Concrete Bi-Modulus Analysis in the Estimation of Stress State of the Normal Sections of Reinforced Bending Element

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Abstract. The article discusses the theory and research of bimodularity of concrete. Noted the results of the works of Professor Volkov, Professor Kakhtzazov, Professor Osize, Professor Belutskii. The peculiarities of work of flexible elements taking into account the differences in the elastic modul in compression and tension. The coefficient of reduction depending on the modulus of elasticity of concrete under compression, tension and bending and the expression for determining the position of the neutral line are determined. Expressions for determination of normal and tangential stresses for compressed and stretched zones with different elastic modules are given. The results of an experiment conducted at the Pacific national University are noted. Consideration of the bimodality of concrete in prestressed structures under the assumption of section integrity is of scientific interest.

1. Introduction

Existing standards and regulations for calculating and designing concrete and reinforced concrete structures assume compression and tension work with the same modulus of elasticity. In this case, the same coefficients of bringing steel to concrete in the compressed and stretched zones are naturally used, and when determining the geometrical characteristics of the cross section the hypothesis of the homogeneous isotropic body is considered to be fully correct. However, in the theory of concrete which is an extremely heterogeneous material, taking these hypotheses as baselines does not always provide strict conformity of practice with theoretical suppositions. This discrepancy generates a variety of coefficients, and most of the mare empirically based. Their purpose is to reconcile the actual and theoretical results.

While evaluating the role of concrete internal structure, G.F. Volkov [1] comes to the conclusion that the deformation characteristics of concrete under compression are determined by the elastic properties of the filler, and when it is stretched - by the elastic properties of the cement stone. Generally the properties mentioned above are not equal. This can be seen in Table 1, introduced by A.L. Kakhtzazov [2], where the values of the elastic moduli for compression (E_{bc}) and the values of elastic moduli for bending (E_{bu}) are given which integrate the work of concrete under compression and tension in a bending element.

Proffesor Kakhtzazov obtained various deformative properties of concrete for several brands. Table 1 shows elastic moduli under compression E_{bc} and the moduli value including the work of concrete under compression and tension at bending E_{bu} are given.

When the tabulated values of E_{bc} and E_{bu} are compared (Table 1), the effect of the stretched zone on the integral magnitude of the elastic modulus E_{bu} becomes obvious.

Table 1. Relationship between concrete brand and elastic moduli under compression and bending

Concrete elastic modulus, kgf/cm^2 (MPa)	Concrete brand					
	350	300	250	200	170	140
E_{bc}	360000 (35280)	340000 (33320)	320000 (31360)	290000 (28420)	260000 (25480)	240000 (23520)
E_{bu}	225000 (22050)	210000 (20580)	200000 (19600)	180000 (17640)	160000 (15680)	140000 (13720)
m_c	2,3	2,37	2,15	2,38	2,42	2,44

Osidze's studies [3] also indicate the difference in the elastic modulus of concrete in the process of compression and tension. According to the theoretical calculations [4], dealing with the peculiarities of bending elements work, taking into account the difference in the moduli of elasticity during compression and tension, and considering the position development in the process [4], analytical relationship between the elastic moduli at compression E_{bc}, E_{bp} tension and bending E_{bu} was obtained [5].

$$E_{bu} = \frac{4E_{bc}}{1+\sqrt{m_c}^2} \tag{1}$$

With the reduction coefficient m$_c$ that is equal to the ratio of the elastic moduli

$$m_c = \frac{E_{bc}}{E_{bp}} \tag{2}$$

After substitution (2) in (1) depending on E_{bc} , E_{bp} , E_{bu} in the form of

$$m_c = \frac{E_{bc}}{E_{bp}} = \ 2\sqrt{\frac{E_{bc}}{E_{bu}}} - 1^{\,2} \tag{3}$$

The structure which allows us to find the coefficient of reduction $m_c=E_{bc}/E_{bp}$ (Table 1) with the help of well-known values of E_{bc} and E_{bu} (Table 1) indicating the inequality of the elastic moduli of concrete under compression (E_{bc}) and tension(E_{bu}) and emphasizes the essential role of the internal structure of concrete in determining its deformation characteristics.

