NORTON-THEVENIN RECEPTANCE COUPLING (NTRC) AS A PAYLOAD ANALYSIS TOOL

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ABSTRACT

The NASA Engineering and Safety Center (NESC) has funded a study of a new method formulated by NASA Engineers called Norton-Thevenin Receptance Coupling (NTRC) to perform coupled loads analysis (CLA).

The problem that NTRC attempts to solve is the dependency of the payload organization to high CLA costs, long schedules, lack of standard capabilities to evaluate multiple configurations and unavailability of loads when needed.

NTRC solves the problem by providing a tool that payload developers can use to obtain loads at a fraction of the cost of a CLA at any time that it is required. While NTRC is not intended to replace the formal load cycles performed by the launch vehicle (LV) provider, it will provide the ability to reduce the conservatism in defining preliminary design loads, assess the impact of design changes between formal load cycles, perform trade studies and parametric or variational CLA [ref. 5] where many different design configurations can be evaluated with a minimum amount of data required from the LV provider.

NTRC condenses all the necessary information into the launch vehicle to payload/s connection points or boundary degrees of freedom (BD). The launch vehicle model is represented by its impedance at its BDs; its forcing functions are represented by the acceleration at those BDs when the payload is absent and the latter is represented by its impedance at the same BDs. Payload responses are represented by transfer functions of selected response to interface BDs.

The methodology has contributed to the Loads and Dynamics discipline advancement and successfully passed Peer Reviews. NTRC is exact in the frequency domain. Time domain replication and accuracy is outstanding. A second phase is envisioned to benchmark the whole set of CLA events for the Agency's most utilized Launch Vehicles, and readiness for operational deployment at NASA.

1. METHODOLOGY DEVELOPMENT

NTRC was inspired from the application of force limiting methods for random vibration since the early 1990s at NASA and Industry [ref. 1] and the flight force measurement project funded by NESC in 2006 [ref. 2]. NTRC is a frequency-response–based substructuring (FBS) method, as opposed to a component mode synthesis (CMS) method. NTRC uses FFTs or Convolution to transform between frequency and time domains. We will use nomenclature is similar to [ref. 4]

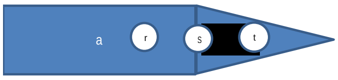


Figure 1 – Launch Vehicle and Payload DoFs

c: coupled system (a+b)
a: source (booster) with internal dofs r
b: load (spacecraft) with internal dofs t
s: connecting dofs
H: accelerance matrix [acceleration/force]
W: impedance matrix [force/acceleration]
A: response vector [EU]
F: Input force vector [force]

There are three subscripts to a matrix, the first one corresponds to the system (a, b or c),the second to the response DoF and the third is the input force DoF. There are two subscripts to a vector, the first one corresponds to the system and the second to the forced or response DoF.

The general CLA expression is: (1)

$$A_c \quad H_c \; F_c$$

Partitioning into booster, interface and spacecraft DoFs: (2)

$$\begin{Bmatrix} A_{cr} \\ A_{cs} \\ A_{ct} \end{Bmatrix} \quad \begin{bmatrix} H_{crr} & H_{crs} & H_{crt} \\ H_{csr} & H_{css} & H_{cst} \\ H_{ctr} & H_{cts} & H_{ctt} \end{bmatrix} \begin{Bmatrix} F_{cr} \\ F_{cs} \\ F_{ct} \end{Bmatrix}$$

For the typical CLA where loads are applied on the booster: (3), (4)

$$A_{cs} \quad H_{csr} \; F_{cr}$$

$$A_{ct} \quad H_{ctr} \; F_{cr}$$

One of the know receptance coupling expressions is: (5)

$$\begin{bmatrix} H_{crr} & H_{crs} & H_{crt} \\ H_{csr} & H_{css} & H_{cst} \\ H_{ctr} & H_{cts} & H_{ctt} \end{bmatrix} \quad \begin{bmatrix} H_{arr} & H_{ars} & 0 \\ H_{asr} & H_{ass} & 0 \\ 0 & 0 & H_{btt} \end{bmatrix} \quad \begin{bmatrix} H_{ars} \\ H_{ass} \\ H_{bts} \end{bmatrix} [H_{ass} \quad H_{bss}]^{1} \begin{bmatrix} H_{ars} \\ H_{ass} \\ H_{bts} \end{bmatrix}^{T}$$

