

The Limiting Values of Moments And Deformations Ratio in Strength Calculations Using Specified Material Diagrams

N I Karpenko¹, V A Eryshev², V I Rimshin³

¹ Research Institute of building physics of RAASN, 127238, Lokomotivniy proezd, 21, Moscow, Russia

² Togliatti State University, 445020, Belorusskaya str., 14, Tolyatti, Russia

³ Corresponding member of Russian Academy of Architecture and Construction Sciences, Doctor of Technical Sciences, Professor, Head of the Institute of urban development of Russian Academy of Architecture and Construction Sciences, Moscow 107031, Russia

E-mail: niisf_lab9@mail.ru, gsx@tltsu.ru, v.rimshin@niisf.ru

Abstract. The authors compare the results of calculating bent reinforced concrete elements strength and deformations using specified concrete and reinforcement diagrams with different outlines and analytical relationships between deformations and stresses with experimental data in the article. In order to improve the deformation calculation methods, the energy laws of the deformation mechanics of solids are used in the proposed technique. From the energy standpoint, concrete and reinforcement in the section under load are stored with potential energy, according to the diagram outline used in the calculation; a stress diagram is formed in the compressed zone concrete. A connection is established between the efforts in concrete and the work spent on the deformations of concrete sample (prism, cylinder) under the load numerically equal to the area used in calculating the specified diagram. Flat cross-section hypothesis is applied in the derivation of the resolving balance equations. The condition of balanced forces in an element section is checked by the method of successive approximation. Element curvature is assumed as a variable approximation parameter.

1. Introduction

Calculation the strength of reinforced concrete elements using the nonlinear deformation model is carried out based on concrete axial compression diagrams, reinforcement expansion and flat cross-section hypothesis. There are different types of diagrams in domestic and foreign normative documents [1,2,3], which are approximating the experimental curves of concrete deformation, steel reinforcement, as curvilinear, simplified piecewise-linear, corresponding to mechanical materials properties. Discussions continue on the establishment of an analytic connection between strains and stresses [4, 5, 6]. The variety of specified material diagrams and the ambiguous values of its limiting parameters necessitate the evaluation of limiting moment's values and its correspondence to experimental data. Deformation models developed on complex modes of loading structures, when the load varies according to some cyclic laws [7-11], numerical methods for solving nonlinear problems are improved [12-14], and research is actively carried out in the field of mechanical safety and survivability of buildings [15-27].



2. Materials and methods

In order to establish the relationship between the values of the limiting moments and deformations for calculation the strength of the reinforced concrete element normal section using a nonlinear deformation model, the authors use the specified diagrams of the concrete under compression state (deformation) $\varepsilon_b - \sigma_b$: bilinear, three-linear (Fig. 1a), curvilinear with a falling branch (Fig.1b) - according to recommendations [1] and similar: simplified bilinear, parabolic-linear, curvilinear - according to recommendations [2]. The changing stresses from deformations laws, depending on the diagrams outline, are presented in the normative documents, as well as in [5, 6, 9, 10]. As a design diagram for the deformation of reinforcing bars under tension $\varepsilon_s - \sigma_s$ (included the class A500), a single bilinear diagram is recommended in [1,2], where the boundary of the elastic part ol is limited by deformations $\varepsilon_{so} = \sigma_{0,2}/E_s$ ($\sigma_{0,2}$ -characteristic value of the yield strength of reinforcing steel). In [11,12], an algorithm for describing the full diagram (the *oeapku* branch) is proposed, where the boundary of the elastic section oe is limited by stresses equal to the reinforcement elastic limit $\sigma_s = \sigma_{s,el}$ (fig.1c). Next, the algorithm for calculating the strength using specified diagrams in [1] is considered, and then it is tested in calculations with diagrams in [2].