Their ratio in a bending element, in particular, made of concrete ($m_c=(E_c/E_p)>1$ при $E_{bc}>E_{bp}$), determines the non-coincidence of the center of gravity in the section with its neutral axis, which position is determined through the heights of the compressed and stretched zones. We find their values by considering stress state of the normal section with the help of the normal stresses of a bending beam of a rectangular cross-section with a width b and height h, Fig.1.
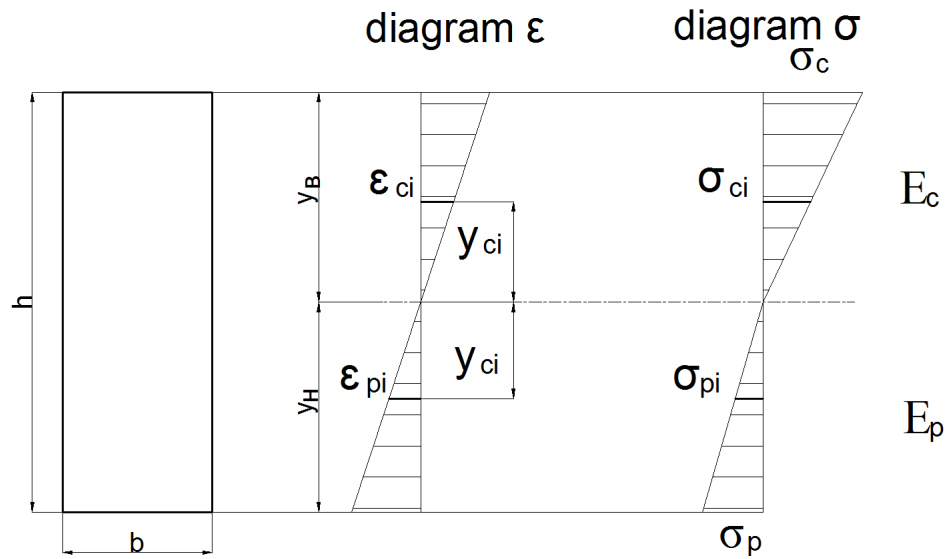
Figure 1. Diagrams of relative deformations and stresses at the operation stage with considering concrete bi-modulus

A distinctive feature of the pattern of normal stresses in the normal section of a bending beam during the beam material's operation in the compressed zone with the one modulus of elasticity E_{bc} and in the stretched zone with another modulus of elasticity E_{bp} that differs from E_{bc}, and as for the concrete bending beam the ratio $E_{bc}>E_{bp}$ is true. A fracture will occur in the pattern of normal stresses in the normal section at the level of the neutral axis.

On this occasion it is interesting to note that even in the period of formation of the theory of reinforced concrete in 1894 E. Coignetand N. Todesco [6] published the paper, where the diagram of the normal stresses was presented with a fracture in the level of the neutral axis, which in fact considered the proposal of the Austrian Melylan on the difference between the elastic moduli of concrete under compression and tension.

The above-mentioned fracture is related to the aspect that the y_{ci} in a condensed area of σ_{ci} stress will be larger than modulus of σ_{pi} stresses in the stretched zone at the level of the $y_{pi}=y_{ci}$.An explanation for this follows from the justice of the law of plane sections. If the levels are equally spaced from the neutral axis in the y_{ci} compressed are and in the y_{pi} stretched zone (Fig. 1) it is obvious that the relative deformation in the ε_{ci} compressed zone and in the ε_{pi} stretched area will be equal as $\varepsilon_{ci}=\varepsilon_{pi}$, and the voltage at designated levels in accordance with the Hooke's law is determined from the expression $\sigma_{ci}=\varepsilon_{ci}E_{bc}$; $\sigma_{pi}=\varepsilon_{pi}E_{bp}$; but as previously established, that $E_{bc}>E_{bp}$, then we can define that $\sigma_{ci}>\sigma_{pi}$.

From the condition of equality of forces of the compressed and stretched zones having the form corresponds that to

$$\frac{1}{2}\sigma_c by_{_в} = \frac{1}{2}\sigma_p by_{_н} \qquad (4)$$

During the change of $y_p=h-y_{_в}$ and reflection of the law of plane sections by the

$$\frac{\sigma_c}{E_{bc}y_{_в}} = \frac{\sigma_p}{E_{bp}y_p} \qquad (5)$$

Follows the quadratic equation in relation to $y_{_в}$ axis

$$y_{_в}^2\left(1-m_c\right)-2hy_{_в}+h^2=0 \qquad (6)$$

With the definition of real root

$$y_{в} = \frac{h}{1 + \sqrt{m_c}} \qquad (7)$$

which points to the mismatch of the position of the neutral axis of the cross section with its center of gravity at $E_{bc} \neq E_{bp}$. It should be noted that the difference between the elastic modules of the compressed and stretched zones in a bending element is considered in work [4] in the evaluation of normal and physical stresses of the compressed and stretched zones.

Considering the obtained value of $y_в$ in (7), the reduced moment of inertia to the material of the compressed zone will be formed as follows:

$$I_{npc} = \frac{by_в^3}{3} + \frac{b\left(h - y_в\right)^3}{3m_c} = \frac{bh^3}{3\left(1 + \sqrt{m_c}\right)^2} \qquad (8)$$

And the normal stress in the limits of compressed and stressed zones defines from the formulas:

$$\sigma_{ci} = \frac{M}{I_{npc}} y \quad npu \quad y \in [0, y_в] \qquad (9)$$

$$\sigma_{pi} = \frac{M}{I_{npc} m_c} y \quad npu \quad y \in [0, h - y_в] \qquad (10)$$

Formulas for the physical stresses in the limits of compressed and stressed zones are:

$$\tau_c = \frac{Q S_c^{omc}}{I_{npc} b} \qquad (11)$$

$$\tau_p = \frac{Q S_p^{omc}}{I_{npc} b m_c} \qquad (12)$$

Where S_c^{omc} and S_p^{omc} are the static moments of truncated part of section, related to the material of compressed and stressed zones.

