Research of Construction Elements of Structure-inhomogeneous Materials

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Abstract. The paper concentrates on strain-stress state research in construction elements of structurally heterogeneous materials which is of current importance. Classical concepts of solid, homogeneous, isotropic, linearly elastic body do not suit the construction practice, as nearly all materials used in construction and technics are structurally heterogeneous. Present article deals with the most typical structurally heterogeneous materials – polycrystalline materials. Finite element model of structurally heterogeneous body – polycrystal is developed. The problem of determination of body minimum volume, which could be endued with averaged properties by averaging elastic properties, is solved. It allows analyzing structurally heterogeneous body at different volumes. Stress concentration is investigated for small-scale plates with circular or ellipse holes and with various stress states considering microstructural aspects of stress concentration. It is shown, that coefficients of stress concentration depending on anisotropy of elastic properties can have values, which considerably differ from values for isotropic body.

1. Introduction

In the further development of construction elements research methods, consideration of materials actual properties is of great importance. Almost all materials used in technics and construction: metals and alloys with heterogeneous polycrystalline structure, concrete, brick, wood, different kinds of reinforced plastics, are composite materials carrying anisotropy of properties [1 - 4]. The polycrystalline metals, which represent conglomerates of differently oriented crystallites (grains) for regions with sufficient extension, can be considered as quasi-isotropic. Although, individual grains constituting the polycrystal are characterized by a high anisotropy of the elastic, plastic, and strength properties along with an anisotropy of hardening. Due to this, the fact of anisotropy influencing stress-strain states of construction elements is significant [5, 6].

The calculation model of a polycrystalline aggregate is developed, which can be used in design of structural elements with geometric factors of stress concentration. This model is generated by handling a polycrystalline aggregate in different volumes [1]: 1) an individual grain; 2) an assembly of grains constituting the smallest polycrystalline volume, which can be assumed to have the averaged properties of a macrovolume; 3) the volume of structural element's typical dimensions. Definition of an elementary volume can be achieved by averaging the elastic properties of polycrystal separate volumes with different numbers of grains when investigating effect of the scale factor on elastic properties [7 - 10]. In this case, the task is to define the number of grains constituting an elementary volume, for which the elastic properties values are close to the elastic properties of polycrystals. For the plane problem, in the elastic theory an elementary volume can be adopted to consist of n^2 equal square grains of the same thickness; for the spatial problem, such a volume consists of n^3 cubic grains. According to Il'yushin [11], the differences in the elastic constants and anisotropies play the major role for the formation of the mechanical properties of the actual materials whilst the shape of grains plays the secondary role [12].

For averaging of the elastic properties, the Hill approximation $<S_{ij}>_H$ was used [7]. For single-phase and two-phase polycrystalline materials, the Voight $<C_{ij}>_V$ and the Reuss $<S_{ij}>_R$ approximations lead to a relatively narrow range, hence, the use of the Hill averaging is sufficient in the vast majority of cases, i.e., the arithmetic mean of the values obtained by the Voight and Reuss procedure is calculated:

$$\left\langle S_{ij}\right\rangle_H = \frac{1}{2}\left[\left\langle C_{ij}\right\rangle_V^{-1} + \left\langle S_{ij}\right\rangle_R\right]. \tag{1}$$

Young's modulus E_H, the shear modulus G_H, and the Poisson ratio ν_H can be calculated, using the known values of the matrix entries $<S_{ij}>_H$ [13]:

$$E_H = \frac{1}{\left\langle s_{11}\right\rangle_H},\ G_H = \frac{1}{2\ \left\langle s_{11}\right\rangle_H - \left\langle s_{12}\right\rangle_H},\ \nu_H = \left|\frac{\left\langle s_{12}\right\rangle_H}{\left\langle s_{11}\right\rangle_H}\right| \tag{2}$$

Equations (1-2) are useful when averaging is performed for separate volumes of polycrystals with finite number of grains. Effect of scale factor on elasticity can be evaluated, regarding dependence of the variation coefficient (figure 1) on n [14]. The mean-square deviations of Young's modulus $\sigma_{E(n)}$, the shear modulus $\sigma_{G(n)}$, and the Poisson ratio $\sigma_{\nu(n)}$, are determined beforehand for the different values of n, which is set to 1, 2, 3, etc. The number of cases of different separate volumes with randomly oriented grains for the adopted value of n is set to 100. Then, the values of the variation coefficient v_E are calculated as the ratio of the mean-square deviation $\sigma_{E(n)}$ to the averaged Young's modulus E, and, analogously, the coefficient v_G for the shear modulus G_H, the coefficient v_ν for the Poisson ratio.

