

Dynamics of the Localized Pulse in Bubbly Liquid

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Abstract. In nature, the movements of multiphase systems with physico-chemical transformations are widespread. The features of such flows are determined by the interaction of hydrodynamic, thermophysical, and also physicochemical phenomena. The most pronounced multiphase effects are manifested in the propagation of pressure waves in such media. In a liquid with bubbles, the rheological properties of a weakly compressible fluid, which is the carrier phase, change drastically with the addition of gas (in the form of bubbles), which is a dispersed phase. The peculiarity of bubble liquids is due to their high static compressibility while maintaining a high mass density close to the density of the liquid, which in turn leads to a low equilibrium speed of sound. An interesting feature of bubble liquid in dynamic processes is the manifestation of fluid inertia when the volume of a mixture changes due to compression or expansion of bubbles. Anomalously strong compressibility and dissipation of wave energy makes it possible to use bubble curtains of liquid to extinguish shock waves in liquids. To date, one-dimensional waves in a bubble liquid have been studied in sufficient detail, the dynamics of two-dimensional and detonation waves in bubble liquids is being actively studied at the moment. This paper presents the results of studies of the dynamics of a pressure pulse localized in a transverse coordinate in a bubble liquid.

1. Introduction

To date, the one-dimensional wave in a bubbly liquid been studied quite extensively [1-10]. At the moment actively studied two-dimensional waves in bubbly liquids and detonation waves in a bubble liquid [11-23]. In this work we investigate the dynamics of localized pulse in a bubbly liquid.

2. Mathematical model

Consider the two-dimensional movement of the bubble medium with the following assumptions. The mixture is monodisperse, i.e. in each elementary volume of all bubbles are spherical and of the same radius. Acoustically compressible liquid viscosity and thermal conductivity are significant only in the interfacial interaction and, in particular, when bubbles pulsation. No mass transfer between the bubbles and the liquid. Based on these assumptions, we write the law of conservation of mass for each phase, the number of bubbles and pulses in one-speed application, and kinematic dependence [4]:



$$\frac{d\rho_i}{dt} + \rho_i \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, \quad (i=l, g) \quad (1)$$

$$\frac{dn}{dt} + n \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, \quad (2)$$

$$\rho \frac{du}{dt} + \frac{\partial p_l}{\partial x} = 0, \quad \rho = \rho_g + \rho_l, \quad \left(\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right), \quad (3)$$

$$\alpha_l + \alpha_g = 1, \quad \rho_i = \rho_i^0 \alpha_i, \quad \alpha_g = \frac{4}{3} \pi n a^3. \quad (4)$$

Here, ρ_i^0 , α_i , v_i , p_l , n , a – respectively the density, the volume content of the i -th phase, the pressure of the carrier liquid, the number and the bubble radius; u and v – are the projection of the velocity on the coordinate axes x and y respectively.

In the description of radial movement in accordance with the clarification proposed [5], we will assume that the speed of radial movement of w consists of two components:

$$w = w_R + w_A, \quad (5)$$

here w_R is described by the equation of Rayleigh-Lamb:

$$a \frac{dw_R}{dt} + \frac{3}{2} w_R^2 + 4v_l \frac{w_R}{a} = \frac{p_g - p_l}{\rho_l^0}. \quad (6)$$

Additive w_A is determined from the solution of the problem of spherical load-relief on a sphere, a in radius, in a carrying liquid in the acoustic approximation:

$$w_A = \frac{p_g - p_l}{\rho_l^0 C_l \alpha_g^{1/3}}. \quad (7)$$

The equation for pressure inside the bubble given the uniformity of the pressure is written as [4]:

$$\frac{dp_g}{dt} = - \frac{3\gamma p_g}{a} w - \frac{3(\gamma-1)}{a} q, \quad (8)$$

where γ – is the specific heat ratio of the gas; q – is the heat transfer intensity or heat flux from liquid to gas per unit of interfacial area. The intensity of interphase heat transfer will take in the form [4]:

$$q = \lambda_g \text{Nu} (T_g - T_0) / 2a, \quad (9)$$

where $T_0 = \text{const}$ – is the temperature of the liquid; Nu - the Nusselt number. When describing the Nusselt number is given as:

$$\text{Nu} = \begin{cases} \sqrt{\text{Pe}}, & \text{Pe} \geq 100 \\ 10, & \text{Pe} < 100. \end{cases} \quad (10)$$

