

# Method for solving problems of optimization of lattice building structures

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**Abstract.** Spatial rod (mesh) structures are widely used in development. The first who used steel wadded structures in architecture was V G Shukhov. Variability of wireframe frames (geodesic dome, hyperboloid shell and other) allows using them for various architectural solutions. Optimization calculations are used to increase the efficiency and profitability of such structures at the stage of search design. Since these constructions have complex geometry, the designer can have difficulties. They are caused by a large number of considered constraints for each element of the construction. Present methods and software of optimal design do not always correspond to the dimension of the problem and the requirements of designers. To solve these problems when designing structures, the authors propose to use a modified simplex search method. The modification consists in the fact that the set of piecewise smooth boundaries of the domain of admissible solutions is replaced by a single convex R-predicate. In addition, a simplex is considered bound to the nearest boundaries by means of elastic bonds, the reactions of which affect the direction of the search. In this paper, the algorithm of this method is presented and the solution of the problem of optimal design of a mesh cylindrical structure is given.

## 1. Introduction

Lattice designs are often used in engineering [1-2] and construction. The first prototype of mesh structures - round structures with a roof in the form of a cone or dome - yurts (Figure 1). V.G. Shukhov was the first to use mesh structures in the industry. The first such construction was the water tower at the All-Russian Exhibition in 1896 (Figure 2). The towers of the Shukhov system have become widespread, since they have the properties of economy, ease and stability [3].

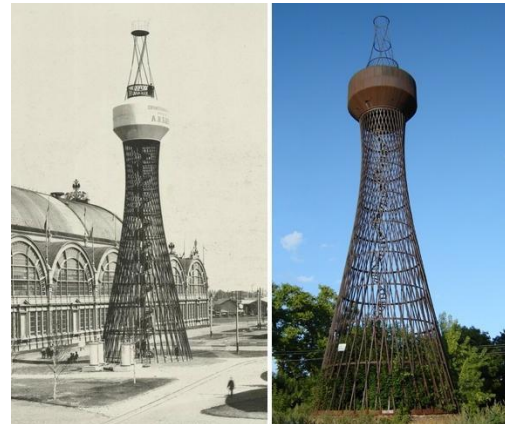
At present, designers and architects widely use mesh constructions of various shapes [4-8] - shell constructions, constructions of an arbitrary shape (Figure 3) and other constructions - in the design of buildings and other building structures. Geodesic dome structures (Figure 4) have received a wide spread in construction. Such structures are used both in industrial and in personal construction [9].

When designing such structures, the optimal design problem may arise [10-12]. Optimal design allows to increase the efficiency and economy of the structure, including by reducing the excess safety margin. Such tasks have a large number of restrictions [13], which must be fulfilled when using such a design. At the same time, as a criterion of optimality, either a minimum of the structural mass or a minimum cost of construction is used.





**Figure 1.** The skeleton of the yurt walls



**Figure 2.** Hyperboloid tower at the Nizhny Novgorod exhibition in 1896 (left) and nowadays (right)



**Figure 3.** Example of projects of reticulated shells of arbitrary configurations



**Figure 4.** Example of geodesic dome structures

## 2. Setting an optimal design problem with a large number of constraints

The problem of optimizing the mass of a multi-element statically indeterminate construction is one-criterion. Limitations on strength, stiffness and stability are expressed through the parameters of the stress-strain state under design loads, which depend on the varying design parameters of the structure.

Since the parameters of the stress-strain state depend on the position of the point (they are fields of displacements, stresses and deformations), limitations on strength and rigidity should be formulated for a sufficiently large number of characteristic points of the construction. Thus, a large number of restrictions can be included in the formulation of the problem. The number of constraints can exceed the number of variable parameters by several orders of magnitude.

Formally, the problem of optimizing the construction by mass can be put in the following form.

It is known:

- initial values of the structural parameters of the model  $X_0$ ,
- a vector of variable effects  $r$ ,
- model of the reaction of the structure to the effects  $q=K(p)r$ .

