % returns the 95% confidence bounds for the
    curve fitting. Level can be change the level between 0 and 1
37 % peak frequency
38 PeakFreq=fitresult.Fo;
39 HigerFreq=ci(2,2);
40 UncertainFreq=(HigerFreq-PeakFreq); % f=f +- UncertainFreq
41 % Linewidth
42 LineWidth=fitresult.Gamma;
43 LowerLW=ci(1,3);
44 UncertainLW=(LineWidth-LowerLW);
45 % Noisefloor
46 NoiseFloor=fitresult.Z;

```

```

47 LowerNF=ci(1,4);
48 UncertainNF=NoiseFloor-LowerNF;
49 % Amplitude
50 Amplitude=fitresult.A;
51 LowerA=ci(1,1);
52 UncertainA=Amplitude-LowerA;

```

APPENDIX C

MATLAB CODE FOR PHONON MODE CENTER FREQUENCY SHIFT DETERMINATION

Main code: 'FindFrequencyShift.m'

```
1
2 close all
3 clear all
4 % First to load the data
5 load('FrequencyTrace_SmoothingSpan20_0s to 55s.mat')
6
7 %% to plot the original CF trace, find peak positions and heights
8 figure(101)
9 CF=(CenterFreq(:,1)-CenterFreq(1,1));
10
11 %moving average smoothing
12 span=20;
13 smoothCF=smooth(CF,span);
14 %Savitzky-Golay FIR smoothing
15 order=5; % change the order of the fitting polynomial. Higher is smoother
16 framelen=441; % change the data interval used for fitting. More is smoother
17 smoothCF_SG=sgolayfilt(CF,order,framelen);
18 plot(TimeValues,CF/1e3,'b', TimeValues,smoothCF_SG/1e3,'r', TimeValues,
      smoothCF/1e3, 'g' )
19
20 set(gca, 'fontsize', 18, 'linewidth',1)
21 hFig = figure(101);
22 set(hFig, 'Position', [0, 550, 1300, 400]);
23 xlabel('Time point, sec', 'fontsize', 20);
24 ylabel('Mechanical frequency shift, kHz', 'fontsize', 20);
25 title(['Original trace, Starting f = ' num2str(CenterFreq(1,1)/1e6) ' MHz'
      '']);
26 legend('Original','Savitzky-Golay','Moving average')
27
28 figure(1001) % to find the positive peak position automatically sing
      findpeaks for the filtered data
29 findpeaks(smoothCF_SG, 'MinPeakHeight',5000, 'MinPeakProminence',1000, '
      Annotate','extents')
30 [~,locs]=findpeaks(smoothCF_SG, 'MinPeakHeight',5000, 'MinPeakProminence'
      ,1000, 'Annotate','extents');
31 set(gca, 'fontsize', 18, 'linewidth',1)
32 hFig = figure(1001);
33 set(hFig, 'Position', [1300, 300, 600, 600]);
34 title(['Positive peaks found']);
```