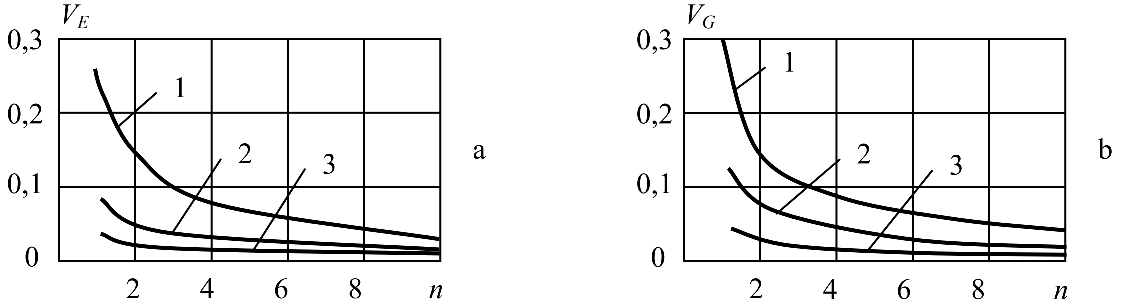

Figure 1. Change in the variation coefficients V_E of Young's modulus (a) and V_G of shear modulus (b): 1 – zinc, 2 – titanium, 3 – magnesium.

For weakly anisotropic magnesium, where n is equal to 4÷5 (which corresponds to 16÷25 grains in the elementary volume of a polycrystal) the values of the elastic constants and the averaged values almost do not differ. For titanium, the value of n is 5÷6, for zinc n is equal to 6÷8, which corresponds to 36÷64 grains. For single-phase metals research, the number of grains constituting the elementary volume assumed to have the averaged properties can be adopted from 25 to 50, depending on the anisotropy of the elastic properties.

Under this approach, the solution to the task of the elastic theory can be divided into two stages: 1) macroscopic; 2) microscopic. On a macroscopic level methods of the classical theory of elasticity are used, the most strained region is determined. The values of strains calculated for this region are adopted as the boundary conditions for the elementary volumes of a polycrystal, which can be assumed to have the averaged properties, i.e., the values of strains obtained are the initial data for calculations on a microscopic level. In the second phase, stress-strain states are calculated in microvolumes, which are equal to the grain size and part of the grain size in a polycrystal, using a finite element method [15, 16].

Composing of the system of equations (3) includes calculation of stiffness matrix for an assembly of grains constituting the elementary volume (4). The matrix is calculated as the sum of the corresponding n members of the stiffness matrix of separate elements (4)

$$K \cdot \delta = F , \tag{3}$$

$$K_{sl} = \sum_{1}^{n} k_{ij} ; \quad k = D^{T} \cdot E_{\varepsilon} \cdot D \; At , \tag{4}$$

where [K] - the stiffness matrix of the elementary volume; {δ} - the displacement vector; {F} - the load vector; [k] - the stiffness matrix of the separate element; [D] - a rectangular matrix, elements of which depend on the kind of a finite element and coordinates of the regarded point; [E_{ε}] - the flexibility matrix; A, t – the cross-sectional area and the thickness of the element, respectively.

Solution to the system of equations (3) allows calculating the strain vector {ε} and the stress vector {σ} as follows:

$$\varepsilon = D \; \delta ; \quad \sigma = E_{\varepsilon} \; \varepsilon . \tag{5}$$

The possibility of applying a finite element method for the analysis of the model of structurally heterogeneous body (polycrystal) depends on development of algorithm solving the flexibility matrix for each grain, which is randomly oriented, constituting an elementary volume of a polycrystal. Relation between stresses and strains of an anisotropic body in the form of the tensor can be defined as follows [19]:

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} ; \qquad \varepsilon_{ij} = s_{ijkl} \sigma_{kl} . \tag{6}$$

Components c'_{ijkl} and s'_{ijkl} are calculated in a laboratory coordinate system, using the law of transformation of the four-rank tensor:

$$c'_{ijkl} = a_{im} a_{jn} a_{kp} a_{lq} c_{mnpq} ; \quad s'_{ijkl} = a_{im} a_{jn} a_{kp} a_{lq} s_{mnpq} . \tag{7}$$

For evaluation of the effect that microstructural aspects of stress concentration have on stress distribution small-scale plates with circular holes made of polycrystalline metals with hexagonal lattice were regarded: magnesium, titanium, and zinc.

