Effect of Gypsum Cement Composition on Tensile Fracture Initiation from a Circular Hole in Compression

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Abstract. The presence of dihydrate gypsum in the original composition of cement plays a significant role in the formation of the microstructure of gypsum in the process of hydration. Any variation in the microstructure not only changes the material's mechanical properties, but it also alters the deformation and fracture behaviour. The effects of the original composition on the initiation and propagation of the tensile cracks from the circular holes of various diameters in gypsum under compression have been studied. Test results have been analyzed in regard to the conventional, non-local, and stress gradient fracture criteria. It has been evidenced that, if the content of dihydrate gypsum in the original composition is low, then tensile crack initiation is equally well described by applying either the stress gradient criterion or the non-local criterion under consideration. However, if a dihydrate gypsum share is significant, then the application of the stress gradient criterion is preferable

1. Introduction

The mechanical properties of heterogeneous materials, including geomaterials (concrete, gypsum) and rocks, are subjected to the size effect, i.e. significantly depend on a loaded volume. This size effect is most evident under stress concentration when the effective loaded volume is defined by the stress concentration zone; the size of which is insignificant compared to the typical dimensions of a deformable body. Here, the application of the conventional approach to the strength design problem that consists in the comparison of internal stresses that are occurring in a deformable body with some critical values is restricted. As conventional fracture criteria do not incorporate the size effect, the non-local and stress gradient criteria gained acceptance as an alternative. The foundations of the non-local criteria were laid in the works of Neuber [1], Novozhilov [2], Lajtai and Nesetova [3, 4], Whitney and Nuismer [5]. Numerous works have been published recently that focused on improving the calculation methods based on the non-local and stress gradient criteria of fracture [6–13].

The introduction of the intrinsic material length defined by its microstructure is common for non-local criteria and allows for the prediction of notch size effects, thus expanding the area of their application as compared to the conventional criteria. Unlike the calculated parameters of the fracture criterion (the maximum value of the equivalent stress and the size of the stress concentration zone), the intrinsic material length is a phenomenological parameter and is defined based on the best fitting of the experimental data on the fracture of notched specimens with the calculation results. The question of how this length is associated with the content, microstructure, and probably some other parameters of a real material has not yet been studied.

This work deals with the effects of the original composition of gypsum cement on the initiation and propagation of tensile cracks from the circular holes of various diameters in gypsum under uniaxial compression. Test results were analyzed regarding the conventional, non-local, and stress gradient criteria of fracture.

2. Material and experimental details

Gypsum mortar that was prepared from a water solution of gypsum cement was used to make the test specimens. The mortar was prepared using the common technique. The cement was produced in the factory by manufacturing technologies, including thermal processing and grinding of natural gypsum rock, with dihydrate gypsum becoming hemihydrate. The gypsum cement thus obtained may contain a certain portion of dihydrate gypsum that did not transform into the hemihydrate state under the thermal treatment or was formed following the thermal treatment while being stored as the cement. On mixing of the gypsum cement, the dihydrate gypsum crystals do not participate in the hydration reaction and practically act as fillers. Dried specimens made from gypsum of various original compositions (with various percentage of hemihydrate and dehydrate gypsum in the cement) have identical chemistry with certain variations of the microstructure depending on the quantity of fillers. This should affect the material's mechanical properties as well as its deformation and fracture behavior.

It is not difficult to calculate the share of hemihydrate gypsum, having defined (after the complete drying of a specimen) the water quantities necessary for hydration. As a result, two specimen parties had been made: one – from gypsum cement with a high (greater than 90%) content of hemihydrate gypsum (gypsum 1) and another – from gypsum cement with a low (60–70 %) content of hemihydrate gypsum in the original composition (gypsum 2).

The formation of tensile cracks in stress concentration zones was examined on specimens with circular holes of various diameters subjected to uniaxial compression. The specimens were made of 30–35 mm thick 200x200 mm square plates. The hole diameter ranged from 3.5 to 25 mm. The tests of the specimens with the same diameter holes included 5 to 19 specimens. Graphite electric conductivity (EC) sensors had been applied to the hole boundary. Fracture initiation had been determined by analyzing the conductivity variations during loading. The experimental technique was previously [14] described.

Figure 1 shows typical EC diagrams obtained while testing the specimens, where a sudden initiation and unstable propagation of long tensile cracks has been illustrated. Two sensors were placed symmetrically in the upper (1) and lower (2) parts of the hole boundary. The diagram contains four characteristic plots marked as I–IV. The moment of a sudden fracture initiation accompanied by a typical click was identified on the border between plots II and III. Initiation of the macro-crack followed the formation of the damage zone (plot II) where the microscopic cracks had risen and grown.
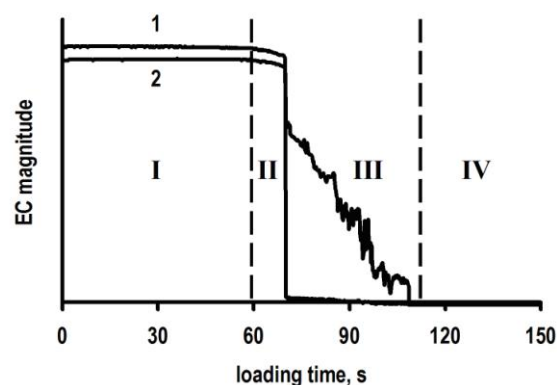

Figure 1. EC diagrams of sudden formation of tensile cracks (1 – upper sensor; 2 – lower sensor).