```

35
36
37 figure(1002) % to find the negative peak position automatically sing
    findpeaks for the filtered data
38 findpeaks(-smoothCF_SG, 'MinPeakHeight',5000, 'MinPeakProminence',100, '
    Annotate', 'extents')
39 [~, locs_neg]=findpeaks(-smoothCF_SG, 'MinPeakHeight',5000, 'MinPeakProminence'
    ,100, 'Annotate', 'extents');
40 set(gca, 'fontsize', 18, 'linewidth',1)
41 hFig = figure(1002);
42 set(hFig, 'Position', [1300, 300, 600, 600]);
43 title(['Negative peaks found']);
44
45
46
47 % next, do Gauss fit to the original noisy data
48 figure(1003)
49 plot(TimeValues,CF, 'b'); hold on
50
51 timewindow=1000; % a x*2=2x data-point fitting window
52 PHeight=zeros(size(locs));
53 dHeight=zeros(size(locs));
54 for i=1:length(locs)
55     xdata=TimeValues(locs(i)-timewindow:locs(i)+timewindow);
56     ydata=CF(locs(i)-timewindow:locs(i)+timewindow);
57     [Height, Uncertain, Peak, Width, Floor, Scale] = gauss1dfit_peaks(xdata,
        ydata );
58     fitdata=(Height*exp(-(xdata-Peak).^2./(2*Width.^2))+Floor)/Scale+min(
        ydata); %Rescale back the fitting
59     PHeight(i)=Height/Scale;
60     dHeight(i)=Uncertain/Scale;
61     plot(xdata, fitdata, 'r')
62 end
63
64 timewindow=1000; % a x*2=2x data-point fitting window. This fitting window
    should be just enough to cover to peak but not to overlap with other
    peak
65 PHeight_neg=zeros(size(locs_neg));
66 dHeight_neg=zeros(size(locs_neg));
67 for i=1:length(locs_neg)
68     xdata=TimeValues(locs_neg(i)-timewindow:locs_neg(i)+timewindow);
69     ydata=-CF(locs_neg(i)-timewindow:locs_neg(i)+timewindow);
70     [Height, Uncertain, Peak, Width, Floor, Scale] = gauss1dfit_peaks(xdata,
        ydata );
71     fitdata=-((Height*exp(-(xdata-Peak).^2./(2*Width.^2))+Floor)/Scale+min(
        ydata)); %Rescale back the fitting
72     PHeight_neg(i)=-Height/Scale;
73     dHeight_neg(i)=Uncertain/Scale;
74     plot(xdata, fitdata, 'r')
75 end
76 set(gca, 'fontsize', 18, 'linewidth',1)
77 hFig = figure(1003);
78 set(hFig, 'Position', [0, 50, 1300, 400]);
79 xlabel('Time point, sec', 'fontsize', 20);

```

```

80 ylabel('Mechanical frequency shift , kHz', 'fontsize', 20);
81 title(['Gaussian fit , Starting f= ' num2str(CenterFreq(1,1)/1e6) ' MHz']);
82
83 FShift=[locs , TimeValues(locs) PHeight,dHeight; locs_neg,TimeValues(locs_neg
        ) PHeight_neg, dHeight_neg]; %1st column time locs , 2nd column frequency
        shift , 3rd column its 95%-bound uncertainty
84 FShift=sortrows(FShift,1); %sort the transit data by time locs
85 save('Frequency Shift Data','FShift') %save the data

```

Function for Gaussian fitting: 'gauss1dfit_peaks.m'

```

1 function [Height,Uncertain,Peak,Width,Floor, Scale] = gauss1dfit_peaks(xdata
    , ydata)
2 Scale=1/(max(ydata)-min(ydata));
3 ydata=(ydata-min(ydata))*Scale; % rescale y from 0 to 1
4 % ydata=smooth(ydata,5);
5 % the minimum span varies with the data, usually around 15. However, the
6 % difference is very small with the result using 5 (<0.5%). We use 5 for
7 % simplicity
8
9
10 opts = fitoptions('Method', 'NonlinearLeastSquares');
11 % vars : A, Miu, Sigma (alphabetical order), Z
12     opts.Lower = [ 0, min(xdata), 0, 0];
13     opts.Upper = [ 1, max(xdata), max(xdata)-min(xdata), 1];
14
15 %     set up the initial fit
16     MiuGuess=xdata(ydata == max(ydata)); % find out the where the
        maximum power is located
17     opts.StartPoint = [0.8, MiuGuess(1), (xdata(end)-xdata(1))/5, 0.1];
        % sometimes there might be more than one maximum
18 %     opts.StartPoint = [0.5, FoLast, (xdata(end)-xdata(1))/10, 0.1]; %
        sometimes there might be more than one maximum
19
20     ftype = fitype('A*exp(-(x-Miu).^2./(2*Sigma.^2))+Z', ...
21         'options', opts, ...
22         'independent', 'x', ...
23         'dependent', 'intensity');
24     fitresult = fit(xdata, ydata, ftype);
25
26 % set up the second fit for refining the fitting
27     opts.StartPoint = [fitresult.A, fitresult.Miu, fitresult.Sigma,
        fitresult.Z];
28     ftype = fitype('A*exp(-(x-Miu).^2./(2*Sigma.^2))+Z', ...
29         'options', opts, ...
30         'independent', 'x', ...
31         'dependent', 'intensity');
32     fitresult = fit(xdata, ydata, ftype);
33
34     ci=confint(fitresult, 0.95); % returns the 95% confidence bounds for the
        curve fitting. Level can be change the level between 0 and 1
35     Peak=fitresult.Miu;
36     Width=fitresult.Sigma;
37     Height=fitresult.A;
38     Floor=fitresult.Z;
39
40 % LowerPeak=ci(1,1);
41 HigerHeight=ci(2,1);
42 Uncertain=(HigerHeight-Height);
43
44 end

```

APPENDIX D

MATLAB CODE FOR FINDING OMFR CENTER LINE

Main code: 'FindCenterLine_Ch1.m'