The rigor and conciseness of these expressions (7), (8), (9), (10), (11), (12) and their obvious adequacy to the usual expressions, characterizing the stressed state of a bending element of rectangular cross-section should be noted.

It is enough to say that in the bending element of the rectangular cross-section at $E_{bc} = E_{bp}$ and $m_c = 1$ the height of the compressed zone of $y_в$ according to (7) is equal to $y_в = 0.5h$ and the neutral axis passes through the middle of the height of the beam, the moment of inertia takes the usual expression $I = bh^3/12$, as well as expressions (9), (10), (11), (12).

According to the author [4], gained formulas in [4], that have been confirmed in Glibovitskii [5] and Belutskii experimental studies, can be used to calculate the elements of wood, plastics and reinforced concrete, while maintaining structural integrity of the sections by the element of reinforced concrete, this clause in relation to the integrity of the structure element is obvious for reinforced concrete elements without pretension, at the same time, for prestressed reinforced concrete elements, for which it is typical to preserve the integrity of the structure of their stress state that can be adequately described by the formulas (9), (10), (11), (12).

Let us refer to the results of the study by professor I. Y. Beletskii in 1972 [5]. Samples (10x10x80cm) of fine-grained concrete in bending were tested. Strain gauges were fixed in the lower and upper fibers of the samples. The ratio of the measured strain increments of the extreme lower fibers to the upper ones corresponds to the ratio of the $E_{bc}/E_{bp} = m_c$ moduli. As a result of processing of the test data, the averaged coefficient of reduction $m_c = 1.48$ was obtained.

In the development of the topical plan of interest to assess the stress state of the reinforced concrete-bending element, we should note that the account of bi-modulus in structures without pre-

stress is not really relevant due to the fact that in accordance with the concept of the calculation method for the limit states, the concrete of the stretched zone is not included in the work in the calculations for the 1st limit state and this is a priority in this method in the current case of calculation.

At the same time, there are fair hypotheses in the calculations of the 2nd group of limit states of structures without prestress of material resistance, and the account of concrete in the stretched zone with the modulus of elasticity different from E_{bc} modulus (namely $E_{bp}<E_{bc}$), will definitely make adjustments to the magnitude of the stress in the stretched zone, will probably change the emphasis in the characterization of fracture toughness of a bending element made of reinforced concrete and maybe, on a more principled basis, will allow us to pay attention to the model of Considera [6] and his hypothesis that increased extensibility of reinforced concrete and to understand the supporters of this hypothesis among the defenders of the ferrocement and steel fiber concrete. It is despite the fact that the concept of bi-modulus and ferrocement, steel fiber concrete, based on the structure of these composites can attain a real basis and practical output in the evaluation of the stress state.

This is not the case related to the prestressed structures, the calculations of which are based on the assumption of the integrity of the cross sections, and the fact that in the stage of operation and in the stage of creating initial stresses, certain zones of a bending element change the signs of stresses, requires its reflection in the calculation models. It is when the principle of independence of forces almost does not allow obtaining an objective picture of the stress state as indicated in the publication [7].

References

[1]   Volkov G F 1962 *The role of the concrete structure and ferrocement Ferrocement and ferrocement constructions* (Moscow:Gosstroiizdat,).

[2]   Kakhtsazov A P 1947 *Bridges on highways*. (Moscow: Dorizdat) p. 420.

[3]   Osidze V I 1965 *Stress-strain modulus of concrete under tension Concrete and reinforced concrete* № 11

[4]   Kulish V I 1970 *Pecularities in beam calculations with different elastic moduli under compression and tension Collection of scientific papers vol. 11* (Khabarovsk Khabarovsk Polytechnic Institute) pp. 28-30.

[5]   Kulish V I, Glibovitskii Y S Belutskii I Y 1971 *Experimental research of hybrid glulam beams Collection of scientific papers vol. 26 chapter General technical science*. (Khabarovsk Khabarovsk Polytechnic Institute) pp. 97-103.

[6]   Lopato A E Loleit A F 1969 *History of domestic reinforced concrete* (Moscow:Gosstroiizdat,). p. 104 .

[7]   Belutsii I Y, Yatsura V G M.I. 2005 *Ryabinovich's concepts of steel reinforced concrete spans as compound beams with antagonistic relations Structural Mechanics and structural engineering* vol. 5. pp. 2 – 7.