The typical EC diagrams are given in figure 2. Resulting from specimen testing, they reveal that the crack initiation and propagation to be gradual. The diagrams lack marked borders between plots II and III that are associated with the moment of macro-cracks initiation. In such cases, the EC diagrams define the moment of crack initiation at the border between plots I and II. Obviously, the value of the applied stress at this moment is the minimum value of the critical stress complying with the macro-crack initiation.
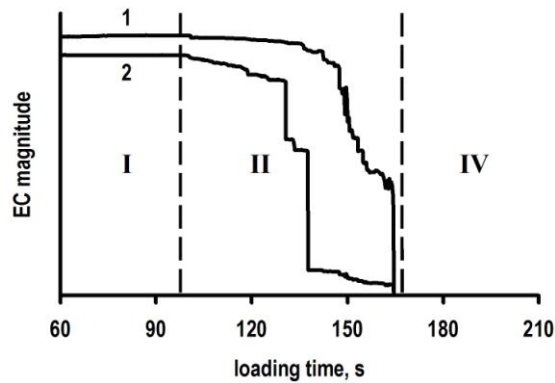

Figure 2. EC diagrams of gradual formation of tensile cracks (1 – upper sensor; 2 – lower sensor).

3. Theoretical approaches

The average stress criterion is most commonly encountered from non-local criteria

$$\langle \sigma_e \rangle_d < \sigma_0, \tag{1}$$

where $\langle \sigma_e \rangle_d$ is the value of the equivalent stress averaged over a distance d in the weakest section:

$$\langle \sigma_e \rangle_d = \frac{1}{d} \int_0^d \sigma_e(x) dx. \tag{2}$$

The ultimate stress σ_0 indicates standard mechanical properties and is believed to be a material constant. The distance of averaging d is taken as a material constant. The necessity of averaging stresses is connected, first of all, with the formation of the damage zone where stress redistribution takes place. In the case of tensile fracture, the equivalent stress is determined by the normal tensile stress. The distribution of the normal stress σ_y along the line of loading (axis x) is presented as follows [15]:

$$\sigma_y = \frac{p}{2} \left(3 \frac{a^4}{x^4} - \frac{a^2}{x^2} \right), \tag{3}$$

where p is the applied compressive stress (pressure), and a is the hole radius. As equation (3) shows, the stress σ_y asymptomatically tends to zero, and with a decrease of the hole diameter l, the average stress also approaches zero, while the critical value of the applied stress p_c, wherein tensile cracks initiate from the hole, accordingly, approaches infinity. In fact, this value is obviously restricted by the material's ultimate compressive strength C_0, whence it follows that, a critical value of the hole diameter $l = l_c$ exists, below which tensile cracks do not occur. In other words, with $l \leq l_c$, the material does not feel the presence of the hole. This conclusion agrees with the current understanding of the real solid as containing primary, inherent defects.

We obtain the critical stress by substituting equation (3) into equation (2) with the range of integration $[a, a + d]$ and equating the result of the integration to σ_0:

$$p_c = \chi C_0 \frac{(1 + 2d/l)^3}{1 + d/l}, \quad l > l_c.$$ (4)

Here, $\chi = \sigma_0 / C_0 = T_0 / C_0$.

Gonano [16] suggested that for gypsum and some rocks, an increase of strength is not associated with stress redistribution but is due to a real rise of local fracture stress. Based on this hypothesis, the stress gradient criterion has been proposed [8], and in compliance with such, the local strength of the material is supposed to be dependent on the size of the zone of stress concentration L_e. If the size L_e is large enough compared to the sizes of the material's microstructural features, such as the grain size or pore size, the value of the local strength insignificantly differs from the value of the ultimate stress σ_0, estimated under the condition of uniform stress distribution. On the contrary, if L_e is comparable with the characteristic size of the microstructural features L_0, their effect on the local strength becomes significant. Moreover, the smaller the size L_e in reference to the size L_0, the greater the effect. Thus, the local strength of the material does not simply depend on the size of the zone of stress concentration L_e, but it is linked with the ratio L_0 / L_e, characterizing the scale in the problem under consideration. In view of this, the condition of the local strength may be presented as follows:

$$\sigma_e < f(\sigma_0, L_0 / L_e).$$ (5)

The function of the local strength is for the problem of tensile fracture initiation under compression [10]:

$$f(\sigma_0, L_0 / L_e) = \sigma_0 \left(1 + \left(\frac{L_0}{L_e} \right)^n \right),$$ (6)

where

$$L_e = \frac{\sigma_e}{|\text{grad}\,\sigma_e|}.$$ (7)

Correspondingly, the critical stress at which tensile cracks initiate from the hole is defined by the expression:

$$p_c = \chi C_0 \left(1 + \left(\frac{L_0}{L_e} \right)^n \right).$$ (8)

4. Results and discussion

4.1. Experimental study of tensile crack initiation and propagation

4.1.1. Gypsum 1. The formation of tensile cracks was sudden for all of the considered diameters of holes, with the crack length extending 5–6 cm at the moment of the crack initiation. An increase in the hole diameter led to a decrease of the critical stress at the moment of the crack initiation, which had been defined by type 1 EC diagrams (figure 1). Further propagation of the crack resulted in the failure of the specimen by being split into two parts.