It is required to determine: the structural parameters of the model  $X \subseteq p$ , for which the constraints of the structural parameters  $F(p) \geq 0$  and the constraints of the state parameters  $\Phi(q) \geq 0$  are satisfied, which ensure the minimum of the objective function  $M(p) \rightarrow \min$ . The mass of the structure is chosen as the objective function  $M(p)$ .

At present, the researchers have sufficiently developed methods for the optimal design of reticular structures of a regular rib structure, provided there is no sheathing [1, 14-15]. However, such methods are difficult to use in the design of complex engineering and buildings that have structural and technological cutouts, reinforcements or sheathing. One of the reasons for the difficulties is the use of the continuum approach in these methods. It is known [16] that such an approach does not give sufficient accuracy for irregular structures. If we talk about building structures, the researchers note [20] that the use of nonlinear programming [17-19] is rare, since it requires engineers to have a deep knowledge of optimization theory.

Existing methods of structural and parametric optimization implemented in packages of design programs are widely used in construction and engineering [21-26]. But complexities can arise when solving multiparameter problems of large dimension. They are associated with a large number of variables and features of production technology.

The optimization algorithm for the criterion of minimum mass with a large number of constraints is shown in [27]. This algorithm uses discrete modeling of structures, which makes it possible to use it on structures of complex structure. The application of this method has been tested to optimize the mesh shell designs of the regular and irregular structure of machine-building designation [28-29].

This method is a modification of the simplex search method. In this method, the set of piecewise smooth boundaries of the domain of admissible solutions is replaced by a single convex R-predicate. In this case, the simplex is considered connected with the nearest boundaries by means of elastic bonds, the reactions of which affect the direction of the search.

### 3. The optimal design algorithm

We represent the optimal design problem as follows: the minimum of the objective function  $z(x_1, x_2, \dots, x_n)$  is sought. The solution is a point  $x$  with coordinates  $(x_1, x_2, \dots, x_n)$  in the area of admissible solutions  $\Omega$ . The area of admissible solutions is determined by the system of inequalities:

$$\Omega: \begin{cases} \omega_1(x_1, x_2, \dots, x_n) \geq 0, \\ \omega_2(x_1, x_2, \dots, x_n) \geq 0, \\ \dots \\ \omega_N(x_1, x_2, \dots, x_n) \geq 0. \end{cases}$$

The space of coordinates  $(x_1, x_2, \dots, x_n)$  is a space of variable (optimized) parameters, the dimension of which can be arbitrary. The objective function  $z(x_1, x_2, \dots, x_n)$  describes the change in the mass of the structure with varying parameters.

The value of each of the functions in (1) determines the measure of the distance of the current (test) point from the corresponding section of the boundary. The problem of looping the algorithm near the sharp corners of the search area and the "multiple" boundaries, which are determined by linearly

dependent constraints, is solved as follows. Each dominant is replaced by an elastic bond that acts on the moving simplex in a manner similar to a spring normal to the surface. The direction of movement of the simplex is corrected taking into account the sum of the reactions of these elastic bonds.

Then, when the auxiliary objective function decreases, the simplex moves at approximately equal distance from the dominants, along the Dirichlet line of the search area. And thus, comes to the desired point on the shortest path. When all the dominants are reduced to a predetermined threshold, a point is determined in which the value of all the dominants is zero, i.e. there is a minimum point of the sum of the squares of the dominants. The search ends when the dimensions of the simplex become less than the specified error value. Finally, as the solution of the problem, the center of gravity of the resulting simplex is chosen.

The testing of the developed optimization algorithm was carried out on control examples and the task of determining the optimal geometric parameters of the cantilever beam. It showed:

- the convergence of the numerical solution to the desired point is obtained in all the examples considered, and the error in calculating the coordinates of the optimum does not exceed the dimensions of the simplex;
- algorithm makes it possible to obtain a solution to the optimization problem in the presence of a "twinning" of boundaries and corner points on the boundary of the range of admissible values;
- the results of the numerical calculation are consistent with the analytical solutions, the sequence of approximate solutions converges to analytical solutions.

When the algorithm is running, the values of the objective function and constraints are calculated at each step. Thus, the design model is rebuilt for each new set of parameters (simplex points) and performs a stress-strain analysis for each model. The use of a discrete approach (for example, the finite element method) is useful for calculating the state parameters of the considered mesh constructions (displacements and stresses in structural elements).