For the Peclet number we will take the expression:

$$\text{Pe} = 12(\gamma-1) \frac{T_0}{|T_g - T_0|} \frac{a|w|}{\kappa_g}, \quad (11)$$

where $\kappa_g = \lambda_g / c_g \rho_{g0}$, λ_g , c_g – is the temperature conductivity, thermal conductivity and heat capacity of the gas, respectively.

The state equation for the carrier phase will take in the acoustic approximation:

$$p_l = p_0 + C_l^2 (\rho_l^0 - \rho_{l0}^0), \quad (12)$$

where subscript “0” refers to initial, unperturbed state; C_l – is the speed of sound in the liquid.

Considering the caloric perfect gas, we write the equation Clapeyron-Mendeleev:

$$p_g = \rho_g^0 R T_g, \quad (13)$$

where R is the gas constant.

The method of numerical calculation is presented [12].

3. Calculations results

Figure 1-3 show the evolution of the bell over the transverse coordinate and time of the wave momentum in the form:

$$u(0, y) = \Delta u \cdot \exp\left[\left(\frac{t - t_*/2}{t_*/6}\right)^2\right] \cdot \exp\left[\left(\frac{y - L_y/2}{y_*}\right)^2\right], \quad (14)$$

in a homogeneous water-air bubbly mixture. In numerical calculations the parameters of the mixture following: $\alpha_{g0} = 10^{-3}$, $a_0 = 10^{-3}$ m, $p_0 = 0.1$ MPa, $\rho_{l0}^0 = 10^3$ kg/m³, $T_0 = 300$ K, $c_g = 1006$ J/K·kg, $\lambda_g = 2.6 \cdot 10^{-2}$ J/K·s·m. There $\Delta u_0 = 600$ m/s, $t_* = 0.1$ ms, $L_y = 0.1$ m. Note that in [12] outrage in bubbly liquid boundary created pressure, and in this paper, the initial momentum created by the influence on the boundary $x = 0$ of the hard drummer.

The results of the calculations are given in the figures differ in that fig. 1 a and fig. 2 the value.

