

Regression Analysis and Calibration Recommendations for the Characterization of Balance Temperature Effects

N. Ulbrich[†] and T. Volden[‡]

Jacobs Technology Inc., Moffett Field, California 94035

Analysis and use of temperature-dependent wind tunnel strain-gage balance calibration data are discussed in the paper. First, three different methods are presented and compared that may be used to process temperature-dependent strain-gage balance data. The first method uses an extended set of independent variables in order to process the data and predict balance loads. The second method applies an extended load iteration equation during the analysis of balance calibration data. The third method uses temperature-dependent sensitivities for the data analysis. Physical interpretations of the most important temperature-dependent regression model terms are provided that relate temperature compensation imperfections and the temperature-dependent nature of the gage factor to sets of regression model terms. Finally, balance calibration recommendations are listed so that temperature-dependent calibration data can be obtained and successfully processed using the reviewed analysis methods.

Nomenclature

A	= matrix used in general global regression problem
AF	= axial force of a strain-gage balance
a	= vector that contains intercept terms
b	= right hand side vector used in general global regression problem
C	= balance calibration coefficient matrix
$c_0, c_1, \dots$	= coefficients of the regression model of a gage output
C'	= extended balance calibration coefficient matrix
C₁	= part of matrix C that consists of linear terms only
C₂	= part of matrix C that consists of absolute value and non-linear terms
C₃	= part of matrix C that consists of temperature-dependent terms
F	= balance load, i.e., force or moment, described relative to absolute load datum of zero load
F	= part of matrix G that consists of linear terms; corrected load vector
G	= load matrix
G'	= extended load matrix
H	= part of load matrix G that consists of absolute value and non-linear terms
<i>i</i>	= bridge output or gage index
<i>j</i>	= index of coefficient or math model term used in fitting function
J	= part of load matrix G that consists of temperature-dependent terms
<i>k</i>	= balance gage/bridge index -or- load component index
<i>m</i>	= total number of math model terms used in fitting function (excluding intercept term)
<i>m'</i>	= total number of temperature-dependent math model terms
<i>n</i>	= total number of bridge outputs
<i>n'</i>	= number of loads (or number of gage outputs) plus the uniform balance temperature
<i>N1</i>	= forward normal force of a force balance
<i>NF</i>	= total normal force of a force balance
<i>p</i>	= total number of calibration points

[†] Aerodynamicist, Jacobs Technology Inc.

[‡] Computer Engineer, Jacobs Technology Inc.

PM	= total pitching moment of a force balance
R	= balance response, i.e., electrical output of a balance gage
$\mathbf{R}$	= balance response vector
$rN1$	= output of the forward normal force gage of a force balance
T	= uniform balance temperature (average of temperature readings on the balance)
T_0	= reference temperature of the balance
x	= unknown vector used in general global regression problem
Γ	= gage factor
Δm	= total number of math terms that are a function of the temperature difference
ΔR	= output change of a strain-gage balance gage
$\Delta \mathbf{R}$	= vector of gage outputs and temperature difference that is an input for the load calculation
ΔT	= difference between uniform balance temperature and reference temperature
ζ	= regression coefficient index
ϑ	= regression coefficient index
λ	= change in length
λ_0	= length
μ	= total number of regression coefficients of method 2 (excluding intercept term)
ζ	= regression coefficient index
ξ	= load iteration step index
ρ	= change in gage resistance (due to strain)
ρ_0	= gage resistance in ohms

I. Introduction

Different methods are used in aerospace testing to predict balance loads during a wind tunnel test from the measured electrical outputs of its strain-gages. The load prediction itself is based on a multivariate regression analysis of balance calibration data that describes the physical behavior of the balance. The *Iterative Method*, for example, first fits outputs of a balance as a function of the tare corrected loads that the balance experienced during the calibration. Then, the resulting regression coefficients are converted to data reduction matrix coefficients so that loads can be predicted from measured gage outputs during a wind tunnel test by using a load iteration scheme (see Refs. [1] to [4] for detailed descriptions and discussions of the *Iterative Method*).

In principle, the *Iterative Method* assumes that the number of “independent” variables, i.e., balance loads, equals the number of “dependent” variables, i.e., strain-gage outputs. This requirement is supported by the *General Theorem on the Inversion of Transformations* that can be applied to strain-gage balance data (see Ref. [5], pp. 4–5, for more details). However, this requirement made it difficult in the past to include temperature-dependent regressors in the analysis if an analyst wanted to apply the traditional load iteration scheme that is described in the literature (see Ref. [1], p. 19, Eq. (3.3.7)). Therefore, at least three different analysis methods were independently developed over the years so that the balance temperature can be used as an input variable for the load prediction. These three methods are discussed in the paper. In addition, physical interpretations of the temperature-dependent regression model terms are provided and calibration recommendations are listed so that “real-world” temperature-dependent calibration data can successfully be processed by using the reviewed analysis methods.

First, important characteristics of the three analysis methods are discussed. Then, physical interpretations of temperature-dependent regression model terms and calibration recommendations are provided.

II. Analysis Methods for Temperature-Dependent Balance Data

A. General Remarks

Three distinctly different methods are discussed in this section that may be used to analyze temperature-dependent strain-gage balance calibration data and predict loads during a wind tunnel test. These methods illustrate different approaches that analysts developed over the years in order to use temperature-dependent balance calibration data within the framework of a wind tunnel’s data processing software.

Basic challenges associated with the influence of temperature effects on the overall accuracy of the load

prediction of a strain–gauge balance are well understood and widely discussed in the literature. For example, *Ewald*, *Polanski*, and *Graewe* summarize associated problems as follows (taken from Ref. [6], p. 2):

... Through the temperature range zero point shifts and sensitivity shifts occur. These effects must be minimized by careful matching of the gages and remaining errors must be calibrated and corrected by a numerical process for satisfying accuracy. ...

Similarly, *Ferris* writes (taken from Ref. [7], p. 2):

... To obtain accurate force data over the large temperature range experienced in the NTF, it is necessary to eliminate or correct for the effects of any thermally–induced output so that the remaining output is a function of the applied load only. These thermally–induced outputs may appear as changes in the zero load output (apparent strain), in the output for a given applied load (sensitivity shift), and in the output due to mechanical deformation caused by thermal transients. ...

Ferris explicitly points out that thermal effects influence the measured electrical outputs of the balance when a set of calibration loads is applied. Therefore, assuming that the *Iterative Method* is applied, it is theoretically possible to include temperature effects in the regression models of the electrical outputs of a balance calibration data set as long as suitable temperature–dependent regression model terms are selected. These terms should model shifts in (i) the zero load outputs and (ii) the sensitivities. The influence of mechanical deformation caused by thermal transients on the outputs is not discussed in the paper. Consequently, the traditional regression model of the outputs (see, for example, Ref. [1], Section 3.3.1, or, Ref. [5], Section 3.2) is simply extended by using temperature–dependent regressors so that residual temperature effects are included.

In principle, it is assumed that all three analysis methods describe the temperature–dependent behavior of a strain–gauge balance by using the difference between the uniform balance temperature (T) and a suitable reference temperature (T_o) as an independent variable. It is critical to assume that the balance itself has a uniform temperature during both calibration and use because, otherwise, the sheer number of required independent variables makes the balance calibration task impractical. The importance of having a uniform temperature of the balance is also highlighted in the literature. *Ewald*, *Polanski*, and *Graewe* summarize the requirement for uniform temperature distribution of the balance as follows (taken from Ref. [6], p. 4):

... Even more important is the fact that copper beryllium has a heat conductivity five times higher than maraging steel. The advantage will be a more uniform temperature distribution in the balance or the same distribution in shorter times. ...

The use of the temperature difference as an independent variable suggests itself because a *Taylor Series* approximation of the functional behavior of a strain–gauge balance is the theoretical basis for the definition of the regression model terms of an electrical output of a balance. Then, the independent variable set for the calibration of the balance is given by the balance loads and the temperature difference. The new independent variable, i.e., the temperature difference, can be expressed as follows

$$\Delta T = T - T_o \quad (1)$$

where T is the assumed “uniform” balance temperature and T_o is the chosen reference temperature for the calibration. Ideally, this reference temperature should be located within the range of all uniform temperatures that the balance is expected to experience during the wind tunnel test.

The balance has to be calibrated at specific loads, load combinations, and temperature differences so that a numerical analysis method can be used to develop a balance load prediction process for the wind tunnel test that will meet a wind tunnel user’s accuracy requirements. Each one of the three potential analysis methods for temperature–dependent balance data is based on specific assumptions that need to be understood so that a suitable calibration design for the balance can be defined. Therefore, basic features and assumptions of the three analysis methods are discussed in the next sections of the paper.

B. Method 1

The first analysis method for temperature–dependent strain–gauge balance calibration data uses an extended set of calibration variables to predict the balance loads. The temperature difference is simply intro-

duced as both an “independent” and “dependent” variable that is used side-by-side with the loads and gage outputs. This approach has the advantage that the traditional load iteration equation defined in Ref. [1] can directly be applied without any modifications (see Refs. [8] to [10] for a detailed description of this approach).