For the typical CLA input forces are generated at the booster side while interface and payload forces are null, and solving for Hcsr: (6)

$$H_{csr} \quad H_{asr} \quad H_{ass}[H_{ass} \quad H_{bss}]^{1} H_{asr}$$

Arriving to a relationship between coupled response and input force: (7) (8)

$$A_{cs} \quad \frac{H_{bss} H_{asr}}{H_{ass} \quad H_{bss}} F_{cr}$$

$$F_{cr} \quad \frac{H_{ass} \quad H_{bss}}{H_{bss} H_{asr}} A_{cs}$$

Solving for Hctr: (9)

$$H_{ctr} \quad H_{bts}[H_{ass} \quad H_{bss}]^{1} H_{asr}$$

Expressing the coupled response as function of input force on the booster: (10)

$$A_{ct} \quad H_{bts} [H_{ass} \quad H_{bss}]^{1} H_{asr} \; F_{cr}$$

By expressing Act as a function of Acs and NOT Fcr we eliminate the R DoFs transfer accelerances (Hasr) from (10), therefore meeting two objectives: a) simplifying the expression for Act and b) does not need to request R DoFs data from the LV. The process is then dependent (11) upon S and T DoFs, which are controlled by the SC, therefore the only LV info needed if at the interface S: Hass and then Aas. (11)

$$A_{ct} \quad H_{bts} \; H_{bss}{}^{1} A_{cs}$$

Now we will use Norton-Thevenin to introduce the free acceleration: A_{as} (12)

$$A_{cs} \quad [H_{ass}{}^{1} \quad H_{bss}{}^{1}]^{1} H_{ass}{}^{1} A_{as}$$

Combining (11) and (12) we arrive at the core NTRC equation relating payload response to free acceleration: (13)

$$A_{ct} \quad H_{bts} H_{bss}{}^{1} [H_{ass}{}^{1} \quad H_{bss}{}^{1}]^{1} H_{ass}{}^{1} A_{as}$$

Anatomy of the core NTRC equation: We can think it as comprised of three contributions:

The Norton-Thevenin scaling matrix: NT (14)

$$NT \quad [H_{ass}{}^{1} \quad H_{bss}{}^{1}]^{1} H_{ass}{}^{1}$$

The LTM*t* (15) or B*bts*

$$H_{bts} H_{bss}{}^{1} \quad \frac{A_{bt}}{F_{bs}} \frac{F_{bs}}{A_{bs}} \quad \frac{A_{bt}}{A_{bs}} \quad B_{bts}$$

This means that the product Hbts Hbss-1 is the LTM [EU/g] corresponding to the item t response for a base input acceleration s. Therefore, the method is generalized to any LTM payload item.

And the free acceleration: A*as*

Time Domain Solutions:

From Equation (13) we can define a transfer function between free acceleration and the desired load response quantity (in this case internal acceleration) as: (16)

$$TF_{ts} \quad H_{bts} H_{bss}{}^{1} [H_{ass}{}^{1} \quad H_{bss}{}^{1}]^{1} H_{ass}{}^{1}$$

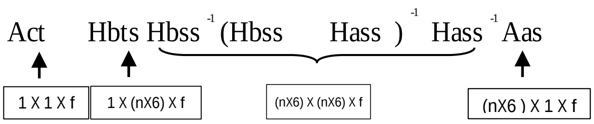
We identified two methods to solve the time domain problem:

a) Multiplication in the frequency domain
b) Convolution

Both methods are equivalent since they are related by the Theorem of Convolution and the Laplace Transform: Laplace [(f*g)(t)]= F(s) G(s), or for s=w (17)

FFT [(f*g)(t)] = F(w)G(w)

For example for a 6 DoF – multipoint (n points) interface: (18)



2. Methodology Validation

The methodology was validated using by incremental steps. It started using simple test cases such as a multi-degree of freedom (DOF) spring-mass system and a simple payload-booster finite element model (FEM) with determinate interface and single-axis input in the frequency and time domain. The second step was executed using an in house Booster and Payload (PL) FEMs resembling the mass, stiffness and complexity of a typical NASA heavy PL mission. The final step was conducted using the SLS Launch Vehicle (LV). Additional runs were also conducted with a commercial US LV.