```
1 % function [p1]=FindCenterLine_Ch1(filename)
2
3 clear all;
4 close all;
5 filename='Background_Channel1_4100.tif';
6 RGB=imread(filename);
7 %% Conver RGB to grayscale
8 % Greyscale=rgb2gray(RGB);
9 % Only use one of the color channel for finding edge line
10 Greyscale=RGB(:, :, 2);
11
12 %% for dark edges (channel 1)
13 % click on two points that bound the left edge
14 [Px_left, Py_left]=FindDarkEdge(Greyscale);
15 % click on two points that bound the right edge
16 [Px_right, Py_right]=FindDarkEdge(Greyscale);
17
18
19 %%
20 figure(301); hold on
21 imshow(Greyscale, 'InitialMagnification','fit')
22 plot(Px_left, Py_left, Px_right, Py_right, 'linewidth', 2)
23 [p1_left, p2_left]=polyfit(Py_left, Px_left, 1);
24 [p1_right, p2_right]=polyfit(Py_right, Px_right, 1);
25 p1(1)=(p1_left(1)+p1_right(1))/2; %average slope
26 p1(2)=(p1_left(2)+p1_right(2))/2; %average intercept
27 save('CenterlineFit_XvsY_Ch1', 'p1')
28
29 [ny, nx]=size(Greyscale);
30 yy=linspace(1, ny, 10000);
31 xx=p1(1)*yy+p1(2);
32 % figure(301);
33 plot(xx, yy, 'linewidth', 2)
```

Main code: 'FindCenterLine_Ch2.m'

```
1 % function [p1]=FindCenterLine_Ch3(filename)
2
3 clear all;
4 close all;
5 filename='Background_Channel2_4100.tif';
6 RGB=imread(filename);
7 % Conver RGB to grayscale
8 % Greyscale=rgb2gray(RGB);
9 % Only use one of the color channel for finding edge line
10 Greyscale=RGB(:, :, 2);
11
12 %% for dark edges (channel 3)
13 % click on two points that bound the left edge
14 [Px_left, Py_left]=FindDarkEdge(Greyscale);
15 % click on two points that bound the right edge
16 [Px_right, Py_right]=FindDarkEdge(Greyscale);
17
18 %%
19 figure (301); hold on
20 imshow(Greyscale, 'InitialMagnification','fit')
21 plot(Px_left, Py_left, Px_right, Py_right, 'linewidth', 2)
22 [p1_left, p2_left]=polyfit(Py_left, Px_left, 1);
23 [p1_right, p2_right]=polyfit(Py_right, Px_right, 1);
24 p1(1)=(p1_left(1)+p1_right(1))/2; %average slope
25 p1(2)=(p1_left(2)+p1_right(2))/2; %average intercept
26 save('CenterlineFit_XvsY_Ch2', 'p1')
27
28
29 [ny, nx]=size(Greyscale);
30 yy=linspace(1, ny, 10000);
31 xx=p1(1)*yy+p1(2);
32 % figure (301);
33 plot(xx, yy, 'linewidth', 2)
```

Function for edge finding: 'FindDarkEdge.m'