The use of the temperature difference as both an “independent” and “dependent” variable is supported by the interpretation of the uniform balance temperature as a “state” variable. This interpretation can be better understood if the balance is placed inside of the “control volume” that is shown in Fig. 1 below.

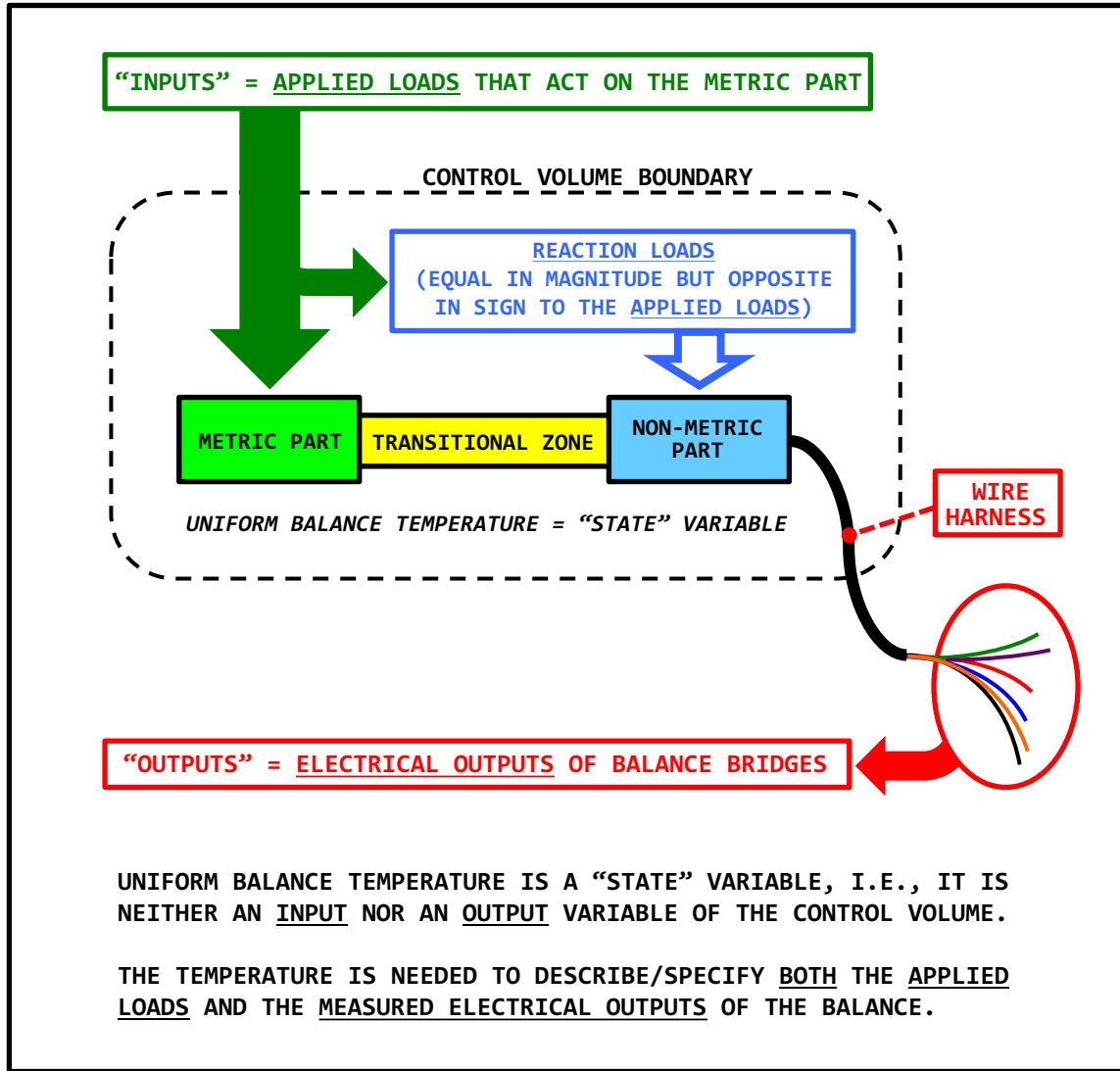


Fig. 1 Control volume analysis of the “inputs” and “outputs” of a strain-gage balance.

The “inputs” into the control volume are the applied loads that act on the metric part of the balance. The “outputs” are the measured electrical outputs at the balance bridges. The balance itself remains at a constant uniform temperature as long as no temperature gradient exists across the control volume boundary. Then, the temperature can be interpreted as a variable that describes the “state” of the balance when (i) the loads were applied to the metric part and (ii) the electrical outputs were measured at the balance bridges. In other words, the temperature, or, more precisely, the temperature difference is a “state” variable that accompanies both the description of the applied loads and measured electrical outputs of a given load configuration of the balance. The resulting “extended” set of “independent” variables of the balance can be defined as follows:

extended set of independent variables of an n -component balance : $\underbrace{F(1), \dots, F(n), \Delta T}_{n+1 \text{ variables}}$

$F(1), F(2), \dots, F(n) \implies$ load components of the balance

$\Delta T \implies$ temperature difference defined in Eq. (1)

The set consists of the n load components of the balance plus the temperature difference. Similarly, the “extended” set of “dependent” variables of an n -component balance can be summarized as follows:

extended set of dependent variables of an n -component balance : $\underbrace{R(1), \dots, R(n), \Delta T}_{n+1 \text{ variables}}$

$R(1), R(2), \dots, R(n) \implies$ gage outputs or bridge outputs of the balance

$\Delta T \implies$ temperature difference defined in Eq. (1)

The set consists of a total of n gage outputs plus the temperature difference. Then, assuming that the balance has a total of $n+1$ “independent” variables, i.e., n load components plus the temperature difference, and a total number of $n+1$ “dependent” variables, i.e., n gage outputs plus the temperature difference, the “extended” regression model of a gage output with index k becomes:

REGRESSION MODEL OF TEMPERATURE-DEPENDENT GAGE OUTPUT

$$R(k) = c_0(k) + c_1(k) \cdot F(1) + \dots + c_k(k) \cdot F(k) + \dots + c_n(k) \cdot F(n) + c_{n+1}(k) \cdot \Delta T + c_{n+2}(k) \cdot |F(1)| + \dots + c_{m-1}(k) \cdot |F^3(n)| + c_m(k) \cdot |\Delta T^3| \quad (2a)$$

where

$$n' = n + 1 \quad (2b)$$

$$1 \leq k \leq n' \quad (2c)$$

$$m = 2 \cdot n' \cdot (n' + 2) \quad (2d)$$

It is important to mention that the regression model described in Eq. (2a) above uses the same ten math term groups that are defined in Eq. (3.1.3) of Ref. [1]. The temperature difference is simply treated as if it is another “independent” load component of the balance. Consequently, after applying Eq. (2d), the maximum number of potential regression coefficients excluding the intercept for a temperature-dependent gage output of a six-component balance becomes:

$$n' = 6 + 1 = 7 \implies m = 2 \cdot n' \cdot (n' + 2) = 126 \quad (3)$$

The actual number of non-zero coefficients of the regression model of the electrical output of a real-world balance is usually substantially smaller than 126. Many terms may simply not be supported by the

calibration data or, if used, may cause massive near-linear dependencies between the regression model terms. In particular, it makes no sense to use terms that contain the absolute value of the temperature difference because, by design, absolute value terms are only to be used with the balance loads themselves to model the “bi-directional” behavior of an output. This restriction can be summarized as follows:

Coefficients of regression model terms that contain $|\Delta T|$, i.e, coefficients of the five term classes $|\Delta T|$, $\Delta T \cdot |\Delta T|$, $F(k) \cdot |\Delta T|$, $|F(k) \cdot \Delta T|$, $|\Delta T^3|$, are explicitly set to zero because, by design, absolute value terms are exclusively to be used with loads in order to model “bi-directional” output behavior (for more detail see Ref. [1], p. 8/9).