3. Frequency Domain Validation

The validation sequence for each one consisted of an initial comprehensive validation in the frequency domain which is the core for the NTRC methodology.

The in house model is presented in figure 2.

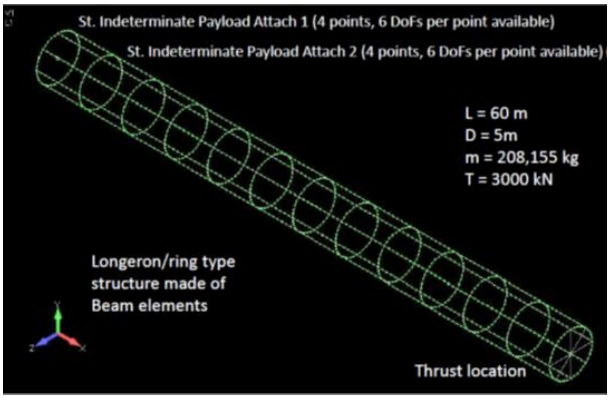


Figure 2 – In House Booster Model

The in house payload model has all the characteristics of a medium-heavy Spacecraft, a weight of 8200 Lb, first lateral modes between 10-20 Hz and first axial mode between 20-40 Hz and off-axis center of gravity (CoG). The payload FEM is shown in Figure 3.

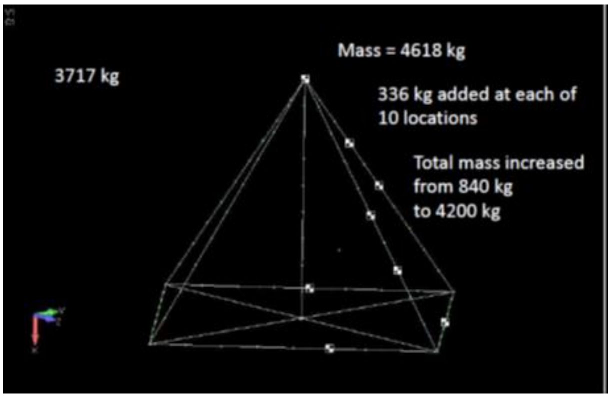


Figure 3 – In House Payload Model

Residual vectors were used same as in a typical CLA to compensate for modal truncation errors that occur when mode shapes are cut-off at a certain frequency. This is done for computational ease.

Frequency domain runs were validated for various cases such as axial thrust only, lateral engine forces only, combined axial/lateral, single and multibody (two payloads) with different damping values and degrees of indeterminate interface to the LV. PL one with 24 DoF tied and PL two with 12 DoF tied (moments released). Axial thrust was set to 3000 kN, Lateral to 5% axial, analysis range 1-100 Hz with a frequency step of 0.2 Hz. LV and payload 1 damping on free modes at 2%, payload 2 damping at 5%.

For each of the benchmark cases, the coupled system and NTRC results were compared for interface forces, interface acceleration, payload internal accelerations, and payload internal stresses. The responses quantities were selected to cover a wide range of numerical values and to look at both acceleration and displacement based responses. As for frequency domain accuracy, there were no differences observed between CLA and NTRC results. NTRC proved to be an exact method of coupled loads analysis (CLA).

Figure 4 shows a typical CLA and NTRC frequency domain overlay

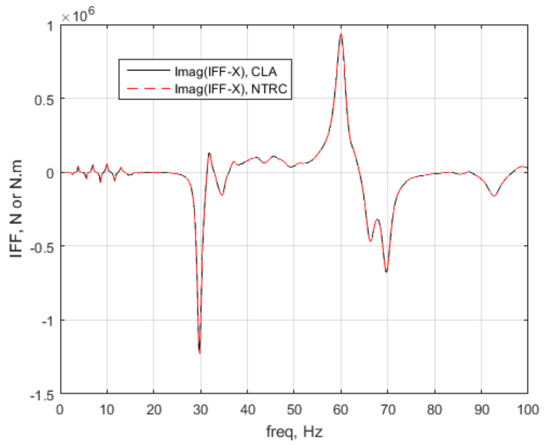


Figure 4 - Interface Force Imaginary Component on Payload 1

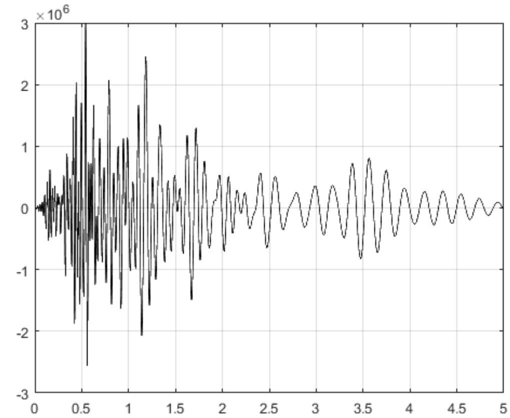
Table 1 shows the NTRC's frequency domain accuracy