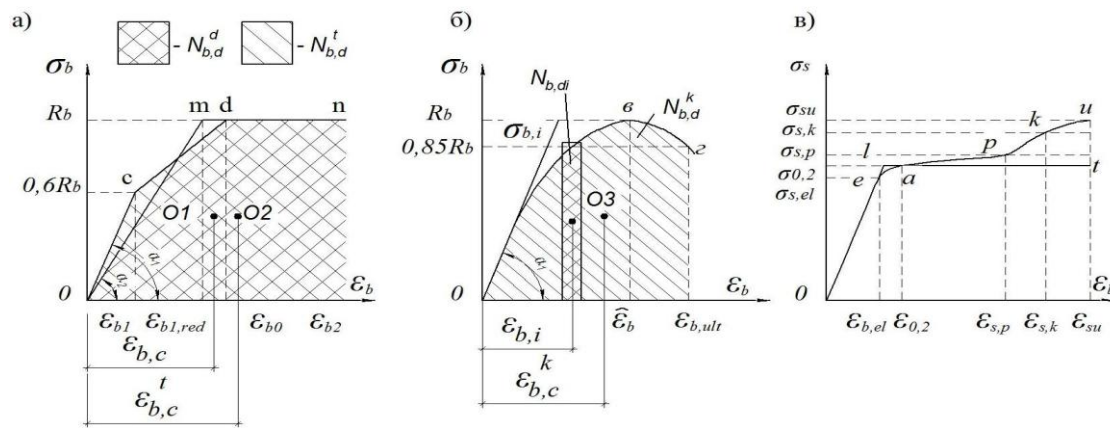


Figure 1. The deformation diagrams of concrete on compression: a - piecewise linear (two-line and three-linear), b - curvilinear; c - reinforcement for tension.

In the complete diagrams of the reinforcement (when $\sigma_s \geq \sigma_{s,el}$) and curvilinear diagrams of concrete, the relationship between strains and stresses in the norms [1] is accordingly taken into the form and curvilinear diagrams of concrete, the relationship between strains and stresses in the norms [1] is accordingly taken in the form

$$\sigma_s = \varepsilon_s \nu_s E_s; \quad \sigma_b = \varepsilon_b \nu_b E_b, \quad (1)$$

where ν_s, ν_b - the variation coefficients of the secant modulus of reinforcement and concrete [6].

For a rectangular cross-section with reinforcement in the lower zone with reinforcement in the area and in the upper zone with square A_s and in the upper zone square A'_s (Fig. 2 a) taking into account the distribution of the relative deformations of concrete and reinforcement by linear law (fig.2 b) stress diagrams are shown in the figure 2 c,d,e. Based on the linear law of distribution relative deformations with respect to the height of the element:

$$\frac{1}{\rho} = \chi = \frac{\varepsilon_{sn}}{h_0 - x} = \frac{\varepsilon_{bn}}{x} = \frac{\varepsilon_{bn} + \varepsilon_{sn}}{h_0} \quad (2)$$

where h_0 – cross-sectional height; x – the compressed zone height; ε_{bn} – relative deformations on the extreme fiber of concrete of the compressed zone; χ – element curvature; ρ – curvature radius; ε_{sn} – relative deformations in the stretched reinforcement.

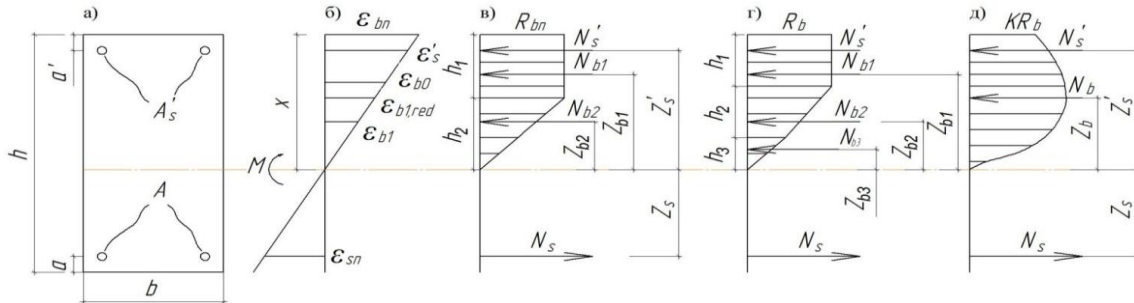


Figure 2. The forces, stresses and deformations schemes in the cross section of a bent non-stressed element for calculation strength using piecewise linear (c-two-linear, d-three-linear) and e-curved diagrams of concrete for compression.

The section strength is checked from the conditions:

$$|\varepsilon_{b,\max}| \leq \varepsilon_{b,ult}; \quad |\varepsilon_{s,\max}| \leq \varepsilon_{s,ult}, \quad (3)$$

where $\varepsilon_{b,\max}, \varepsilon_{s,\max}$ – maximum relative strain from external load; $\varepsilon_{b,ult} = \varepsilon_{b2} = 3,5 \text{‰}$ – limiting relative deformations of compressed concrete (for a curvilinear diagram $\varepsilon_{b,ult}$ is calculated at a stress level $\eta = 0,85$); $\varepsilon_{s,ult} = \varepsilon_{s2} = 25 \text{‰}$ – limiting relative deformations of stretched reinforcement. The corresponding element of concrete or the reinforcement rod is switched off because of deformations above the limit.