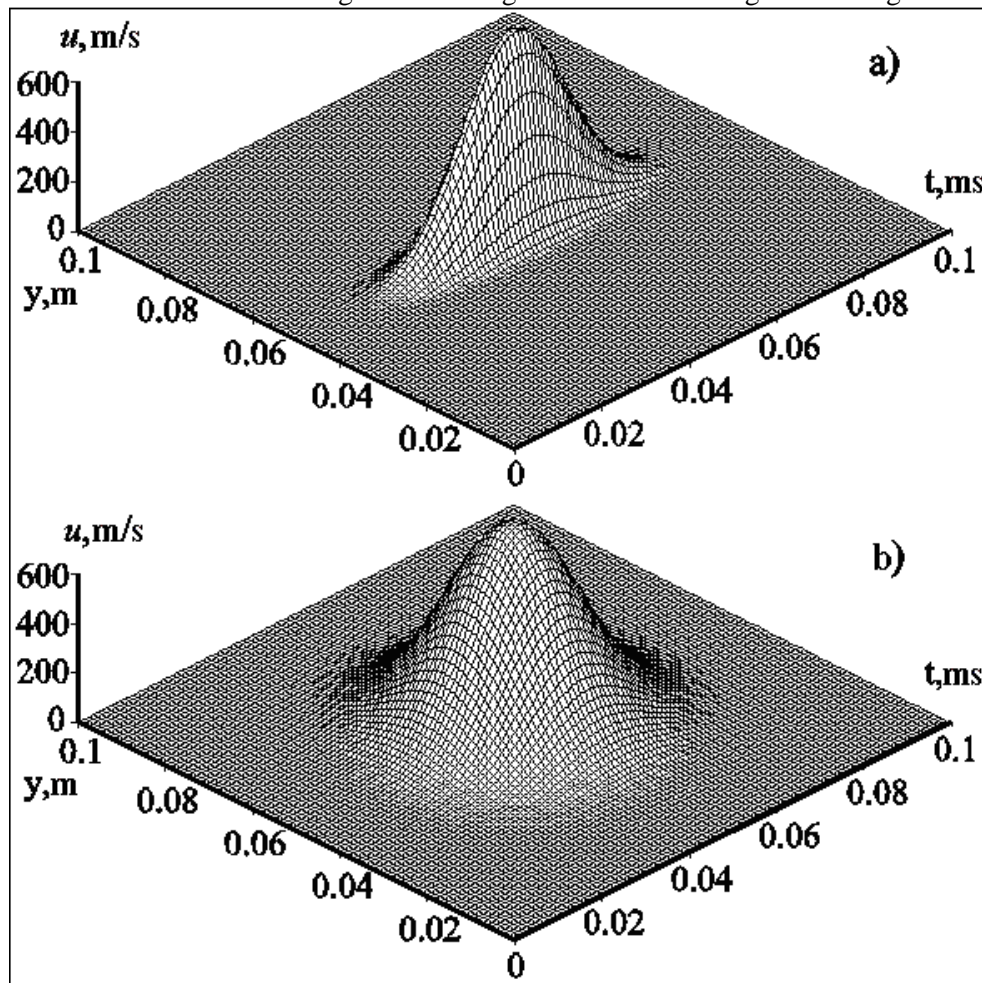


Figure 1. The shape of the initial wave pulse

$y_* = 0.004\text{m}$, fig. 1 *b* and 3 – $y_* = 0.016\text{m}$. In fig.1 shows the initial profiles of the velocity components for the kick, which affects the computational domain at the boundary. In Fig. 2 and 3 (fragments (a)) presents the pressure distribution at the time of 3.6 ms. Since the wave pattern is symmetrical about the line $y = L_y / 2$, so the drawings only for one of the half-planes separated by line $y = L_y / 2$. On fragments of *b* also presents plots of pressure at different points of time (time is indicated above the lines in ms), the curves on the left correspond to the line $y = L_y / 2$, the line curves to the right $y = L_y / 4$. Note that since the initial momentum is localized not only on time but also on the coordinate y , the attenuation is determined not only by energy dissipation in bubbly liquid, but also two-dimensional spreading. From the comparative analysis of fig. 2 and 3 shows that in both cases the motion is oscillatory in nature, due to radial inertia bubble liquid. In addition, it is seen that the leading edge longer (along the y coordinate) of the pulse ($y_* = 0.016\text{m}$, fig.3) has the form of a solitary wave, and the remainder is distributed as a package of waves, having a pulsating character, like the solitons [24]. More “short” pulse ($y_* = 0.004\text{m}$, fig.2) is distributed as u-pack waves with a characteristic wavelength $\lambda \approx C / \omega_M$, where C is the equilibrium sound velocity in bubbly liquids; ω_M – frequency Minnaert [4]. From fig. 2 shows that the pressure distribution along has $y = L_y / 4$ the form of a solitary wave, i.e. the wave packet practical invisible.