Prob4D - Peak Interface Force (IMAGINARY Component)				
Item Description (N, N.m)	CLA	NTRC	Abs Diff	% Diff
100001-X	1224503.3158906400	1224503.3158906400	0.0000E+00	0.0000%
100001-Y	163720.0304134590	163720.0304134580	9.8953E-10	0.0000%
100001-Z	208865.9033263400	208865.9033263390	9.8953E-10	0.0000%
100001-RX	190190.1795275590	190190.1795275590	0.0000E+00	0.0000%
100001-RY	389627.3549821530	389627.3549821520	9.8953E-10	0.0000%
100001-RZ	348075.0507594150	348075.0507594150	0.0000E+00	0.0000%
100023-X	1232014.2086318000	1232014.2086318000	0.0000E+00	0.0000%
100023-Y	324201.9944722090	324201.9944722090	0.0000E+00	0.0000%
100023-Z	258681.9606893850	258681.9606893840	1.0186E-09	0.0000%
100023-RX	189979.0996452940	189979.0996452930	9.8953E-10	0.0000%
100023-RY	117208.4746738830	117208.4746738830	0.0000E+00	0.0000%
100023-RZ	566133.1615680290	566133.1615680290	0.0000E+00	0.0000%
100045-X	641414.2895235530	641414.2895235520	9.3132E-10	0.0000%
100045-Y	231560.6210827130	231560.6210827130	0.0000E+00	0.0000%
100045-Z	218237.8213779690	218237.8213779680	9.8953E-10	0.0000%
100045-RX	78518.2816605897	78518.2816605889	8.0036E-10	0.0000%
100045-RY	351692.2684003100	351692.2684003100	0.0000E+00	0.0000%
100045-RZ	383499.9880144130	383499.9880144130	0.0000E+00	0.0000%
100067-X	396749.8938183080	396749.8938183070	9.8953E-10	0.0000%
100067-Y	184382.8930486970	184382.8930487020	-5.0059E-09	0.0000%
100067-Z	171333.1959620670	171333.1959620670	0.0000E+00	0.0000%
100067-RX	57904.0043261669	57904.0043261668	9.4587E-11	0.0000%
100067-RY	244697.3395450000	244697.3395450000	0.0000E+00	0.0000%
100067-RZ	220072.2822611140	220072.2822611130	9.8953E-10	0.0000%

Table 1 – Interface Force Imaginary Component on Payload 1

4. Time Domain Validation

The time domain validation with in house models was done with in house forcing functions used were modally rich and representative of a complex real case CLA. Figure 5, 6 and 7 show these forcing functions.



