

Mathematical Description of the Process of Resistance to the Thermal Impact of Reinforced Concrete Columns

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Abstract. The article presents the results of analysis of experimental data on the variation in the degree of section reinforcement, the intensity of power stresses, the width of the column cross-section, the thermal diffusion of concrete under the conditions of exposure to a standard fire, the change in the compressive strength of concrete - on the basis of which a mathematical model of the process of reducing the load-carrying capacity of a column was developed; a mathematical description of the onset of the limiting state of a reinforced concrete column on the basis of a loss of bearing capacity in the conditions of a fire; the poly-parametric equation for evaluating the design fire resistance of the reinforced concrete column has been derived, and the influence of the distinctive design parameters (the degree of reinforcement of the section, the hollowness, the depth of the reinforcement, the longitudinal bending) on the design limit of the fire resistance of the reinforced concrete column of the building is taken into account. Analytical formulas are obtained for estimating the indices of fire resistance of a column from the value of the standard resistance of concrete to compression and the index of its thermal diffusion.

1. Introduction

The study is aimed to reveal the influence of basic structural parameters of compressed reinforced concrete elements on the value of their actual fire resistance. It also presents a mathematical description of the process of reinforced concrete columns resistance to combined influence of external load and high temperature as well as a substantiation and introduction of mathematical dependencies for analytical calculation of design limits of fire resistance on the basis of load bearing capacity loss.

2. Materials and methods

2.1 Factors influencing columns fire resistance properties

The fire resistance limit of linear compressed concrete and reinforced concrete elements is influenced by the intensity of stresses from the current (test) load, section dimensions, heating conditions, the type of concrete and its compression strength and the degree of section reinforcement. That is:

$$\tau_u = f(J_\sigma; \alpha_{br}; b; n; R_{bn}; \alpha_\mu; \alpha_s) \quad (1)$$

where J_σ is initial stresses intensity, $(0 \div 1)$; α_{br} is concrete temperature diffusivity, cm^2/h ; b is the minimum section width of the element, cm ; n is the number of section sides at heating; R_{bn} is



normative concrete compression breaking strength, MPa ; α_μ is the degree of section reinforcement; α_s is the value of the working reinforcement fire resistance(axial distance, laying depth, sm).

The influence of these parameters on the compressed elements stress state in the conditions of combined impact of the test load and high temperature can be taken into account by the initial intensity of power stresses in the cross section J_σ and indicators of heated concrete and reinforced concrete working conditions: $k_a; k_h; k_n; k_R; k_\mu$ etc. Here k_a is a coefficient that takes into account concrete temperature diffusivity; k_h a coefficient of the element's section mass; k_n is a coefficient of heating conditions of the element section; k_R is a coefficient, taking into account the class of concrete according to its compression breaking strength; k_μ and $k_{\mu k}$ are coefficients, taking into account the degree of reinforcement of the element cross-section by longitudinal and transverse (indirect) reinforcement; k_a – a coefficient, taking into account the degree of working reinforcement fire protection by concrete.

2.2. The column loading degree

The intensity of stresses from the test load to the fire effect (the column loading degree) is determined by the following formula:

$$J_\sigma = \frac{N_n}{N_o} \quad (2)$$

where N_n is the test load on the column, kH ; N_o is the column bearing capacity before the fire, kH .

The load bearing capacity of the reinforced concrete column at the longitudinal force action which is applied with random eccentricity $e_o = e \leq h/30$ at $e_o \leq 20/h$ with is determined by the condition (119) [9]:

$$N_o = (R_{bn} \cdot A_b + R_{scn} \cdot A_s) \cdot \varphi \quad (3)$$

where R_{bn} is concrete normative resistance to axial compression, MPa ; A_b is the concrete element cross-section area, cm^2 ; R_{scn} is reinforcement normative resistance to compression, MPa ; A_s is the area of longitudinal reinforcement in the cross-section of the element, cm^2 ; φ is a coefficient that takes into account the load duration, the flexibility and the nature of the element reinforcement, and is determined by the formula (120) [9].

2.3. The Mass of the column cross-section and conditions of its section heating

With the increase of reinforced concrete columns massiveness, their fire resistance also increases. The heating conditions of the section are characterized by the degree of section heating. At one-sided heating of a section the maximum fire resistance is higher, than it is at quadrilateral.

Table I shows the values of the limits of the columns fire resistance depending on the section width [9], p. 218.

