

# Stress-Strain State of Steel Structural Details Under Cyclic Loading

O V Mkrtychev<sup>1</sup>, A E Mkrtychev<sup>1</sup>

<sup>1</sup>NRU MGSU, Yaroslavskoye highway, 26, Moscow, 129337, Russian Federation

Email: mkr.artur@gmail.com

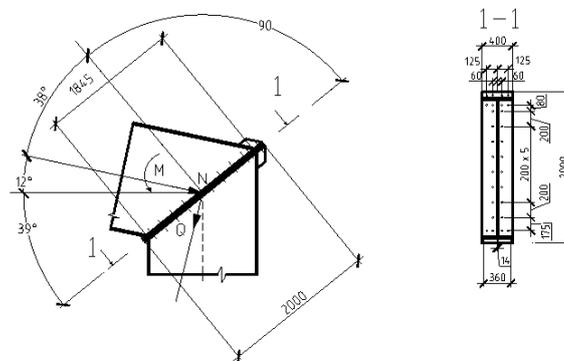
**Abstract.** Article shows general principles of elastic plastic continuum mechanics, solid body dynamics and time integration methods. The methods for finite element modeling of steel structure details with high strength bolts and their computation using time domain integration under cyclic loading are considered with due regard to various types of nonlinearities and strain rate effects. Comparison of nonlinear static and nonlinear dynamic dynamics methods that implement explicit and implicit time integration methods of dynamic equilibrium equations is made.

## 1. Introduction

When designing, constructing and carrying out verification calculations, in some cases, it becomes necessary to perform the calculation of a detailed model of steel structural detail for cyclic and vibration effects.

Calculation of flange connections with high-strength bolts is of great interest. The stress diagram in detail can be alternating or of the same sign. In flange connections with high-strength bolts that have an alternating stress diagram, for correct modeling, it is necessary to take into account bolt tension, contact conditions between flanges, interaction of bolt heads with the flange surface, as well as the material dynamic hardening, depending on strain rate.

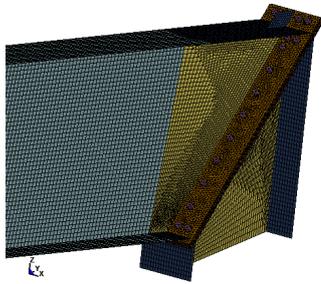
Consider the cornice frame node (Figure 1), main parameters of which were selected in [1]. Let us investigate this detail's operation using the LS-Dyna computational complex, in which nonlinear static and nonlinear dynamic calculation methods are realized.



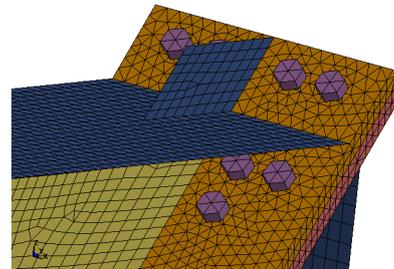
**Figure 1.** The scheme for applying forces and the stress diagram in the flange.



The flanges were modeled by the second order volume pyramidal finite elements with rotational degrees of freedom in the nodes. Rack's shelves and walls and frame's crossbar, forming the detail – by the shell's spatial finite elements. The bolt heads were modeled by volume elements. The flange thickness was adopted to be 30 mm [1].