A detailed review of the remaining temperature-dependent math model term options shows that a term is either constructed exclusively from the temperature difference (i.e., ΔT , ΔT^2 , ΔT^3) or it is a combination of a single load component with the temperature difference (i.e., $F(1) \cdot \Delta T$, $F(2) \cdot \Delta T$, ...). This conclusion can be expressed by using the following alternate description of Eq. (2a):

REGRESSION MODEL OF TEMPERATURE-DEPENDENT GAGE OUTPUT

$$\begin{aligned}
 R(k) = & \dots + \underbrace{c_{n+1}(k) \cdot \Delta T}_{\text{linear term}} + \dots + \underbrace{c_{\zeta}(k) \cdot \Delta T^2}_{\text{quadratic term}} + \dots \\
 & + \underbrace{c_{\vartheta}(k) \cdot F(k) \cdot \Delta T}_{\text{cross-product term}} + \dots + \underbrace{c_{\zeta}(k) \cdot \Delta T^3}_{\text{cubic term}} + \dots
 \end{aligned} \tag{4}$$

Equation (2a) and its alternate description given by Eq. (4) describe a set of $n + 1$ linear equations that define the least squares problem of the temperature-dependent balance calibration data set. Fortunately, the regression coefficients of the additional “dependent” variable, i.e., $R(n + 1) = R(n') \equiv \Delta T$ of Eq. (2a), are explicitly known as ΔT appears on both the left and right-hand side of the regression model for $R(n')$. Consequently, the regression coefficient $c_{\vartheta}(n')$ is “1.0” for $\vartheta = n'$ and $c_{\vartheta}(n')$ is “0.0” for $\vartheta \neq n'$. This important conclusion can be expressed as follows:

REGRESSION MODEL OF TEMPERATURE DIFFERENCE

$$R(n') \equiv \underbrace{\Delta T}_{\text{dep.}} = 0 + \dots + 0 \cdot F(n) + 1.0 \cdot \underbrace{\Delta T}_{\text{indep.}} + 0 \cdot |F(1)| + \dots \tag{5a}$$

or

$$c_{\vartheta}(n') = \begin{cases} 1.0 & \text{if } \vartheta = n' \\ 0.0 & \text{if } \vartheta \neq n' \end{cases} \tag{5b}$$

The simple addition of the “trivial-but-correct” relationship defined in Eq. (5a) above to the system of linear equations given by Eq. (2a) makes it ultimately possible to construct a load iteration scheme that (i) uses temperature-dependent gage outputs as input and (ii) does not require a modification of the original load iteration equation that is defined in Ref. [1]. This load iteration equation has the following general

format (taken from Ref. [1], p. 19, Eq. (3.3.7))

$$\mathbf{F}_\xi = \left[\mathbf{C}_1^{-1} \right]_{n' \times n'} \cdot \Delta \mathbf{R} - \left[\mathbf{C}_1^{-1} \mathbf{C}_2 \right]_{n' \times (m-n')} \cdot \mathbf{H}\{\mathbf{F}_{\xi-1}\} \quad (6)$$

where vector $\mathbf{F}$ of the extended “independent” variable set is defined as

$$\mathbf{F}_{n' \times 1} = \begin{bmatrix} F(1) \\ F(2) \\ \vdots \\ F(n) \\ \Delta T \end{bmatrix} \quad (7a)$$

and vector $\Delta \mathbf{R}$ of the extended “dependent” variable set is defined as

$$\Delta \mathbf{R}_{n' \times 1} = \begin{bmatrix} R(1) - c_0(1) \\ R(2) - c_0(2) \\ \vdots \\ R(n) - c_0(n) \\ \Delta T \end{bmatrix} \quad (7b)$$

and matrices $\mathbf{C}_1^{-1}$ and $\mathbf{C}_1^{-1} \mathbf{C}_2$ have the coefficients of the data reduction matrix that results from the fit of the calibration data. The symbol $\mathbf{H}(\mathbf{F}_{\xi-1})$ represents a rectangular matrix that only depends on load estimates of the previous iteration step and the temperature difference (see Ref. [1], p. 18, for a definition of the matrix). It is the part on the right-hand side Eq. (6) that changes with each iteration step.

In the next section of the paper a second method for the analysis of temperature-dependent balance calibration data is discussed that uses an extended load iteration equation.

C. Method 2

Method 2, i.e., the second analysis approach that uses the temperature difference as an independent variable for the prediction of balance loads, is very similar to Method 1. Method 2 was rigorously developed by *Lynn, Commo, and Parker* in 2012 who defined a balance load iteration scheme that uses both temperatures and pressures as additional independent variables of the gage outputs of a balance (for more detail see Ref. [11], pp. 564–565, Eq. (A5)). Method 2, similar to Method 1, also performs an analysis that fits all “ n ” outputs as a function of (i) the “ n ” load components of the balance and (ii) the temperature difference. However, Method 2 no longer requires that the number of “independent” variables matches the number of “dependent” variables. Therefore, Method 2 has to apply an “extended” load iteration scheme for the prediction of loads from the measured outputs and the temperature difference that (i) depends on the specific choice of temperature (and/or pressure) dependent regression model terms and (ii) is not identical with the traditional load iteration equation that is given in Ref. [1].

A new derivation of Method 2, i.e., of *Lynn, Commo, and Parker’s* extended load iteration scheme, is given in App. 1 and 2 of the current paper in order to allow for a direct comparison of Method 1 with Method 2. Therefore, only the most important characteristics of Method 2 are summarized in this section.

Method 2 uses a regression model of the gage outputs that is very similar to the regression model of Method 1 that is defined in Eq. (2a). Method 2 simply “appends” the chosen temperature-dependent regression model terms to the basic regression model term set that is defined in Eq. (3.1.3) of Ref. [1]. Then, for example, using linear, quadratic, and cubic temperature terms in combination with cross-product terms that are constructed from the loads and the temperature difference, the following extended regression model of a gage output with index “ k ” is obtained:

EXTENDED REGRESSION MODEL OF GAGE OUTPUT

$$\begin{aligned}
 R(k) = & \underbrace{c_0(k) + c_1(k) \cdot F(1) + c_2(k) \cdot F(2) + \dots + c_m(k) \cdot |F^3(n)|}_{\text{assumed to be identical with regression model given by Eq. (3.1.3) of Ref. [1]} \\
 & + \underbrace{c_{m+1}(k) \cdot \Delta T + c_{m+2}(k) \cdot \Delta T^2 + c_{m+3}(k) \cdot \Delta T^3}_{\text{temperature-dependent linear, quadratic, and cubic terms}} \\
 & + \underbrace{\sum_{i=1}^n \{ c_{m+3+i}(k) \cdot F(i) \cdot \Delta T \}}_{\text{temperature-dependent cross-products}}
 \end{aligned} \tag{8a}$$

... or, assuming an auxiliary index μ is introduced, we can write ...

$$R(k) = c_0(k) + c_1(k) \cdot F(1) + c_2(k) \cdot F(2) + \dots + c_\mu(k) \cdot F(n) \cdot \Delta T \tag{8b}$$

where

$$\text{total number of math terms (excluding intercept)} \implies \mu = m + m' \tag{8c}$$

$$\text{total number of temperature-dependent terms} \implies m' = 3 + n \tag{8d}$$

Method 2 simply fits the gage outputs of the balance calibration data using the extended regression model given in Eq. (8a) above. Then, after (i) the fitted coefficients c_0 to c_μ have been obtained and (ii) the iteration equation defined in Eq. (47) of App. 2 is constructed from them, the following extended iteration equation needs to be applied to predict the loads from the measured outputs:

$$\mathbf{F}_\xi = \left[\mathbf{C}_1^{-1} \right]_{n \times n} \cdot \Delta \mathbf{R} - \left[\mathbf{C}_1^{-1} \mathbf{C}_2 \right]_{n \times (\mu - n - m')} \cdot \mathbf{H}\{\mathbf{F}_{\xi-1}\} - \left[\mathbf{C}_1^{-1} \mathbf{C}_3 \right]_{n \times m'} \cdot \mathbf{J}\{\mathbf{F}_{\xi-1}; \Delta T\} \tag{9}$$

where vector $\mathbf{F}$ of the “independent” variable set is defined as

$$\mathbf{F}_{n \times 1} = \begin{bmatrix} F(1) \\ F(2) \\ \vdots \\ F(n) \end{bmatrix} \tag{10a}$$

and vector $\Delta \mathbf{R}$ of the “dependent” gage outputs is defined as

$$\Delta \mathbf{R}_{n \times 1} = \begin{bmatrix} R(1) - c_0(1) \\ R(2) - c_0(2) \\ \vdots \\ R(n) - c_0(n) \end{bmatrix} \tag{10b}$$

Key differences between the iteration equation of Method 1 and the iteration equation of Method 2 become obvious if Eqs. (6), (7a), and (7b) are compared with Eqs. (9), (10a), and (10b). The temperature-dependent data reduction matrix coefficients of Method 1 are a “subset” of the coefficients that are contained in the rectangular matrix $\mathbf{C}_1^{-1} \mathbf{C}_2$. The load contribution $\mathbf{C}_1^{-1} \Delta \mathbf{R}$ of Eq. (6) is not temperature-dependent because the last column of $\mathbf{C}_1^{-1}$ equals $\{0 \ 0 \ \dots \ 0 \ 1\}^T$. The temperature-dependent data reduction matrix coefficients of Method 2, on the other hand, are the coefficients that are contained in the rectangular matrix

$\mathbf{C}_1^{-1} \mathbf{C}_3$ (see Eq. (9) or (47)). In addition, only the iteration equation of Method 1, i.e., Eq. (6), matches the traditional iteration equation that is defined in Ref. [1].

D. Method 3

Method 3 also first fits the gage outputs as a function of the balance loads and, afterwards, uses the traditional load iteration equation of Ref. [1] for the prediction of balance loads during a wind tunnel test. However, Method 3 treats some of the regression coefficients no longer as constants. It makes the “prime sensitivity” of a balance gage, i.e., the regression coefficient of the primary gage load, be a function of temperature in order to model the potential “sensitivity shift” that is reported in the literature (see, e.g., the discussion of this phenomenon in Refs. [6] and [7]).