Table I data analysis makes it possible to determine the coefficient of the column section massivity. Let us take $B_1 = 40 \text{ cm}$, $\tau_1 = 2,5 \text{ h}$, $B_2 = 30 \text{ cm}$, $\tau_2 = 2 \text{ h}$, and calculate the following ratio:

$$m = \frac{\lg(\tau_1/\tau_2)}{\lg(a_{br1}/a_{br2})} = \frac{\lg(2,5/2)}{\lg(40/30)} = 0,776 \quad (4)$$

Table 1. Limits of columns fire resistance depending on the section width

Type of concrete	Number of sides of heated sections, n	Fire resistance limit τ_u , hour, depending on the column width, B, cm					
		0.5	1	1.5	2	2.5	3
Heavy	4	15	20	24	30	40	45
	1	10	12	14	16	20	24
Light	4	15	16	19	24	32	36
	1	10	10	12	13	16	19
Axial distance, a, cm		1	2.5	3.5	4	4	4

Taking $B = 300 \text{ cm}$, $B_1 = 100 \text{ cm}$, let us calculate the coefficient of the column section massivity:

$$k_h = \left(\frac{B - B_1}{B_o} \right)^m = \left(\frac{B - 10}{30} \right)^{0.75} \quad (5)$$

or

$$k_h = 7.8 \cdot 10^{-2} \cdot (B - 10)^{0.75} \quad (6)$$

where B is the width of the column section, cm.

2.4 Account of concrete temperature diffusivity

The coefficient, taking into account the concrete temperature diffusivity, is represented as:

$$k_a = f \cdot (\alpha_{br} / \alpha_o)^m \quad (7)$$

and is determined according to the calculation of professor A.I. Yakovlev [5].

For reinforced concrete columns of 30-KO type, we have: $\alpha_{br1} = 13.26 \text{ cm}^2/\text{h}$; $\tau_1 = 1.55 \text{ h}$; $\alpha_{br2} = 11.8 \text{ cm}^2/\text{h}$; $\tau_2 = 2.03 \text{ h}$; in this case, the proportion is

$$m = \frac{\lg(\tau_1 / \tau_2)}{\lg(\alpha_{br1} / \alpha_{br2})} = \frac{\lg(1.55 / 2.03)}{\lg(13.26 / 11.8)} = 2.31;$$

According to data given in Paper [4], with $B = 300 \text{ mm}$, we have: $\alpha_{br1} = 13.26 \text{ cm}^2/\text{h}$; $\tau_1 = 2 \text{ h}$; $\alpha_{br2} = 11.8 \text{ cm}^2/\text{h}$; $\tau_2 = 2.5 \text{ h}$; in this case, the value is

$$m_2 = \frac{\lg(2 / 2.5)}{\lg(13.26 / 11.8)} = 1.91;$$

Let us calculate the average value:

$$m = \frac{m_1 + m_2}{2} = -2.11 \cong -2.0;$$

Therefore, the coefficient, taking into account the type of concrete according to its temperature diffusivity (at $\alpha_o = 13.53 \text{ cm}^2/\text{h}$) is presented as the function:

$$k_a = \left(\frac{\alpha_{br}}{\alpha_{bo}} \right)^{-2} = \left(\frac{\alpha_{br}}{13.53} \right)^{-2} = 183 \cdot \alpha_{br}^{-2} \quad (8)$$

2.5 Impact of concrete strength on its compression breaking strength

The coefficient, taking into account the strength of concrete for compression, is defined by the data of Paper [5]. For columns of 40KC type, made of B45 and B25 concrete, we have: $\tau_1 = 1,17h$; $R_{bn} = 32 MPa$; $\tau_2 = 1,33h$; $R_{bn} = 18,5 MPa$; let us find the dependence:

$$m = \frac{\lg(\tau_1/\tau_2)}{\lg(R_{bn1}/R_{bn2})} = \frac{\lg(1,17/1,33)}{\lg(32/18,5)} = 0,234 ;$$

Therefore, the coefficient that takes into account the concrete strength on its compression breaking strength is:

$$k_a = \left(\frac{R_{bn}}{R_o} \right)^{-0,234} = \left(\frac{R_{bn}}{18,5} \right)^{-0,234} = 1,98 \cdot R_{bn}^{-0,234} \quad (9)$$

or

$$k_a = 2,1 \cdot R_{bn}^{-0,25} \quad (10)$$

where R_{bn} is concrete compression breaking strength, MPa .