In the general case, when the branches of a diagram are described by nonlinear equations (fig.1b), separate small sections are deposited along the axis of deformations $\Delta\varepsilon_{b,i}$ (i – area number) using computer modeling. Relative deformations in diagrams $\Delta\varepsilon_{b,i}$ in the compressed zone of the element corresponds to the height of the elementary section $\Delta h_{b,i} = \Delta\varepsilon_{b,i} / \chi$, with the amount of stresses $\sigma_{b,i}$. The square of i - th section of the diagram is determined by the formula: $A_{b,i} = \Delta\varepsilon_{b,i} \sigma_{b,i}$. The equation of balanced forces in the section of the reinforced concrete element is written in the form:

$$N_b + N'_s - N_s = 0 \quad (4)$$

The deformations of the reinforcement are calculated from the expressions:

$$\varepsilon'_s = \varepsilon_{bn} - \chi a', \quad \varepsilon_s = \chi h_0 - \varepsilon_{bn} \quad (5)$$

Value of effort N_b , perceived concrete concave zone in the limiting state for a strip of unit width ($b = 1$), is calculated by the formula

$$N_b = N_{b,d} / \chi, \quad (6)$$

where in the general case $N_{b,d} = \sum_{i=1}^n A_{b,i} = \sum_{i=1}^n \sigma_{b,i} \Delta \varepsilon_{b,i}$ - is the work expended on the deformation of the sample under load up to their limiting values, numerically equal to the sum of the squares of elementary sections in the area bounded by the branches of the concrete compression diagrams. Taking into account the obtained dependences, the balance equation (4) with width b is written

$$\frac{N_{b,d}b}{\chi} + \sigma'_s A'_s - \sigma_s A_s = 0, \quad (7)$$

The verification of the balance equation (7) is carried out by the method of successive approximations (by the iteration method). At the first approximation, the limiting values of deformations are taken on the extreme fiber concrete of the compressed zone and the stretched reinforcement $\varepsilon_{bn}^{(1)} = \varepsilon_{b,ult}$; $\varepsilon_{sn}^{(1)} = \varepsilon_{s,ult}$. Accordingly to the accepted signs, two cases arise before the terms on the left-hand side of equation (7): 1 - the left-hand side of equation (7) is greater than zero, that indicates a lack of section reinforcement; 2 - the left-hand side of equations (7) is less than zero, which means - a re-cross section. When the first case arises, it is necessary to reduce, at constant values $\varepsilon_{s,ult}$, the deformations of the first approximation $\varepsilon_{bn}^{(1)}$ by $\Delta \varepsilon_b^{(k)}$ until the specified accuracy of approximation is reached:

$$\Delta \varepsilon_b^{(k)} \leq 0.01 \varepsilon_{bn}^{(1)} \quad (8)$$

When the second case is realized, that is, when the left-hand side of the equation is less than zero, the algorithm for checking the balance equation (7) is satisfied in the same sequence. However, the deformations in the reinforcement, adopted in the first approximation $\varepsilon_{sn}^{(1)} = \varepsilon_{s2} = 25 \text{‰}$, decrease in the second iteration cycle by the increment value $\varepsilon_s^{(2)} = \varepsilon_{sn}^{(1)} - \Delta \varepsilon_s^{(1)}$ at constant values of deformations on the extreme fibres of the compressed zone of concrete $\varepsilon_{b,ult} = 3,5 \text{‰}$. Calculations are performed until a sufficient (predetermined) accuracy is achieved for condition $\Delta \varepsilon_s^{(k)}$. Distances between efforts in reinforcement N'_s , N_s and efforts in concrete N_b to the neutral axis, respectively, are: to the neutral axis, respectively, are:

$$z'_s = \frac{\varepsilon_b^{(k)} - a' \chi^{(k)}}{\chi^{(k)}}; \quad z_s = \frac{\chi^{(k)} h_0 - \varepsilon_b^{(k)}}{\chi^{(k)}}; \quad z_b = \frac{S_{b,d}}{\chi^{(k)} N_{b,d}} = \frac{\varepsilon_{b,c}}{\chi^{(k)}}, \quad (9)$$

where $S_{b,d} = \sum_{i=1}^n A_{b,i} \varepsilon_{b,i} = \sum_{i=1}^n \sigma_{b,i} \Delta \varepsilon_{b,i} \varepsilon_{b,i}$ - a moment numerically equal to the sum of the products of the areas of elementary areas in the concrete diagrams by the distances of its centers of gravity to the stress axis σ_b ; $\varepsilon_{b,c} = S_{b,d} / N_{b,d}$ - distance from stress axis, σ_b ($\varepsilon_{b,c}^d, \varepsilon_{b,c}^t, \varepsilon_{b,c}^k$ - in Figure 1a, b) diagrams of concrete to its gravity centers O_1, O_2, O_3 ; $\chi^{(k)}$ - curvature of the element after the condition (8) is satisfied at the k -th iteration.

