

Accelerometer Optimization Placement Using Improved Particle Swarm Optimization Algorithm Based on Structural Damage Identification

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Abstract. In this paper, an Accelerometer optimization arrangement method based on Structural Damage Identification (SDI) is proposed. Firstly, the method constructs a fitness function according to the selection of the optimal measuring point of the displacement mode based on SDI, which selects a group of points with the largest displacement. Secondly, the efficient intelligent optimization is adopted. The algorithm, the improved particle swarm optimization algorithm, searches for the optimal placement position of the sensor. Finally, the feasibility and effectiveness of the proposed method for sensor optimal placement and damage identification are verified by an example of the wheel frame structure. Simulation and experimental results demonstrate the effectiveness of the proposed method.

1. Introduction

When a mechanical structure needs to be monitored, people always want to use sensors as few as possible to get as much machine information as possible. Finding the best sensor placement can be the basis for condition monitoring of the manufacturing machine, which can reduce the number of sensors and thus reduce costs.

Accelerometers play an important role in vibration testing, data acquisition and health monitoring of engineering structures. Reasonable sensor arrangement is directly related to the sensor's sensing efficiency and the costs. Damage identification is the core of structural health monitoring research. The analysis data mainly comes from structural response information obtained by sensors installed in various parts of the structure.[1-2]

Due to the economic and structural conditions and other constraints, the installation of sensors in all structural degrees of freedom is impossible. In recent years, extensive research has been conducted on the optimal placement of sensors for structural damage identification and many methods have been proposed.

In order to use a limited number of sensors to detect damage information, Cobb[3] et al. proposed a sensor placement method based on structural eigenvector sensitivity analysis. Shi[4] chose the large degree of freedom that contributes to the rank of the Fisher information matrix as the location of sensor. The damage position of the structure is determined by using modal information of limited measuring points and the method of multiple damage location assurance criteria. On the basis of Shi's work, Zeng Guohua[5] proposed a correction method for the optimal placement of sensors for structural damage identification, taking into account the difference in noise level of each measurement data. Bruggi[6] et al used topology optimization method to the best placement of the sensor, and identified the damage of the flexible plate structure. Moore[7] combined with genetic algorithm and the



steepest descent optimization method to optimize the sensor configuration, and through the experimental data to find the crack location of aluminum plate. Beygzadeh S[8] et al. proposed optimal sensor placement for damage detection based on a geometrical viewpoint. Although the above methods have been used to study the optimal placement of sensors for damage identification, some achievements have been achieved. However, there are still too many complicated sensor layout theories and low optimization efficiency.

2. Basic Theory of Structural Damage Identification

From the structural dynamics, we can get that if an n-degree-of-freedom structural system is subjected to external load, its kinetic equation is:

$$M\ddot{\delta}(t) + C\dot{\delta}(t) + K\delta(t) = P(t) \quad (1)$$

Where: M, C and K are $n \times n$ order mass matrix, damping matrix and stiffness matrix; $\ddot{\delta}(t)$, $\dot{\delta}(t)$ and $\delta(t)$ are the n-dimensional acceleration, velocity and displacement column vectors respectively; $P(t)$ is the external load vector.

If you ignore the impact of damping when a small deflection occurs in the stiffness, the characteristic equation can be expressed as:

$$(K - \lambda M)\Phi = 0 \quad (2)$$

Where: λ and Φ respectively represent the eigenvalue of the structure and its mode shape matrix.

Under normal circumstances, the damage caused only to reduce the stiffness of the structure, while the quality of the structure remains unchanged. According to the perturbation theory, the characteristic equation of the damaged structure is:

$$[(K - \Delta K) - (\lambda - \Delta\lambda)M](\Phi - \Delta\Phi) = 0 \quad (3)$$

Where: ΔK , $\Delta\lambda$, $\Delta\Phi$ represent the small changes in stiffness, eigenvalues and mode shapes caused by damage, respectively.