A “hidden” connection between the temperature-dependent regression coefficients used by Method 3 and the regression models used by both Method 1 and Method 2 exists that can be understood if the coefficient of the primary gage load of a gage is analyzed in more detail. First, it is assumed that the regression model of the gage output is not a function of the temperature. The corresponding regression model is given by the following relationship (equals Eq. (3.1.3) of Ref. [1]):

REGRESSION MODEL OF GAGE OUTPUT
 (assumed to be identical with regression model given by Eq. (3.1.3) of Ref. [1])

$$R(k) = c_0(k) + c_1(k) \cdot F(1) + \dots + \underbrace{c_k(k)}_{\text{sensitivity}} \cdot F(k) + \dots + c_m(k) \cdot |F^3(n)| \quad (11)$$

The regression model of the gage output given in Eq. (11) above has the following total number of potential regression model terms if a six-component balance is used (not counting the intercept):

$$\text{six-component balance} \implies n = 6 \implies m = 96 \quad (12)$$

It is assumed that the load component $F(k)$ in Eq. (11) above is the primary load associated with gage output $R(k)$. Then, the “prime sensitivity” of the balance gage with index “ k ” can be defined as follows:

DEFINITION OF THE GAGE SENSITIVITY

$$c_k(k) = \frac{\partial R(k)}{\partial F(k)} \equiv \frac{\text{change of primary gage output}}{\text{change of primary gage load}} \quad (13)$$

The regression coefficient $c_k(k)$ in Eq. (11) above is the least squares approximation of the “prime sensitivity” of the balance gage with index “ k ”. Now, the assumption is made that the prime sensitivity of the gage is also a function of temperature. We get:

$$\text{assumption} \implies c_k(k) \neq \text{const.} \implies c_k(k, T) \quad (14)$$

Then, after replacing $c_k(k)$ with $c_k(k, T)$, Eq. (11) can be written as follows:

REGRESSION MODEL WITH TEMPERATURE-DEPENDENT SENSITIVITY

$$R(k) = c_0(k) + c_1(k) \cdot F(1) + \dots + \underbrace{c_k(k, T)}_{\text{sensitivity}} \cdot F(k) + \dots + c_m(k) \cdot |F^3(n)| \quad (15)$$

It is important to point out that the regression coefficients of Eq. (15) above, i.e., $c_0(k)$, $c_1(k)$, $\dots$, $c_k(k, T)$, $\dots$, $c_m(k)$, have to be obtained by using two separate and completely independent least squares fits because the “prime sensitivity” $c_k(k, T)$ is no longer treated as a constant. The result of these two independent fits has to be superimposed in order to predict the balance loads during the wind tunnel test. The first least squares fit uses calibration data as input that is obtained by applying both single- and multi-component loadings while keeping the temperature at a constant reference temperature T_o . This least squares fit determines all coefficients that are independent of the balance temperature. The second least squares fit, on the other hand, only uses single-component loadings as input that were obtained while varying the balance temperature. This least squares fit is used to determine the temperature-dependent regression coefficients $c_k(k, T)$, i.e., the temperature-dependent sensitivities of each gage.

It is possible to approximate the temperature-dependent regression coefficient $c_k(k, T)$ by using a *Taylor Series* that is developed near a fixed reference temperature T_o . Then, we get:

$$\textit{Taylor Series} \implies c_k(k, T) \approx c_k(k, T_o) + \left[\frac{d c_k}{d T} \right]_{T_o} \cdot \Delta T + \frac{1}{2} \cdot \left[\frac{d^2 c_k}{d T^2} \right]_{T_o} \cdot \Delta T^2 + \dots \quad (16a)$$

Then, after dropping all higher order terms on the right-hand side of Eq. (16a) above, we get the following linear approximation of the temperature-dependent sensitivity:

$$\textit{linear approximation} \implies c_k(k, T) \approx c_k(k, T_o) + \left[\frac{d c_k}{d T} \right]_{T_o} \cdot \Delta T \quad (16b)$$

Finally, after replacing $c_k(k, T)$ in Eq. (15) with the right-hand side of Eq. (16b), a linear approximation of the temperature-dependent regression model with temperature-dependent sensitivity is obtained. We get:

**LINEAR APPROXIMATION OF REGRESSION MODEL
WITH TEMPERATURE-DEPENDENT SENSITIVITY**

$$R(k) \approx \dots + c_k(k, T_o) \cdot F(k) + \underbrace{\left[\frac{d c_k}{d T} \right]_{T_o} \cdot F(k) \cdot \Delta T}_{\text{cross-product term}} + \dots \quad (17)$$

Several observations can be made after analyzing Eq. (17) above. First, Method 3 only supports modeling of a temperature-dependent gage factor as the cross-product term $F(k) \cdot \Delta T$ associated with the primary gage load and the temperature difference is used ($\implies$ it is shown in a later section of the current paper that a temperature-dependent gage factor is directly related to the cross-product term $F(k) \cdot \Delta T$). In addition, the linear approximation of the regression model of Method 3 is, by design, “non-hierarchical” as the lower order linear term ΔT is not included. Finally, the regression model of Method 3 cannot model residual temperature compensation imperfections of the gage as terms like ΔT , ΔT^2 , and ΔT^3 are missing.

It is necessary to make a few remarks regarding the load iteration equation that has to be used with Method 3. In principle, two options exist. The first option assumes that the outputs of the calibration data are fitted using Eq. (15). Then, the traditional load iteration scheme given in Ref. [1] (or its variation given in Ref. [3]) have to be applied to predict the balance loads. In this case, the coefficient $c_k(k, T)$ of the primary gage load, i.e., the primary gage sensitivity, is updated before the load iteration itself takes place assuming that the temperature-dependent nature of $c_k(k, T)$ is known. The second option assumes that the outputs are fitted using Eq. (17), i.e., the linearized approximation of Eq. (15). Then, a variation of the extended load iteration equation given by Eq. (9) has to be applied where (i) $\mathbf{C}_3$ is a $n \times n$ diagonal matrix that has the derivatives dc_k/dT on the principle diagonal and (ii) $\mathbf{C}_1^{-1}\mathbf{C}_3$ is a $n \times n$ matrix that is multiplied with the $n \times 1$ vector $\mathbf{J}\{\mathbf{F}, \Delta T\}$.

It is useful to compare characteristics of the three analysis methods that may be used to process temperature-dependent balance data. Results of this comparison are summarized in the next section.

E. Comparison of Analysis Methods

Three analysis methods were discussed in the previous sections that may be used to process temperature-dependent strain-gage balance data. The successful implementation of a chosen method in both the analysis software package of a balance calibration laboratory and/or the data system of a wind tunnel facility depends on a good understanding of its key characteristics and limitations. In addition, it is useful to understand the connection between the three methods so that advantages and disadvantages of an implementation choice can better be evaluated.

Analysis Method 1 and Method 2 are very similar. They essentially use identical regression models of the outputs for the least squares fit of the calibration data and only differ in the definition of the load iteration equation that is used to predict the temperature-dependent balance loads. Therefore, by design, the predicted loads for Method 1 and Method 2 will show almost “perfect” agreement as long as the same regression model terms are used for the regression models of the electrical outputs of the balance.

Method 1 has the advantage that it applies the widely used traditional load iteration equation that is defined in the literature (Ref. [1], Eq. (3.3.7)). Therefore, Method 1 can immediately be used with an existing data analysis software package as long as (i) the software’s load iteration equation matches the traditional iteration equation defined in Ref. [1], and, (ii) the software can handle matrices with more than six independent variables (for completeness, it needs to be mentioned that Method 1 will also work with the variation of the traditional iteration equation that is described by Eq. (26) of Ref. [3]).

Method 2, on the other hand, requires the implementation of a new load iteration scheme in the data processing software. This new iteration scheme can be viewed as an extension of the traditional load iteration equation that is described in the literature. An additional contribution is simply included on the right-hand side of the iteration equation that depends on (i) load estimates from the previous iteration step and (ii) the temperature difference (e.g., compare Eq. (9) with Eq. (6)).

Both Method 1 and Method 2 perform a multivariate linear regression analysis of the balance data. Therefore, statistical metrics like the *variance inflation factor* and the *p-value of the t-statistic* may be used to assess if (i) the chosen temperature-dependent regression model terms have massive near-linear dependencies in the regression model and if (ii) they are statistically significant.

Method 3 uses either the traditional iteration equation that is defined in Ref. [1] or a variation of the extended iteration equation that is defined in Eq. (9). The choice of temperature-dependent regression model terms with Method 3 is more limited when compared with Method 1 and Method 2 as the linearized description of Method 3, i.e., Eq. (17), only supports a single temperature-dependent term for each gage. It is the cross-product term of the primary gage load $F(k)$ with the temperature difference ΔT . Fortunately, it can rigorously be shown that this specific cross-product term choice is related to the temperature dependency of the gage factor (this physical interpretation of the cross-product term is discussed in great detail in the next section of the paper). It is also important to point out that Method 3 uses a “non-hierarchical” regression model of the gage outputs. Therefore, residual temperature compensation imperfections of the gage itself cannot be modeled as terms like ΔT , ΔT^2 and ΔT^3 are missing in Eq. (17).