The results obtained show the nonuniformity of normal stress distribution due to the interaction of differently oriented grains possessing anisotropy of elastic properties (figure 2). It may be noted that the degree of the strain nonuniformity depends on the anisotropy degree of the material being investigated. For magnesium and titanium, both of the polycrystalline materials have close degrees of anisotropy, hence, close degrees of the stress nonuniformity take place. For weakly anisotropic magnesium, range of values that tensile stresses take in the area far removed from the hole is from 19.4 MPa to 29.0 MPa at mean stress equal to 25 MPa (figure 2, a). Strongly anisotropic zinc has more significant stress nonuniformity (figure 2, b): tensile stresses change from 19.7 MPa to 32.9 MPa.

It should be noted that there is a significant increase in stress concentration factors compared to solutions achieved for isotropic fine grained material, along with dependences of stress concentration factors on the degree of anisotropy of elastic properties [17, 18]. According to table 1, when performing uniaxial tension the most significant increase in stress concentration factor takes place for a plate made of zinc – by 2.35 times in comparison to isotropic solution. The stress concentration factors for magnesium and titanium are close to one another and increase by 2 times in comparison to isotropic solution.

In practice cases, increase in values of stress concentration factors can be lower, since for ductile materials stress redistribution takes place because of the occurrence of the local plastic flows. For brittle materials, in stress concentration regions there will be micro destructions with the following overall destruction [20]. The results of research in construction elements of structurally heterogeneous materials show expediency and prospects of carrying out further investigation and development of physico-mechanical models of structurally heterogeneous bodies with different configurations influencing stress concentration and at different kinds of stress-strain states.

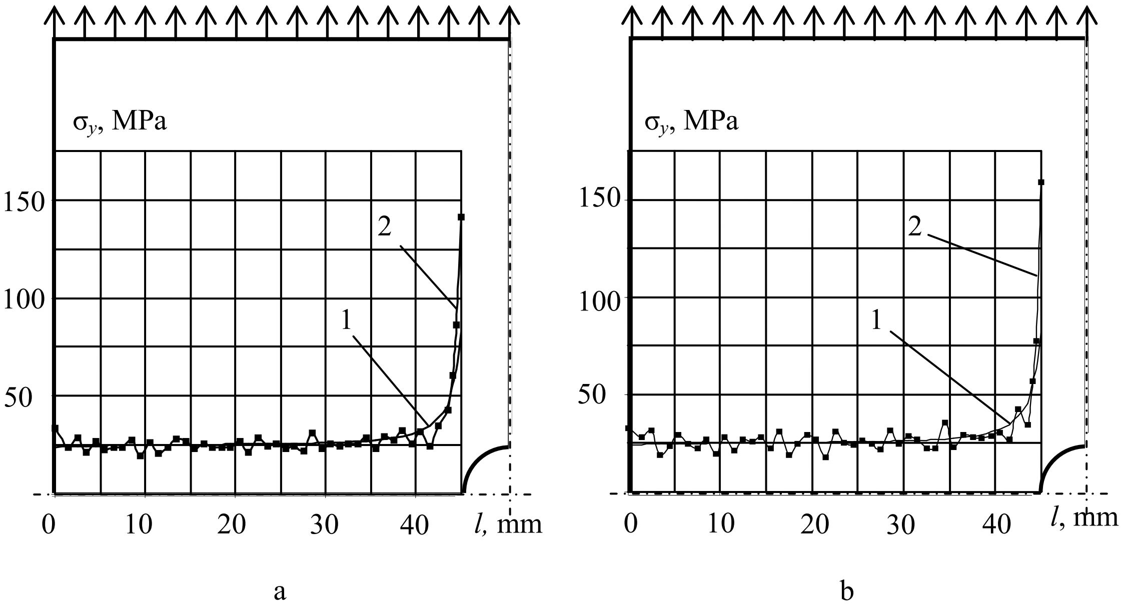

Figure 2. Curves of σ_y constructed for plates with circular holes under uniaxial tension and made of: a– magnesium, b – zinc; 1 – isotropic material, 2 – polycrystalline material.

Table 1. Stress concentration factors considering microstructural aspects of stress concentration for plates with circular holes when uniaxial tension is performed.

Metal	α_σ	
	Isotropic fine grained material	Polycrystalline material
Magnesium	3.0	6.07
Titanium		6.15
Zinc		7.05

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