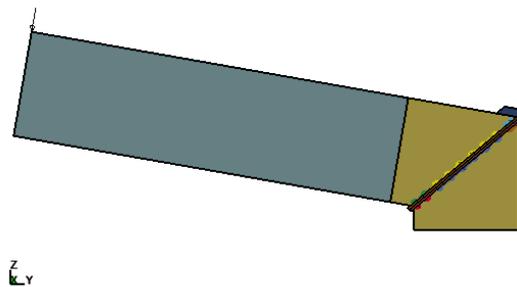


**Figure 2.** The model of the cornice detail (together with the part of the crossbar)



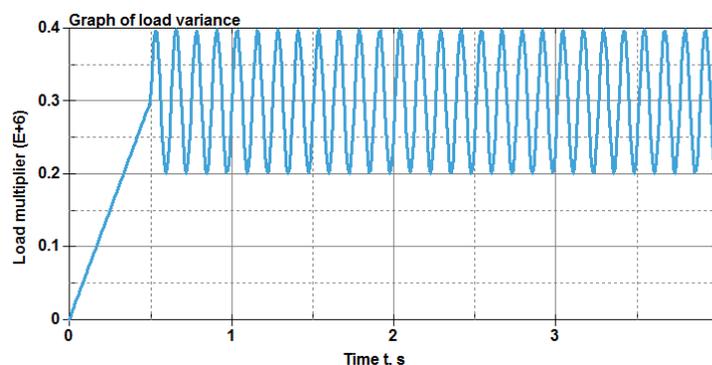
**Figure 3.** Model of bolts, detail's wall and booms.

The tension of bolts was carried out using the selected temperature values. Between the flanges, a one-way frictional contact was established using the penalty function algorithm. The contact points of bolts and outer surfaces of flanges were modeled by rigid nodal bodies.



**Figure 4.** A scheme for applying a cyclic load to the detail.

Before applying a cyclic load, the thermal load was applied to the bolt bodies to set the bolts tension. Load was applied in two stages: during the first stage – increasing from zero to the final value, and during the second stage – according to the harmonic law. Point of load application and direction is shown in Figure 5. The cyclic loading graph is shown in Figure 4. The design load is applied to the structure after setting the tension of bolts.



**Figure 5.** Graph of load variance over time. The ordinate shows the values of load multiplier  $n$  to its target value.

Let us consider three methods of calculation: nonlinear static, nonlinear dynamic (implicit scheme), nonlinear dynamic (explicit scheme). At the same time, physical, geometric, and constructive nonlinearities were taken into account.

## 2. Method for solving nonlinear static problems

In the LS-Dyna computational complex, the Newton method and its derivatives are implemented for solving nonlinear static problems. The required vector of nodal displacements from the system of motion equation  $u_{n+1}$  is found at the next time step from the condition:

$$K_t(u^n)\Delta u_0 = P(u^n)^{n+1} - F(u^n),$$

where  $K_t(u^n)$  is a positive definite stiffness matrix at a timepoint n (nonphysical time is implied);

$\Delta u_0$  - the desired increment of displacements;

$P(u^n)^{n+1}$  - the vector of external loads at a timepoint n+1 with geometry at a timepoint n;

-  $F(u^n)$  is the vector of internal forces at a timepoint n

The displacement vector is calculated in the next step:

$$u_1^{n+1} = u_0 + s_0 \Delta u_0;$$

The iterative process consists in computing the vector Q:

$$Q_i^{n+1} = K_{tj}(u_i^n)\Delta u_0 = P(u_i^n)^{n+1} - F(u_i^n)$$

where the index i denotes the iteration number,  $i < j$ , the parameter  $s_0$  takes a value from 0 to 1. After each iteration, the convergence is checked, which is considered achieved when the following conditions are met:

$$\frac{\|\Delta u_i\|}{u_{\max}} < \varepsilon_d \quad \text{and} \quad \frac{\|\Delta u_i^t Q_i\|}{\|\Delta u_0^t Q_0\|} < \varepsilon_e$$

If the convergence conditions are not satisfied, then the desired displacement vector

$u_{i+1}^{n+1} = u_0 + s_i \Delta u_i$  is updated and the iteration is repeated. If the discrepancy is observed within a

given number of iterations, then the stiffness matrix  $K_t$  is recalculated. If a discrepancy is observed within a given number of stiffness matrix reformation, the calculation is interrupted.

## 3. The method for solving nonlinear dynamic problems using implicit schemes of motion equation integration.

To solve dynamic problems by a direct method, the Newmark method is applied implementing an implicit scheme of time integration:

$$M\ddot{u}_t + C\dot{u}_t + Ku_t = f_t^a,$$

where  $M$  is the diagonal mass matrix;

$C$  is the dissipation matrix;

$K$  is the stiffness matrix;

$f$  is the vector of external loads;

$u$  is the vector of nodal displacements.