The use of Method 3 in combination with the linearized approximation of the regression model of the gage output, i.e., Eq. (17), has an advantage over the use of a temperature-dependent regression coefficient, i.e., Eq. (15). Only this linearized approximation allows an analyst to directly use statistical metrics like the *variance inflation factor* and the *p-value of the t-statistic* to assess if (i) the chosen temperature-dependent regression model term has massive near-linear dependencies in the regression model and if (ii) the term is statistically significant. Finally, assuming that balance data is given in its “design” format (e.g., force balance data in force balance format), the linearized approximation of Method 3 is identical with Method 2 for the special case when the regression models of Method 2 only use a single temperature-dependent term, i.e., the cross-product of the primary gage load with the temperature difference, for each gage.

The most important characteristics of each one of the three load prediction methods for temperature-dependent strain-gage balance data are summarized in Table 1 below.

Table 1: Characteristics of load prediction methods for temperature–dependent balance data.

METHOD	REGRESSION MODEL OF GAGE OUTPUT	ITERATION EQUATION
Method 1	$R(k) = c_0(k) + \dots + c_m(k) \cdot \Delta T^3 $ $\implies \text{Eqs. (2a), (4)}$ Indep. Var. $\equiv F(1), \dots, F(n), \Delta T$ Dep. Var. $\equiv R(1), \dots, R(n), \Delta T$	$\mathbf{F}_\xi = \dots - \mathbf{C}_1^{-1} \mathbf{C}_2 \mathbf{H}\{\mathbf{F}_{\xi-1}\}^\dagger$ $\implies \text{Eq. (6)}$ (matches Eq. (3.3.7) of Ref. [1])
Method 2	$R(k) = c_0(k) + \dots + c_\mu(k) \cdot F(n) \cdot \Delta T$ $\implies \text{Eqs. (8a), (8b)}$ Indep. Var. $\equiv F(1), \dots, F(n), \Delta T$ Dep. Var. $\equiv R(1), \dots, R(n)$	$\mathbf{F}_\xi = \dots - \mathbf{C}_1^{-1} \mathbf{C}_3 \mathbf{J}\{\mathbf{F}_{\xi-1}; \Delta T\}$ $\implies \text{Eqs. (9), (47)}$
Method 3	$R(k) = \dots + c_k(k, T) \cdot F(k) + \dots$ $\implies \text{Eq. (15)}$ Indep. Var. $\equiv F(1), \dots, F(n), T$ Dep. Var. $\equiv R(1), \dots, R(n)$	$\mathbf{F}_\xi = \dots - \mathbf{C}_1^{-1} \mathbf{C}_2 \mathbf{H}\{\mathbf{F}_{\xi-1}\}^\dagger$ $\implies \text{Eq. (6)}$ $\implies$ coefficients $c_k(k, T)$ have to be updated <u>before</u> iteration starts
Method 3 (linearized)	$R(k) \approx \dots + c_k(k) \cdot F(k) \cdot \Delta T + \dots$ $\implies \text{Eq. (17)}$ Indep. Var. $\equiv F(1), \dots, F(n), \Delta T$ Dep. Var. $\equiv R(1), \dots, R(n)$ Method 3 (linearized) $\equiv$ Method 2 with <u>single</u> temperature–dependent term (assuming balance data is given in its “design” format)	$\mathbf{F}_\xi = \dots - \mathbf{C}_1^{-1} \mathbf{C}_3 \mathbf{J}\{\mathbf{F}_{\xi-1}; \Delta T\}$ $\implies \text{Eqs. (9), (47)}$

[†]Iteration equation $\mathbf{F}_\xi = \mathbf{B}_1^{-1} \Delta \mathbf{R} - \mathbf{B}_1^{-1} \mathbf{B}_2 \mathbf{F}_{\xi-1} - \mathbf{B}_1^{-1} \mathbf{C}_2 \mathbf{H}\{\mathbf{F}_{\xi-1}\}$ may also be used (Ref. [3], Eq. (26)).

III. Physical Interpretation of Temperature–Dependent Terms

A. General Remarks

In the previous sections four different types of temperature–dependent regression model terms for the analysis of balance calibration data were introduced (linear, quadratic, cubic, and cross–product terms). These four types can be split into two groups that model specific physical characteristics of the temperature–dependent nature of strain–gage balance data. The first group consists of the linear, quadratic, and cubic

terms. They may be used to model residual temperature compensation imperfections (“bias” shifts) of the gage output. The second group consists of the temperature–dependent cross–product terms. Their use is related to the modeling of the temperature–dependent nature of the gage factor. Both term groups are more rigorously discussed in the next two sections.

B. Residual Temperature Compensation Imperfections $\Longleftrightarrow \Delta T, \Delta T^2, \Delta T^3$

In general, strain–gages of a wind tunnel balance are temperature compensated so that the gage outputs are significantly less sensitive to temperature effects. However, no temperature compensation is “perfect” and a residual “bias” shift of the output may occur at certain test conditions. This temperature–dependent “bias” shift may be modeled in the regression model of a strain–gage output by simply using linear, quadratic, and cubic terms of the temperature difference as regression coefficients. It is expected that most of the “bias” shift will be characterized by the linear and quadratic terms. Therefore, the cubic term of the temperature difference may probably be omitted in most practical situations.

C. Temperature Dependency of the Gage Factor $\Longleftrightarrow F(k) \cdot \Delta T$

It can rigorously be shown that the temperature–dependent cross–product terms defined in Eqs. (4), (8a), and (17) are related to the temperature–dependent nature of the gage factor. This connection is supported by a detailed analysis of parameter that influence the magnitude of the gage factor. In general, the gage factor Γ of a specific balance gage with index “ k ” is defined as follows (from Ref. [12], Eq. (10))

$$\text{Gage Factor} \implies \Gamma(k) = \frac{\rho(k) / \rho_o}{\lambda(k) / \lambda_o} \quad (18)$$

where ρ is a gage resistance change, ρ_o is the gage resistance, λ is a length change, and λ_o is the reference length. Then, solving Eq. (18) for the gage resistance change, we get:

$$\rho(k) = \Gamma(k) \cdot \lambda(k) \cdot \frac{\rho_o}{\lambda_o} \quad (19)$$

It is possible to use a *Taylor Series* approximation for the relationship between the gage resistance change and the output change of the gage. Then, we get:

$$\text{Taylor Series} \implies \rho(k) = 0 + \left[\frac{d \rho}{d R} \right] \cdot \Delta R(k) + \frac{1}{2} \cdot \left[\frac{d^2 \rho}{d R^2} \right] \cdot \Delta R(k)^2 + \dots \quad (20a)$$

Now, after dropping all higher order terms, we get the following linear approximation:

$$\text{linear approximation} \implies \rho(k) \approx \varphi \cdot \Delta R(k) \quad \text{where} \quad \varphi = \frac{d \rho}{d R} \equiv \text{const.} \quad (20b)$$

Similarly, it is possible to use a *Taylor Series* approximation for the relationship between the length change and the absolute load that is applied at the gage location. Then, we get:

$$\text{Taylor Series} \implies \lambda(k) = 0 + \left[\frac{d \lambda}{d F} \right] \cdot F(k) + \frac{1}{2} \cdot \left[\frac{d^2 \lambda}{d F^2} \right] \cdot F(k)^2 + \dots \quad (21a)$$

Again, after dropping all higher order terms, we get the following linear approximation:

$$\text{linear approximation} \implies \lambda(k) \approx \psi \cdot F(k) \quad \text{where} \quad \psi = \frac{d \lambda}{d F} \equiv \text{const.} \quad (21b)$$

In the next step, after using the right–hand sides of Eqs. (20b) and (21b) in order to substitute ρ and λ in Eq. (19), we get the following approximation:

$$\varphi \cdot \Delta R(k) \approx \Gamma(k) \cdot \psi \cdot F(k) \cdot \frac{\rho_o}{\lambda_o} \quad (22a)$$

Then, after solving the above equation for the output change of the gage, we get:

$$\Delta R(k) \approx \Gamma(k) \cdot F(k) \cdot \frac{\psi \cdot \rho_o}{\varphi \cdot \lambda_o} \quad (22b)$$

The gage factor is typically a function of the uniform temperature of the balance (see related discussion in Ref. [13], pp. 10–11). This relationship can be described as a *Taylor Series* that is developed near the reference temperature T_o of the gage. Then, we get:

$$\textit{Taylor Series} \implies \Gamma(k, T) = \Gamma(k, T_o) + \left[\frac{d \Gamma}{d T} \right]_{T_o} \cdot \Delta T + \frac{1}{2} \cdot \left[\frac{d^2 \Gamma}{d T^2} \right]_{T_o} \cdot \Delta T^2 + \dots \quad (23a)$$