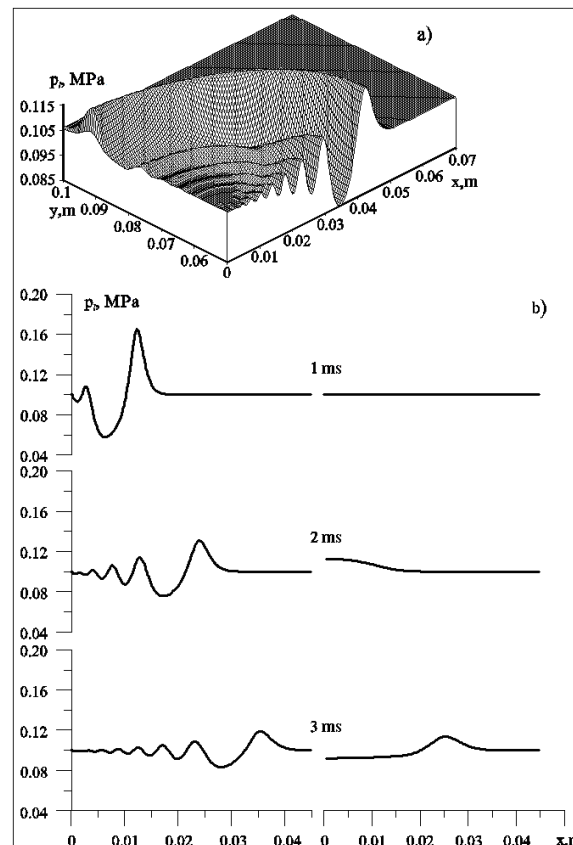


Figure 2. The dynamics of the localized pulse in a bubbly liquid. The spatial distribution of pressure $y_* = 0.004\text{m}$ at time 3.6 ms (a).

For fragment (b) correspond to the left boundary line $y = L_y / 2$,
the curves on the right lines $y = L_y / 4$

In fig. 4 presents the evolution of the wave initiated by the impact at the boundary $x = 0$ of the hard drummer in a homogenous water-air mixture. The speed of the drummer on the y -coordinate is changing parabolic law

$$u(0, y) = \Delta u \cdot \frac{4}{L_y^2} \left(y - \frac{L_y}{2} \right)^2 \quad (15)$$

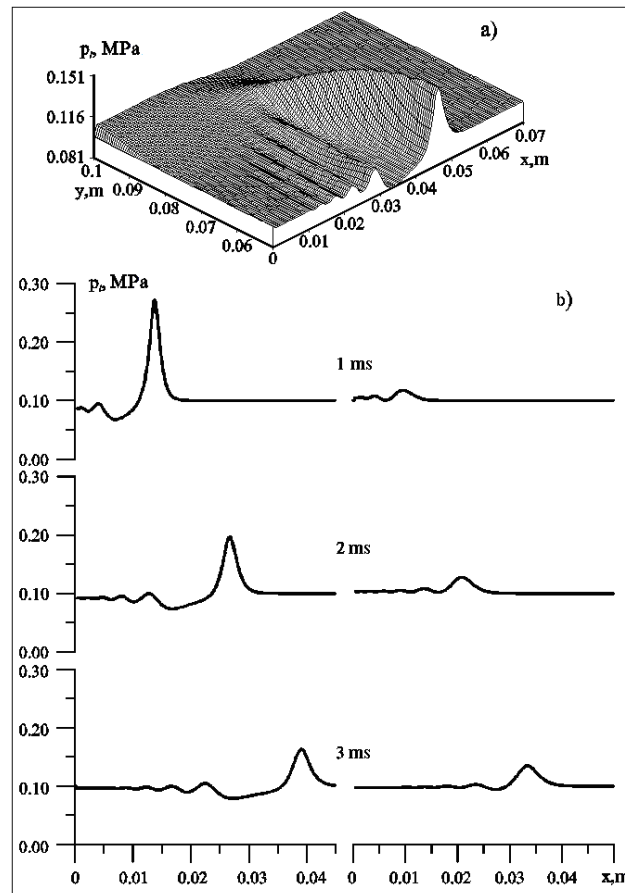


Figure 3. The dynamics of the localized pulse in a bubbly liquid. The spatial distribution of pressure $y_* = 0.016$ m at time 3.6 ms (a).

For fragment (b) correspond to the left boundary line $y = L_y / 2$, the curves on the right lines $y = L_y / 4$

where $L_y = 0.06$ m, the other parameters are the same as for fig. 2. In fig. 4 presents the pressure distribution along coordinate axes at various points in time. From fig. 4 shows that by the time 1.6 ms (fragment a) at the water-air mixture is formed wave with a parabolic profile at the boundary $x = 0$, and, since the amplitude of the wave is non-uniform in y and symmetric along the line $y = L_y / 2$, there is a convergence of the wave along the line $y = L_y / 2$. Note that the front wave profile has oscillational structure associated with the radial inertia of the bubble liquid; the amplitude of the pressure by the time 1.6 ms reaches 8 atm. By the time 2.7 ms (fig.4b) is a collision of waves propagating to the center of the computational domain. The collision of the waves is accompanied by the appearance of the tower-like bursts of amplitude of pressure; time 2.7 ms amplitude like spike reaches 10 atm. It is seen

that towering burst pressure "framed" pulsatile perturbations due to the addition of oscillations on the front lines of the propagating wave. With the further evolution of the waves (fragment *c*) to the towering bursts of damped due to the two-dimensional spreading and dissipation in bubble liquid.