Ignoring the effects of higher order terms, expand and organize the Eq. (3):

$$(K - \lambda M)\Phi - (K - \lambda M)\Delta\Phi - (\Delta K - \Delta\lambda M)\Phi = 0 \quad (4)$$

Contrast formula (2), formula (4), the first item is the structure of the characteristic equation without damage. Right both sides of Eq. (4) by the same time by Φ_i^T , were:

$$\Phi_i^T(K - \lambda_i M)\Delta\Phi + \Phi_i^T(\Delta K - \Delta\lambda M)\Phi = 0 \quad (5)$$

Because stiffness and mass matrix are symmetric square matrix, that is $K^T = K$, $M^T = M$, so the transpose of Eq. (2) shows that the first term of Eq. (5) is zero. Assuming that the mode shape satisfies the normalized condition of mass, $\Phi_i^T M \Phi = 1$, Eq. (5) can be simplified as:

$$\Phi_i^T \Delta K \Phi = \Delta\lambda \quad (6)$$

Eq. (6) expresses the relationship between the change of structural stiffness matrix before and after damage and the measured modal parameters of the damaged structure.

Since both the vibration-based and the finite element model-based damage identification techniques require the measurement of vibration modes, the finite element model has the same degree of freedom, so Eq. (6) must satisfy the requirement of measuring the complete vibration mode.

Structural damage causes local changes in stiffness. In the finite element model, the proportional damage model can be used to represent the change of the stiffness matrix. That is, ΔK can be expressed as the sum of the product of each element stiffness matrix and the damage factor. Specifically expressed as:

$$\Delta K = \sum_{j=1}^{n_e} \alpha_j K_j, \quad 0 \leq \alpha_j \leq 1 \quad (7)$$

Where: K_j , α_j represent the stiffness matrix of the j-th unit and the corresponding damage coefficient; n_e is the total number of units. The damage coefficient α_j is the parameter to be identified

defined at the unit level. The size of the value can not only indicate the damage degree of the unit, but also directly indicate the damage position of the unit. Therefore, the damage coefficient α_j can be used as the structural damage recognition factor to identify the structure damage location and extent. When $\alpha_j = 0$, it indicates that the unit is not damaged. When $\alpha_j = 1$, it indicates that the unit has been completely damaged. When α_j is between 0 and 1, it indicates that the unit is damaged to a certain extent.

Substituting Eq. (7) into Eq. (6), Finishing equations can be:

$$\sum_{j=1}^{n_e} \varphi_i^T K_j \varphi_i \alpha_j = \Delta \lambda_i \quad (8)$$

Eq. (8) is the governing equation with unknown damage factor α_j . Where i can range from 1 to r , where r is the order of the measured mode shapes.

3. Particle Swarm Optimization(PSO)

To optimize the placement of sensors, it is necessary to determine a reasonable and can meet the design requirements of the optimization guidelines and the selection of simple and efficient optimization methods. PSO [10-12] simulates bird flocking Society uses three simple rules to manipulate individual particles: (1)fly away from the nearest individual to avoid collisions; (2) fly to the target; (3) fly to the center of the population.

Common PSO algorithm expression is as follows:

$$v_{id}(t+1) = \omega v_{id}(t) + c_1 r_1 (p_{id} - x_{id}(t)) + c_2 r_2 (p_{gd} - x_{id}(t)) \quad (9)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (10)$$

When $v_{id} > v_{\max}$, take $v_{id} = v_{\max}$; then $v_{id} < -v_{\max}$, take $v_{id} = -v_{\max}$;

Where: a is the number of particles that make up the group; $d = 1, 2, \dots, D$ is the dimension of the target search space; c_1 and c_2 are non-negative constants; r_1 and r_2 are uniformly distributed random numbers subject to $[0, 1]$; $x_{id}(t)$ is the current position of the first particle; p_{id} is the optimal position searched by the first particle so far; p_{gd} for the entire particle group to search the best location; v_{id} is the current velocity of the first particle; v_{\max} is the maximum speed limit, non-negative number; t are the current evolution algebra respectively; ω is inertia weight, usually take

$$\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min})t/t_{\max} \quad (11)$$

Particle swarm optimization algorithm can be used to process discrete variables. The improved algorithm is often called discrete multi-objective optimization based on the particle swarm optimization [13].