Now, after dropping all higher order terms, we get the following linear approximation:

$$\textit{linear approximation} \implies \Gamma(k, T) \approx \Gamma(k, T_o) + \left[\frac{d \Gamma}{d T} \right]_{T_o} \cdot \Delta T \quad (23b)$$

Then, after using the right-hand side of Eq. (23b) in order to replace the gage factor in Eq. (22b), we get for the output change of the gage the following approximation:

$$\Delta R(k) \approx \left[\Gamma(k, T_o) + \left[\frac{d \Gamma}{d T} \right]_{T_o} \cdot \Delta T \right] \cdot F(k) \cdot \frac{\psi \cdot \rho_o}{\varphi \cdot \lambda_o} \quad (24a)$$

Finally, after rearranging Eq. (24a) and assuming that the output change $\Delta R(k)$ equals $R(k)$, i.e., the difference between the output of the gage and its output at zero absolute load, we get the following result:

$$R(k) = \Delta R(k) \approx \underbrace{\left\{ \Gamma(k, T_o) \cdot \frac{\psi \cdot \rho_o}{\varphi \cdot \lambda_o} \right\} \cdot F(k)}_{\textit{load-dependent contribution}} + \underbrace{\left\{ \left[\frac{d \Gamma}{d T} \right]_{T_o} \cdot \frac{\psi \cdot \rho_o}{\varphi \cdot \lambda_o} \right\} \cdot F(k) \cdot \Delta T}_{\textit{temperature-dependent contribution}} \quad (24b)$$

It can be seen that the right-hand side of Eq. (24b) above has two parts. The first part depends on the primary gage load. Its multiplier equals the prime sensitivity of the gage at constant temperature. The second part depends on both the primary gage load and the temperature difference. Therefore, it is concluded that the temperature-dependent nature of the gage factor can be modeled by using a cross-product term that is constructed from the primary gage load and the temperature difference.

The analysis above has an interesting consequence as far as the selection of temperature-dependent cross-product terms for the regression model of a gage output is concerned: the choice of temperature-dependent cross-product terms depends on an analyst's definition of the balance loads. Let us assume, for example, that an analyst chooses to process balance data in its original design format (e.g., force balance in force balance format). Then, the regression model of a gage output may only need a single temperature-dependent cross-product term that is defined as the product of the primary gage load with the temperature difference (e.g., term $N1 \cdot \Delta T$ may be needed in the regression model of the forward normal force gage $rN1$ of a force balance). Now, let us assume that an analyst elects to process force balance data in direct-read format. Then, the regression model of a gage output may need two temperature-dependent cross-product terms because two load components may have a strong influence on the given gage output (e.g., the cross-product terms $NF \cdot \Delta T$ and $PM \cdot \Delta T$ may be needed in the regression model of the forward normal force gage output $rN1$ of a force balance if the calibration data is processed in direct-read format).

IV. Calibration Recommendations

The application of the three previously discussed analysis methods requires that temperature-dependent balance calibration data is obtained such that the coefficients of the most important regression model terms can be computed with confidence. Therefore, the authors recommend to perform the calibration by using the following steps.

Step 1: Calibration data should be collected at a minimum of three distinctly different temperatures. Data at one of the temperatures, i.e., at the chosen reference temperature, should be obtained by applying all desired load and load combinations that are needed to describe the overall behavior of the balance. Data at the remaining temperatures, on the other hand, only needs to be obtained by applying single-component loads as the regression models of the gage outputs of the different analysis methods (i) only support the temperature-dependent cross-product term $F(i) \cdot \Delta T$ and (ii) do not have third order cross-product terms like $F(i) \cdot F(j) \cdot \Delta T$.

Step 2: The natural zeros, i.e., the gage outputs at zero absolute load, should be determined separately for each one of the chosen temperatures. The natural zeros are needed in order to obtain the tare corrected loads of the balance calibration data.

Step 3: The tare load iteration should be performed for each one of the three temperature-dependent data sets separately by using the natural zeros of the given temperature as output datum. The result of this analysis step are the tare corrected loads of all three temperature-dependent calibration data sets.

Step 4: The final temperature-dependent balance calibration data input file should be assembled by combining the tare corrected load sets of Step 3 with output differences that are computed relative to the natural zero of the chosen global reference temperature.

Step 5: The regression analysis of the tare corrected calibration data should be performed by using the chosen analysis method (apply regression model term reduction as needed).

V. Summary and Conclusions

Three methods for analysis and use of temperature-dependent strain-gage balance data were presented. The first method uses an extended set of independent variables to process the data and predict balance loads. The second method uses an extended load iteration equation for the analysis of balance calibration data (an alternate derivation of the second method is provided in the appendices of the paper for reference). The third method uses temperature-dependent sensitivities for the analysis. Differences between the three methods were discussed so that an analyst can choose a method for implementation that best meets given data processing software constraints.

Physical interpretations of the suggested temperature-dependent regression model terms are provided that relate (i) residual temperature compensation imperfections of the gage and (ii) the temperature-dependent nature of the gage factor to sets of regression model terms. This analysis showed that only two classes of regression model terms are needed to model temperature-dependent strain-gage balance data. The first class consists of the term groups ΔT , ΔT^2 , and ΔT^3 . These terms can be used to represent a temperature-dependent zero load gage output shift. The second class consists of the term group $F(k) \cdot \Delta T$. These cross-product terms can be used to represent a temperature-dependent sensitivity shift of the gage output. Table 2 below summarizes the temperature-dependent regression model term classes that the reviewed analysis methods for strain-gage balance data support.

Table 2: Supported temperature-dependent regression model terms.

METHOD	REGRESSION MODEL TERMS
Method 1	$\Delta T, \Delta T^2, \Delta T^3; F(k) \cdot \Delta T$
Method 2	$\Delta T, \Delta T^2, \Delta T^3; F(k) \cdot \Delta T$
Method 3	$c_k(k, T)$
Method 3 (linearized)	$F(k) \cdot \Delta T$

It is important to point out that the number of required temperature-dependent cross-product terms per gage depends on the chosen load format of the balance. One cross-product term of the type $F(k) \cdot \Delta T$ is needed per gage if balance loads are described in the design format of the balance (e.g., the term $N1 \cdot \Delta T$ is needed for the regression model of the forward normal force gage output $rN1$ if the loads of a force balance are expressed in force balance format). On the other hand, two cross-product terms of the type $F(k) \cdot \Delta T$ are needed per gage if balance loads are not described in the design format of the balance (e.g., the terms

$NF \cdot \Delta T$ and $PM \cdot \Delta T$ are needed for the regression model of the forward normal force gage output $rN1$ if the loads of a force balance are expressed in direct-read format).

Finally, balance calibration recommendations are made so that temperature-dependent balance data can successfully be processed by using any one of the three analysis methods.

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Appendix 1: Coefficients of Extended Load Iteration Scheme

Lynn, Commo, and Parker rigorously demonstrated in 2012 how to obtain a more general version of the traditional balance load iteration equation that uses both temperatures and pressures as additional independent variables of the gage outputs of a strain–gage balance (for a detailed description of their approach see Ref. [11], pp. 564–565). However, they derived their temperature and pressure–dependent load iteration equation using nomenclature that does not match the nomenclature used in Ref. [1] for the description of the traditional load iteration equation. In addition, their iteration equation, i.e., Eq. (A5) of Ref. [11], defines the load vector as a $1 \times n$ vector which is the transpose of the load vector definition that is used in Ref. [1]. Finally, Eq. (A5) of Ref. [11] has a “misprint” because the inverse matrix $(\hat{\beta}_1^*)^{-1}$ should be on the right–hand side of the square brackets. Therefore, an alternate derivation of *Lynn, Commo, and Parker’s* iteration equation was developed in the two appendices of the present paper so that the connection between their extended iteration equation and the traditional iteration equation used in Ref. [1] becomes more obvious. This new derivation is also intended to be used as an alternate reference for anyone who (i) already uses the iteration equation of Ref. [1] and (ii) wants to extend it to both temperatures and/or pressures.

For simplicity, the alternate derivation will omit any pressure–dependent regression model terms as the current paper focuses on the characterization of balance temperature effects. In addition, the regression model of a gage output from Ref. [1], i.e., Eq. (3.1.3), is used as a subset of the regression model that uses temperature–dependent regression model terms. The resulting extended regression model is described in detail in Eq. (8a) of the present paper.