The equation for calculating the limiting bending moment takes the form:

$$M_{ult} = N_b b z_b + \sigma_s A_s z_s + \sigma'_s A'_s z'_s. \quad (10)$$

In order to determine the bending moment M_{ult} use the values $\varepsilon_{bu} = \varepsilon_b^{(\kappa)}$ - for the first case, $\varepsilon_{su} = \varepsilon_s^{(\kappa)}$ - for the second case, $\chi^{(\kappa)}$, obtained at the last iteration cycles, after condition (8) is satisfied.

A comparative analysis of the calculated strength parameters and its experimental values was made for reinforced concrete elements of rectangular cross-section with height $h = 18$ cm, width $b = 12$ cm, span $l = 194$ cm. The samples were made from one concrete composition. The compressive strength of concrete was $\sigma_b = 30,6$ MPa, elastic modulus $E_b = 3,07 \cdot 10^4$ MPa, which values were determined from the test results of standard prism samples with a proportional increase in the load before failure. In the compressed and stretched zone, two diameters of an A400 class reinforcement were installed. By saturation with the reinforcement, the samples are divided into three series: B-8 with percent reinforcement $\mu = \mu'_s = 0,52\%$; B-10 with $\mu = \mu'_s = 0,82\%$; B-12 with $\mu = \mu'_s = 1,18\%$. The procedure of successive approximation during checking the balance equation (7) using piecewise linear and nonlinear diagrams of normative documents [1, 2] was performed according to the developed program on a computer. The calculated deformation parameters of the reinforced concrete section, using, as an example, curvilinear diagrams in [1, 2] after checking the balance equation (7) in the k -th approximation: $\varepsilon_{bn}^{(k)}$ - deformation on the extreme fiber of concrete of the compressed zone, $\chi^{(k)}$ - the height of the compressed zone, and calculations: M_{ult} - by the formula (10), f - deflection in the middle of the span of beam structures according to the formulas of structural mechanics, experimental values M_{ult} are presented in Table 1.

Table 1. The calculated and experimental parameters in the limiting state of the element

μ %	National Codes and Standards of Belarus [2]				Set of rules [1]				Experience
	$\varepsilon_{bn}^{(k)}$ ‰	$\chi^{(k)}$, cm	M_{ult} , knm	f , mm	$\varepsilon_s^{(k)}$ ‰	$\chi^{(k)}$, cm	M_{ult} , knm	f , mm	M_{ult} , knm
0,52	1,56	2,16	7,28	30,6	25	1,84	7,41	73,5	7,6
0,82	2,02	2,69	12,2	31,2	21	2,3	12,2	63,5	12,1
1,18	2,31	3,0	16,5	32,0	18	2,6	16,6	56,0	16,8

In the calculations, taking into account the notations adopted in the norms, the limiting deformations of concrete for specified diagrams in [1, 2] were assumed to be equal to $\varepsilon_{b2} = \varepsilon_{cu} = 3,5$ ‰, the limiting deformations of the reinforcement in the diagrams [1] is $\varepsilon_{s,ult} = \varepsilon_{s2} = 25$ ‰, and in the norms [2], respectively $\varepsilon_{s,ult} = \varepsilon_{su} = 10$ ‰. The limiting moments in the reinforcement range and in the specified diagrams of concrete and reinforcement in accordance with the recommendations of the norms [1, 2] differ by 2-3%, and correspond to their experimental values. The limiting deformations in the diagrams of reinforcement and concrete determines the general deformations (deflections) of the element. The procedure for reducing the force in an element cross section differs according to the norms. If in norms [1], balance is achieved by reducing the limiting deformations in reinforcement with constant limiting deformations in concrete, then in the norms of [2] the limiting deformations in the reinforcement are retained, increasing reinforcement percentage, deformations increase on the extreme concrete fiber of the compressed zone (increases the completeness of using diagrams of concrete deformation). The limiting deformations in the diagrams

of concrete and reinforcement: in norms [1], equivalence is achieved with the percentage of reinforcement $\mu=0,6\%$, the limiting moment is $M_{ult}=8,56$ Knm; in the norms [2], respectively $\mu=5,36\%$ и $M_{ult}=69,2$ Knm with the plastic character of the cross section destruction.

3. Conclusions

The strength of the reinforced concrete element normal section in the deformation model calculations does not depend on the shape of the concrete deformation diagrams. Increasing in the specified values of limiting deformations in the reinforcement results in inefficient use of the strength and deformation properties of concrete.

The deformation model, taking into account the specified values of the limiting deformations, can be used in calculations of statically determinate structures under the effects of extreme loads of a natural and technogenic character, while retaining its limited work capacity. In order to use the diagrams in practical calculations of statically indeterminate systems, taking into account the redistribution of internal forces, it is necessary to limit boundary values of deformations in the deformation diagrams of concrete for axial compression and reinforcement for axial extension.

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