The classical Newmark method is based on the decomposition of displacements  $u(t_i + \delta t)$  and velocities  $\dot{u}(t_i + \delta t)$  in power series by  $\delta t$  in the point vicinity  $t_i$ . To calculate the velocities and accelerations in the next step by time  $t_{i+1} = t_i + \Delta t$ , we obtain the following expressions:

$$\dot{u}_{i+1} = \Delta u / \gamma \Delta t + (1 - 1/\gamma) \dot{u}_i + (1 - 1/2\gamma) \ddot{u}_i \Delta t,$$

$$\ddot{u}_{i+1} = \Delta u / \alpha \Delta t^2 - \dot{u}_i / \alpha \Delta t + (1 - 1/2\alpha) \ddot{u}_i$$

Substituting these expressions into the motion equations, we obtain the basic system of equations for calculating the vector  $u_{i+1}$ :

$$A \times u_{i+1} = b_{i+1},$$

where:

$$\ddot{u}_{i+1} = 1/\alpha \Delta t^2 M + \dot{u}_i / \gamma \Delta t C + K$$

$$b_{i+1} = f_{i+1} + M \cdot \left( \Delta u / \alpha \Delta t^2 + \dot{u}_i / \alpha \Delta t + (1/2\alpha - 1) \ddot{u}_i \right) + \\ + C \cdot \left( \Delta u / \gamma \Delta t + (1 - 1/\gamma) \dot{u}_i + (1/2\gamma - 1) \ddot{u}_i \Delta t \right)$$

In the expressions, there are parameters of integration  $\alpha$  and  $\gamma = \alpha/\beta$ .

The solution is unconditionally stable when  $\alpha \geq 0,25 \cdot (\beta + 0,5)^2$ ,  $\beta \geq 0,5$

The following parameter values are accepted  $\alpha = 0,25$ ,  $\beta = 0,5$

If the problem is linear, then the matrix  $A$  containing the initial stiffness matrix  $K$  should be inverted only once. Otherwise, it is required to perform the procedure for checking the convergence at each step and, if necessary, to reduce the time step or to transform the stiffness matrix.

#### 4. The method for solving nonlinear dynamic problems using explicit schemes of motion equation integration.

To solve nonlinear dynamic problems, the method of central differences is applied in the calculation, which implements an explicit scheme of motion equation integration. To determine the displacement, an expression with time lag is used:

$$M \ddot{u}_t + C \dot{u}_t + K u_t = f_t^a,$$

where  $M$  is the mass matrix;

$C$  is the dissipation matrix;

$\mathbf{K}$  is the stiffness matrix;

$\mathbf{f}$  is the vector of external loads;

$\mathbf{u}$  is the vector of nodal displacements.

The peculiarity of explicit methods is that nodal accelerations and velocities are included in the calculation as unknowns (in the number of nodal degrees of freedom) and are calculated directly, and not by numerical differentiation of displacements.

Explicit methods use recurrence relations that express the displacements, velocities, and accelerations at a given step through their values in the previous steps.

Acceleration vector:

$$\mathbf{a}_t = \mathbf{M}^{-1} (\mathbf{f}_t^{ext} - \mathbf{f}_t^{int}),$$

where  $\mathbf{f}_t^{ext}$  is the vector of applied external and volume forces;

$\mathbf{f}_t^{int}$  is the vector of internal forces.