N vs seconds

Figure 5 – LV axial Fx forcing function

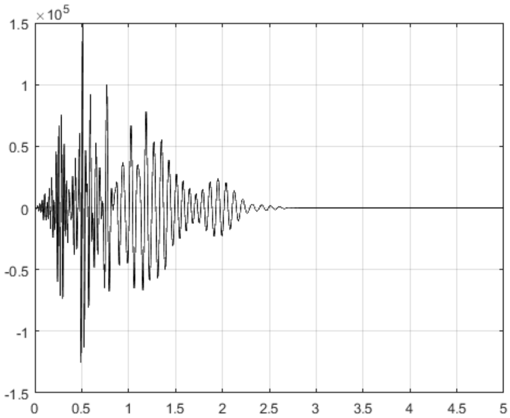


Figure 6 – LV lateral Fy forcing function

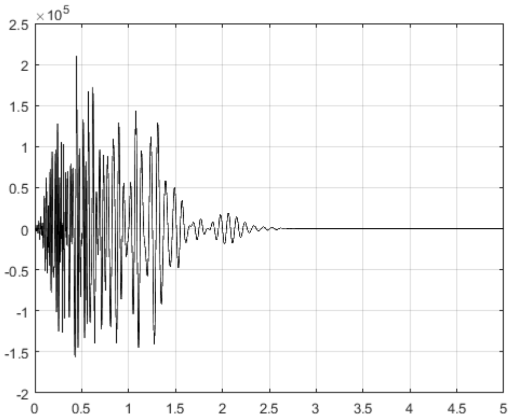
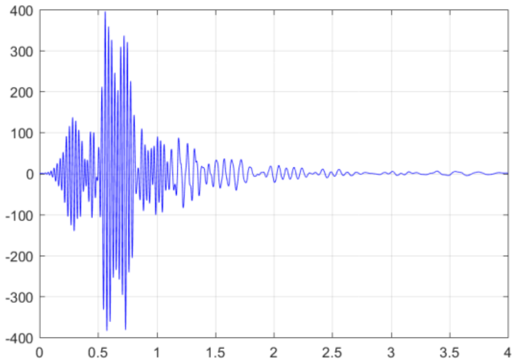


Figure 7 – LV lateral Fz forcing function

Figure 8 shows a typical unloaded or free acceleration

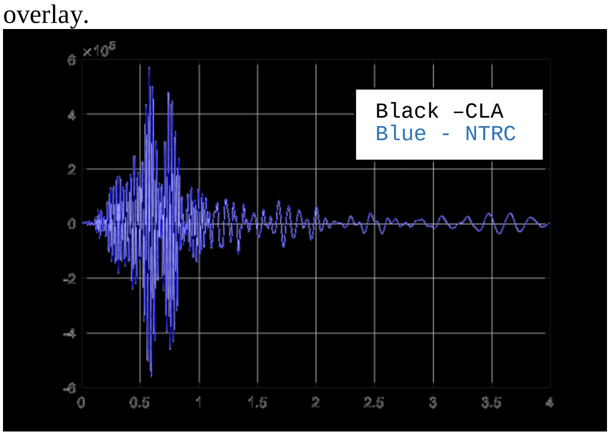


m/sec^2 or rad/sec^2 vs seconds

Figure 8– LV free acceleration DoF 1

As for accuracy, NTRC time domain analysis showed outstanding results which is in general in the 0.5 % with some outliers into 1 to 3 %. The NTRC team thinks that there is additional room for improvement here by tuning NTRC parameters to improve accuracy, however due to the fact that the accuracy goals were met, the overlays were outstanding and schedule constraints this was not further explored. This type of fine tuning can take place when benchmarking a specific LV CLA event.

Figure 9 shows a typical CLA and NTRC time domain overlay.



N or N.m vs seconds

Figure 9– LV free acceleration DoF 1

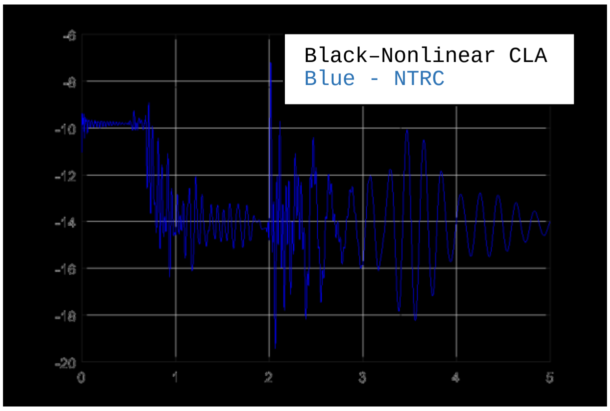
Table 2 shows the NTRC's time domain accuracy

.