4.1.2. Gypsum 2. The character of the tensile crack formation differed as compared to that of small and large holes. Cracks formed suddenly from the small-diameter holes (up to 5 mm inclusive) with an initial length of 5–6 cm. Cracking occurred when the stress-strain curve showed the maximum stress, and its value practically coincided with the value   defined on the plane specimens without holes. The initiation and propagation of the cracks from the large-diameter holes (more than 5 mm) were gradual, and with the formation of remote cracks, their opening diminished and they ceased to grow. They did not affect the further failure of the specimen. The typical EC diagrams are given in figure 2. The decrease in the critical stress value with an increase of the hole diameter has been evidenced.

Thus, alterations in the material's microstructure associated with the presence of abundant filler (dihydrate gypsum) in the original composition of the gypsum cement lead to an increase in the material resistance to the formation of primary (propagating from the hole) tensile cracks.

4.2. Critical stress calculations by various criteria

4.2.1. Average stress criterion. Figure 3 shows the experimental data (dark points) on the applied pressure at the tensile fracture initiation from the hole against the hole diameter obtained from the specimens of gypsum 1 as well as the results of calculating the critical stress (curve 1) by equation (4). The dashed line is calculated according to the conventional approach. Here, the experimental data (light points) and calculation results (curve 2) for gypsum 2 are also given. Figure 3 illustrates a significant hole-size effect on the critical stress. Such a behavior is qualitatively described by the average stress criterion. However, a good quantitative correspondence of the calculated and experimental data has been obtained only for gypsum 1.
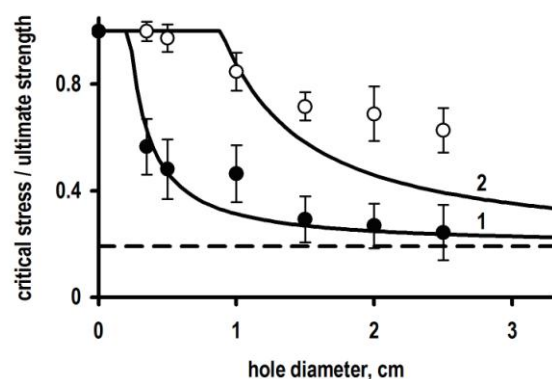

Figure 3. Relationship of critical stress with hole diameter. Calculated by average stress criterion.

4.2.2. Stress gradient criterion. Figure 4 presents the experimental data (dark points) on the load value at the moment of the tensile fracture initiation from the hole in the relationship with its diameter obtained from the specimens of gypsum 1 as well as the results of calculating the critical stress (curve 1) performed by equation (8). The dashed line is calculated in accordance with the conventional approach. Also, the experimental data (light points) and calculation results (curve 2) for gypsum 2 are given. The experimental data are well described by the stress gradient criterion.
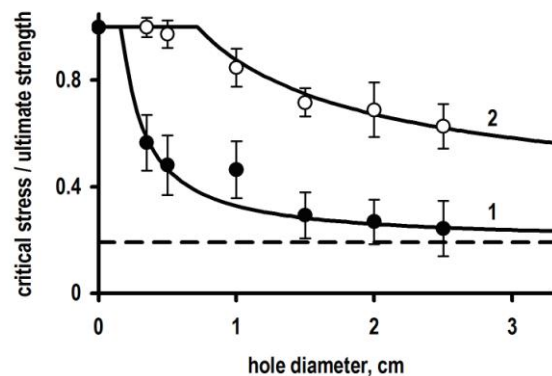
Figure 4. Relationship of critical stress with hole diameter. Calculated by stress gradient criterion.

5. Conclusions

From experimental results, it has been determined that peculiarities in the microstructure formation of gypsum of various compositions significantly affect the initiation and propagation of tensile cracks from the circular holes under uniaxial compression. In particular, a decrease in the share of hemihydrate gypsum in the original composition results in a lowered sensitivity to the stress concentration and an increased critical stress at the moment of tensile crack initiation from the hole. Conditions of loading have certain effects on the fracture behavior. Under similar loading conditions, tensile fracture initiation in the stress concentration zone is well described by the stress gradient criterion. In reference to some materials (gypsum 1), the non-local criterion of average stress similar to the stress gradient criterion can be successfully applied to describe the hole size effects on the tensile fracture initiation. However, its application to other materials (gypsum 2) makes it possible to obtain only a qualitative assessment of the critical stress.

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