4. Sensor Optimization Layout Methods

A good sensor optimization arrangement should make the measurement results most sensitive to changes in parameters and provide reliable information on potential damage to the structure. [14] The optimal placement of sensors is essentially a special type of traveling salesman problem in which a given number of sensors are placed at optimal positions in order to obtain as much structural dynamic characteristic information and response data as possible.

4.1. Optimization of Accelerometers Based on Particle Swarm Optimization

The problem of selecting the location of a measuring point is essentially an optimization problem that satisfies a criterion or goal. Figure 1 shows the structure of the application of PSO in sensor layout optimization. First, the machine structure is analysed using finite element analysis, and all possible measurement points can be determined based on shape and application. From the results of the above steps, all vibration displacement patterns can be calculated. Then, enter all of this data into the PSO to

find the best sensor location that can be sent to the design and management centre. Depending on the results, the staff can make it is easy to monitor the machine with high precision and correctness.

4.2. The Fitness Function

Modal analysis is a very important method of condition monitoring. Equipment failures such as cracks, shaft looseness and fatigue are often accompanied by changes in physical parameters such as natural frequency, modal damping, vibration mode and frequency response function. Researchers can diagnose faults based on these changes. The machine’s vibration is supposed to be an n degree of freedom linear time-invariant system which differential function can be expressed by the general formula [16]:

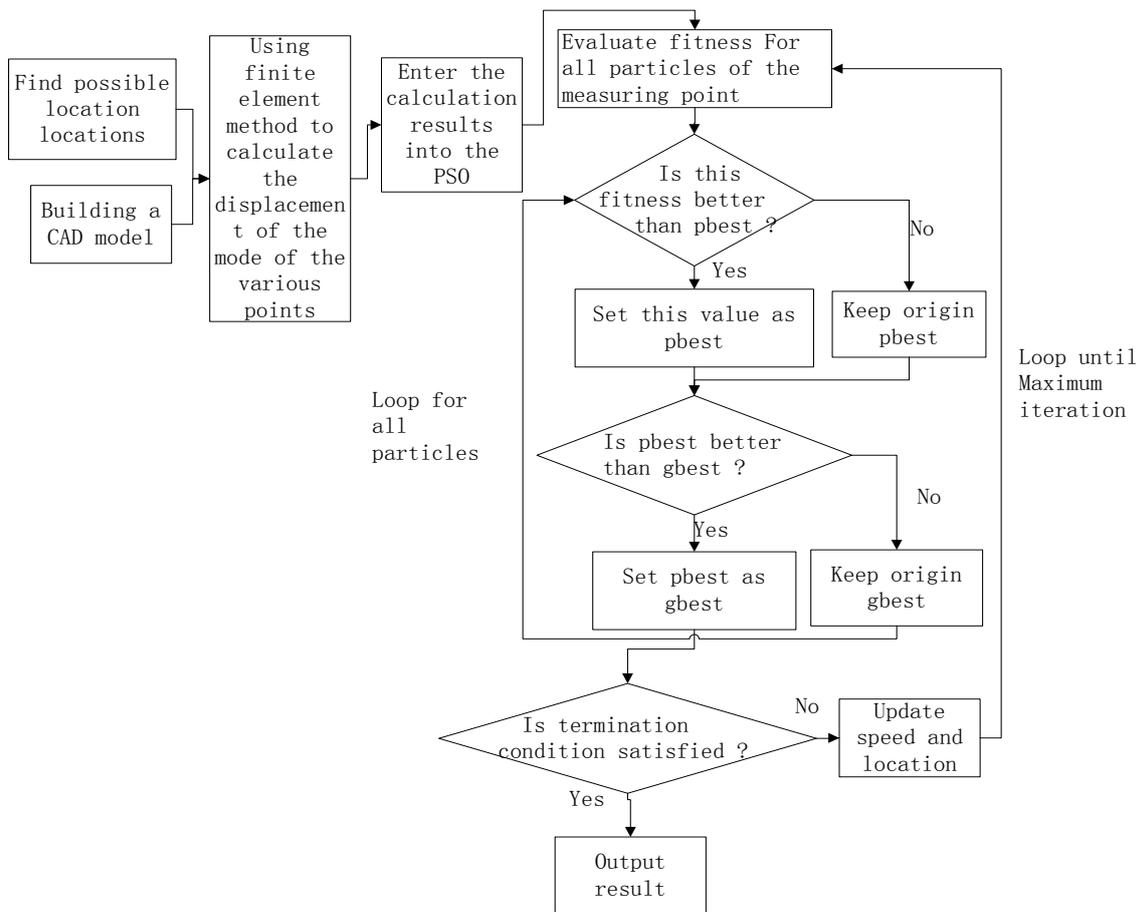


Figure 1. PSO acceleration sensor placement process

$$M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = f(t) \tag{12}$$

Where: M, C, and K are mechanical system mass, damping, and stiffness matrices, respectively, which are $n \times n$ matrices. $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ are the n-order response vectors of the system acceleration, velocity and displacement, respectively. $f(t)$ is the excitation force vector. Then obtain the frequency displacement response function by Fourier transform and set $x(t) = xe^{j\omega t}$ to:

$$x(\omega) = H(\omega)F(\omega) \tag{13}$$

$H(\omega)$ is the frequency response function matrix of displacement.

$$H_{ij}(w) = \sum_{r=1}^n \frac{\phi_{jr} \phi_{ir}}{-\omega^2 M_r + j\omega C_r + K_r} \quad (14)$$

Where M_r , C_r , K_r and ϕ_r represent mass, damping, stiffness and per-order vibration mode vectors respectively. Eq. (30) shows the relationship between the transfer function and the modal parameters. For a machine, the value of $(-\omega^2 M_r + j\omega C_r + K_r)$ is always the same because it depends only on the frequency and Damping ratio. Therefore, the value of the frequency response function depends on the vibration mode vector of the points i and j .

Let $\phi = [\phi_1, \phi_2, \dots, \phi_n]$ (taking n modes) be a displacement mode, where ϕ_i is an N -dimensional vector, where N is the degree of freedom of the machine structure. Let m be the number of sensors (or the number of measurement points) mounted on the mechanical structure, and $o = N - m$ be the non-measurement point. According to the previous description, the fitness function can be:

$$f_1 = \sum_{i=1}^n \sum_{j=1}^n \left| \sum_{r \in o} \phi_{ri} \phi_{rj} \right| \quad (15)$$

Where ϕ_{ri} denotes the r^{th} component of the j^{th} vibration mode, and $r \in o$ denotes that all calculation vectors are non-measurement points. Comparing equation (14) with Eq. (15), the key task is to find the minimum of the optimal distribution of the sensor Eq. (15). Therefore, the optimal position of the sensor is found by calculating the fitness function. In order to optimize the sensor layout, considering the characteristics of wheel frame structure, acceleration sensors are mainly used in the process of wheel frame system monitoring. In this paper, f_1 is the fitness function, the fitness value is the smaller the better.

The initial placement of the sensor is shown in Figure 2. The finite element mesh of the CAE model of the wheel carrier system is shown in Figure 3.