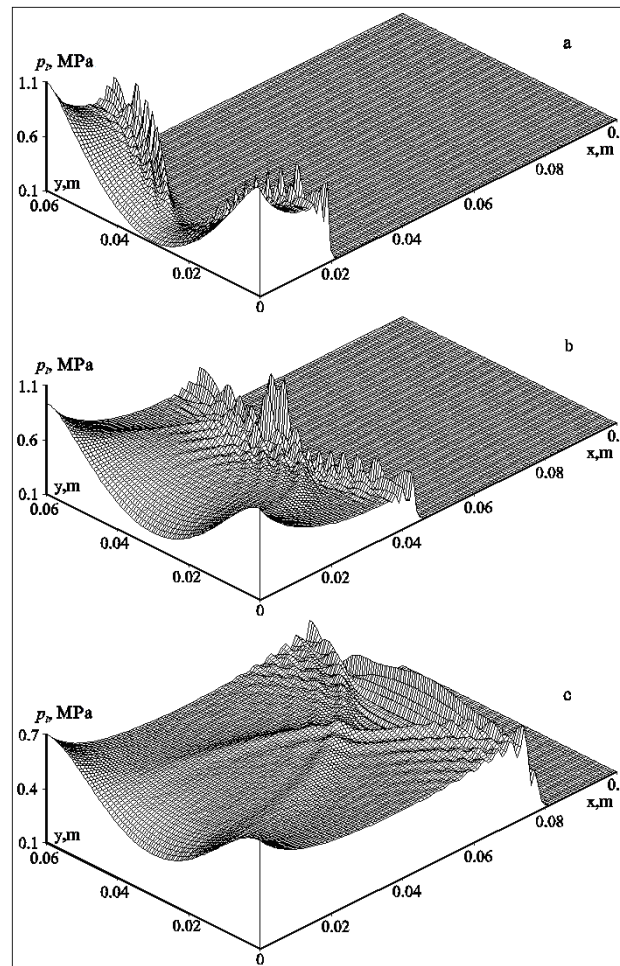


Figure 4. The pressure distribution when the impact of a rigid striker on the boundary $x = 0$.

Fragments *a*, *b* and *c* correspond to the points 1.6, 2.7 and 5 ms

Conclusion and perspectives

We study numerically the propagation of two-dimensional waves in water-air bubble liquid. Initial perturbation initiated by impact of a hard drummer. The study established the following facts. When the evolution of the passive in a uniform bubbly fluid bell-shaped over the transverse coordinate of a pulse signal of the attenuation is determined not only by energy dissipation in bubbly liquid, but also two-dimensional spreading. The leading edge bell-shaped over the transverse coordinate of the pulse has the form of a solitary wave, and the remainder is distributed as a collection of waves having pulsating character, the shorter the signal is distributed as a horseshoe pack waves. When exposed to flat bubble liquid drummer with a parabolic profile over the transverse coordinate is established that two-dimensional effects is the focus of a wave along a line of symmetry.