In the particular case:

$$\mathbf{f}_t^{int} = \sum \left( \int_{\Omega} \mathbf{B}^T \sigma d\Omega + \mathbf{f}_t^{cont} \right)$$

where  $\mathbf{B}$  is the deformation-displacement matrix;

–  $\sigma$  is the stress vector;

–  $\mathbf{f}_t^{cont}$  is the vector of contact forces

The velocity and displacement vectors in the corresponding time step are defined as follows:

$$\mathbf{u}_{t+\Delta t} = \mathbf{u}_t + \mathbf{v}_{t+\Delta t/2} \cdot \Delta t_{t+\Delta t/2}$$

where the velocity vector  $\mathbf{v}_{t+\Delta t/2} = \mathbf{v}_{t-\Delta t/2} + \mathbf{a}_n \cdot \Delta t_t$  is calculated at an intermediate timepoint  $t + \Delta t / 2$ :  $\Delta t_{n+1/2} = 0,5 \cdot (\Delta t_n + \Delta t_{n+1})$ .

In the case of using a diagonal mass matrix, it is possible to calculate the inverse matrix, thereby simplifying the calculation and decreasing the time of one iteration. This shows that explicit methods are not connected with the solution of systems of algebraic equations. The most laborious operation is the calculation of the internal force vector  $\mathbf{f}_t^{int}$ , which takes into account all types of nonlinearities.

## 5. Allowance for strain rate effects

To account for the kinematic hardening characteristic of metals, the Cooper-Simonds model was used. In this case, an elastoplastic model of deformation is used. The dynamic yield point without taking into account the temperature dependence was calculated from the expression:

$$\sigma_d = R_y (1 + \dot{\epsilon}/C)^{1/P}$$

where  $R_y$  is the static yield point;

$\dot{\epsilon}$  is the deformation rate;

$C$  and  $P$  are parameters describing the nature of dynamic hardening.

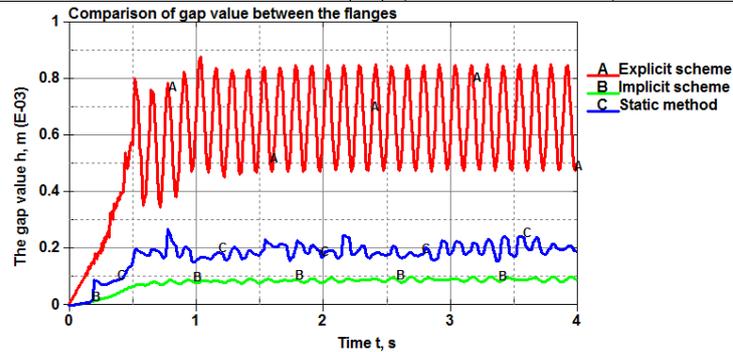
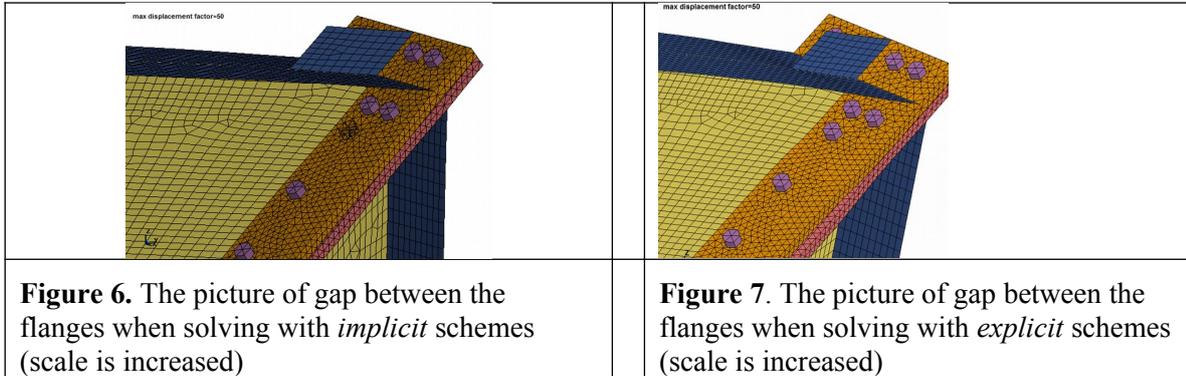
The material parameters used in the problem solution are given in Table 1.