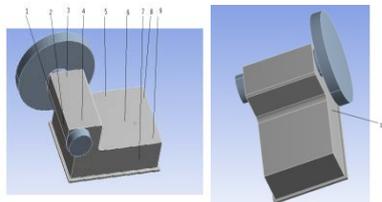


Figure 2. Sensor Layout The initial measurement point selection diagram

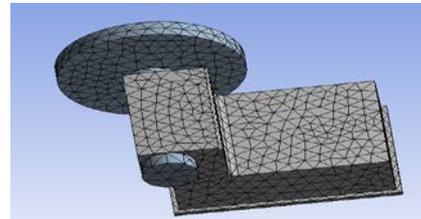


Figure 3. Wheel carrier system CAE model finite element meshing schematic

5. Application of PSO in Accelerometer Placement Optimization

To optimize In order to verify the effectiveness of the method proposed in this paper, the grinding wheel frame of a large crankshaft grinder was selected for analysis. Select the possible 10 measurement points for analysis. The 3D solid model of the wheel frame was built using 3D software Pro/E, and finite element analysis and modal analysis were performed. The study calculated the total 10th-order natural frequency and obtained 10 vibrational modal shapes for the wheel frame.

The analysis and calculation results are shown in Table 1, 2 and 3. Figure 4 is the fitness curve with the number of sensors.

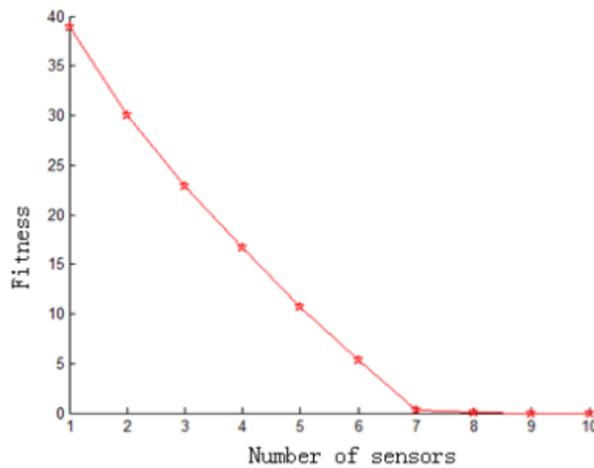


Figure 4. The fitness value varies with the number of sensors

Table 1. Total modal displacement of measuring point (mm)

| Modal order Point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 0.374556 | 0.19026 | 0.156093 | 0.134211 | 0.097629 | 0.42588 | 0.683802 | 0.222348 | 0.093135 | 0.073689 |
| 2 | 0.342678 | 0.202986 | 0.128226 | 0.131628 | 0.092526 | 0.387387 | 0.646716 | 0.207753 | 0.085008 | 0.085701 |
| 3 | 0.368277 | 0.458829 | 0.270858 | 0.224574 | 0.091455 | 0.814191 | 0.640794 | 0.24213 | 0.085239 | 0.071064 |
| 4 | 0.363993 | 0.277704 | 0.174867 | 0.164031 | 0.095508 | 0.543900 | 0.670635 | 0.231756 | 0.089922 | 0.054957 |
| 5 | 0.656838 | 0.087171 | 0.206115 | 0.11004 | 0.099897 | 0.238791 | 0.33642 | 0.393288 | 0.095214 | 0.257481 |
| 6 | 0.051093 | 0.047166 | 0.020945 | 0.028203 | 0.013073 | 0.115878 | 0.075495 | 0.065499 | 0.006359 | 0.066108 |
| 7 | 0.52857 | 0.020042 | 0.181041 | 0.066465 | 0.076566 | 0.219618 | 0.300468 | 0.402528 | 0.190176 | 0.267792 |
| 8 | 0.01231 | 0.003864 | 0.001453 | 0.00353 | 0.004238 | 0.025662 | 0.052605 | 0.020614 | 0.00129 | 0.009278 |
| 9 | 1.25E-09 | 5.49E-10 | 5.8E-10 | 1.6E-10 | 4.22E-11 | 4.1E-10 | 9.03E-10 | 1.07E-10 | 5.39E-11 | 1.87E-10 |
| 10 | 0.67095 | 0.127722 | 0.216993 | 0.134715 | 0.112749 | 0.435477 | 0.333102 | 0.51408 | 0.090867 | 0.354942 |