```
1 function [Px, Py]=FindDarkEdge(Greyscale)
2
3
4 figure (3301); hold on
5 imshow(Greyscale, 'InitialMagnification', 130)
6 title('Click on 2 points that bound the left edge or the right edge')
7
8 % ask the user to draw two points that bound the left edge
9 [Xclick, Yclick]=ginput(2); % xclick = column, yclick = row;
10 Xclick=round(Xclick); % the position of click in the original view
11 Yclick=round(Yclick);
12 % Make a horizontal line based on the two points
13 Yclick(2)=Yclick(1);
14 % Make a square based on the two points
15 Xclick(3)=Xclick(2);
16 Xclick(4)=Xclick(1);
17 Yclick(3)=Yclick(2)+400;
18 Yclick(4)=Yclick(1)+400;
19 Xclick(5)=Xclick(1);
20 Yclick(5)=Yclick(1);
21 % plot the square for verification
22 figure (3301); hold on
23 plot(Xclick, Yclick)
24
25 % plot the cropped view
26 figure (3302);
27 zdata=Greyscale(Yclick(2):Yclick(3), Xclick(1):Xclick(2));
28 % remember, for a image, the first data is row and second is column
29
30 imshow(zdata(), 'InitialMagnification', 'fit')
31 zdata=double(zdata);
32 [ny, ~]=size(zdata); % Find out the dimensions
33 px=zeros(ny, 1);
34 py=zeros(ny, 1);
35
36 % next we find out the min intensity pixel's coordinates in the cropped
37 % view
38 for ii=1:ny;
39     py(ii)=ii;
40     pxx=find(zdata(ii, :)==min(zdata(ii, :)));
41     px(ii)=pxx(1);
42 end
43
44 Px=Xclick(1)+px;
45 Py=Yclick(1)+py;
46
47 figure (3301);
48 plot(Px, Py)
49 close (3301)
50 close (3302)
```

APPENDIX E

MATLAB CODE FOR FINDING PARTICLE POSITION IN PROJECTION IMAGES

Main code: 'FindParticlePosition.m'

```
1 % This file help find the position of particle in a image.
2 clear all
3 close all
4
5 %% First we do channel 1
6 Background='Background-Channel1_5000.tif'; %load the background
7 list=dir('Ch1*.tif'); % reading all the images one by one
8 Px=zeros(length(list),2); % x position , ch1 and ch2
9 Py=zeros(length(list),2); % y position , ch1 and ch2
10 dx=zeros(length(list),2); % x position uncertainty , ch1 and ch2
11 dy=zeros(length(list),2); % y position uncertainty , ch1 and ch2
12 Sigmax=zeros(length(list),2); % x fitting sigma , ch1 and ch2
13 Sigmay=zeros(length(list),2); % y fitting sigma , ch1 and ch2
14 index=zeros(length(list),1);
15
16 for i = 1:length(list)
17 filename = list(i).name;
18 index(i)=str2double(filename(14:15));
19 RGB=imread(filename);
20 [Px(i,1),Py(i,1),dx(i,1),dy(i,1),Sigmax(i,1),Sigmay(i,1)]=gauss2dfit_noBgd(
    filename,Background);
21 % pause;
22 pause(2); % wait one second to check the fit
23 end
24
25 %% Then we do channel 2
26 Background='Background-Channel2_5770.tif'; %load the background
27 list=dir('Ch2*.tif'); % reading all the images one by one
28 for i = 1:length(list)
29 filename = list(i).name;
30 RGB=imread(filename);
31 [Px(i,2),Py(i,2),dx(i,2),dy(i,2),Sigmax(i,2),Sigmay(i,2)]=gauss2dfit_noBgd(
    filename,Background);
32 % pause;
33 pause(2); % wait one second to check the fit
34 end
35
36 save('Silica_0623_Position_Data', 'index','Px','Py','dx','dy','Sigmax','
    Sigmay')
```

Function for Gaussian 2d fit: 'gauss2dfit_noBgd.m'