**Table 1** Material Characteristics.

	$R_y$ (MPa)	$C$	$P$
<b>C 245 (column and crossbar)</b>	240	80	4
<b>C345 (flanges)</b>	320	80	4

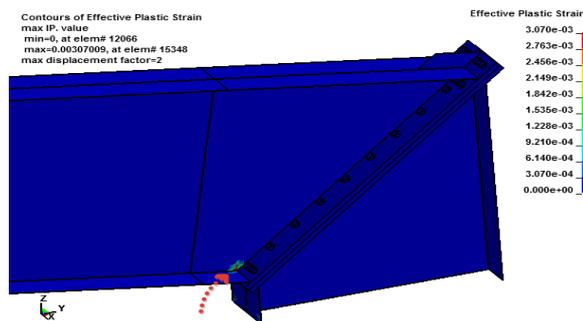
The following is a comparative analysis of the results of the study.

In the upper part of the detail, in the zone of tension, one can observe the gap between the flanges with an increased scale of displacements. As can be seen from Figures 6 and 7 and graphs (Fig. 8), there is a difference in the value of gap opening when solving a problem by using explicit and implicit schemes.

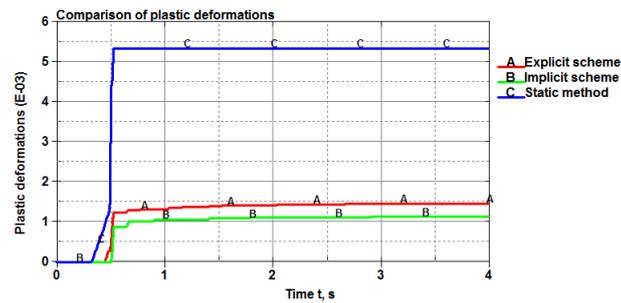


**Figure 8.** Graphs of the gap value h when calculated using different methods

The most loaded detail area is the joint of the wall and the lower compressed crossbar flange, in which plastic deformations develop (Figure 9). Figure 10 shows the graphs of the nonlinear deformations development.

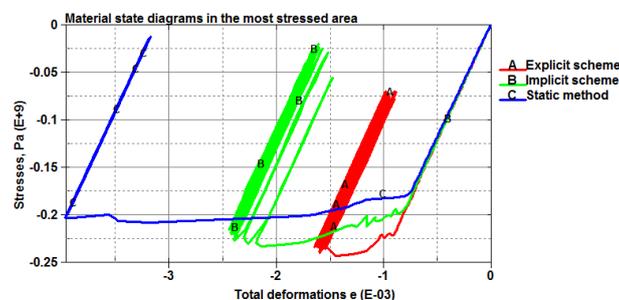


**Figure 9.** Intensity isofields of plastic deformations  $e_{pl}$



**Figure 10.** Comparison of the intensity development graphs of plastic deformations

When accounting for the strain rate effects, as well as inertial forces and viscous damping, the magnitude of plastic deformations occurring in the elements is several times lower. The stepwise increase and asymptotic deformation approximation to a certain value are also clearly noticeable when solving the problem in a dynamic formulation.

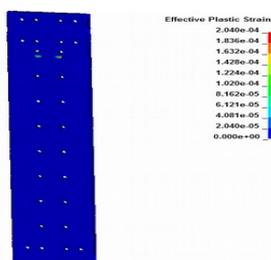


**Figure 11.** Comparison of the material state diagrams in the joint of the wall and the lower compressed crossbar boom

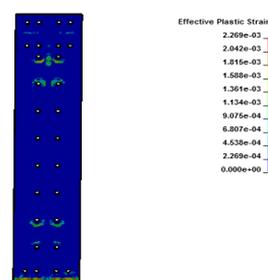
Figure 11 shows the material state diagrams, on which the effect of dynamic hardening and dynamic yield point is clearly visible.

When solving the problem in a dynamic formulation by the Newmark method, plastic deformations in the flange are not observed.