2.6 Influence of reinforcement fire resistance degree

The coefficient taking into account the degree of working reinforcement fire resistance is determined by the dependence:

$$k_a = 1 - 0,1 \cdot \frac{\alpha_n - \alpha}{\alpha_n} = 1 - 0,1 \cdot \left(1 - \frac{\alpha}{\alpha_n} \right) \quad (11)$$

where a and a_n are the actual and normative values of axial stretching, respectively, cm :

$$\alpha = u + 0,5 \cdot d \quad (12)$$

u is the thickness of concrete protective layer, cm ; d is the diameter of working reinforcement, cm .

2.7 Influence of the degree of the column section reinforcement

With the growth of the reinforcement coefficient of the reinforced concrete column cross-section, its resistance to the effect of fire also reduces (see Fig. 3).

Coefficient, taking into account the degree of the column section reinforcement with longitudinal reinforcement:

$$k_{\mu s} = 1 - 0,17 \cdot a_{\mu s} \quad (13)$$

$$\alpha_{\mu s} = \frac{R_{sn} \cdot A_s}{R_{bn} \cdot b \cdot n} = \frac{R_{sn} \cdot A_s}{R_{bn} \cdot A_b} = \frac{R_{sn}}{R_{bn}} \cdot \mu_s \quad (14)$$

The use of indirect reinforcement increases the limit of the column fire resistance. In this case, the coefficient taking into account the indirect reinforcement of the section is equal to:

$$k_{\mu k} = 1 - 0,6 \cdot \mu_k \quad (15)$$

where μ_k and μ_s are indirect and normal reinforcement of the column cross-section, respectively, %.

The reference value $\alpha_{\mu o}$ is defined for a column of 30 KC type 30, made of B15-class concrete ($R_{bn} = 11 MPa$) with A 400-class reinforcement ($R_{sn} = 400 MPa$)

$$\alpha_{\mu} = \frac{R_{sn}}{R_{bn}} \cdot \mu_s = \frac{400}{11} \cdot \mu_s = 36,36 \cdot \mu_s ;$$

from that we have

$$\alpha_{\mu o} = 3,63 \cdot 10^{-2} \cdot \mu_s \quad (16)$$

then

$$\alpha_{\mu} = 3,63 \cdot 10^{-2} \cdot \mu_s \cdot \frac{R_{sn}}{R_{bn}} \quad (17)$$

Therefore, the coefficient, taking into account the degree of the column section reinforcement with longitudinal reinforcement:

$$k_{\mu} = (1 - 0,17 \cdot \alpha_{\mu o}) = \left(1 - 0,17 \cdot 3,63 \cdot 10^{-2} \cdot \mu_s \cdot \frac{R_{sn}}{R_{bn}} \right) \quad (18)$$

or

$$k_{\mu} = 1 - 6 \cdot 10^{-3} \cdot \mu_s \cdot \frac{R_{sn}}{R_{bn}} \quad (19)$$

where μ_s is longitudinal reinforcement of the section, %; R_{sn} is normative compression breaking strength of reinforcement, MPa; R_{bn} is normative compression breaking strength of concrete, MPa.

3. Results

3.1 Essence of analytical calculation of reinforced concrete columns fire resistance

The limit of the column fire resistance is determined by Value τ_u , hour, depending on the intensity of initial stresses J_{σ} and the slope angles M of the load bearing capacity reducing line of the heated concrete element in fire conditions.

The mathematical description of the process of reducing the load bearing capacity of the reinforced concrete column in fire conditions is given below.

3.2 Substantiation of fire resistance of concrete column calculation

The reducing line of concrete elements load bearing capacity can be represented mathematically in the form of equation in the following segments [9]:

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (20)$$

The group of right lines, characterizing reduction of a concrete column load bearing capacity depending on compression breaking strength of concrete, is presented in the form:

$$\frac{\tau_i}{a_i} + \frac{J_{\sigma o}}{b} = 1 \quad (21)$$

The processing of experimental data gives the following characteristics for different concrete columns:

Right line equation, for which $a_i = 9,8$; $b_i = 1$, can be presented as

$$\frac{\Delta\tau_1}{\alpha_1} + \frac{J_{\sigma_0}}{b_1} = 1; \quad \frac{\Delta\tau_1}{9,8} + \frac{J_{\sigma_0}}{1} = 1 \quad (22)$$

from that we have

$$\Delta\tau_1 = 10 \cdot (1 - J_{\sigma_0}) \quad (23)$$

Table 2. Normative compression strength of concrete

Class of concrete according to its compression breaking strength	15	25	35
Right line inclination	5	5.5	6
Normative compression strength of concrete R_{bn} , MPa.	11.4	10.4	9.8