In general, following the basic approach given in Ref. [1], it is convenient to express the balance calibration analysis algorithm using matrices (a column vector is simply considered to be a matrix with a single column). When needed, a subscript is added to a matrix in the present derivation that describes the matrix size. Then, assuming, e.g., that $\mathbf{A}$ is a matrix, the following convention is used:

$$\mathbf{A}_{rows \times columns}$$

A balance calibration has to be performed in order to find a mathematical relationship between applied balance loads, balance temperature differences, and measured responses. A single output of a strain–gage balance may be a function of all loads and the temperature difference that are applied to the balance. Then, assuming that the balance response (output) may be expressed as a function of the balance loads and the temperature difference, we get the following generalized regression model for the output of a gage with index “ k ” (see also Eq. (8a) for a more complete description of the regression model)

$$R(k) = c_0(k) + c_1(k) \cdot F(1) + c_2(k) \cdot F(2) + \cdots + c_\mu(k) \cdot F(n) \cdot \Delta T \quad (25)$$

where index “ μ ” equals the total number of coefficients that are used with the balance loads and the temperature difference. The unknown coefficients of the regression model above may be computed by applying global regression to data measured during a balance calibration. This global least squares problem can be solved if Eq. (25) is written as a matrix equation. Then, after introducing the row and column vectors

$$\mathbf{R}_{1 \times 1} = [R(k)] \quad (26a)$$

$$\mathbf{C}'_{1 \times (\mu+1)} = [c_0(k) \ c_1(k) \ c_2(k) \ \cdots \ c_\mu(k)] \quad (26b)$$

$$\mathbf{G}'_{(\mu+1) \times 1} = \begin{bmatrix} 1 \\ F(1) \\ F(2) \\ \vdots \\ F(n) \cdot \Delta T \end{bmatrix} \quad (26c)$$

we get the following matrix formulation of Eq. (25):

$$\mathbf{R}_{1 \times 1} = \mathbf{C}'_{1 \times (\mu+1)} \cdot \mathbf{G}'_{(\mu+1) \times 1} \quad (27)$$

Equation (27) may easily be extended from a single gage output to multiple gage outputs. Then, assuming that the balance has a total of “ n ” gages, we get:

$$\mathbf{R}_{n \times 1} = \mathbf{C}'_{n \times (\mu+1)} \cdot \mathbf{G}'_{(\mu+1) \times 1} \quad (28)$$

Finally, assuming that a total number of “ p ” calibration points were recorded, the left hand side of Eq. (28) becomes a matrix with “ p ” columns. We get:

$$\mathbf{R}_{n \times p} = \mathbf{C}'_{n \times (\mu+1)} \cdot \mathbf{G}'_{(\mu+1) \times p} \quad (29)$$

Equation (29) is the global least squares fit problem for given balance calibration data that is described by the measured loads, temperature difference, and the measured gage outputs. The solution of this regression problem can easily be derived from a general solution of a global least squares problem that is discussed in the literature (see, e.g., Ref. [14]). First, let us consider the global least squares problem for a single gage output in order to develop the desired general solution of Eq. (29). In this case, the left-hand side of Eq. (29) becomes a vector with a single row and “ p ” columns. We get:

$$\mathbf{R}_{1 \times p} = \mathbf{C}'_{1 \times (\mu+1)} \cdot \mathbf{G}'_{(\mu+1) \times p} \quad (30)$$

Now, taking the transpose of Eq. (30) and using the formula $(\mathbf{A} \cdot \mathbf{B})^T = \mathbf{B}^T \cdot \mathbf{A}^T$ (Ref. [15], p. 334), we get:

$$\mathbf{R}_{p \times 1}^T = \mathbf{G}'_{p \times (\mu+1)}^T \cdot \mathbf{C}'_{(\mu+1) \times 1}^T \quad (31)$$

It can be shown that Eq. (31) is identical with the general global least squares problem that is discussed in Ref. [14]. The solution of this general least squares problem is developed in Ref. [14] assuming that “ $\mathbf{A}$ ” is a rectangular matrix that contains a set of measurements, “ $\mathbf{b}$ ” is a vector that has to be fitted in the least squares sense, and “ $\mathbf{x}$ ” is the solution vector of the least squares problem. Then, the general least squares problem can be written as:

$$\mathbf{A}_{p \times (\mu+1)} \cdot \mathbf{x}_{(\mu+1) \times 1} = \mathbf{b}_{p \times 1} \quad (32a)$$

The solution of the least squares problem is given in Ref. [14] by the following matrix equation:

$$\mathbf{x}_{(\mu+1) \times 1} = \left[\mathbf{A}^T \cdot \mathbf{A} \right]_{(\mu+1) \times (\mu+1)}^{-1} \cdot \left[\mathbf{A}^T \cdot \mathbf{b} \right]_{(\mu+1) \times 1} \quad (32b)$$

Comparing Eqs. (31) and (32a), we see that

$$\mathbf{A} = \mathbf{G}'^T \quad (33a)$$

$$\mathbf{A}^T = \mathbf{G}' \quad (33b)$$

$$\mathbf{b} = \mathbf{R}^T \quad (33c)$$

$$\mathbf{x} = \mathbf{C}'^T \quad (33d)$$

Then, using Eq. (32b) to solve the least squares problem posed by Eq. (31) above, we get the following solution of the global least squares problem for a single gage output:

$$\mathbf{C}'_{(\mu+1) \times 1}^T = \left[\mathbf{G}' \cdot \mathbf{G}'^T \right]_{(\mu+1) \times (\mu+1)}^{-1} \cdot \left[\mathbf{G}' \cdot \mathbf{R}^T \right]_{(\mu+1) \times 1} \quad (34)$$

Taking the transpose of Eq. (34) and using again the formula $(\mathbf{A} \cdot \mathbf{B})^T = \mathbf{B}^T \cdot \mathbf{A}^T$, we get:

$$\mathbf{C}'_{1 \times (\mu+1)} = \mathbf{R}_{1 \times p} \cdot \mathbf{G}'_{p \times (\mu+1)}^T \cdot \left[\left[\mathbf{G}' \cdot \mathbf{G}'^T \right]_{(\mu+1) \times (\mu+1)}^{-1} \right]^T \quad (35a)$$

We also know that $(\mathbf{A}^{-1})^T = (\mathbf{A}^T)^{-1}$ (Ref. [15], p. 334). Then, we get

$$\mathbf{C}'_{1 \times (\mu+1)} = \mathbf{R}_{1 \times p} \cdot \mathbf{G}'_{p \times (\mu+1)}{}^T \cdot \left[\left[\mathbf{G}' \cdot \mathbf{G}'^T \right]^T \right]_{(\mu+1) \times (\mu+1)}^{-1} \quad (35b)$$

Using again the formula $(\mathbf{A} \cdot \mathbf{B})^T = \mathbf{B}^T \cdot \mathbf{A}^T$ and the formula $(\mathbf{A}^T)^T = \mathbf{A}$, we get:

$$\mathbf{C}'_{1 \times (\mu+1)} = \mathbf{R}_{1 \times p} \cdot \mathbf{G}'_{p \times (\mu+1)}{}^T \cdot \left[\mathbf{G}' \cdot \mathbf{G}'^T \right]_{(\mu+1) \times (\mu+1)}^{-1} \quad (35c)$$

Finally, after extending the solution of the least squares problem given Eq. (35c) from a single gage output to a total of “ n ” gage outputs, we get

$$\begin{aligned} \mathbf{C}'_{1 \times (\mu+1)} &\implies \mathbf{C}'_{n \times (\mu+1)} \\ \mathbf{R}_{1 \times p} &\implies \mathbf{R}_{n \times p} \end{aligned}$$

Then, Eq. (35c) becomes:

$$\mathbf{C}'_{n \times (\mu+1)} = \mathbf{R}_{n \times p} \cdot \mathbf{G}'_{p \times (\mu+1)}{}^T \cdot \left[\mathbf{G}' \cdot \mathbf{G}'^T \right]_{(\mu+1) \times (\mu+1)}^{-1} \quad (36)$$

Equation (36), i.e., the regression coefficient matrix $\mathbf{C}'$, is the solution of the global least squares problem that is defined in Eq. (29). It contains the regression coefficients of the fitted gage outputs. In the next step, a load iteration equation has to be developed using the regression coefficients described in Eq. (36) above as input in order to compute loads for a given set of gage outputs and the temperature difference. This process is described in detail in Appendix 2 of the paper.

Appendix 2: Iteration Equation of Extended Load Iteration Scheme

It is possible to develop an extended load iteration scheme from Eq. (36) of App. 1 that follows *Lynn, Commo, and Parker's* approach of 2012 (see Ref. [11], Eq. (A5)). The coefficients of the regression models of the gage outputs are known after the measured outputs are fitted as a function of the loads and the temperature difference using global regression (they are given by Eq. (36) of App. 1). During a wind tunnel test loads need to be determined using the measured gage outputs and the temperature difference of the balance. Therefore, as the regression coefficient matrix may contain coefficients that are, for example, related to products of loads, an iterative process has to be developed in order to relate measured responses and the temperature difference to predicted loads. It is first necessary to split the regression coefficient matrix $\mathbf{C}'$, i.e., Eq. (36), into two parts so that an iteration procedure can be developed:

$$\mathbf{C}'_{\mathbf{n} \times (\mu+1)} \implies \mathbf{a}_{\mathbf{n} \times 1}, \mathbf{C}_{\mathbf{n} \times \mu}$$

The coefficients of vector $\mathbf{a}$ and matrix $\mathbf{C}$ are simply given as:

$$a(i) = c_0(i) = C'(i, 1) \quad \text{for } 1 \leq i \leq n \quad (37a)$$

$$C(i, j) = C'(i, j+1) \quad \text{for } 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq \mu \quad (37b)$$