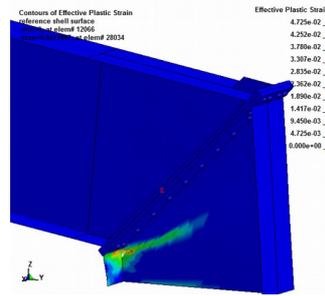
The results of the analysis show that the solutions obtained using static and dynamic methods of calculation differ. It should be noted that the use of explicit methods for motion equation integration allows to obtain stable solutions at large deformations and displacements, which allows to investigate the failure behavior and to determine the actual load-bearing capacity of the detail elements. In this case, depletion of the unit's bearing capacity occurs as a result of destruction of the joint zone of the compressed boom and the column wall when load increment goes above the design values.



**Figure 12.** Intensity isofields of plastic deformations in the flange when solving by using the *explicit method*



**Figure 13.** Intensity isofields of plastic deformations in the flange when solving the static problem



**Figure 14.** The intensity of plastic deformations  $e_{pl}$  (the detail destruction pattern)

## 6. Conclusions:

- The method of modeling the steel structure details, in particular, flange joints, is developed in the article. An approach is considered in which the contact surfaces of nodes, in particular flanges, are modeled by volume elements, and the detail elements that are not subject to contact – by using the spatial elements of the finite shell elements.
- The problem was solved in nonlinear static and dynamic formulations, which allows to calculate structure details under dynamic effects (seismic [2-8], wind, shock, etc.) with due regard to the development of nonlinear deformations. The solutions of nonlinear static and dynamic problems have been compared.
- The proposed method allows to simulate the steel structure details with the necessary degree of detailization.
- When calculating dynamic loads of junctions of steel elements undergoing large deformations and displacements, it is expedient to use implicit schemes of time integration.
- With the use of explicit schemes, it is possible to investigate the failure behavior and determine real assurance coefficients of detail bearing capacity. At the same time, elements with a sufficiently small side or edge size should be used to model structure detail, it is required to choose a very small integration step in time to obtain a stable solution in accordance with the Courant-Levy criterion.
- When the strain rate hardening is taken into account, the yield point depends on the strain rate and is somewhat higher than the static one. Accounting for inertial forces, viscous damping, as well as strain rate hardening reduces the conservatism of the solution obtained.

## 7. References:

- [1] "On the problem of calculating the flange joints for strength under an alternating stress diagram". "Promyshlennoe i grazhdanskoe stroitelstvo", 2009/2.
- [2] Investigation of a large-span structure for reliability under accidental seismic actions. V.L. Mondrus, O.V. Mkrtychev, A.E. Mkrtychev. (MGSU Harbinger, 2012/5.)
- [3] A.P. Falaleev 2012 *Modeling of plastic deformations of two-phase steels of body elements*. (UDK 629.113.011, 2012.)
- [4] I.N. Borodin 1, A.E. Mayer 2, Yu.V. Petrov 1,3, A.A. Gruzdkov 2014 *Maximum yield point under quasistatic and high-speed plastic deformation of metals*. Solid state physics, vol. 56, no. 12.
- [5] J. Klemenc 2016 *Estimating the Strain-Rate-Dependent Parameters of the Cowper-Symonds and Johnson-Cook Material Models using Taguchi Arrays*. Andrej Škrlec. University of Ljubljana, Faculty of Mechanical Engineering, Slovenia.
- [6] D. R.Abashev 2016 *Development of the elastoplastic deformation model, fatigue criteria and methods for identifying the material parameters of structural alloys*. Korolev.
- [7] Compressive Behavior of AISI-416 Stainless Steel at Different Rates of Loading. Ajay K.

Behera, Nilamber K. Singh, and Maloy K. Singha. Ajay K. Behera, Nilamber K. Singh, and Maloy K. Singha

- [8] D B Solovev 2015 Instrument current transducers with Rogowski coils in protective relaying applications *International Journal of Electrical Power and Energy Systems* vol. 73 pp. 107-113. [Online]. Available: <http://dx.doi.org/10.1016/j.ijepes.2015.04.011>.

### **Acknowledgements**

This study was performed with the support of the RF Ministry of Education and Science, grant №7.1524.2017/Project Part.