#### **4. Solution of the problem of optimal design of a cylindrical shell of a regular structure without skin**

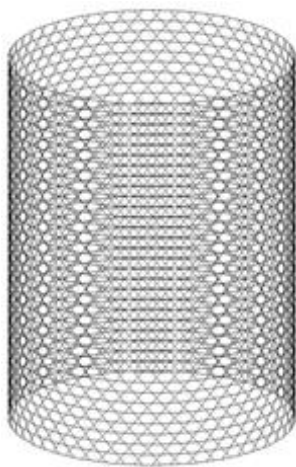
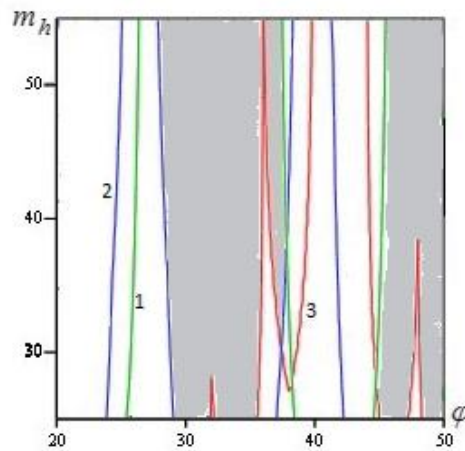
The developed algorithm is applied to optimization of the mass of mesh structures of cylindrical shape. Typical lattice construction [1] is characterized by the thickness of the mesh structure  $h$ , the thicknesses of the spiral  $\delta_s$  and annular ribs  $\delta_c$ , the distances between the spiral ribs  $a_s$  (along the normal to the axis of the rib) and between the annular ribs  $a_c$  and the angle of inclination of the spiral ribs  $\varphi$  (relative to the generatrix).

The optimization results for the mass of the mesh-shaped cylindrical structure are shown in Table 1. The number 1 indicates the results of the design calculation for analytical dependencies on the choice of optimal parameters under the action of compressive loads [30], number 2 - the results of the calculation of parameters by the method of search by known analytical dependences [1]. Numbers 3 and 4 are the results of applying the algorithm under consideration with constraints [1] for a fixed and variable number of pairs of spiral edges, respectively. The discrepancy between the results can be explained by the fact that, for optimal design, the value of the width of the section of the annular rib was replaced by its expression from the annular rib sectional area (the calculation was made from the applied load and the tensile strength).

Areas of the plane constraints for fixed values of the geometric dimensions of the fin sections of the net structure and the topological structure of the grid model are shown in Figures 5 and 6. In Figure 6, a gray area indicates the range of acceptable values. The optimum is at the point with the coordinates (38.5, 49), taking into account the antigradient of the objective function. The dominant limitation in this case is the local stability of the structure.

**Table 1.** Optimal design results

Number	Results of optimization calculation						Stress-strain state	
	$\varphi$	$m_h$	$h$ (mm)	$\delta_h$ (mm)	$\delta_c$ (mm)	Volume V, (m <sup>3</sup> )	$\sigma_{\min}$ , (kgf/mm <sup>2</sup> )	$\sigma_{\max}$ , (kgf/mm <sup>2</sup> )
1	24.5	32	21	6	3	429.9	-26.4	7.1
2	36	32	14	4	1	665.,9	-67.9	50.8
3	39	32	4	8	3.5	130.9	-70.4	55.33
4	38.5	49	4	5.5	3	177.8	-44.8	39.5

**Figure 5.** Topological structure of a lattice cylindrical shell**Figure 6.** Area of admissible values

1 - restrictions on strength, 2 - restrictions on general stability,  
3 - restrictions on local stability

## 5. Conclusions

The developed method and the algorithm of the elastic simplex are applicable to the solution of the problem of optimal design of mesh designs of an irregular edge structure with a large number of constraints. The algorithm can use a lot of different quantitative design parameters, which allows taking into account the mutual effect of the parameters on each other. This allows us to apply this algorithm to solve problems of optimal design of mesh structures in construction and engineering.

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