Similarly, load vector $\mathbf{G}'$ may be split into two corresponding parts:

$$\mathbf{G}'_{(\mu+1) \times 1} \implies \mathbf{I}_{1 \times 1}, \mathbf{G}_{\mu \times 1}$$

where $\mathbf{I}$ is the “identity” matrix with only a single row and column and $\mathbf{G}$ is a vector that has the form:

$$\mathbf{G}_{\mu \times 1} = \begin{bmatrix} F(1) \\ F(2) \\ \vdots \\ F(n) \cdot \Delta T \end{bmatrix} \quad (37c)$$

Then, by inspection, Eq. (28) of App. 1 may be expressed as follows for a total of “ n ” bridge outputs:

$$\begin{aligned} \mathbf{R}_{n \times 1} &= \mathbf{C}'_{\mathbf{n} \times (\mu+1)} \cdot \mathbf{G}'_{(\mu+1) \times 1} \\ &= \mathbf{a}_{\mathbf{n} \times 1} \cdot \mathbf{I}_{1 \times 1} + \mathbf{C}_{\mathbf{n} \times \mu} \cdot \mathbf{G}_{\mu \times 1} \\ &= \mathbf{a}_{\mathbf{n} \times 1} + \mathbf{C}_{\mathbf{n} \times \mu} \cdot \mathbf{G}_{\mu \times 1} \end{aligned} \quad (38)$$

It is convenient to introduce a “delta bridge output vector”. This vector is simply defined as the difference between the gage output and the intercept term, i.e., the regression coefficient for zero absolute load. We get:

$$\Delta \mathbf{R}_{n \times 1} = \mathbf{R}_{n \times 1} - \mathbf{a}_{n \times 1} \quad (39)$$

The “delta bridge output vector” is needed, because gage output differences are used to determine the loads. Then, combining Eqs. (38) and (39), we get the following equation for the “delta bridge output vector”:

$$\Delta \mathbf{R}_{n \times 1} = \mathbf{C}_{n \times \mu} \cdot \mathbf{G}_{\mu \times 1} \quad (40)$$

The coefficient matrix $\mathbf{C}$ may be split into three parts in order to develop a load iteration process. The first part includes the first n columns of matrix $\mathbf{C}$ (index range $\implies 1 \leq j \leq n$). The second part includes all remaining columns with the exception of the m' columns that are related to temperature-dependent terms (index range $\implies n+1 \leq j \leq \mu - m'$; see also Eq. (8d) in the body of the text). Finally, the third part

includes columns that are related to the temperature-dependent terms (index range $\implies \mu - m' + 1 \leq j \leq \mu$). Then, we get

$$\mathbf{C}_{n \times \mu} \implies \mathbf{C}_1_{n \times n}, \mathbf{C}_2_{n \times (\mu - n - m')}, \mathbf{C}_3_{n \times m'}$$

where coefficients of the linear load terms are saved in matrix $\mathbf{C}_1$. This matrix is defined as:

$$C_1(i, j) = C(i, j) \quad \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq n \quad (41a)$$

Similarly, all temperature-independent terms are saved in matrix $\mathbf{C}_2$ which is defined as:

$$C_2(i, j) = C(i, j + n) \quad \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq \mu - n - m' \quad (41b)$$

Finally, all temperature-dependent terms are saved in matrix $\mathbf{C}_3$ which is defined as:

$$C_3(i, j) = C(i, j + \mu - m') \quad \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m' \quad (41c)$$

It is also necessary to split the vector $\mathbf{G}$ into three corresponding parts. We get

$$\mathbf{G}_{\mu \times 1} \implies \mathbf{F}_{n \times 1}, \mathbf{H}_{(\mu - n - m') \times 1}, \mathbf{J}_{m' \times 1}$$

where

$$F(i, 1) = G(i, 1) \quad \text{for} \quad 1 \leq i \leq n \quad (42a)$$

$$H(i, 1) = G(i + n, 1) \quad \text{for} \quad 1 \leq i \leq \mu - n - m' \quad (42b)$$

$$J(i, 1) = G(i + \mu - m', 1) \quad \text{for} \quad 1 \leq i \leq m' \quad (42c)$$

Combining Eqs. (40), (41a), (41b), (42a), and (42b), we get for the “delta bridge outputs”:

$$\Delta \mathbf{R}_{n \times 1} = \mathbf{C}_1_{n \times n} \cdot \mathbf{F}_{n \times 1} + \mathbf{C}_2_{n \times (\mu - n - m')} \cdot \mathbf{H}_{(\mu - n - m') \times 1} + \mathbf{C}_3_{n \times m'} \cdot \mathbf{J}_{m' \times 1} \quad (43)$$

Then, after solving the matrix equation for the balance load vector $\mathbf{F}$, we get:

$$\mathbf{F}_{n \times 1} = \mathbf{C}_1^{-1}_{n \times n} \cdot \left[\Delta \mathbf{R}_{n \times 1} - \mathbf{C}_2_{n \times (\mu - n - m')} \cdot \mathbf{H}_{(\mu - n - m') \times 1} - \mathbf{C}_3_{n \times m'} \cdot \mathbf{J}_{m' \times 1} \right] \quad (44)$$

The load vector $\mathbf{F}$ can only be found using an iterative process, because (i) vector $\mathbf{H}$ is a function of $\mathbf{F}$ and (ii) vector $\mathbf{J}$ is a function of $\mathbf{F}$ and ΔT :

$$\mathbf{H} = \mathbf{H}\{\mathbf{F}\} \quad (45a)$$

$$\mathbf{J} = \mathbf{J}\{\mathbf{F}, \Delta T\} \quad (45b)$$

Therefore, introducing the iteration index ξ , we get the following iteration equation for the balance loads expressed as a function of the measured gage outputs and the temperature difference

$$\mathbf{F}_\xi = \mathbf{C}_1^{-1} \cdot \left[\Delta \mathbf{R} - \mathbf{C}_2 \cdot \mathbf{H}_{\xi-1} - \mathbf{C}_3 \cdot \mathbf{J}_{\xi-1} \right] \quad (46a)$$

$$\mathbf{H}_{\xi-1} = \mathbf{H}\{\mathbf{F}_{\xi-1}\} \quad (46b)$$

$$\mathbf{J}_{\xi-1} = \mathbf{J}\{\mathbf{F}_{\xi-1}, \Delta T\} \quad (46c)$$

where the initial guesses of vectors $\mathbf{H}$ and $\mathbf{J}$ are given as:

$$\mathbf{H}_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (46d)$$

$$\mathbf{J}_o = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (46e)$$

Equation (46a) above is the desired load iteration equation that uses both gage outputs and the temperature difference as input. As expected, it is the transpose of *Lynn, Commo, and Parker's* iteration equation who described the balance loads as a $1 \times n$ vector instead of an $n \times 1$ vector (see Ref. [11], p. 565, Eq. (A5); equation (A5) has a “misprint” because the matrix $(\hat{\beta}_1^*)^{-1}$ should be on the right-hand side of the square brackets). Sometimes, it is convenient to express Eq. (46a) in the following alternate form which more clearly shows its connection to the load iteration equation that is used in Ref. [1]:

$$\mathbf{F}_\xi = \underbrace{\mathbf{C}_1^{-1} \cdot \Delta \mathbf{R} - \mathbf{C}_1^{-1} \mathbf{C}_2 \cdot \mathbf{H}\{\mathbf{F}_{\xi-1}\}}_{\text{matches Eq. (3.3.7) of Ref. [1]}} - \underbrace{\mathbf{C}_1^{-1} \mathbf{C}_3 \cdot \mathbf{J}\{\mathbf{F}_{\xi-1}, \Delta T\}}_{\text{temperature-dependent part}} \quad (47)$$

It can be observed that the first two terms on the right-hand side of Eq. (47) above are identical with the iteration equation that is given in the literature (see Ref. [1], p. 19, Eq. (3.3.7)). The third term is the extension of the iteration equation that is needed so that the temperature difference can be included in the regression models of the gage outputs.

It is also important to mention that the convergence of the iterative process requires that the 2-Norm (also described as the *Euclidean Norm* or $\|_2$), of the first constant term on the right-hand side of Eq. (47) above is significantly larger than the 2-Norm of the two load- and temperature- dependent terms (the 2-Norm of a vector equals the square root of the sum of squares of its component values). These two requirements can be expressed as follows:

$$\| \mathbf{C}_1^{-1} \cdot \Delta \mathbf{R} \|_2 \gg \| \mathbf{C}_1^{-1} \mathbf{C}_2 \cdot \mathbf{H}\{\mathbf{F}_{\xi-1}\} \|_2 \quad (48a)$$

$$\| \mathbf{C}_1^{-1} \cdot \Delta \mathbf{R} \|_2 \gg \| \mathbf{C}_1^{-1} \mathbf{C}_3 \cdot \mathbf{J}\{\mathbf{F}_{\xi-1}, \Delta T\} \|_2 \quad (48b)$$

The convergence characteristics of Eq. (47) can more rigorously be tested for a given set of data reduction matrix coefficients by using the Lipschitz constant (for more detail see Ref. [16], App. B).