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Sensor Gain-Phase Errors Estimation for an Improved Data Model

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Abstract. Gain-phase errors calibration is important for systems using sensor array. In this paper, we consider an improved data model, and propose a low-complexity algorithm for sensor gain-phase errors estimation using an auxiliary source with known direction. The proposed algorithm has good performance in low signal-to-noise ratio (SNR). Moreover, it performs independently of phase errors. Computer simulations are presented to show the efficacy and performance of the proposed algorithm.

1. Introduction

Direction finding is an important task of radar, sonar and communication systems. Many high resolution algorithms have been proposed to estimate the direction-of-arrival (DOA), such as maximum like (ML) [1], multiple signal classification (MUSIC) [2], rotational invariance techniques (ESPRIT) [3]. In these methods, the good performance is obtained on the premise that the array manifold is exactly known. However, the performance may be degraded severely due to the sensor gain-phase errors. In this paper, we consider an improved data model, which is different from the conventional data model. In the conventional data model, the influence of gain-phase errors on channel noise is ignored. In the case of low SNR level, it is reasonable. However, in the improved data model, the influence of gain-phase errors on channel noise should be considered. In [4], the authors propose a calibration algorithm for gain-phase errors based on the improved data model. The method only needs some simple matrix operations and no prior information about the direction of auxiliary source is required, but it is applicable only to uniform linear array. In [5], a performance analysis of MUSIC algorithm in the presence of gain-phase errors based on the improved data model is studied, and a quadratic equation for solving the SNR resolution threshold is presented. A joint iteration method is proposed for the improved data model to compensate gain-phase errors in [6], but many computations and long convergent time are required. In [7], a low-complexity algorithm is proposed based on the improved data model, which estimates gain-phase errors by using the first-order statistical property of receiving data with an auxiliary source in known direction. Few computations is required in the method, however, it is only suitable to the case that SNR is high (SNR >10dB) while the improved data model is more suitable to be applied to the case of low SNR.

Inspired by [7], we propose an algorithm to estimate gain-phase errors based on the improved data model using an auxiliary source in known direction. In [7], the gain-phase errors are obtained by using first-order statistical property (observation data). However, in our algorithm, we use second-order statistical property (covariance matrix of observation data). In this way, the contribution of noise in the data covariance matrix is taken into account to extend the performance assessment to low SNR level. In addition, the estimation accuracy of the algorithm is not affected by phase errors.

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Notation: In this paper, superscripts $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ represent the conjugate and conjugate transpose operations, respectively. $E\{\cdot\}$ stands for mathematical expectation and $diag\{\cdot\}$ denotes a diagonal matrix.

2. Data Model

We consider a planar array composed of M omni-directional antenna elements. Assume that these is one far-field auxiliary source at direction (θ,γ) impinging on the array. θ and γ are the azimuth and elevation angle of the auxiliary source, respectively. The array manifold vector $\mathbf{a}(\theta,\gamma)$ at direction (θ,γ) can be expressed as

$$\mathbf{a}(\theta,\gamma)=[a_1,a_2,\ldots,a_M] \quad (1)$$

In the presence of gain-phsae errors, the gain-phase errors matrix $\mathbf{\Gamma}$ is introduced to modify the array manifold vector as

$$\tilde{\mathbf{a}}(\theta,\gamma)=\mathbf{\Gamma}\mathbf{a}(\theta,\gamma) \quad (2)$$

Here, $\mathbf{\Gamma}$ is a $M\times M$ diagonal matrix given by

$$\begin{aligned}\mathbf{\Gamma}&=diag\{\Gamma_1,\Gamma_2,\ldots,\Gamma_M\}\\&=diag\{\rho_1 e^{j\varphi_1},\rho_2 e^{j\varphi_2},\ldots,\rho_M e^{j\varphi_M}\}\end{aligned} \quad (3)$$

where ρ_m and φ_m for $m=1,2,\ldots,M$ are the gain errors and phase errors of the m-th sensor. We take the first sensor as the reference sensor, thus we have $a_1=1$, $\rho_1=1$ and $\varphi_1=0$. The conventional data model is expressed as

$$\mathbf{X}_c(t)=\mathbf{\Gamma}\mathbf{a}(\theta,\gamma)s(t)+\mathbf{n}(t) \quad (4)$$

where $\mathbf{n}(t)$ is the vector of zero-mean white additive noise, which is uncorrelated with $s(t)$. However, in low SNR level, the influence of gain-phase errors on noise can not be ignored. As [4-7], the observation data of the improved data model is given by

$$\mathbf{X}_i(t)=\mathbf{\Gamma}(\mathbf{a}(\theta,\gamma)s(t)+\mathbf{n}(t)) \quad (5)$$