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References

- [1] Iordanskii, S.V. 1960 *J. Appl. Mech. Tech. Phys.*, pp. 102-110.
- [2] Kogarko, B.S. 1961 *Dokl. Akad. Nauk SSSR*, 137, pp. 1331-1333.
- [3] Wijngaarden, L.V. 1968 *Journal of Fluid Mechanics*, 33 (3), pp. 465-474.
- [4] Nigmatulin R I 1987 *Dynamics of multiphase media Part 1* (Moscow: Nauka), 464 p.
- [5] Nigmatulin R I, Shagapov V Sh, Vakhitova N 1989 *Dokl. academy of sciences of the USSR №5*, pp. 1077-1081.
- [6] Gubaidulin A. A. , Kutrunov A. V., Rustyumova O. Sh., and Yakovleva T. K. 1994 *In: Summary of Investigations of the Inst. Mech. Multiphase Systems SB RAS, N5 [in Russian], Ufa Sci. Center, Tyumen'*, pp. 23-31.
- [7] Shagapov, V. S., & Gimaltdinov, I. K. 1998 *Xinxing Jianzhu Cailiao/New Building Materials*, (12), pp. 987-992.
- [8] Lauterborn, W., Kurz, T., Mettin, R., Ohl, C.D. 1999 *Advances in Chemical Physics*, 110, pp. 295-380.
- [9] Gimaltdinov, I. K., Nigmatulin, R. I., & Shagapov, V. S. 2001 *Fluid Dynamics*, 36(3), pp. 458-466. 10.1023/A:1019244303834
- [10] Shagapov, V. S., Gimaltdinov, I. K., Khabeev, N. S., & Bailey, S. S. 2003 *Shock Waves*, 13(1), pp. 49-56. 10.1007/s00193-003-0194-1
- [11] Nigmatulin R I, Shagapov V Sh, Gimaltdinov I K, Galimzyanov M N 2001 *Physics Doklady № 1*, pp. 763-765.
- [12] Galimzyanov M N, Gimaltdinov I K, Shagapov V Sh 2002 *Fluid Dynamics* 7, pp. 139-147
- [13] Nigmatulin, R. I., Shagapov, V. S., Gimaltdinov, I. K., & Akhmadulin, F. F. 2003 *Doklady Physics*, 48(2), pp.75-79. 10.1134/1.1560735
- [14] Kedrinskiĭ, V.K., Vshivkov, V.A., Dudnikova, G.I., Shokin, Yu.I., Lazareva, G.G. 2004 *Journal of Experimental and Theoretical Physics*, 98 (6), pp. 1138-1145. doi: 10.1134/1.1777626
- [15] Kedrinskiĭ, V.K., Vshivkov, V.A., Dudnikova, G.I., Shokin, Yu.I., Lazareva, G.G. 2004 *Journal of Experimental and Theoretical Physics*, 98 (6), pp. 1138-1145. doi: 10.1134/1.1777626
- [16] Vshivkov V A, Lazareva G G, Kedrinskii V K 2005 *J. of applied mechanics and technical physics* pp. 746-753.
- [17] Lepikhin, S. A., Galimzyanov, M. N., & Gimaltdinov, I. K. 2010 *High Temperature*, 48(2), pp. 215-221. 10.1134/S0018151X10020124
- [18] Bayazitova, A. R., Gimaltdinov, I. K., & Shagapov, V. S. 2006 *Fluid Dynamics*, 41(3), pp. 381-390. 10.1007/s10697-006-0054-7
- [19] Shagapov, V. S., Gimaltdinov, I. K., Bayazitova, A. R., & Spevak, D. S. 2009 *High Temperature*, 47(3), pp. 424-431. 10.1134/S0018151X0903016X
- [20] Gimaltdinov, I. K., & Kucher, A. M. 2014 *High Temperature*, 52(3), pp. 411-416. 10.1134/S0018151X14020096
- [21] Topolnikov, A. S., & Gimaltdinov, I. K. 2014 *Thermophysics and Aeromechanics*, 21(4), pp. 489-498. 10.1134/S086986431404009X
- [22] Gimaltdinov, I. K., Arslanbekova, R. R., & Levina, T. M. 2016 *Thermophysics and Aeromechanics*, 23(3), pp. 355-367. 10.1134/S0869864316030057
- [23] Gimaltdinov, I. K., & Levina, T. M. 2017 *Bulletin of the Tomsk Polytechnic University, Geo Assets Engineering*, 328(8), pp. 55-63.
- [24] Petviashvili V I, Sologub O Y 1978 *Dokl. academy of sciences of the USSR № 3*, pp. 1321-1324/