```

1 function [Px,Py,dx,dy,sigmax,sigmay]=gauss2dfit_noBgd(filename,Background)
2 % input: filename should be the in 'RGB_Ch#_Frame.tif'
3 % Background: choose a frame from the video without any particle in it
4 % When the greyscale figure pops out, click on the center of the particle
5
6 % output: Px +- dx, Py +- dy is the fitted Gaussian locations from in the
7 % original image.
8
9 % Fitting 2d gaussian function on to a 2d array of RGB data from a frame
10 % selected manually from video.
11
12 % First step: use implay('vedioname.avi') to open the video. Save the frame
13 % of interest to RGB_Ch#_Frame#.tiff
14
15 %%
16 RGB=imread(filename); % read the picture with the particle
17 Bgd=imread(Background); % read the background picture
18 % Conver RGB to grayscale
19 % Greyscale=rgb2gray(RGB);
20
21 figure(401);
22 imshow(RGB,'InitialMagnification',130)
23 title('Click on the center of the particle')
24 % note that imshow draws the matrix from top left corner as (0, 0)
25 [Xclick,Yclick]=ginput(1);
26 Xclick=round(Xclick); % the position of click in the original view
27 Yclick=round(Yclick);
28
29 RGB=double(RGB);
30 Bgd=double(Bgd);
31 % ginput will return the row and column to x and y from the click
32 % close(figure)
33 xwindow=70;
34 xwindow=xwindow+mod(xwindow,2); % force the window to be even
35 ywindow=60;
36 ywindow=ywindow+mod(ywindow,2); % force the window to be even
37
38 Substraction=RGB-Bgd;
39 Greyscale=0.2989 * Substraction(:,:,1) + 0.5870 * Substraction(:,:,2) +
    0.1140 *Substraction(:,:,3);
40 Greyscale=255-Greyscale;
41 % the window size can be changed depending on the situation
42 zdata=Greyscale((Yclick-ywindow/2):(Yclick+ywindow/2),(Xclick-xwindow/2):(
    Xclick+xwindow/2));
43 % zdata=double(zdata);
44 % Crop and flip the window near by the particle. Note that the y index needs
    to come first for images.
45 % figure(402);
46 hFig = figure(402);
47 set(hFig,'Position',[100,100,500,500]);
48 hdata=surf(zdata);
49 colormap hsv; view(2); % Plotting data

```

```

50 set(hdata, 'FaceAlpha', 0.6, 'EdgeColor', 'none');           % Changing figure
    appearance
51 [ny, nx] = size(zdata);                                     % Find out the dimensions
52
53 z = reshape(zdata, ny*nx, 1);                               % Convert 2d array into 1d array, one
    column by another
54 [~, imax] = max(z);                                         % Determine the max value
55
56 ix = floor(imax(1)/ny)+1;                                    % Find the x element of peak
57 iy = mod((imax(1)-1), ny)+1;                                 % Find the y element of peak
58 xAxis = 1:nx;                                                % Create the x-Axis gridding
59 yAxis = 1:ny;                                                % Create the y-Axis gridding
60
61 [px, dx, sigmax] = gauss1dfit(xAxis, zdata(iy, :));
62 % 1d gaussian fit to the x-axis of data, px is the fitted x position in the
63 % cropped view
64 [py, dy, sigmay] = gauss1dfit(yAxis, zdata(:, ix)); % 1d gaussian fit to the
    y-axis of data
65
66 xclick=xwindow/2+1; % position of the click in the cropped view
67 yclick=ywindow/2+1; % position of the click in the cropped view
68 Px=Xclick+(px-xclick);
69 Py=Yclick+(py-yclick);
70 % position of the fitted position in the cropped view in the original view
71
72 %% for testing
73 figure (401); hold on
74 % imshow(Greyscale, 'InitialMagnification', 'fit')
75 errorbar(Px, Py, dy, 'rx');
76 herrorbar(Px, Py, dx, 'rx');
77 hold off

```

Function for Gaussian 1d fit: 'gauss1dfit.m'