The covariance matrix of $\mathbf{X}_i(t)$ can be written as

$$\mathbf{R}_x=E\{\mathbf{X}_i(t)\mathbf{X}_i^{\mathrm{H}}(t)\}=\mathbf{\Gamma}\mathbf{a}(\theta,\gamma)\sigma_s^2\mathbf{a}(\theta,\gamma)\mathbf{\Gamma}^{\mathrm{H}}+\sigma_n^2\mathbf{\Gamma}\mathbf{\Gamma}^{\mathrm{H}} \quad (6)$$

where σ_s^2 is the power of calibration signal and σ_n^2 is the noise power. The problem we focus on is to estimate sensor gain-phase errors matrix $\mathbf{\Gamma}$ when the incident angle and the observation data are known.

3. The Proposed Algorithm

To make use of the second-order statistical property (covariance matrix of observation data), we write (6) as

$$\begin{aligned}\mathbf{R}_x&=E\{\mathbf{X}_i(t)\mathbf{X}_i^{\mathrm{H}}(t)\}=\mathbf{\Gamma}\mathbf{a}(\theta,\gamma)\sigma_s^2\mathbf{a}^{\mathrm{H}}(\theta,\gamma)\mathbf{\Gamma}^{\mathrm{H}}+\sigma_s^2\mathbf{\Gamma}\mathbf{\Gamma}^{\mathrm{H}}\\&=\sigma_s^2\begin{bmatrix}\Gamma_1\Gamma_1^*a_1a_1^* & \Gamma_1\Gamma_2^*a_1a_2^* & \cdots & \Gamma_1\Gamma_M^*a_1a_M^*\\ \Gamma_2\Gamma_1^*a_2a_1^* & \Gamma_2\Gamma_2^*a_2a_2^* & \cdots & \Gamma_2\Gamma_M^*a_2a_M^*\\ \vdots & \vdots & \ddots & \vdots\\ \Gamma_M\Gamma_1^*a_Ma_1^* & \Gamma_M\Gamma_2^*a_Ma_2^* & \cdots & \Gamma_M\Gamma_M^*a_Ma_M^*\end{bmatrix}+\sigma_n^2\begin{bmatrix}\Gamma_1\Gamma_1^* & & & \\ & \Gamma_2\Gamma_2^* & & \\ & & \ddots & \\ & & & \Gamma_2\Gamma_2^*\end{bmatrix}\end{aligned} \quad (7)$$

The elements in the main diagonal of $\mathbf{R}_x$ are given by

$$[\mathbf{R}_x]_{mm} = \sigma_s^2 \mathbf{\Gamma}_m \mathbf{\Gamma}_m^* a_m a_m^* + \sigma_n^2 \mathbf{\Gamma}_m \mathbf{\Gamma}_m^* \tag{8}$$

Respecting the first sensor is the reference sensor with $a_1 = 1$, $\rho_1 = 1$ and $\varphi_1 = 0$, we have

$$\begin{aligned} \frac{[\mathbf{R}_x]_{mm}}{[\mathbf{R}_x]_{11}} &= \frac{\sigma_s^2 \Gamma_m \Gamma_m^* a_m a_m^* + \sigma_n^2 \Gamma_m \Gamma_m^*}{\sigma_s^2 \Gamma_1 \Gamma_1^* a_1 a_1^* + \sigma_n^2 \Gamma_1 \Gamma_1^*} \\ &= \frac{\sigma_s^2 \rho_m^2 + \sigma_n^2 \rho_m^2}{\sigma_s^2 + \sigma_n^2} \\ &= \rho_m^2 \end{aligned} \tag{9}$$