Table 2. Natural frequency of each step (Hz)

| Modal order | Natural frequency | Modal order | Natural frequency |
|-------------|-------------------|-------------|-------------------|
| 1 | 138.29 | 6 | 388.56 |
| 2 | 162.25 | 7 | 434.63 |
| 3 | 181.06 | 8 | 586.52 |
| 4 | 233.96 | 9 | 627.04 |
| 5 | 250.56 | 10 | 630.72 |

Table 3. Sensor Arrangement Results

| Measuring points | Fitness | Sensor position | Measuring points | Fitness | Sensor position |
|------------------|----------|-----------------|------------------|---------------|----------------------|
| 1 | 38.90521 | 3 | 6 | 5.335314 | 1 2 3 4 5 10 |
| 2 | 29.95556 | 3 10 | 7 | 0.2581050 | 1 2 3 4 5 7 10 |
| 3 | 22.84121 | 3 4 10 | 8 | 1.818260E-002 | 1 2 3 4 5 6 7 10 |
| 4 | 16.68459 | 3 4 5 10 | 9 | 1.801762E-017 | 1 2 3 4 5 6 7 8 10 |
| 5 | 10.67423 | 1 3 4 5 10 | 10 | 0 | 1 2 3 4 5 6 7 8 9 10 |

Taking into account the fitness value and the sensitivity of each measuring point to the signal, it can be seen from Figure 4. 8 that the optimal measuring point number is 7, since the effect of adding

measuring points is not obvious. Considering the symmetry and economy, The optimal location for selecting 6 sensors is {1 2 3 4 5 10} or {1 2 3 4 5 7}.

6. Experimental Study

According to the measuring point arrangement of the above research, six acceleration sensors are used to monitor the condition of the grinding wheel frame of the large crankshaft grinding machine. The experimental data was collected using the INV3060 system of China Orient Institute of Noise & Vibration.

Figure 7 is a signal in which the acceleration sensor at the position 1 is in a normal state, and Figure 8 is a signal at which the acceleration sensor at the position 1 is subjected to grinding flutter. In summary, the sensor arranged by the sensor arrangement method proposed in the present invention can obtain a better recognition effect.



Figure 5. The large crankshaft grinding machine



Figure 6. The grinding wheel frame of the large crankshaft grinding machine.

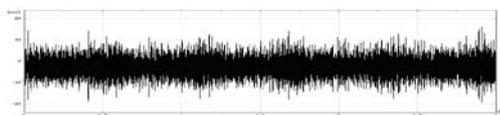


Figure 7. Normal state of acceleration signal

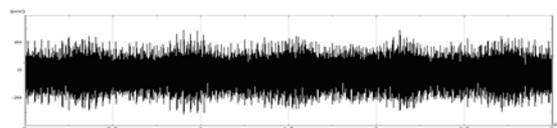


Figure 8. Acceleration signal when grinding flutter

7. Conclusion

Aiming at the problem of optimal placement of accelerometers, this paper uses the improved particle swarm optimization algorithm to solve multi-objective optimization, obtains a better sensor layout scheme, and conducts case study on the monitoring of the grinding wheel frame of large crankshaft grinding machine, and obtains better monitoring results.

1. Combined with the damage condition of the mechanical structure, the modal test effect of the health monitoring can be maximized by adding a small number of sensors at a reasonable position.

2. The selection of the optimal measuring point of the displacement mode of the mechanical equipment is transformed into a multi-objective optimization problem of finding the position and number of a group of points with large displacement.

3. The multi-objective optimization algorithm adapts to different number of sensors, can calculate the relative number of sensors and their arrangement position, and provide good technical support for the acceleration sensor arrangement on the large crank wheel frame to meet the needs of long-term health monitoring.

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