By substituting a coefficient that takes into account concrete compression breaking strength, let us find

$$\tau_1 = 10 \cdot (1 - J_{\sigma_0}) \cdot k_r = 10 \cdot (1 - J_{\sigma_0}) \cdot 2,1 \cdot R_{bn}^{-0,25} \cong 21 \cdot (1 - J_{\sigma_0}) \cdot R_{bn}^{-0,25} \quad (24)$$

Therefore, to calculate the fire resistance limit of concrete columns with 40x40 cm cross-section (4-sided heating), made of heavy concrete of various classes on its compression breaking strength, let us use the formula:

$$\tau_R = 21 \cdot (1 - J_{\sigma_0}) \cdot R_{bn}^{-0,25}; \quad (25)$$

where J_{σ_0} is the initial intensity of stresses in the section of the concrete column; R_{bn} is normative concrete compression breaking strength, MPa.

Therefore, with account of concrete temperature diffusivity, the massiveness of the element, the condition of section heating the class of concrete according to its compression breaking strength and the intensity of initial stresses, the duration of resistance to the thermal impact of concrete columns can be expressed with the following mathematical dependence:

$$\tau_{u,b} = k_{\alpha} \cdot k_h \cdot k_n \cdot k_R \cdot (1 - J_{\sigma_0}) = M \cdot (1 - J_{\sigma_0}) \quad (26)$$

where

$$k_{\alpha} \cdot k_h \cdot k_n \cdot k_R = M \quad (27)$$

Therefore, while using equations 5, 10 and 24, we get

$$\tau_{u,b} = 183 \cdot \alpha_{br}^{-2} \cdot 7,8 \cdot 10^{-2} \cdot (B - 100)^{0,75} \cdot 2,1 \cdot R_{bn}^{-0,25} \cdot 10 \cdot (1 - J_{\sigma_0}) \quad (28)$$

or

$$\tau_{u,b} = 300 \cdot \alpha_{br}^{-2} \cdot (B - 100)^{0,75} \cdot R_{bn}^{-0,25} \cdot (1 - J_{\sigma_0}); \quad (29)$$

At $J_{\sigma_0} = 0$ the highest value is

$$\tau_{b,max} = 300 \cdot (b - 10)^{0,75} \cdot \alpha_{br}^{-2} \cdot R_{bn}^{-0,25} \quad (30)$$

where b is the element section width, cm; α_{br} is the coefficient of concrete temperature diffusivity, cm^2/h ; R_{bn} is normative concrete compression breaking strength, MPa; J_{σ_0} is the intensity of initial stresses in the cross section of the concrete element from the external load.

3.3 Substantiation of fire resistance of reinforced concrete column calculation

The reduction of the reinforced column load bearing capacity in fire conditions can be described by the following function:

$$\tau_u = f \cdot (\alpha_\mu; J_\sigma; \tau_{b,max}) \quad (31)$$

The limit of KC-40 columns fire resistance (heavy concrete on crushed limestone, $R_{bn} = 11 \text{ MPa}$; $\alpha_{br} = 11 \text{ cm}^2/\text{h}$; is axial distance as $= 3,5 \text{ cm}$, is the section dimensions $40 \times 40 \text{ cm}$, $1 \leq \mu \leq 3\%$) can be defined by the formula:

$$\tau_u = \tau_{b,max} \cdot (1 - 0,17 \cdot \alpha_\mu) \cdot (1 - J_\sigma)^2 \quad (32)$$

where $\tau_{b,max}$ is duration of resistance to fire of an unloaded concrete element, *hour*; α_μ is the degree of reinforcement of cross-section with longitudinal reinforcement; J_σ is intensity of initial stresses in cross-section of reinforced concrete column from test load.

3.4 Results of analytical description of the reinforced concrete column fire resistance calculations

Let us transfer "old-fashioned" measuring units into modern ones: $\text{cm} - \text{mm}$; $\tau_u, h - F_u, \text{min}$; $\alpha_\mu, \text{cm}^2/\text{h} - D_{bm}, \text{cm}^2/\text{min}$ ($1 \text{ cm}^2/\text{h} = 1,67 \text{ cm}^2/\text{min}$), and calculate the value of reinforced columns fire resistance $F_{u(R)}, \text{min}$ using analytical equations

$$F_{U(R)} = \frac{5 \cdot B_{min}^2 \cdot (1 - J_{\sigma 3}^2) \cdot (1 - 0,6 \cdot \alpha_{\mu s}) \cdot k}{D_{bm}^2 \cdot R_{bn}^{0,2}} \quad (33)$$