Interface Forces	MAXIMUM				MINIMUM			
Item Description (N, N.m)	CLA	NTRC	Abs Diff	% Diff	CLA	NTRC	Abs Diff	% Diff
100001-X	570.46	572.14	-1.68	-0.2944%	-554.80	-561.45	6.65	-1.1990%
100001-Y	138.18	137.63	0.55	0.3998%	-138.82	-138.76	-0.06	0.0421%
100001-Z	190.53	191.56	-1.03	-0.5430%	-187.39	-187.00	-0.39	0.2082%
100001-RX	41.04	41.45	-0.42	-1.0174%	-42.00	-42.38	0.38	-0.8937%
100001-RY	190.07	192.18	-2.10	-1.1073%	-180.98	-182.98	2.00	-1.1035%
100001-RZ	45.99	45.89	0.10	0.2166%	-46.33	-46.21	-0.13	0.2730%
100023-X	440.19	449.92	-9.73	-2.2105%	-470.60	-478.58	7.98	-1.6965%
100023-Y	153.21	153.29	-0.08	-0.0516%	-148.64	-146.86	-1.78	1.2005%
100023-Z	230.21	230.47	-0.26	-0.1136%	-231.71	-233.02	1.31	-0.5642%
100023-RX	73.48	73.22	0.26	0.3495%	-77.11	-77.19	0.08	-0.1049%
100023-RY	145.74	143.32	2.41	1.6568%	-150.59	-148.49	-2.10	1.3965%
100023-RZ	118.83	117.62	1.20	1.0136%	-106.06	-107.06	1.00	-0.9435%
100045-X	420.19	422.00	-1.81	-0.4299%	-470.30	-473.09	2.79	-0.5940%
100045-Y	116.48	116.08	0.40	0.3401%	-136.97	-136.02	-0.95	0.6961%
100045-Z	280.23	281.82	-1.60	-0.5694%	-278.99	-275.55	-3.44	1.2329%
100045-RX	13.43	13.91	-0.49	-3.6153%	-11.76	-11.85	0.09	-0.7834%
100045-RY	113.87	113.95	-0.07	-0.0645%	-101.48	-101.01	-0.47	0.4624%
100045-RZ	33.78	34.29	-0.52	-1.5304%	-30.92	-31.66	0.74	-2.3981%
100067-X	369.28	379.01	-9.72	-2.6330%	-395.85	-403.43	7.57	-1.9132%
100067-Y	219.69	223.44	-3.75	-1.7083%	-220.84	-221.78	0.93	-0.4220%
100067-Z	163.05	161.84	1.21	0.7419%	-158.87	-161.44	2.57	-1.6156%
100067-RX	20.47	21.07	-0.60	-2.9207%	-16.75	-17.03	0.28	-1.6519%
100067-RY	61.90	61.81	0.10	0.1556%	-62.86	-62.32	-0.54	0.8627%
100067-RZ	51.18	50.97	0.20	0.3972%	-44.68	-45.31	0.63	-1.4010%

Table 2 – Interface Force Max/Min accuracy

Time domain validation included the evaluation of non-zero initial conditions, steady-state acceleration and a comparison to a non-linear Henkel-Mar Lift off case. Typical results are shown in Figure 10.



Units: m/s^2 or rad/s^2

Finally NTRC was evaluated for a Delta-2 lift off case using in-house forcing functions and also to an actual SLS lift off case, resulting in the same performance as with the in house models and forcing functions.

The SLS to payload interface consisted of 144 DoFs. Time replication remained outstanding for all recovery items, being large forces or moments or small accelerations, displacements and stresses. NTRC max/min accuracy remained consistent.

5. Concluding remarks

A new multibody coupled loads analysis method has been developed by the NASA Engineering Safety Center (NESC) and validated against standard launch vehicle coupled loads analyses (CLA). NTRC was inspired by force limiting methods for random vibration used at GSFC during the last 24 years such as Norton-Thevenin (NT) and Neubert's Impedance Analysis methods [ref. 3] which led to the use of receptance coupling (RC) in the methodology.

NTRC CLA method deals with at least an order opf magnitude less DoFs than the traditional shaped based CLA, hence computational time is significantly reduced and enables parametric or variational CLA and/or fast turn-around times to assess multiple payload manifests.

NTRC is exact in the frequency domain and time domain replication and accuracy is outstanding. NTRC has been demonstrated on a number of complex problems such as heavy payloads, indeterminate boundaries, numerous connection DoFs, dual payload/multibody, steady state initial conditions, and matching a nonlinear Henkel-Mar pad separation case. In addition, an actual SLS liftoff case was matched.

6. ABBREVIATIONS AND ACRONYMS

NTRC: Norton-Thevenin Receptance Coupling
NESC: NASA Engineering and Safety Center
CLA: coupled loads analysis
LV: launch vehicle
FBS: frequency-response–based substructuring
CMS: component mode synthesis
DoF: degree of freedom
CoG: center of gravity
PL: payload
SLS: Space Launch Vehicle

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