Therefore, the gain error of the m-th sensor is obtained as

$$\rho_m = \sqrt{\frac{[R_x]_{mm}}{[R_x]_{11}}} \, (m = 1, 2, \ldots, M) \tag{10}$$

Observe the first column of the covariance matrix in (7), we have

$$\begin{aligned} [R_x]_{m1} &= \sigma_s^2 \Gamma_m \Gamma_1^* a_m a_1^* \\ &= \sigma_s^2 \rho_m e^{j\varphi_m} a_m \, (m = 2, 3, \ldots M) \end{aligned} \tag{11}$$

Thus, the phase error of the m-th sensor is obtained as

$$\varphi_m = angle\left(\frac{[\mathbf{R}_x]_{m1}}{a_m}\right)(m = 2, 3, \ldots, M) \tag{12}$$

The algorithm is quite simple. Unlike the method in [7], the derivational process of the proposed algorithm does not require approximation. Therefore, regarding the estimation accuracy, the proposed algorithm has an advantage over the EAIDM in [7]. Respecting (7) to (12), we can find the proposed algorithm performs independently of phase errors. No matter how large the phase errors are, the estimation accuracy does not affected by them.

In summary, the proposed algorithm consists of the following steps:

Step1: Obtain the observation data $\mathbf{X}_i(t)$ and compute the covariance matrix $\mathbf{R}_x = E\{\mathbf{X}_i(t)\mathbf{X}_i^{\mathrm{H}}(t)\}$ in (6).

Step2: Use the elements in the main diagonal of $\mathbf{R}_x$ to estimate the gain errors in (10).

Step3: Use the elements in the first column of $\mathbf{R}_x$ to estimate the phase errors in (12).

4. Simulation Results

In this section, we present some computer simulations to show the performance of the proposed algorithm. Being the same as [7], [8] and [9], we generate the gain-phase errors as

$$\begin{cases} \rho_m = 1 + \sqrt{12}\sigma_\rho \kappa_m \\ \varphi_m = \sqrt{12}\sigma_\varphi \zeta_m \end{cases} \tag{13}$$

where σ_ρ and σ_φ are the standard deviations of gain errors and phase errors, κ_m and ζ_m are independent random variables distributed uniformly over $[-0.5, 0.5]$. The noise is a complex Gaussian process with zero mean.

4.1. Performance Comparison with EAIDM in [7]

In this section, we show the RMSE curves to compare the performance with EAIDM in [7]. We consider a uniform circular (UCA) composed of 12 sensor elements with radius $r = \lambda / (4\sin(\pi/12))$, the calibration source is placed at direction $(45°, 45°)$. The standard deviation of gain errors σ_φ is

0.9064 and the number of snapshots is 500. Based on 2000 Monte Carlo experiments, the RMSE curves versus SNR are shown in Fig. 1. The gain-phase errors are first averaged over 12 sensors and then averaged over 2000 Monte Carlo experiments.

It can be seen that the estimation performance of the proposed algorithm is better than EAIDM, especially when SNR is lower than 10dB.