$$J_{\sigma 3} = \mu_0^{0,25} \cdot J_H = \mu_0^{0,25} N_0 / N_{cc} \quad (34)$$

$$\alpha_{\mu s} = \mu \cdot R_{sc} / R_{bn} \quad (35)$$

$$k = \gamma_u \cdot k_\alpha \cdot k_f \cdot m_{o\sigma} \cdot \varphi \quad (36)$$

$$k_\alpha = [1 - 0,1 \cdot (\alpha_u - \alpha) / \alpha_u] \quad (37)$$

$$k_f = 1 - (A_{nym} / A)^2 \quad (38)$$

$$m_{o\sigma} = (P / P_0)^{1,2} \quad (39)$$

$$\varphi = 0,9 - 0,7 \cdot 10^{-2} \cdot [(l_o / B_{min}) - 10] \quad (40)$$

where $J_{\sigma 3}$ is power stresses in the section of the column intensity; $N_o; N_{cc}$ is the test load and corresponding bearing capacity of the column, kH ; $\alpha_{\mu s}$ is the degree of the column section reinforcement; μ is the coefficient of the column section reinforcement; R_{sc} and R_{bn} is the resistance limit of reinforcement to compression and, accordingly, the normative strength of concrete to axial compression, MPa ; k is the integral indicator of the column safety; γ_u is the indicator of the responsibility level of the column; k_α is the indicator of the working reinforcement depth; α and α_u are the actual and normative depth of reinforcement, mm ; k_f is the indicator of the solidity of the hollow section; A and A_{nym} is the section area and accordingly the area of emptiness, mm^2 ; $m_{o\sigma}$ is the indicator of heating conditions of the column section; P and P_0 is the section perimeter and the length of the heated section, mm ; φ is the coefficient of longitudinal bending of the reinforced

concrete column; l_0 and B_{min} is the calculated length of the column and the minimum size of its rectangular section, mm.

4. Conclusion

1. The study analyzes load bearing reinforced concrete elements on exposure to heat flow in conditions of fire. It identifies a sufficient number of constructive parameters influencing the limit of reinforced concrete columns fire resistance.

2. For reinforced concrete columns, the basic parameters are accepted as follows: intensity of power stresses in the section of the column from the current load before the beginning of the fire effect; width of the column section and a degree of its reinforcement, degree of fire reinforcement by concrete, its thermo-diffusion indicator.

3. The paper further proposes a mathematical description of indicators, that take into account the value of concrete thermo-diffusion, the scheme of heating and arrangement of reinforcement cores in the section of the column.

4. It also presents quantitative analysis of the influence of the section width and the degree of its reinforcement on reinforced concrete column fire resistance limit describing a mathematical dependence of the indicator taking these parameters into account.

5. The research produces analytical formulas for indices of fire resistance of a column assessment from the value of concrete normative resistance to compression and the indicator of its thermo-diffusion.

6. By using the derived mathematical dependencies of change for the basic structural parameters of reinforced concrete columns, the researchers obtain poly-parametrical dependencies for calculating their designed fire resistance.

5. References

- [1] Bushev V P , Pchelintsev V A , Fedorenko V S , Yakovlev A I et al 1970 *Buildings Fire Resistance* p 261
- [2] *Instruction of actual limits of fire resistance calculation for reinforced concrete structures on the basis of computer application* 1975 ed V P Bushev, A I Yakovlev
- [3] *Manual on fire resistance limits of reinforced concrete structures and groups of flammable materials determination* 1985 Ed I G Romanenkov, A I Yakovlev and V V Solomonov, et al.
- [4] Milovanov A F 1986 *Fire resistance of reinforced concrete structures* p 224
- [5] Yakovlev A I 1988 *Calculation of building constructions fire resistance* p 143
- [6] Ilyin N A and Panfilov D A 2017 *Assessment of fire resistance of designed reinforced concrete structures of buildings*. Monograph, p 186
- [7] Ilyin N A and Panfilov D A 2017 *Resistance of building structures in conditions of fire. textbook* p 76
- [8] Patent 2 281482 Ru, MPC C 01 N 25/50 Method of fire resistance determination of compressed reinforced concrete elements of building constructions / Ilyin N A, Butenko S A, Esmont S V, By SGASU: 06.09.04; pub. 10.08.06. Bul. 22
- [9] *Manual for the design of concrete and reinforced concrete structures made of heavy concrete without PN reinforcement*. (NIIZHB) 2005
- [10] Filchakov P F 1974 *Handbook of Higher Mathematics*, p 743