```

1 function [Peak,Uncertain,Width] = gauss1dfit(xdata, ydata)
2
3 xdata=xdata'; %x an y should be a column
4 ydata=((ydata-min(ydata))/(max(ydata)-min(ydata)))'; % rescale y from 0 to 1
5 ydata=smooth(ydata,5);
6 % the minimum span varies with the data, usually around 15. However, the
7 % difference is very small with the result using 5 (<0.5%). We use 5 for
8 % simplicity
9
10
11 opts = fitoptions('Method', 'NonlinearLeastSquares');
12     % vars : A, Miu, Sigma (alphabetical order), Z
13     opts.Lower = [ 0, min(xdata), 0, 0];
14     opts.Upper = [ 1, max(xdata), 1000, 1];
15
16 %     set up the initial fit
17     MiuGuess=xdata(ydata == max(ydata)); % find out the where the
18         maximum power is located
19     opts.StartPoint = [0.5, MiuGuess(1), (xdata(end)-xdata(1))/10, 0.1];
20         % sometimes there might me more than one maximum
21     opts.StartPoint = [0.5, FoLast, (xdata(end)-xdata(1))/10, 0.1]; %
22         sometimes there might me more than one maximum
23
24 % Initial guess
25 % Amplitude: half of the amplitude
26 % Center frequency: frequency at the maximum power
27 % Linewidth: Frequency range/10
28 ftype = fitype('A*exp(-(x-Miu).^2./(2*Sigma.^2))+Z', ...
29     'options', opts, ...
30     'independent', 'x', ...
31     'dependent', 'intensity');
32 fitresult = fit(xdata, ydata, ftype);
33
34 ci=confint(fitresult, 0.95); % returns the 95% confidence bounds for the
35     curve fitting. Level can be change the level between 0 and 1
36 Peak=fitresult.Miu;
37 Width=fitresult.Sigma;
38
39 LowerPeak=ci(1,2);
40 HigerPeak=ci(2,2);
41 Uncertain=(HigerPeak-Peak); % f=f +- UncertainFreq
42
43 end

```

Function for labeling horizontal error bar: 'herrorbar.m'

```
1 function hh = herrorbar(x, y, l, u, symbol)
2 %HERRORBAR Horizontal Error bar plot.
3 % HERRORBAR(X,Y,L,R) plots the graph of vector X vs. vector Y with
4 % horizontal error bars specified by the vectors L and R. L and R contain
   the
5 % left and right error ranges for each point in X. Each error bar
6 % is L(i) + R(i) long and is drawn a distance of L(i) to the right and R(i)
   )
7 % to the right the points in (X,Y). The vectors X,Y,L and R must all be
8 % the same length. If X,Y,L and R are matrices then each column
9 % produces a separate line.
10 %
11 % HERRORBAR(X,Y,E) or HERRORBAR(Y,E) plots X with error bars [X-E X+E].
12 % HERRORBAR(...,'LineStyle') uses the color and linestyle specified by
13 % the string 'LineStyle'. See PLOT for possibilities.
14 %
15 % H = HERRORBAR(...) returns a vector of line handles.
16 %
17 % Example:
18 %     x = 1:10;
19 %     y = sin(x);
20 %     e = std(y)*ones(size(x));
21 %     herrorbar(x,y,e)
22 % draws symmetric horizontal error bars of unit standard deviation.
23 %
24 % This code is based on ERRORBAR provided in MATLAB.
25 %
26 % See also ERRORBAR
27
28 % Jos van der Geest
29 % email: jos@jasen.nl
30 %
31 % File history:
32 % August 2006 (Jos): I have taken back ownership. I like to thank Greg
   Aloe from
33 % The MathWorks who originally introduced this piece of code to the
34 % Matlab File Exchange.
35 % September 2003 (Greg Aloe): This code was originally provided by Jos
36 % from the newsgroup comp.soft-sys.matlab:
37 % http://newsreader.mathworks.com/WebX?50@118.fdnxaJz9btF^1@.eea3ff9
38 % After unsuccessfully attempting to contact the original author, I
39 % decided to take ownership so that others could benefit from finding it
40 % on the MATLAB Central File Exchange.
41
42 if min(size(x))==1,
43     npt = length(x);
44     x = x(:);
45     y = y(:);
46     if nargin > 2,
47         if ~isstr(1),
48             l = l(:);
49         end
```