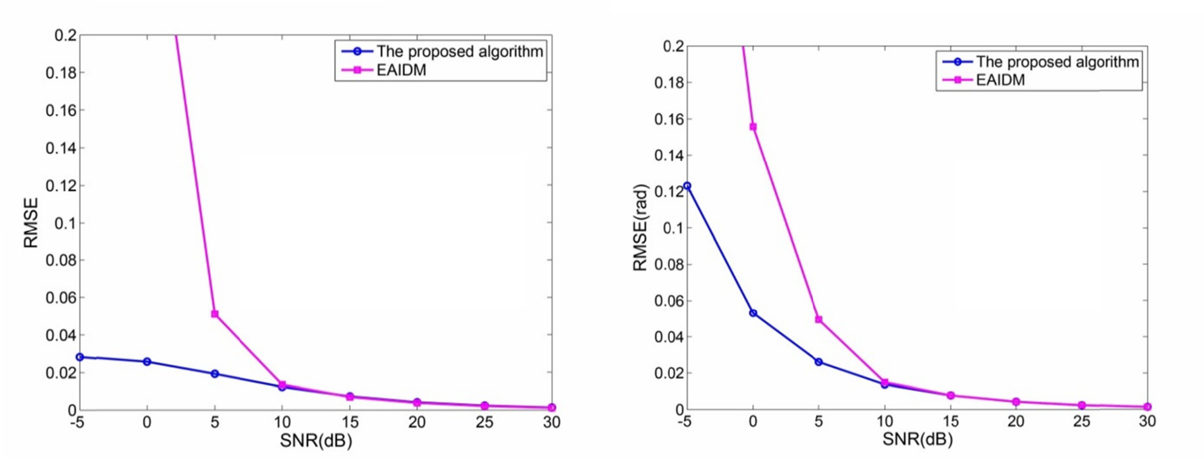

Figure 1. (a) RMSE curves of gain errors estimates versus SNR.
(b) RMSE curves of phase errors estimates versus SNR.

4.2. Influence of Phase Errors

In this section, we examine if the proposed algorithm performs independently of phase errors. We consider a uniform circular array composed of 8 sensor elements with radius $r = \lambda / (4\sin(\pi / 8))$, the calibration source is placed at direction $(45°, 45°)$. The standard deviation of gain errors σ_ρ is 0.3 and the number of snapshots is 100. Based on 2000 Monte Carlo experiments, the RMSE curves are shown in Fig. 2. The gain-phase errors are first averaged over the 8 sensors and then averaged over the 2000 Monte Carlo experiments.

It can be seen that the RMSEs of gain errors and phase errors are changeless when the standard deviation of phase errors change from 01. to 0.9.

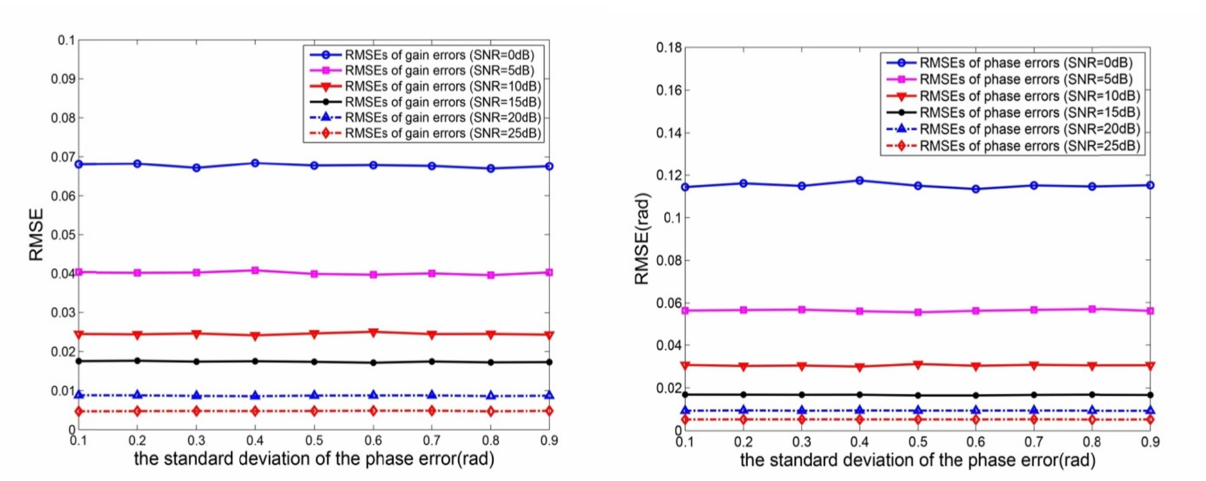

Figure 2. (a) RMSE curves of gain errors estimates versus the standard deviation of phase errors σ_φ when SNR is 0, 5, 10, 15, 20, 25dB.
(b) RMSE curves of phase errors estimates versus the standard deviation of phase errors σ_φ when SNR is 0, 5, 10, 15, 20, 25dB.

5. Conclusion
In this paper, we present an improved data model in the presence of sensor gain-phase errors. Based on this improved data model, we proposed a low-complexity algorithm, which uses the second-order statistical property (covariance matrix of observation data) to estimate the gain errors and phase errors. The proposed algorithm has good performance in low SNR level and performs independently of phase errors.

6. Acknowledge
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7. References
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