```

50         if nargin > 3
51             if ~isstr(u)
52                 u = u(:);
53             end
54         end
55     end
56 else
57     [npt,n] = size(x);
58 end
59
60 if nargin == 3
61     if ~isstr(l)
62         u = l;
63         symbol = '-';
64     else
65         symbol = l;
66         l = y;
67         u = y;
68         y = x;
69         [m,n] = size(y);
70         x(:) = (1:npt)'*ones(1,n);;
71     end
72 end
73
74 if nargin == 4
75     if isstr(u),
76         symbol = u;
77         u = l;
78     else
79         symbol = '-';
80     end
81 end
82
83 if nargin == 2
84     l = y;
85     u = y;
86     y = x;
87     [m,n] = size(y);
88     x(:) = (1:npt)'*ones(1,n);;
89     symbol = '-';
90 end
91
92 u = abs(u);
93 l = abs(l);
94
95 if isstr(x) | isstr(y) | isstr(u) | isstr(l)
96     error('Arguments must be numeric.')
97 end
98
99 if ~isequal(size(x),size(y)) | ~isequal(size(x),size(l)) | ~isequal(size(x),
    size(u)),
100     error('The sizes of X, Y, L and U must be the same.');
```

```

103 tee = (max(y(:))-min(y(:)))/100; % make tee .02 x-distance for error bars
104 % changed from errorbar.m
105 xl = x - l;
106 xr = x + u;
107 ytop = y + tee;
108 ybot = y - tee;
109 n = size(y,2);
110 % end change
111
112 % Plot graph and bars
113 hold_state = ishold;
114 cax = newplot;
115 next = lower(get(cax, 'NextPlot'));
116
117 % build up nan-separated vector for bars
118 % changed from errorbar.m
119 xb = zeros(npt*9,n);
120 xb(1:9:end,:) = xl;
121 xb(2:9:end,:) = xl;
122 xb(3:9:end,:) = NaN;
123 xb(4:9:end,:) = xl;
124 xb(5:9:end,:) = xr;
125 xb(6:9:end,:) = NaN;
126 xb(7:9:end,:) = xr;
127 xb(8:9:end,:) = xr;
128 xb(9:9:end,:) = NaN;
129
130 yb = zeros(npt*9,n);
131 yb(1:9:end,:) = ytop;
132 yb(2:9:end,:) = ybot;
133 yb(3:9:end,:) = NaN;
134 yb(4:9:end,:) = y;
135 yb(5:9:end,:) = y;
136 yb(6:9:end,:) = NaN;
137 yb(7:9:end,:) = ytop;
138 yb(8:9:end,:) = ybot;
139 yb(9:9:end,:) = NaN;
140 % end change
141
142
143 [ls,col,mark,msg] = colstyle(symbol); if ~isempty(msg), error(msg); end
144 symbol = [ls mark col]; % Use marker only on data part
145 esymbol = ['- ' col]; % Make sure bars are solid
146
147 h = plot(xb,yb,esymbol); hold on
148 h = [h;plot(x,y,symbol)];
149
150 if ~hold_state, hold off; end
151
152 if nargin>0, hh = h; end

```

APPENDIX F

MATLAB CODE FOR FINDING PARTICLE RADIAL LOCATION

Main code: 'RadialLocationFinding.m'

```
1 clear all
2 close all
3 load('Fluo d3 Particle Position Data.mat')
4 load('Frequency Shift Data.mat')
5
6 %% Ch1
7 load('CenterlineFit_XvsY_Ch1.mat');
8 offset(1)=11 %offset the centerline by n pixel
9 p1(2)=p1(2)+offset(1);
10 d(:,1)=(-1*Px(:,1)+p1(1)*Py(:,1)+p1(2))/(1+p1(1)^2)^0.5; % pixel distance,
    channel 1, the left of centerline is negative, which is reversed later
    in this code
11
12
13 %% Ch2
14 load('CenterlineFit_XvsY_Ch2.mat');
15 offset(2)=-3; %offset the centerline by n pixel
16 p1(2)=p1(2)+offset(2);
17 d(:,2)=(-1*Px(:,2)+p1(1)*Py(:,2)+p1(2))/(1+p1(1)^2)^0.5; % pixel distance,
    channel 2, the left of centerline is negative, which is reversed later
    in this code
18
19 %% Convert pixel distance to um
20 ActPix=480; % 480 pixels in the y direction
21 ActLength_Ch1=220e-6; % the length of the image in y direction
22 ActLength_Ch3=226e-6; % the length of the image in y direction
23 PixSize_Ch1=ActLength_Ch1/ActPix;
24 PixSize_Ch3=ActLength_Ch3/ActPix;
25 d(:,1)=-d(:,1)*PixSize_Ch1; % adjust the direction of positive x
26 d(:,2)=-d(:,2)*PixSize_Ch3; % adjust the direction of positive x
27 CameraAngle=78.5;
28
29 %% Find the radial distance after remove the lensing effect. Assume far-
    field imaging, ie, only the parallel refracted output light matters.
30
31 OR=55e-6/2; % assume the outer diamter is 55 um.
32 x=d/OR; % normalization, assume the cross section is a circle
33 y=-(1-x.^2).^0.5;
34 n_air=1.0003;
```

```

35 n_water=1.35; % n_water is 1.33, n_silca=1.45
36
37 % Channel 1:
38 n1=[x(:,1), y(:,1)];
39 theta1=asind(n1(:,1));
40 alpha1=atan2d(abs(n1(:,2)), -n1(:,1));
41 sind(theta1)
42 gammal=asind(n_air*sind(theta1)/n_water);
43 beta1=alpha1-gammal;
44
45 % Channel 2:
46 n2=[x(:,2), y(:,2)];
47 theta2=asind(n2(:,1));
48 alpha2=atan2d(abs(n2(:,2)), -n2(:,1));
49 gamma2=asind(n_air*sind(theta2)/n_water);
50 beta2=alpha2-gamma2;
51
52 % transform the incident line from coordinate 1 to 2
53 beta1=beta1-CameraAngle;
54 trans=[cosd(CameraAngle), sind(CameraAngle); -sind(CameraAngle) cosd(
    CameraAngle)];
55 n1=(trans*n1')';
56
57 % Find the intersection between two incident lines in coordinate 1
58 a1=tand(beta1); %slope of line 1
59 b1=n1(:,2)-a1.*n1(:,1); %intercept of line 1
60 a2=tand(beta2); %slope of line 2
61 b2=n2(:,2)-a2.*n2(:,1); %intercept of line 2
62 x_inter=(b2-b1)/(a1-a2);
63 y_inter=(a1.*b2-a2.*b1)/(a1-a2);
64 LLL=(x_inter.^2+y_inter.^2).^0.5*OR;
65
66 %% plotting
67 figure(102)
68 plot(LL*1e6, FShift/1e3, 'rx', 'markers', 8, 'linewidth', 2)
69 set(gca, 'fontsize', 18, 'linewidth', 1)
70 hFig = figure(102);
71 set(hFig, 'Position', [200, 200, 800, 550]);
72 xlabel('Particle radial position (\mm)', 'fontsize', 20)
73 ylabel('Mechanical frequency shift (kHz)', 'fontsize', 20)

```

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